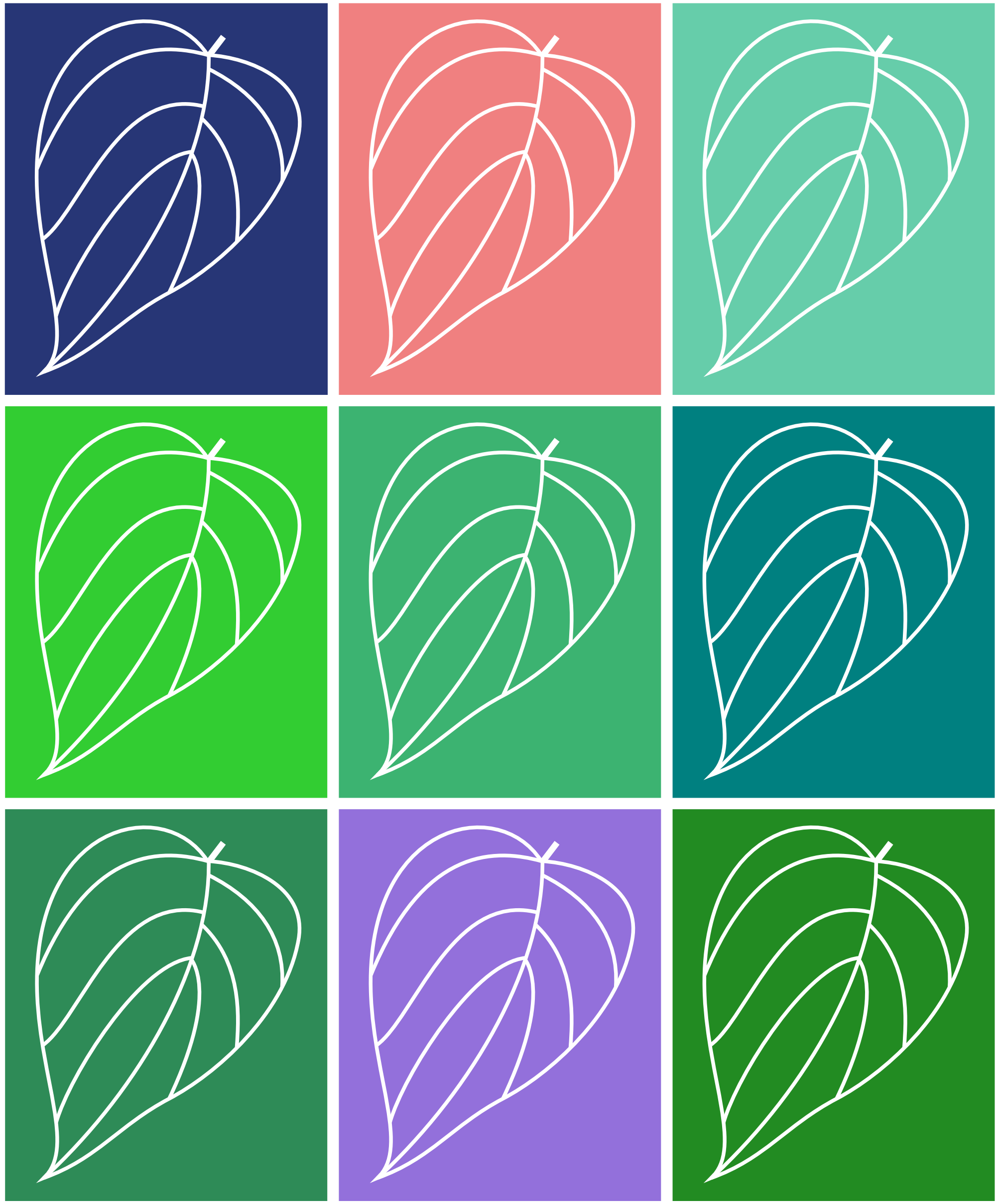
Francesco Tosoni

Toward Greener Matrix Operations by Lossless Compressed Formats

Learning from large, complex and structured data: advances in methods and applications

4th June 2025
Aula Magna, Sant'Anna School of Advanced Studies, Pisa



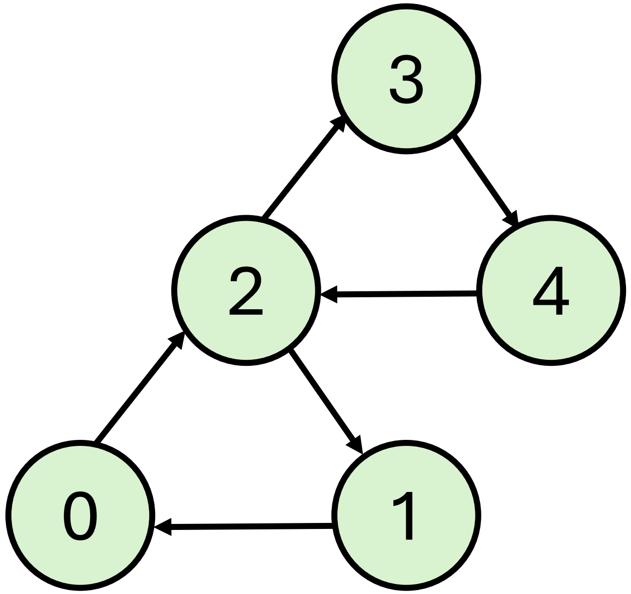
File:Betelblatt.svg originally uploaded to Commons by User:Furfur

Introduction

Sparse Matrix-Vector Multiplication (**SpMV**) are relevant in ML, scientific computing, and graph algorithms.

Research focus: Investigate **space**, **time**, and **energy** efficiency of SpMV.

We challenge prevailing assumptions about a straightforward linear correlation between time and energy.



$$\overset{\mathbf{M}}{\begin{array}{|c|c|c|c|c|}\hline 0 & 0 & 1 & 0 & 0 \\ \hline 1 & 0 & 0 & 0 & 0 \\ \hline 0 & 1 & 0 & 1 & 0 \\ \hline 0 & 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & 0 & 0 \\ \hline \end{array}} \times \overset{\mathbf{x}}{\begin{array}{|c|}\hline 2.1 \\ \hline 1.5 \\ \hline 3.8 \\ \hline 0.7 \\ \hline 4.4 \\ \hline \end{array}} = \overset{\mathbf{y}}{\begin{array}{|c|}\hline 3.8 \\ \hline 2.1 \\ \hline 2.2 \\ \hline 4.4 \\ \hline 3.8 \\ \hline \end{array}}$$

Motivation

POPULARITY OF ML ALGORITHMS (E.G. CHATGPT) AND DATA GENERATION SURPASSING MOORE'S LAW

AI NOT JUST ON SERVER MACHINES BUT ALSO ON EDGE AND IOT DEVICES → BATTERY DURATION AS CRITICAL CONCERN.

ENERGY AS A LEADING DESIGN CONSTRAINT IN COMPUTING DEVICES; HOWEVER, GREEN SOFTWARE ENGINEERING IS STILL IN ITS INFANCY.

OVERSIMPLIFIED ENERGY COMPLEXITY MODELS FAIL TO CAPTURE REAL-WORLD DYNAMICS → NEED FOR COMPREHENSIVE REFERENCE MODELS

Platforms

Raspberry Pi 4 model B @1.5GHz (4 cores) and a Fluke 8845A benchtop **multimeter**

vs.

Intel® Core™ i9-7960X CPU @2.80GHz (16 cores, 2-way hyperthreading) and a **power meter** + the **RAPL** energy profiler

Research questions

How does compression affect the efficiency of space, time, and energy? Some matrix formats are more space-efficient but consume order of magnitude more energy than others.

What trade-offs exist between time and energy optimisation? Often the energy-optimal parallelism degree is lower than the time-optimal one.

Which runtime metrics impact energy efficiency? The number of L1 and L3 cache accesses impact performances.

Compressed matrix formats

VLDB 2022

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Compressed matrix formats

We compare *three* computation-friendly compression schemes for **large** yet sparse **binary matrices**:

- Google's Zuckerli (Versari *et al.*, 2020)
- k^2-tree (Brisaboa, Ladra, and Navarro 2014)
- RePair-compressed matrices [mm-repair] (Ferragina *et al.*, 2022)

Why lossless?

Lossy compression solutions for space reduction:

- Low-precision storage (e.g. FP32)
- Sparsification
- Quantisation (binary, ternary)

→careful & manual application

Lossless compression is a better "automated" alternative

- data independent
- no need a priori knowledge about the input data.

Computation-friendly compression

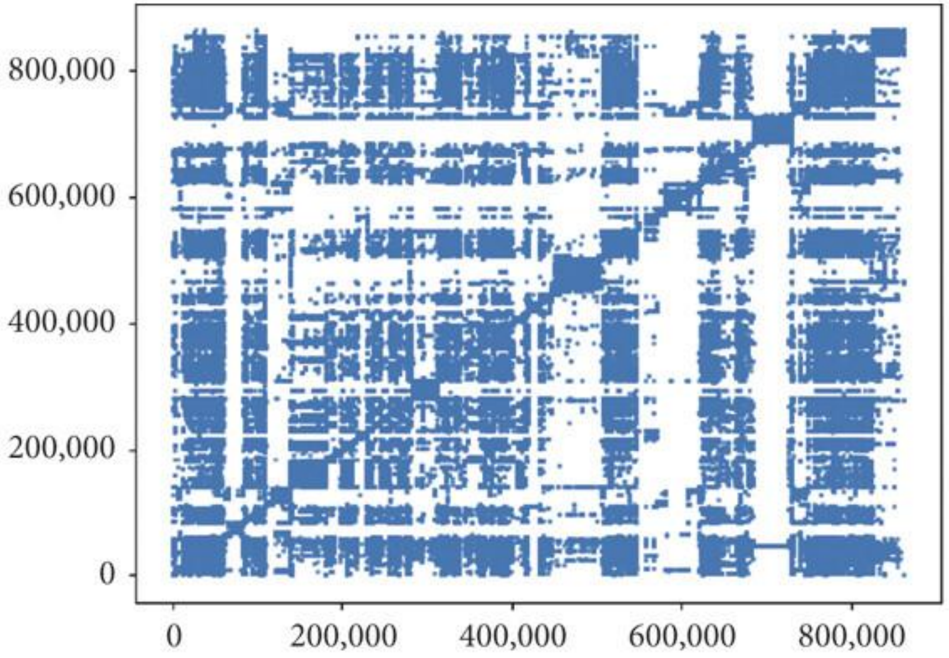
All formats we tested are **computation-friendly** for matrix-vector multiplications:

- Exploit more than mere sparsity
- Enable direct operations on data without prior decompression
- Allow operating in time proportional to the size of the compressed representation (*win-win*!)

WebGraph & Zuckerli (1/2)

Exploit redundancies of outgoing links within the same domain.

- **Webgraph** (2004) exploit the copying property of consecutive adjacency lists to compress each list based on a reference list (Java, c++, Rust).
- Google's **Zuckerli** (2020) applies novel compression heuristics on top of Zuckerli.



Adjacency matrix for eu-2005. Figure taken from DOI: 10.1155/2020/2354875

WebGraph & Zuckerli (2/2)

These graph formats allow for *compression-friendly* matrix-to-vector multiplications (Francisco *et al.* 2022).

Idea: exploit deltas between similar adjacency lists.

We implement multiplications on top of **Zuckerli**.

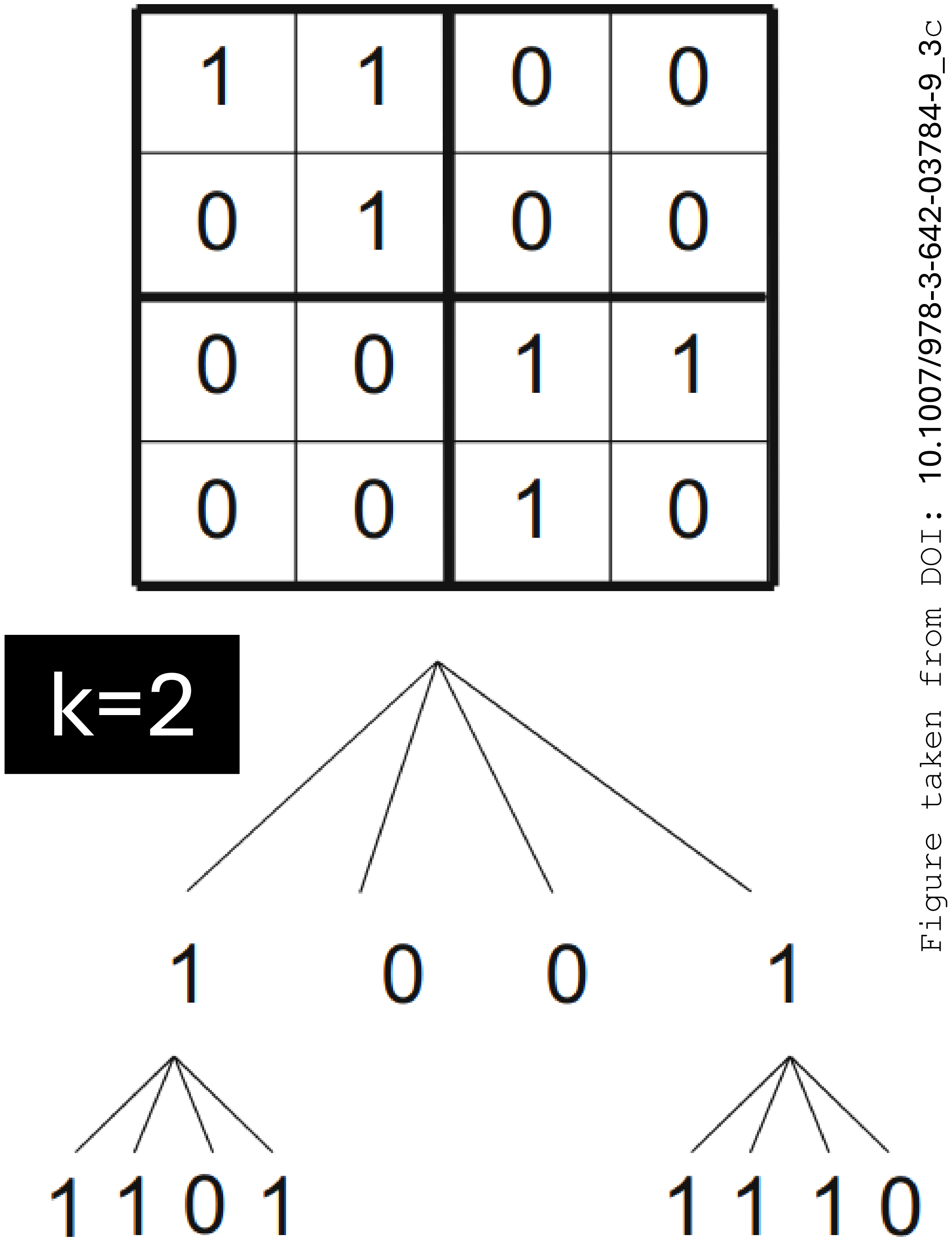
Node	Outdegree	Successors
…	…	…
15	11	13, 15, 16, 17 , 18, 19, 23, 24, 203, 315, 316, 317, 3041
16	10	15, 16, 17, 22, 23, 24, 315, 316, 317, 3041
17	0	
18	5	13, 15, 16, 17, 50
…	…	…

k^2-tree (1/3)

Exploit sparsity and clustering of **0**'s.

Submatrices are recursively split into k^2 smaller submatrices.

- **0**: empty submatrix;
- **1**: non-empty ones.



1	1	0	0
0	1	0	0
0	0	1	1
0	0	1	0

k=2

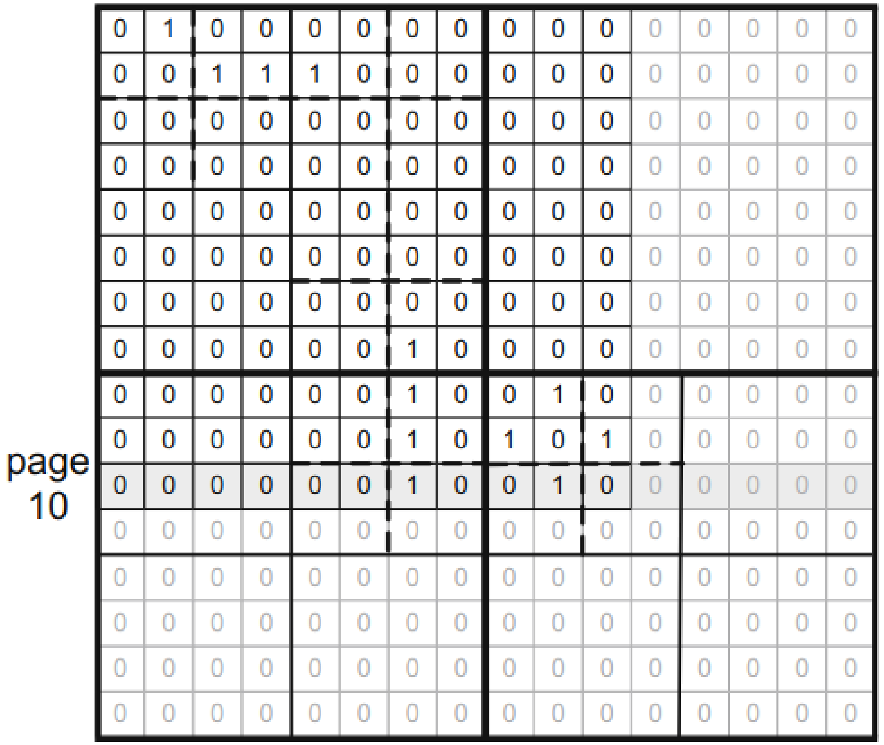
Figure taken from DOI: 10.1007/978-3-642-03784-9_3c

k^2-tree (2/3)

Height is always $\log_k n$ → access to each single cell $\mathcal{O}(\log_k n)$.

Matrix-vector **multiplication** = visit to the tree

Retrieving neighbours for page 10



	0	1	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	1	1	1	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	1	0	1	0	1	0	0	0	0	0
page 10	0	0	0	0	0	0	1	0	0	1	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

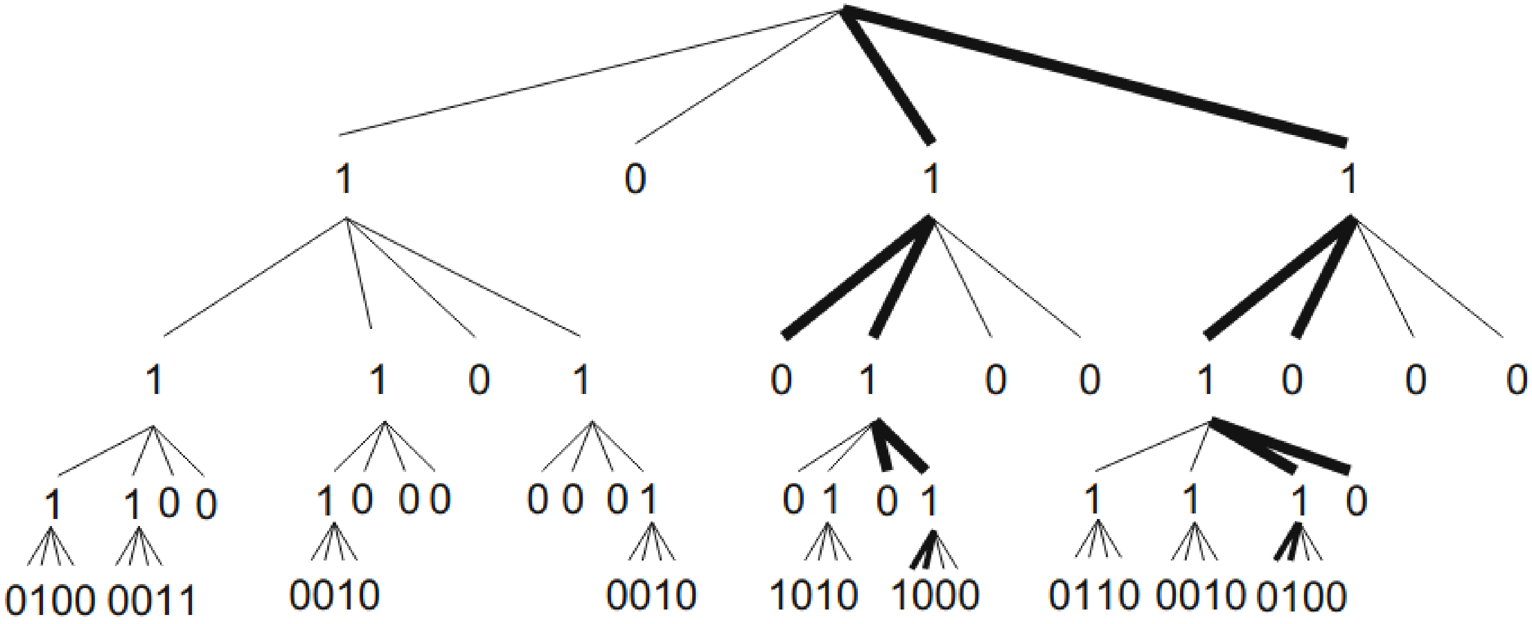


Figure taken from DOI: 10.1007/978-3-642-03784-9_3c

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Millennium Institute
Foundational
Research on Data

simongog/**sdsl-lite**

Succinct Data Structure Library 2.0

38 Contributors
68 Issues
2k Stars
350 Forks

k^2-tree (3/3)

Implementations: University of A Coruña, SDSL library, and one by the University of Chile/Millennium Institute.

Matrix-to-vector multiplications can be implemented by a tree traversal (in any order). There's **no need of `rank`** and `select` data structures.

mm-repair ~ Step #1: exploit sparsity

$$\begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 & \quad 4 \end{matrix} \\ \begin{bmatrix} 4.1 & 5.3 & 0 & 5.3 \\ 4.1 & 0 & 2.0 & 4.1 \\ 0 & 5.3 & 2.0 & 5.3 \\ 4.1 & 5.3 & 2.0 & 4.1 \\ 4.1 & 5.3 & 2.0 & 5.3 \end{bmatrix} \end{array}$$

$$V = \begin{array}{c} \begin{matrix} 1 & \quad 2 & \quad 3 \end{matrix} \\ \begin{pmatrix} 2.0 & 4.1 & 5.3 \end{pmatrix} \end{array}$$

Set of nonzeros

$S =$

$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$

$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$

$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$

Sequence of pairs

Compute **the CSRV representation of a matrix**, a modification of the CSR representation.

$ is the EOL character

mm-repair ~ Step #1: exploit sparsity

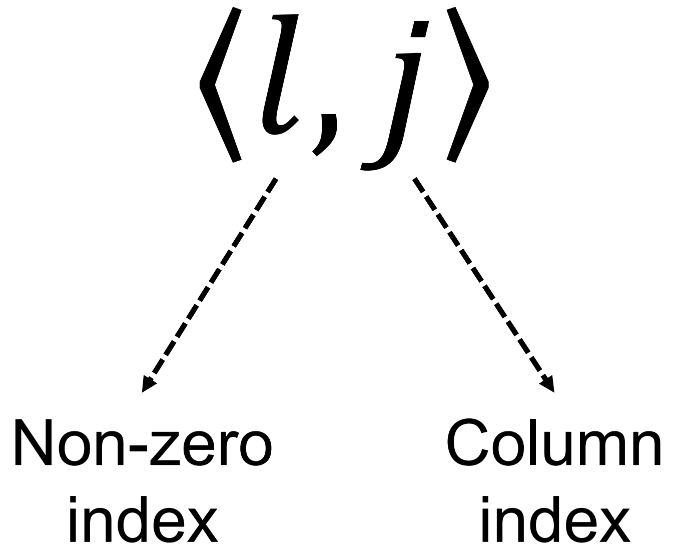
$$\begin{array}{c} \begin{array}{cccc} 1 & 2 & 3 & 4 \end{array} \\ \begin{bmatrix} 4.1 & 5.3 & 0 & 5.3 \\ 4.1 & 0 & 2.0 & 4.1 \\ 0 & 5.3 & 2.0 & 5.3 \\ 4.1 & 5.3 & 2.0 & 4.1 \\ 4.1 & 5.3 & 2.0 & 5.3 \end{bmatrix} \end{array}$$

$$\Downarrow$$

$$V = \begin{array}{ccc} 1 & 2 & 3 \\ (2.0 & 4.1 & 5.3) \end{array}$$

$S =$
$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$
$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$
$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$

Generate column-value index pairs



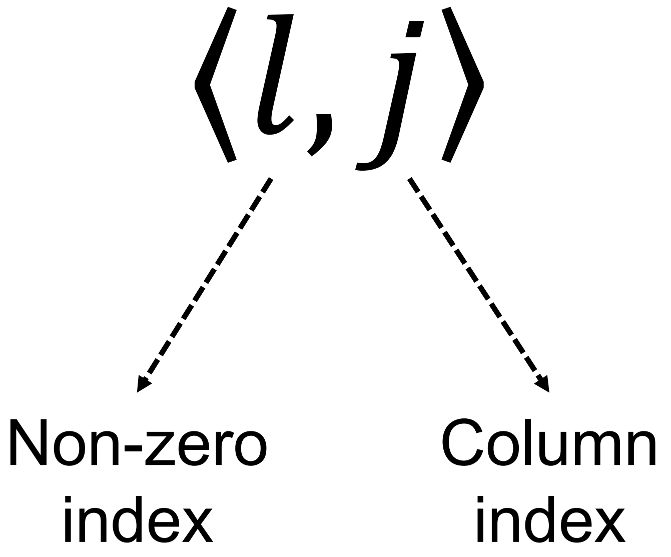
mm-repair ~ Step #1: exploit sparsity

$$
\begin{array}{cccc}
1 & 2 & 3 & 4 \\
\left[\begin{array}{cccc}
4.1 & 5.3 & 0 & 5.3 \\
4.1 & 0 & 2.0 & 4.1 \\
0 & 5.3 & 2.0 & 5.3 \\
4.1 & 5.3 & 2.0 & 4.1 \\
4.1 & 5.3 & 2.0 & 5.3
\end{array}\right]
\end{array}
$$

$$
\begin{array}{cccc}
 & 1 & 2 & 3 \\
V = (& 2.0 & 4.1 & 5.3)
\end{array}
$$

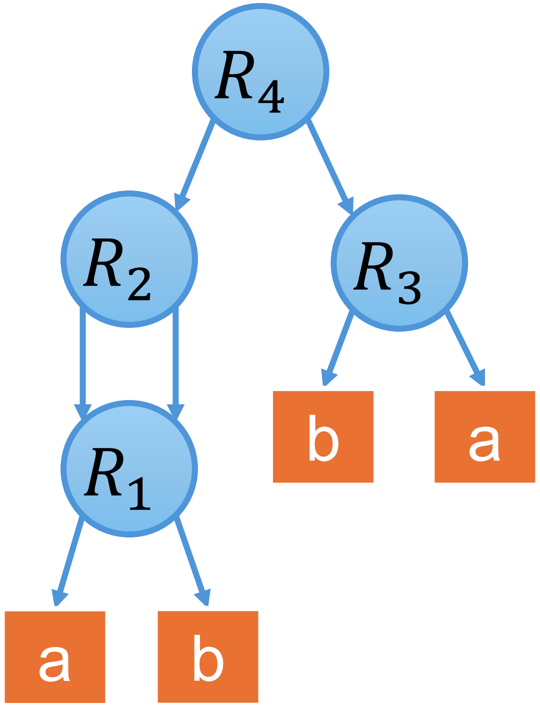
$S =$
$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$
$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$
$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$

Generate column-value index pairs



mm-repair ~ Step #2: grammar-compress

In the following, as a (lossless) compressor we use RePair, which is a **grammar compressor** based on **straight-line programs (SLP).**

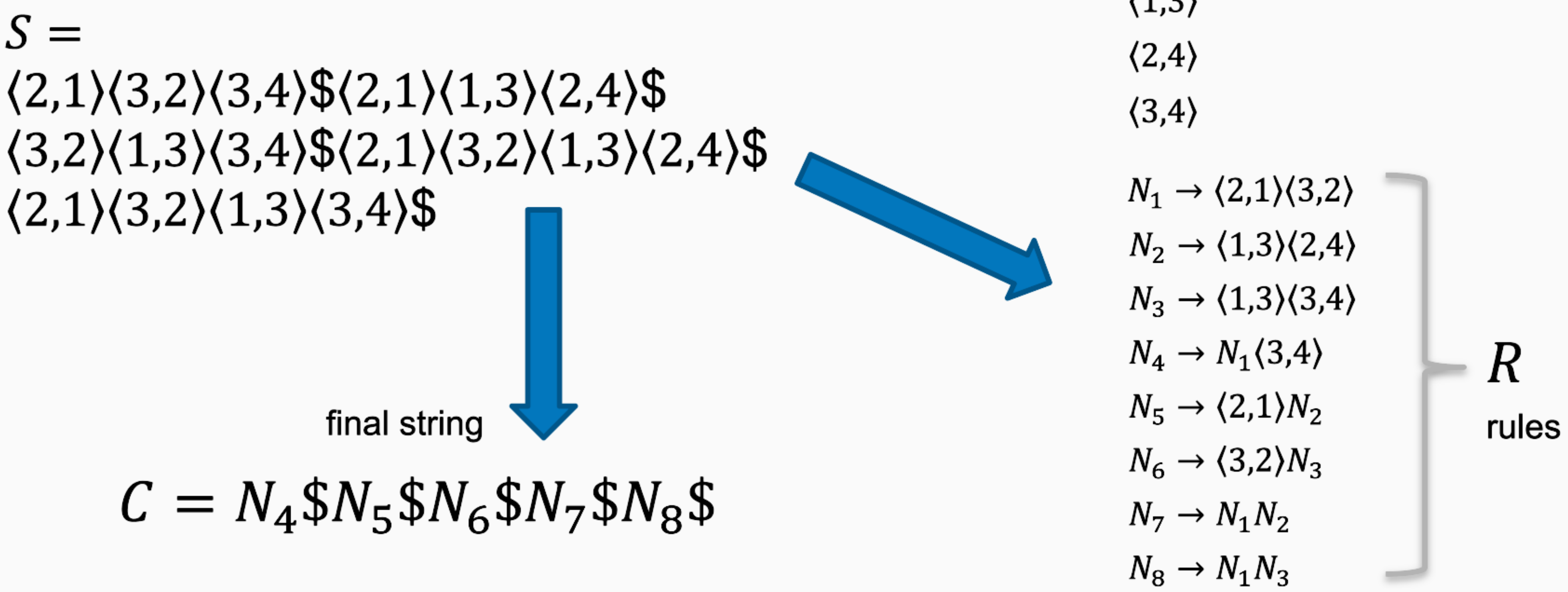


$$S = ababba$$

$$R_1 \rightarrow ab$$
$$R_2 \rightarrow R_1 R_1$$
$$R_3 \rightarrow ba$$
$$R_4 \rightarrow R_2 R_3$$

mm-repair ~ Step #2: grammar-compress

$$V = (\overset{1}{2.0}\quad \overset{2}{4.1}\quad \overset{3}{5.3})$$



$$S =$$

$$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$$
$$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$$
$$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$$

$\langle 2,1\rangle$

$\langle 3,2\rangle$

$\langle 1,3\rangle$

$\langle 2,4\rangle$

$\langle 3,4\rangle$

$$\left.\begin{array}{l}
N_1 \rightarrow \langle 2,1\rangle\langle 3,2\rangle \\
N_2 \rightarrow \langle 1,3\rangle\langle 2,4\rangle \\
N_3 \rightarrow \langle 1,3\rangle\langle 3,4\rangle \\
N_4 \rightarrow N_1\langle 3,4\rangle \\
N_5 \rightarrow \langle 2,1\rangle N_2 \\
N_6 \rightarrow \langle 3,2\rangle N_3 \\
N_7 \rightarrow N_1 N_2 \\
N_8 \rightarrow N_1 N_3
\end{array}\right\} R$$

rules

final string

$$C = N_4\$N_5\$N_6\$N_7\$N_8\$$$

mm-repair ~ Step #2: grammar-compress

$$V = (\overset{1}{2.0} \quad \overset{2}{4.1} \quad \overset{3}{5.3})$$

$S =$

$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$

$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$

$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$

$\langle 2,1\rangle$

$\langle 3,2\rangle$

$\langle 1,3\rangle$

$\langle 2,4\rangle$

$\langle 3,4\rangle$

$$\left.\begin{array}{l} N_1 \to \langle 2,1\rangle\langle 3,2\rangle \\ N_2 \to \langle 1,3\rangle\langle 2,4\rangle \\ N_3 \to \langle 1,3\rangle\langle 3,4\rangle \\ N_4 \to N_1\langle 3,4\rangle \\ N_5 \to \langle 2,1\rangle N_2 \\ N_6 \to \langle 3,2\rangle N_3 \\ N_7 \to N_1 N_2 \\ N_8 \to N_1 N_3 \end{array}\right\} R$$

rules

final string

$$C = N_4\$N_5\$N_6\$N_7\$N_8\$$$

mm-repair ~ Step #2: grammar-compress

nonzeros

$$V = \begin{array}{ccc} 1 & 2 & 3 \\ (2.0 & 4.1 & 5.3) \end{array}$$

~~$S =$~~
~~$\langle 2,1\rangle\langle 3,2\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 1,3\rangle\langle 2,4\rangle\$$~~
~~$\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 2,4\rangle\$$~~
~~$\langle 2,1\rangle\langle 3,2\rangle\langle 1,3\rangle\langle 3,4\rangle\$$~~

$\langle 2,1\rangle$

$\langle 3,2\rangle$

$\langle 1,3\rangle$

$\langle 2,4\rangle$

$\langle 3,4\rangle$

$$\left.\begin{aligned}
N_1 &\rightarrow \langle 2,1\rangle\langle 3,2\rangle \\
N_2 &\rightarrow \langle 1,3\rangle\langle 2,4\rangle \\
N_3 &\rightarrow \langle 1,3\rangle\langle 3,4\rangle \\
N_4 &\rightarrow N_1\langle 3,4\rangle \\
N_5 &\rightarrow \langle 2,1\rangle N_2 \\
N_6 &\rightarrow \langle 3,2\rangle N_3 \\
N_7 &\rightarrow N_1 N_2 \\
N_8 &\rightarrow N_1 N_3
\end{aligned}\right\} R$$

rules

final string

$$C = N_4\$N_5\$N_6\$N_7\$N_8\$$$

mm-repair ~ Step #3: multiply

nonzeros

$$\begin{array}{cccc} & 1 & 2 & 3 \\ V = & (2.0 & 4.1 & 5.3) \end{array}$$

$$y = M \begin{array}{c} x \\ \begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix} \end{array}$$

$$\langle l, j \rangle \quad \Rightarrow \quad eval_x(\langle l, j \rangle) = V[l] \cdot x[j]$$

$$N_i \to AB \quad \Rightarrow \quad eval_x(N_i) = eval(A) + eval(B)$$

$\langle 2{,}1 \rangle$

$\langle 3{,}2 \rangle$

$\langle 1{,}3 \rangle$

$\langle 2{,}4 \rangle$

$\langle 3{,}4 \rangle$

$$\left.\begin{array}{l} N_1 \to \langle 2{,}1 \rangle \langle 3{,}2 \rangle \\ N_2 \to \langle 1{,}3 \rangle \langle 2{,}4 \rangle \\ N_3 \to \langle 1{,}3 \rangle \langle 3{,}4 \rangle \\ N_4 \to N_1 \langle 3{,}4 \rangle \\ N_5 \to \langle 2{,}1 \rangle N_2 \\ N_6 \to \langle 3{,}2 \rangle N_3 \\ N_7 \to N_1 N_2 \\ N_8 \to N_1 N_3 \end{array}\right\} R$$

rules

mm-repair ~ Step #3: multiply

nonzeros

$$\begin{array}{cccc} & 1 & 2 & 3 \\ V = (& 2.0 & 4.1 & 5.3) \end{array}$$

$$y = M \overset{x}{\begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix}}$$

$$\langle l, j \rangle \quad \Rightarrow \quad eval_x(\langle l, j \rangle) = V[l] \cdot x[j]$$

$$N_i \rightarrow AB \quad \Rightarrow \quad eval_x(N_i) = eval(A) + eval(B)$$

8.2	$\langle 2,1 \rangle$
0	$\langle 3,2 \rangle$
4.0	$\langle 1,3 \rangle$
16.4	$\langle 2,4 \rangle$
21.2	$\langle 3,4 \rangle$

8.2	$N_1 \rightarrow \langle 2,1 \rangle \langle 3,2 \rangle$
20.4	$N_2 \rightarrow \langle 1,3 \rangle \langle 2,4 \rangle$
25.2	$N_3 \rightarrow \langle 1,3 \rangle \langle 3,4 \rangle$
29.4	$N_4 \rightarrow N_1 \langle 3,4 \rangle$
28.6	$N_5 \rightarrow \langle 2,1 \rangle N_2$
25.2	$N_6 \rightarrow \langle 3,2 \rangle N_3$
28.6	$N_7 \rightarrow N_1 N_2$
33.4	$N_8 \rightarrow N_1 N_3$

R

rules

mm-repair ~ Step #3: multiply

nonzeros

$$\begin{array}{lccc} & 1 & 2 & 3 \\ V = (& 2.0 & 4.1 & 5.3) \end{array}$$

$$\begin{array}{cc} & x \\ y = M & \begin{pmatrix} 2 \\ 0 \\ 2 \\ 4 \end{pmatrix} \end{array}$$

8.2	$\langle 2,1 \rangle$
0	$\langle 3,2 \rangle$
4.0	$\langle 1,3 \rangle$
16.4	$\langle 2,4 \rangle$
21.2	$\langle 3,4 \rangle$
8.2	$N_1 \rightarrow \langle 2,1 \rangle \langle 3,2 \rangle$
20.4	$N_2 \rightarrow \langle 1,3 \rangle \langle 2,4 \rangle$
25.2	$N_3 \rightarrow \langle 1,3 \rangle \langle 3,4 \rangle$
29.4	$N_4 \rightarrow N_1 \langle 3,4 \rangle$
28.6	$N_5 \rightarrow \langle 2,1 \rangle N_2$
25.2	$N_6 \rightarrow \langle 3,2 \rangle N_3$
28.6	$N_7 \rightarrow N_1 N_2$
33.4	$N_8 \rightarrow N_1 N_3$

R

rules

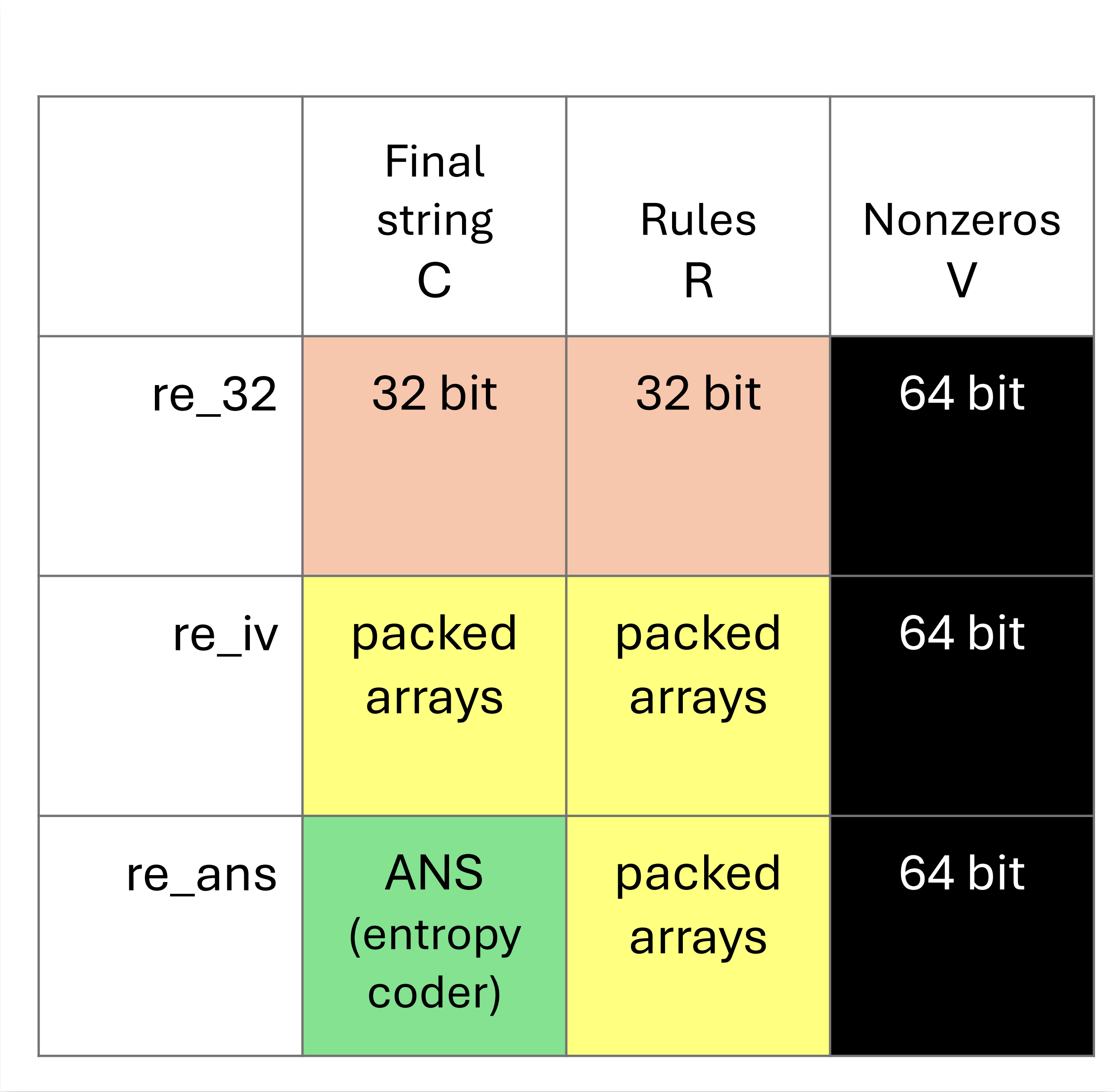
$$\langle l, j \rangle \quad \Rightarrow \quad eval_x(\langle l, j \rangle) = V[l] \cdot x[j]$$

$$N_i \rightarrow AB \quad \Rightarrow \quad eval_x(N_i) = eval(A) + eval(B)$$

$O(|R|)$ extra space

$O(|R| + |C|)$ time

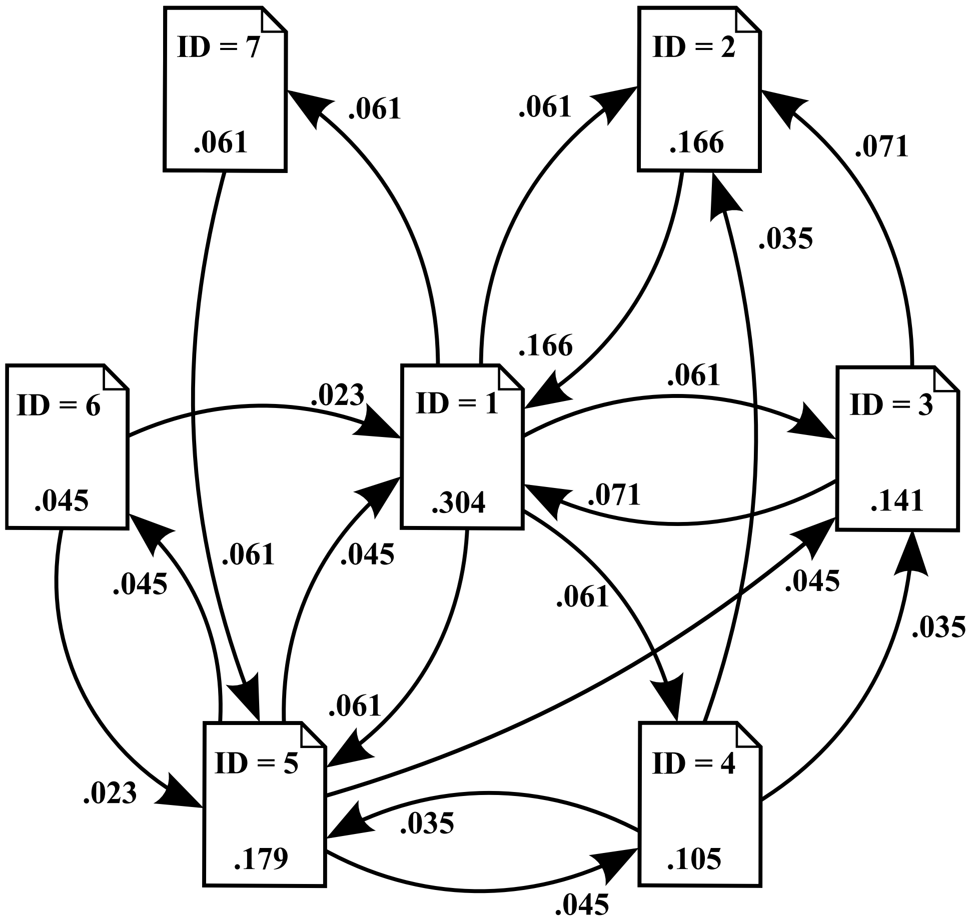
mm-repair ~ physical representations



	Final string C	Rules R	Nonzeros V
re_32	32 bit	32 bit	64 bit
re_iv	packed arrays	packed arrays	64 bit
re_ans	ANS (entropy coder)	packed arrays	64 bit

Different space–time tradeoffs

Experimental setup



PageRank

A classical algorithm in graph analysis.

Teleporting parameter (α=0.15)

$$\vec{\pi}_t = \alpha \cdot \vec{\pi}_0 + (1-\alpha) \cdot \vec{\pi}_{t-1} \cdot M$$

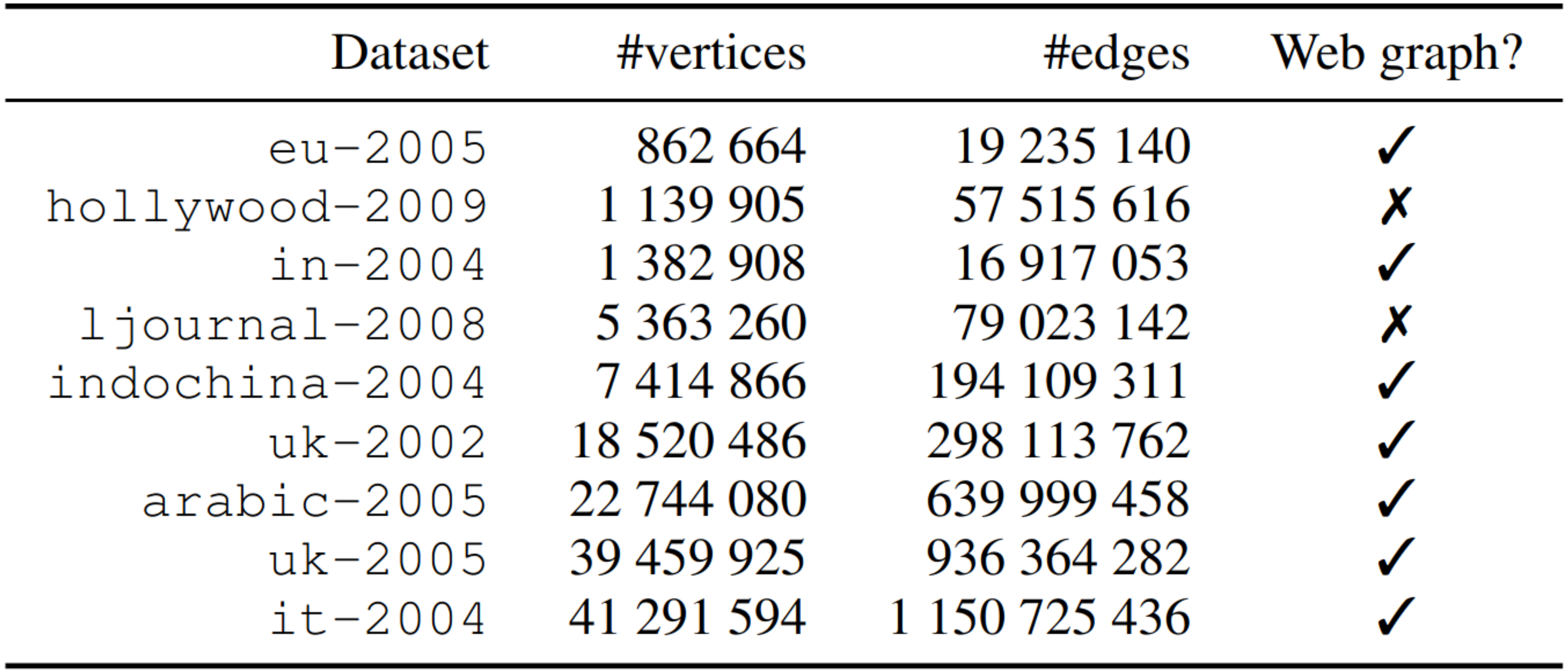
Initial probability distribution

Left matrix-vector multiplication

File:Rhacophorus nigropalmatus.jpg originally uploaded to Commons by User:Rushenb

File:Linkstruct2.svg uploaded to Commons in SVG format by User:RobertG

Datasets



Dataset	#vertices	#edges	Web graph?
eu-2005	862 664	19 235 140	✓
hollywood-2009	1 139 905	57 515 616	✗
in-2004	1 382 908	16 917 053	✓
ljournal-2008	5 363 260	79 023 142	✗
indochina-2004	7 414 866	194 109 311	✓
uk-2002	18 520 486	298 113 762	✓
arabic-2005	22 744 080	639 999 458	✓
uk-2005	39 459 925	936 364 282	✓
it-2004	41 291 594	1 150 725 436	✓

From the WebGraph framework, via the SuiteSparse Matrix Collection. Most graphs are derived from **web crawls**, with vertices ordered by the reversed URL lexicographically.

We also included two **social network** graphs:

- `hollywood-2009`: An undirected graph representing movie actors, where edges connect actors who co-starred in films.
- `ljournal-2008`: A directed graph illustrating asymmetric friendships in the LiveJournal social network.

Specifications 📜

Intel® Core™ i9-7960X

- **CPU**: single-socket Intel® Core™ i9-7960X @2.80GHz
- **Core**: 16 physical cores (2-way Intel® Hyper-Threading)

Memory:

- Total RAM: 384 GB (12 x 32 GB DDR4)
- Velocità: 2666 MT/s

Operating system:

- Ubuntu 22.04.3 LTS (64 bit)

Cache architecture:

- L1d: 32 KiB (8-way set associative)
- L1i: 32 KiB (8-way set associative)
- L2: 1024 MiB (16-way set associative)
- L3: 22 MiB (11-way set associative)

Raspberry Pi 4 Model B

- **CPU**: 4 x ARM Cortex-A72 @ 1.5GHz
- **Cores**: 4 physical cores

Memory:

- Total RAM: 4 GB LPDDR4 SDRAM

Operating system:

- Ubuntu Server 24.04 LTS (64-bit)

Cache architecture:

- L1d: 128 KiB per core; L1i: 192 KiB per core
- L2: 1 MB shared (16-way set associative)
- No L3!

Datasets Used: $\leq 10^7$ vertices

Code setup

Preprocessing steps

Transposed the matrix

Compress via Zuckerli, k2-tree, mm-repair

Store out-degree array O[1, n] for each vertex

Experiment execution

Executed 100 iterations of PageRank

Repeated for each compression format and dataset

Code optimisation

All codes written in C/C++

Compiled with -O3 flag for maximum optimization → energy savings (<43% compared to –O0)

Data parallelism

$$\left[\begin{array}{rrrrr}
\hline
1.2 & 3.4 & 5.6 & 0 & 2.3 \\
2.3 & 0 & 2.3 & 4.5 & 1.7 \\
\hline
1.2 & 3.4 & 2.3 & 4.5 & 0 \\
3.4 & 0 & 5.6 & 0 & 2.3 \\
\hline
2.3 & 0 & 2.3 & 4.5 & 0 \\
1.2 & 3.4 & 2.3 & 4.5 & 3.4 \\
\hline
\end{array}\right]$$

b=3

We exploit multicore architectures with **data-parallel versions**.

We divide each matrix into *b* row blocks. *y*=M*x* consists of *b* independent multiplications over a single block.

We use C/C++ with POSIX Threads (Pthreads) and fork-join mechanisms

Power measurement on the Intel® Core™ i9-7960X

Measurement of **power** consumption using a **PZEM-016** multimeter (~€20). Within the power range of our machine, it provides estimations with an error of 0.5%.

Intel® RAPL (Running Average Power Limit) interface, accessible via the Linux `perf` profiler (version 5.15.149).

- Package-level energy estimation: includes cores, memory controller, last-level cache, and other components.
- **Accuracy is disputed.** Integrated with tools such as Scaphandre, CodeCarbon, and Green Metrics Tool, but recent contributions in literature have shown that RAPL can make errors of up to 150%.
- Integrated into perf, which also allows for counting CPU cycles, cache hits/misses, and instruction counts.

Power Measurement on Raspberry Pi 4

The voltage (5V) is constant ⇒power is proportional to current.

We sample supply **current** via a **Fluke 8845A** benchtop multimeter (~€1000).

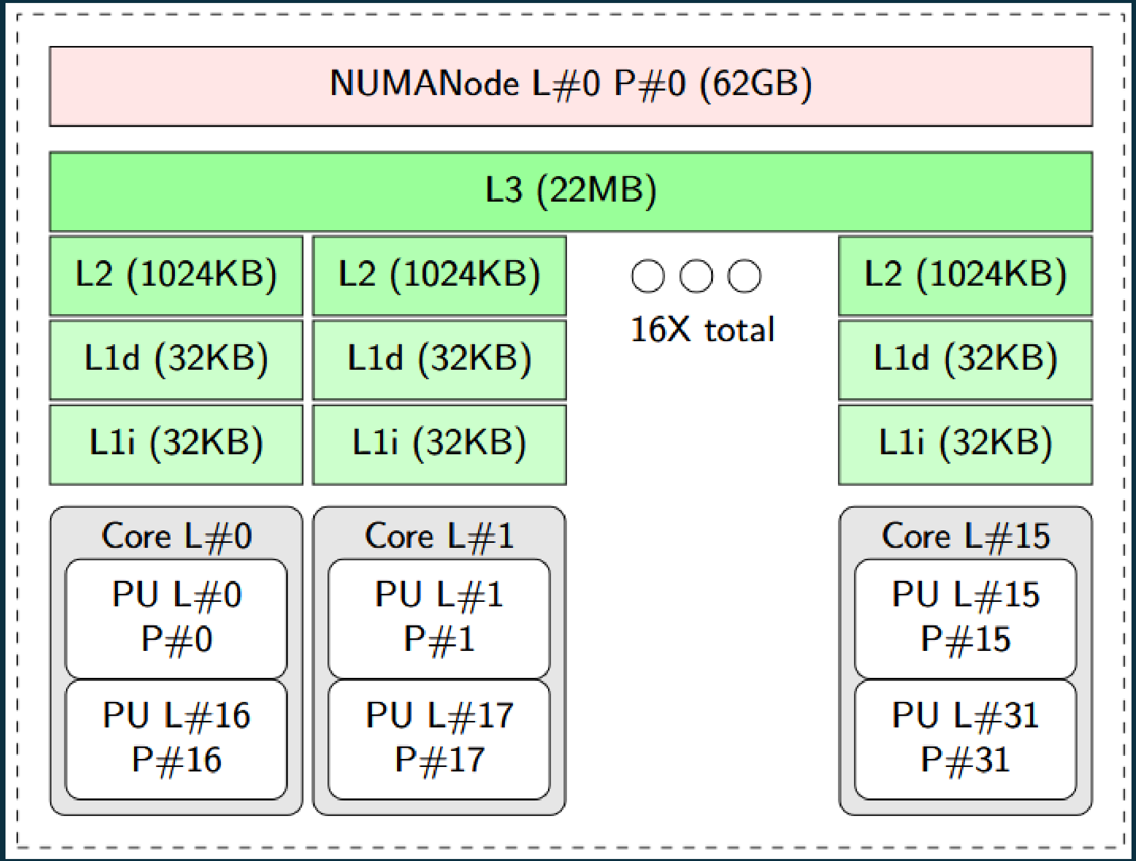
- The multimeter is connected in series with the USB-C power cable.
- Range: $R = 10\text{A}$
- Resolution: 4½ digits
- Measurement uncertainty: $\epsilon = k_1 i + k_2 R$ with $k_1 = 0.15\%$ and $k_2 = 0.0080\%$
- Sampling frequency: $3s^{-1}$.

Reproducibility Setup

- Dedicated machines
- Problem size limited to main memory (RAM) to avoid swapping ⇒ smaller datasets on Raspberry Pi
- CPU affinity: `pthread_setaffinity_np` to assign threads to logical cores ⇒ maximise parallelism, including through Hyper-Threading.
- Fixed-speed fans: maximum speed for constant power consumption.
- Fixed CPU frequency via DVFS `powersave`.
- Verification of PageRank result consistency across different matrix formats.

Objective: Reliable and reproducible measurements of our PageRank algorithm's energy consumption."

Architectures: `lstopo`



NUMANode L#0 P#0 (62GB)

L3 (22MB)

L2 (1024KB) L2 (1024KB) L2 (1024KB)

L1d (32KB) L1d (32KB) L1d (32KB)

L1i (32KB) L1i (32KB) L1i (32KB)

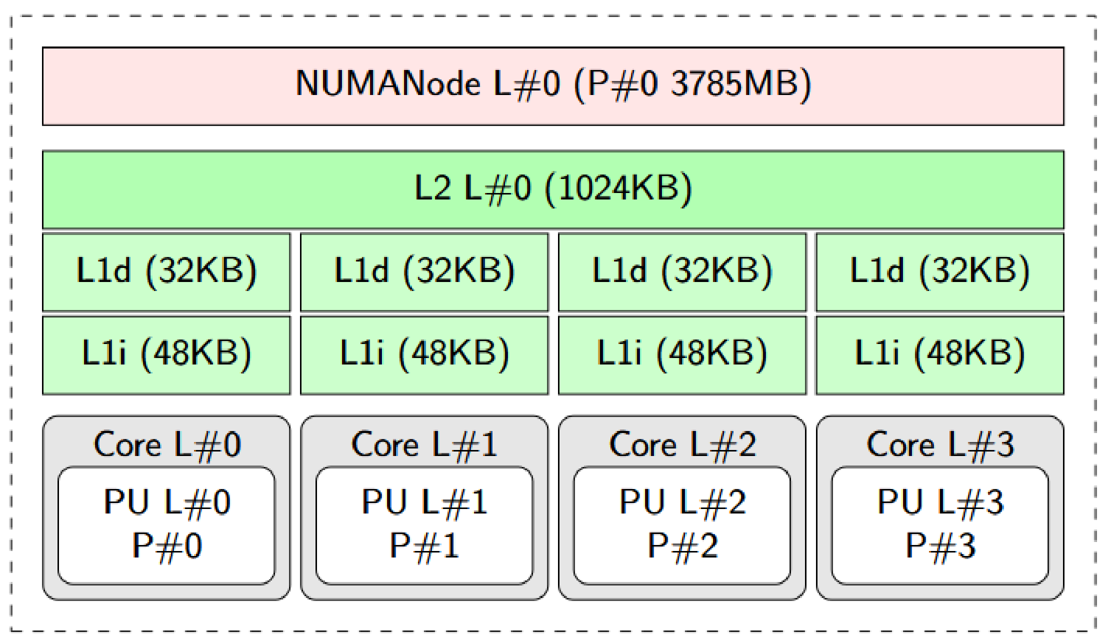
○○○
16X total

Core L#0
PU L#0
P#0
PU L#16
P#16

Core L#1
PU L#1
P#1
PU L#17
P#17

Core L#15
PU L#15
P#15
PU L#31
P#31

Intel® Core™ i9-7960X



NUMANode L#0 (P#0 3785MB)

L2 L#0 (1024KB)

L1d (32KB) L1d (32KB) L1d (32KB) L1d (32KB)

L1i (48KB) L1i (48KB) L1i (48KB) L1i (48KB)

Core L#0
PU L#0
P#0

Core L#1
PU L#1
P#1

Core L#2
PU L#2
P#2

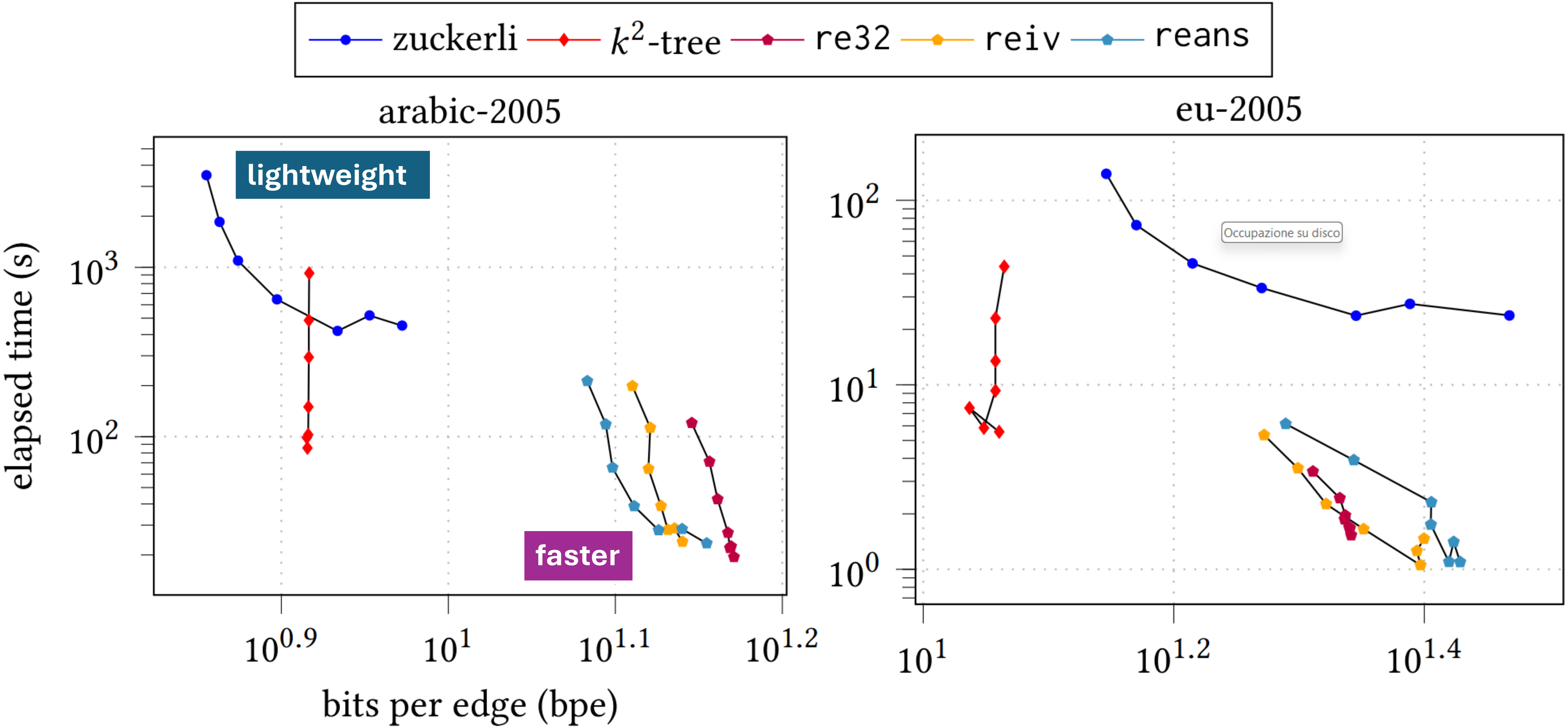
Core L#3
PU L#3
P#3

Raspberry Pi 4

Results and discussion

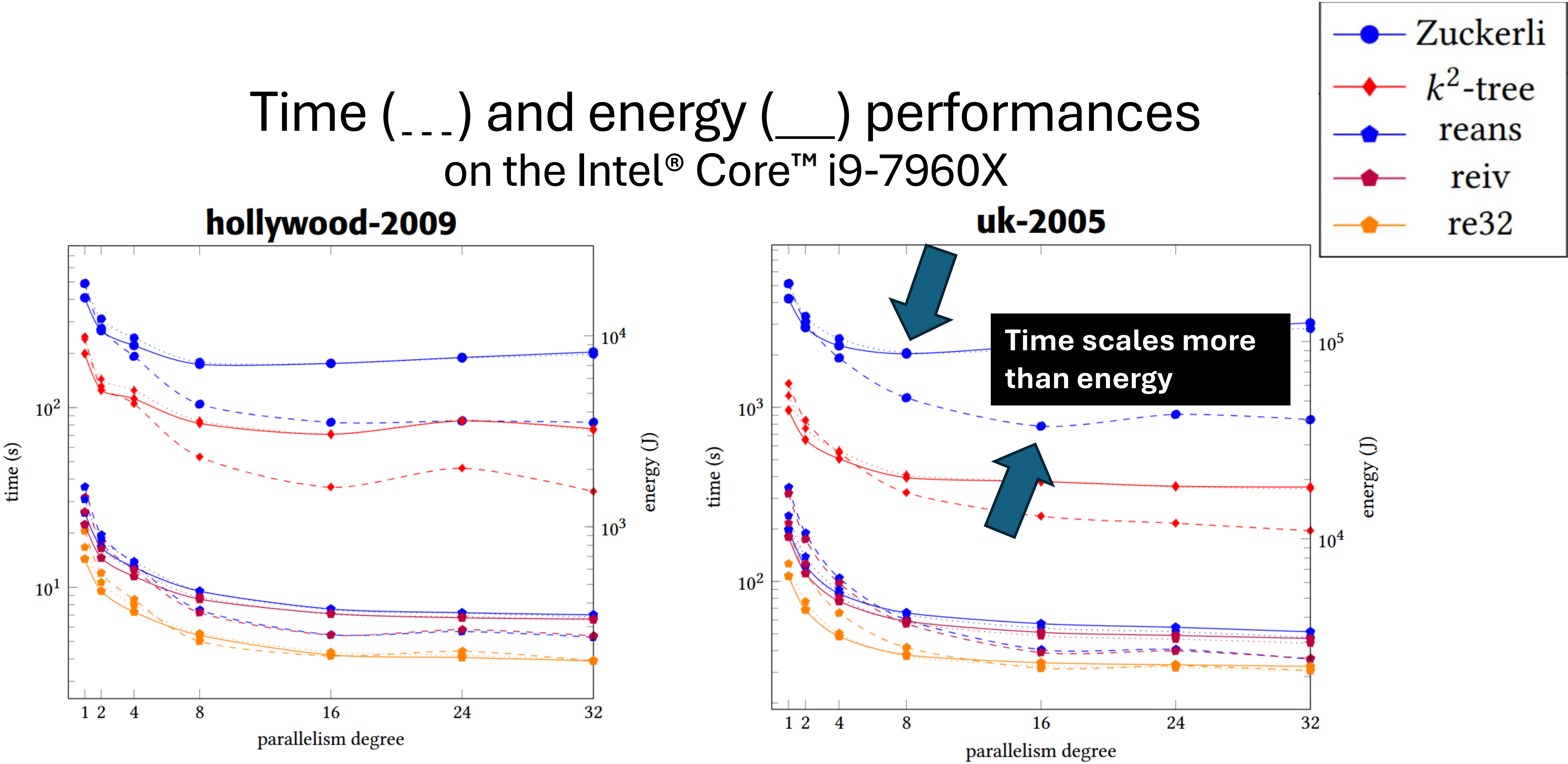
Space-time Performances

on the Intel® Core™ i9-7960X



Time (_ _ _) and energy (___) performances

on the Intel® Core™ i9-7960X



Time scales more than energy

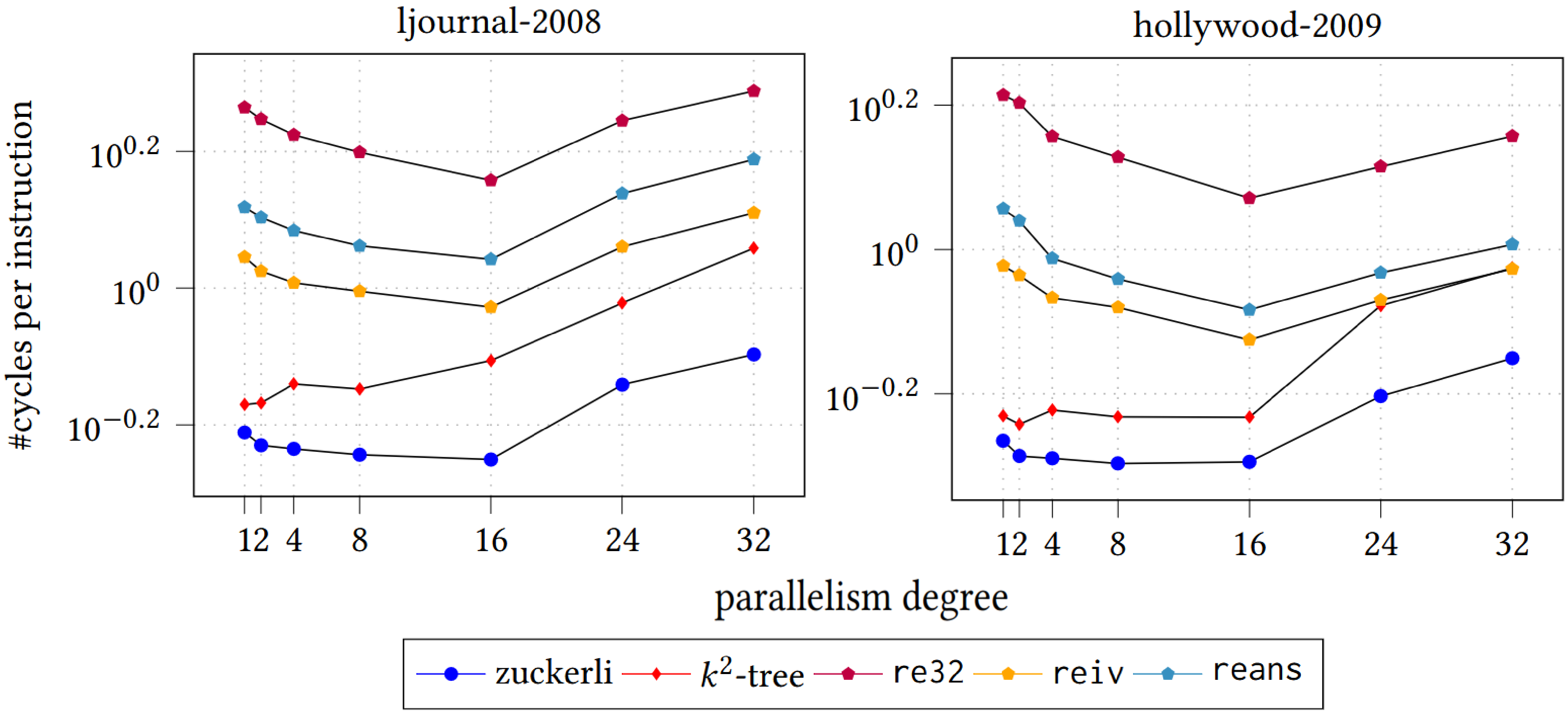
Multi-criteria approach in multi-threaded applications
(e.g., selecting the fastest solution within energy constraints)

Instruction throughput

on the Intel® Core™ i9-7960X

Zuckerli and the k^2-tree exhibit better throughputs

better



L1d/L3 cache access patterns

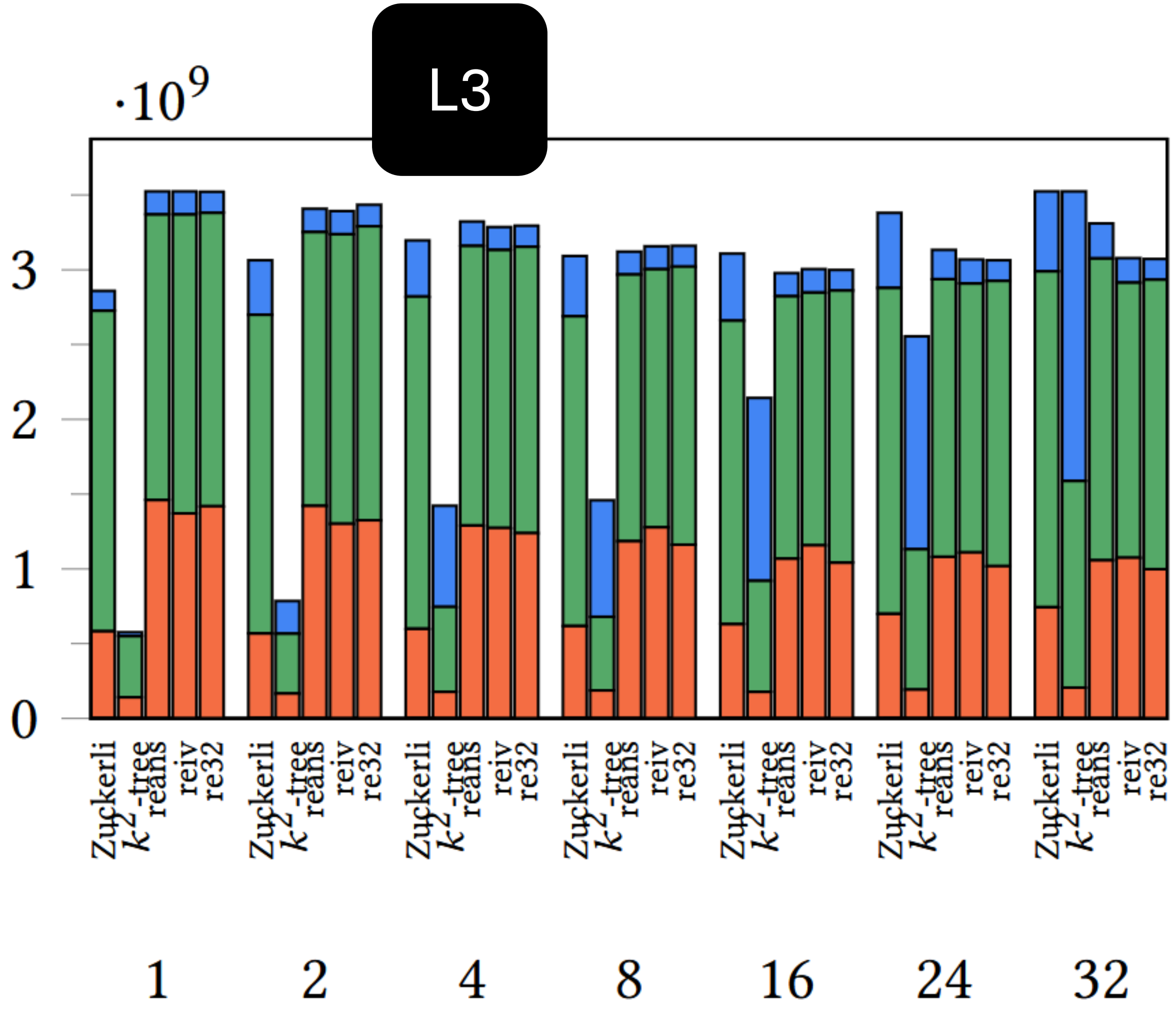
on the Intel® Core™ i9-7960X

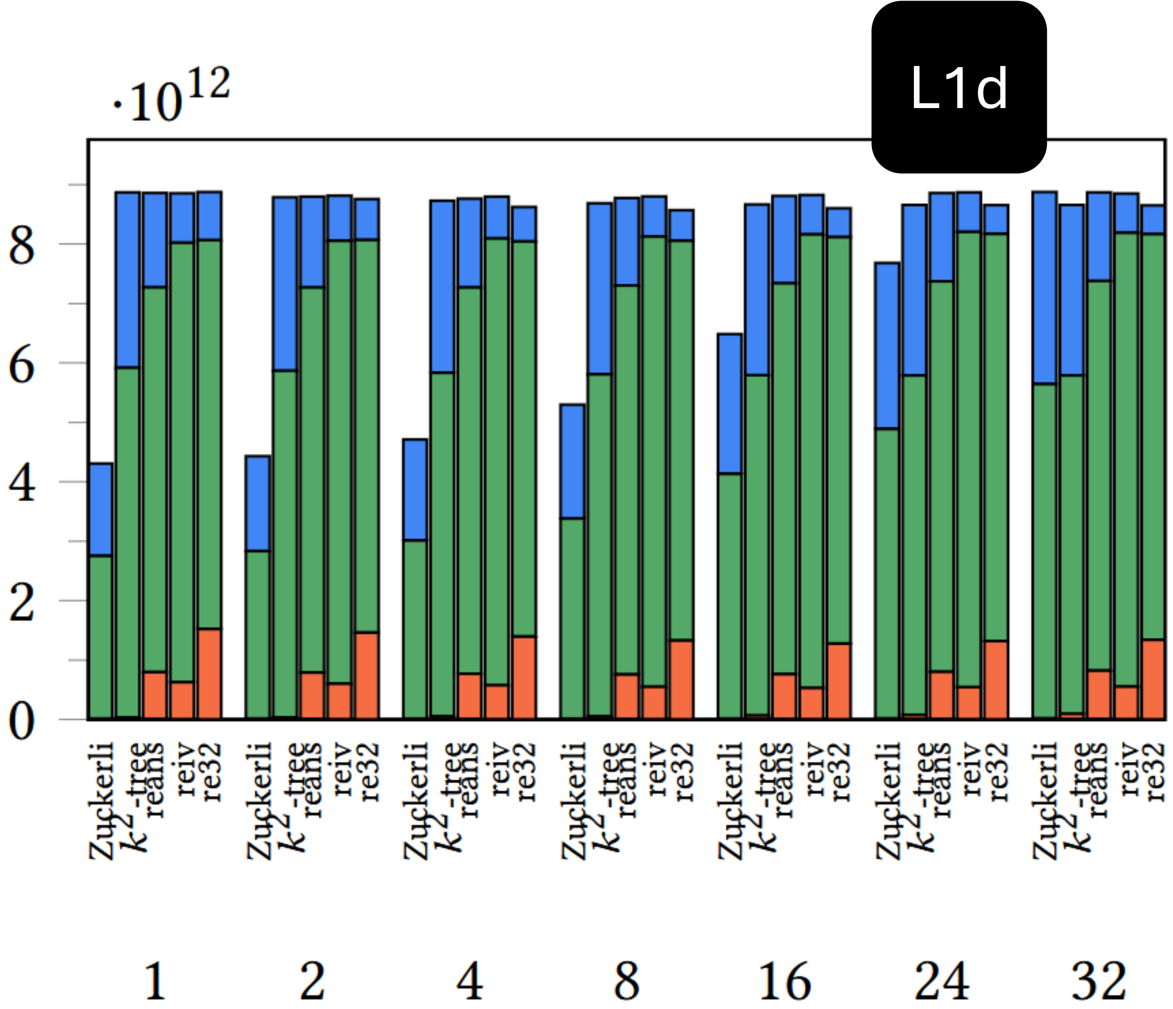
Dataset: ljournal-2008

load cache misses,

load cache hits,

operazioni di store.





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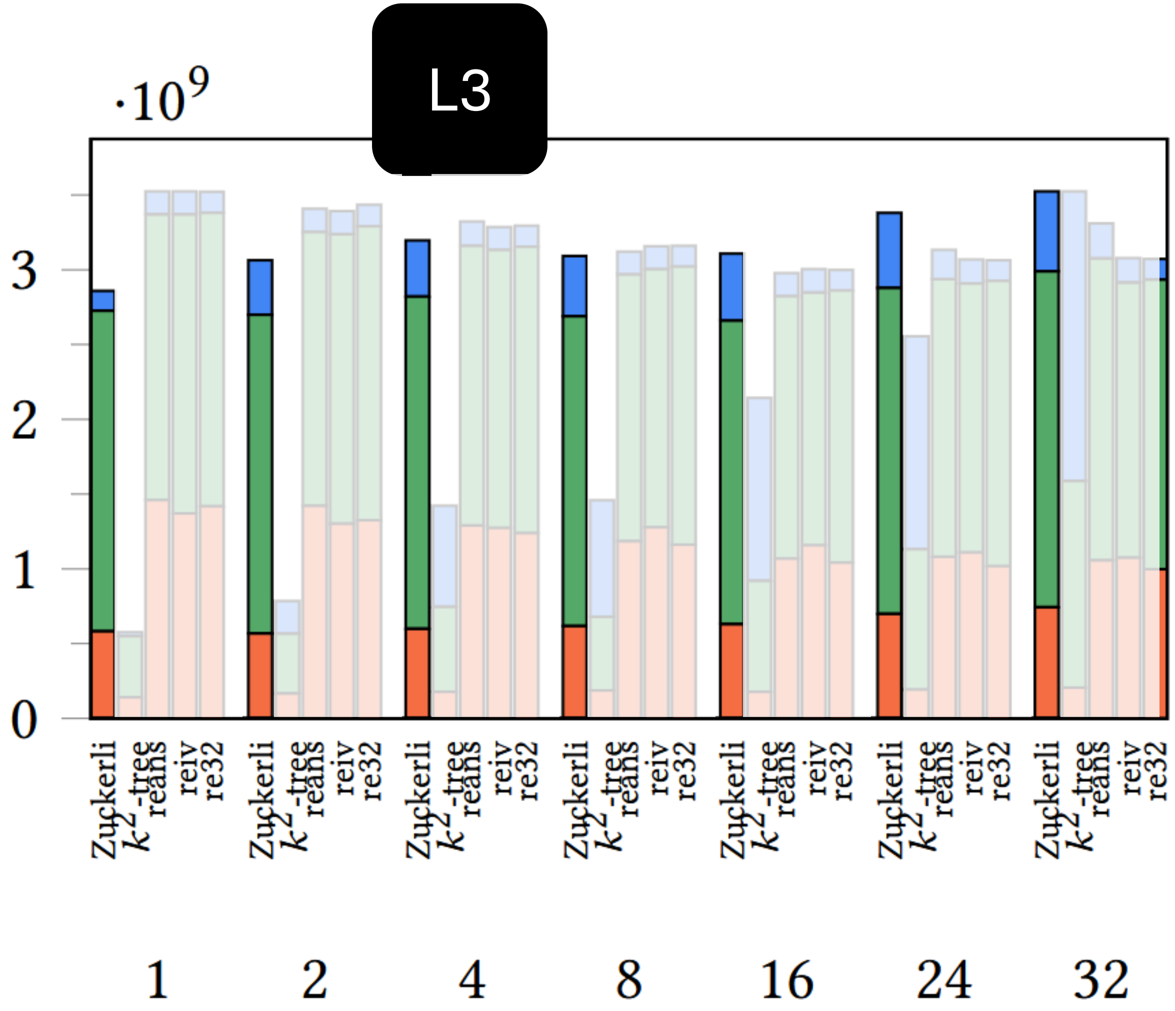

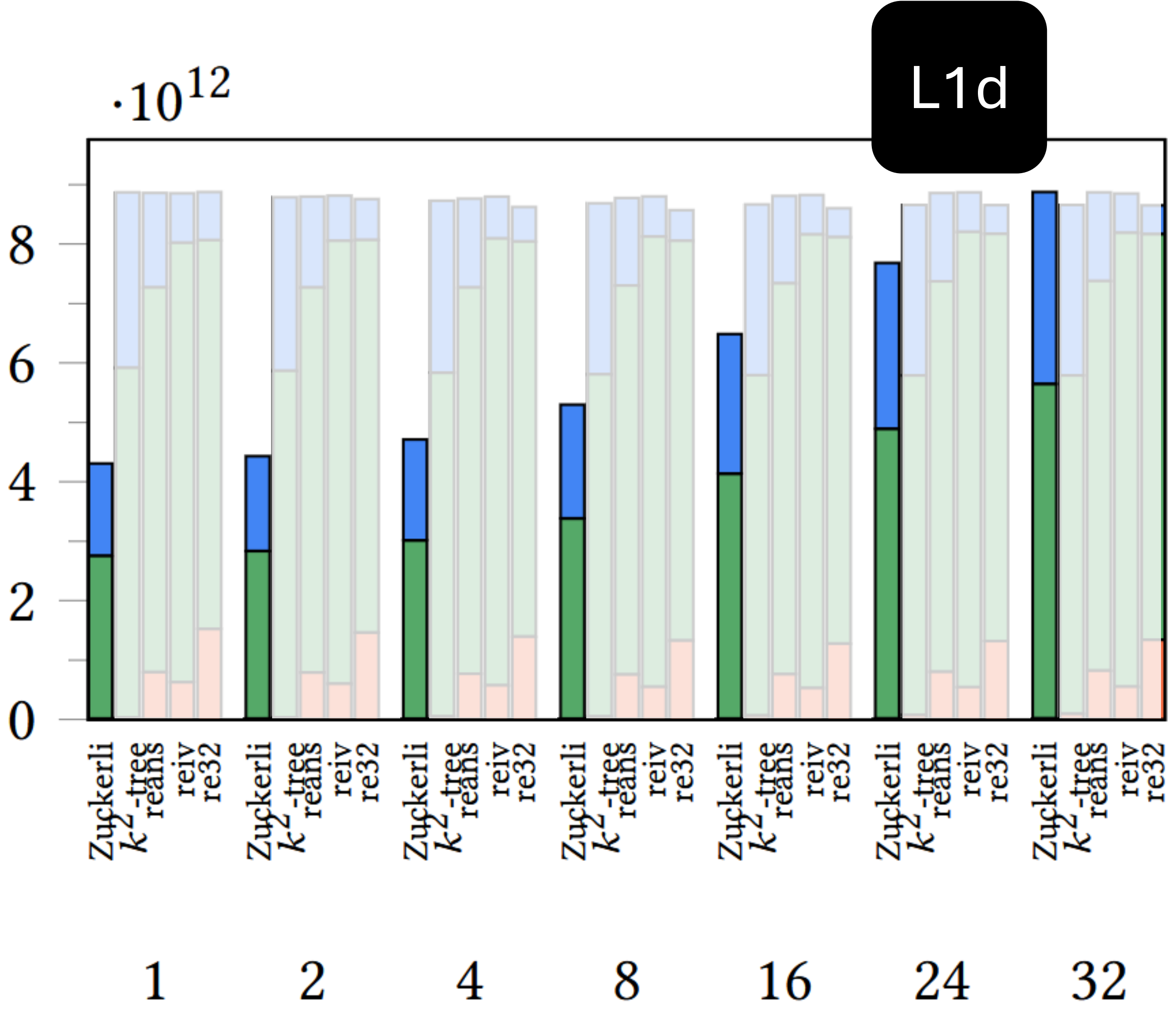

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on the Intel® Core™ i9-7960X
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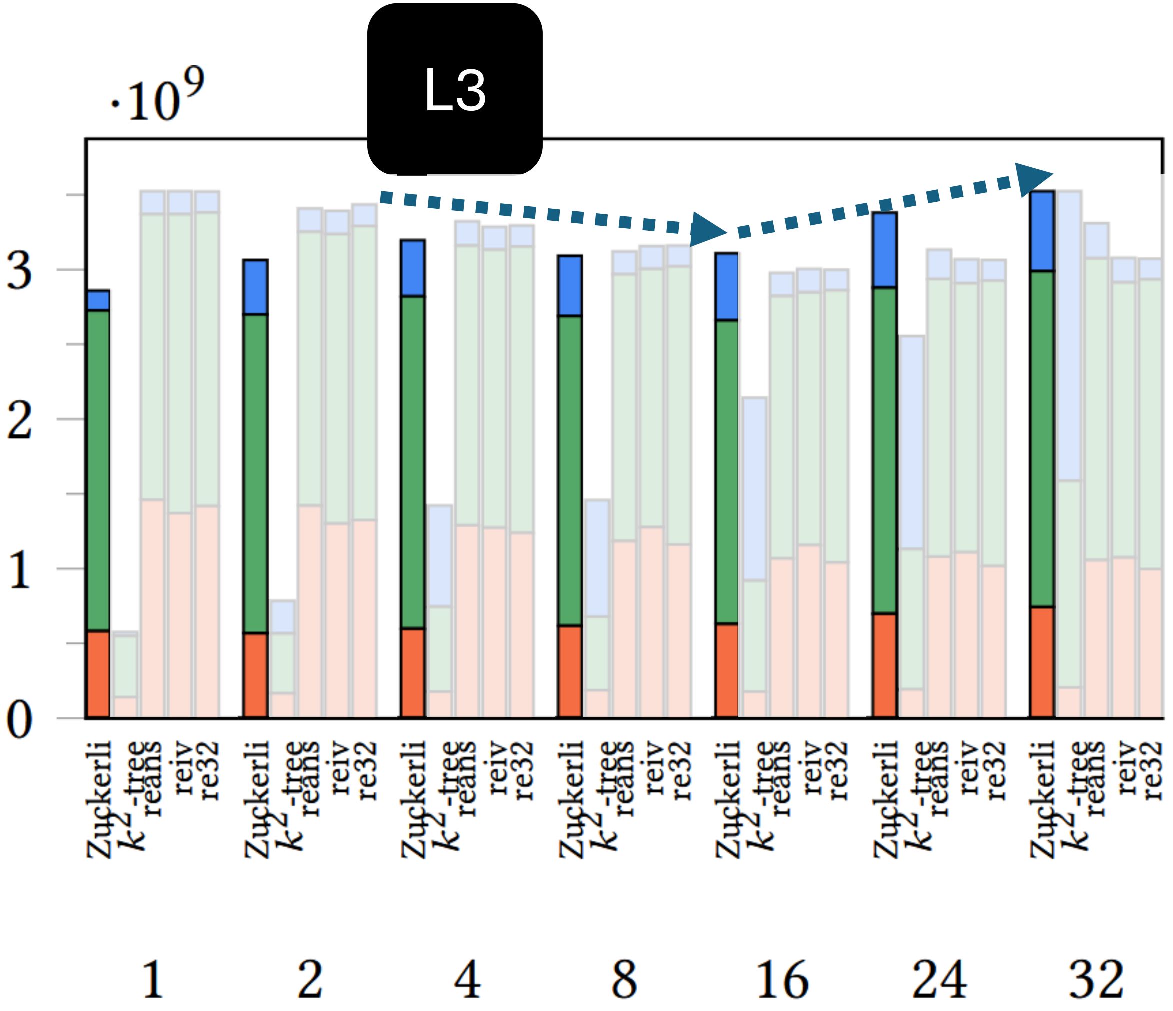

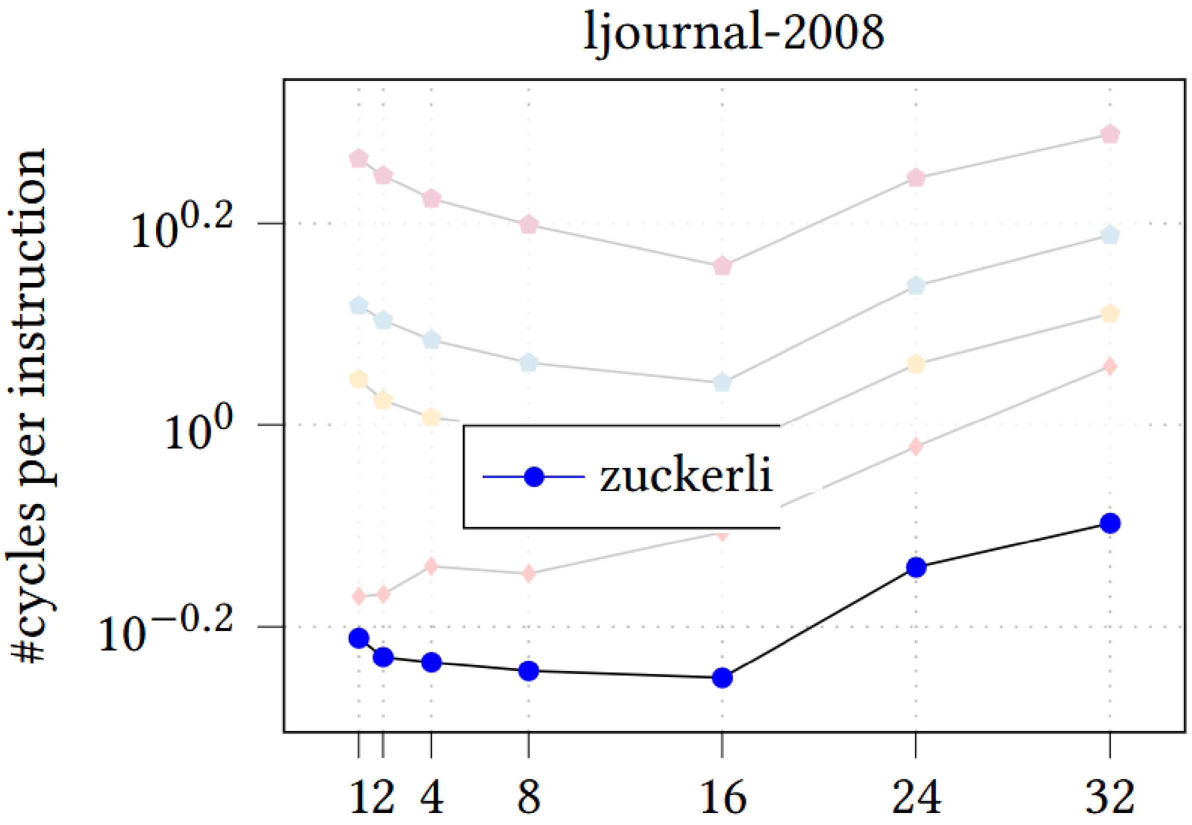

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on the Intel® Core™ i9-7960X

Dataset: ljournal-2008

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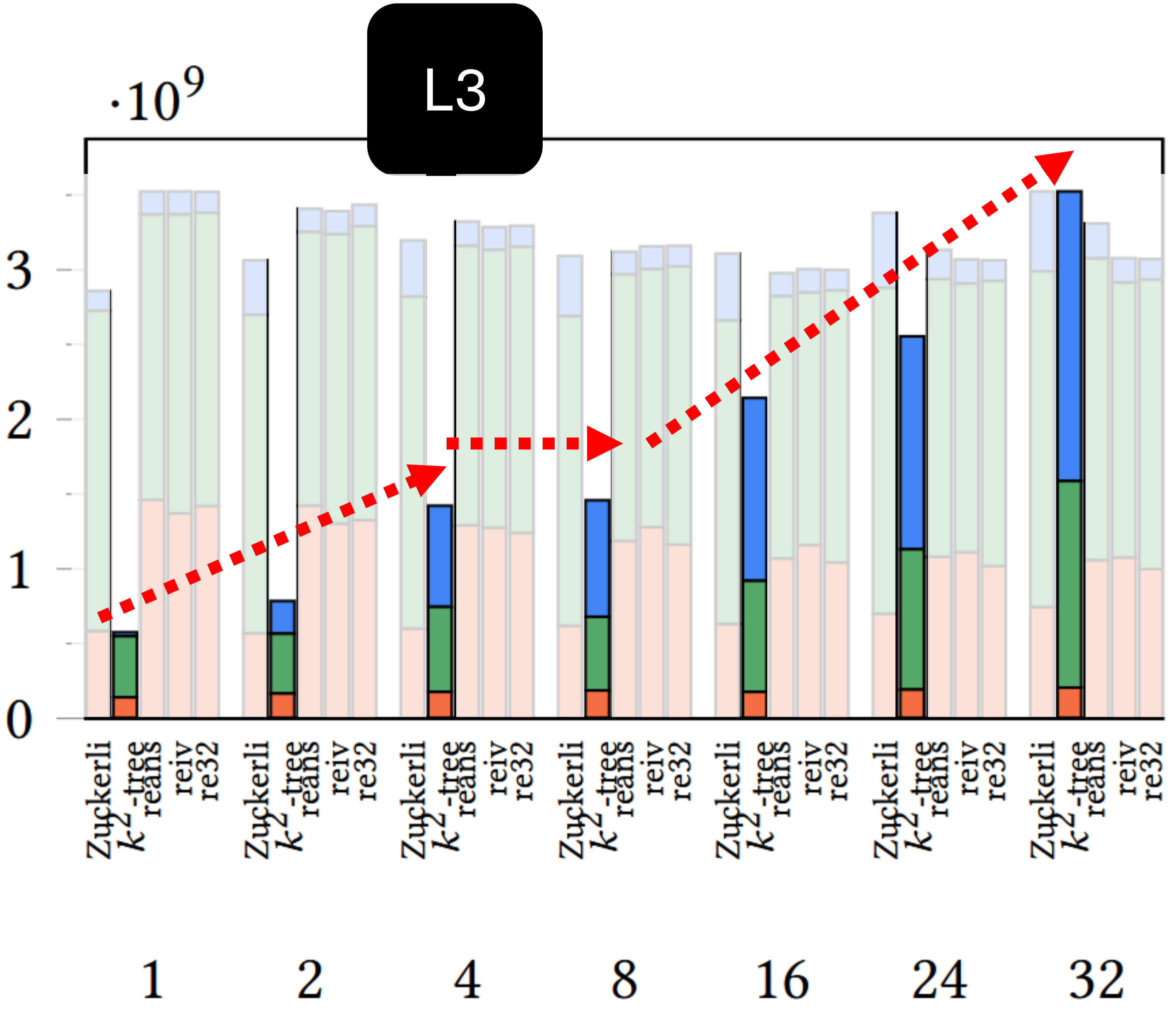
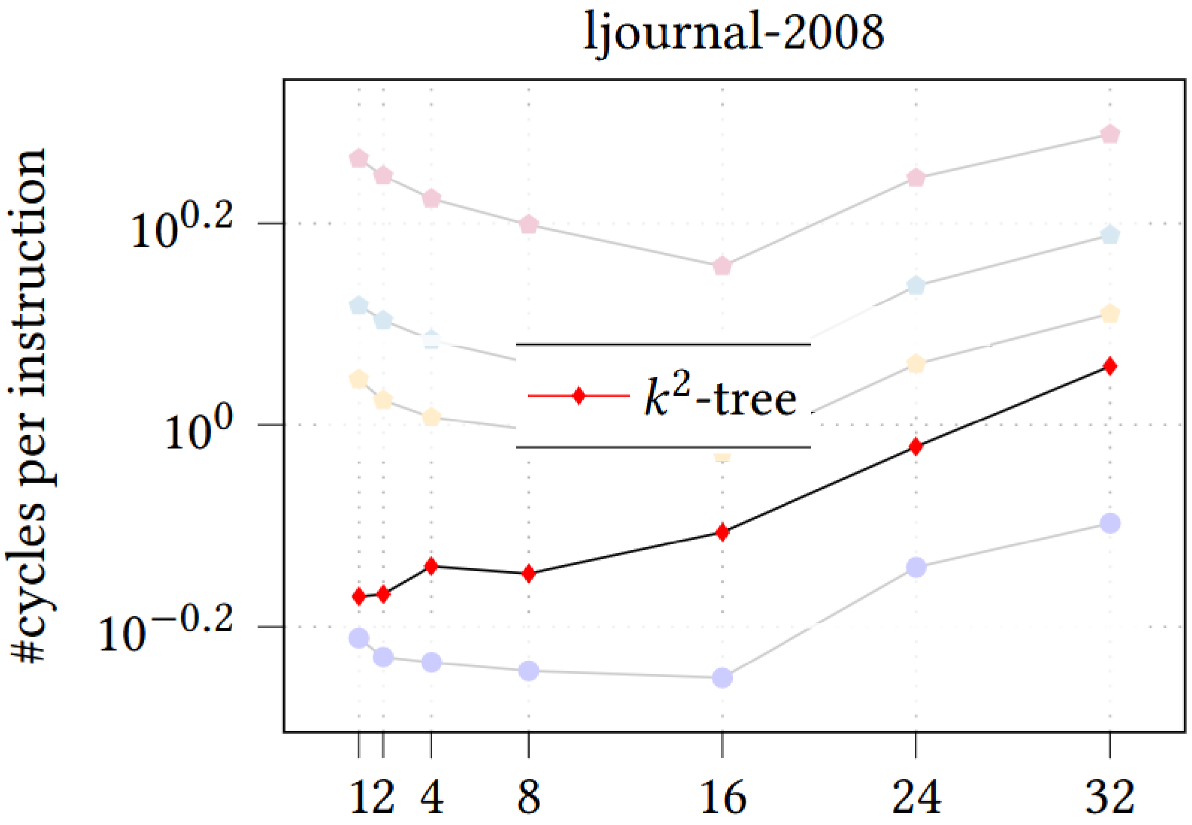


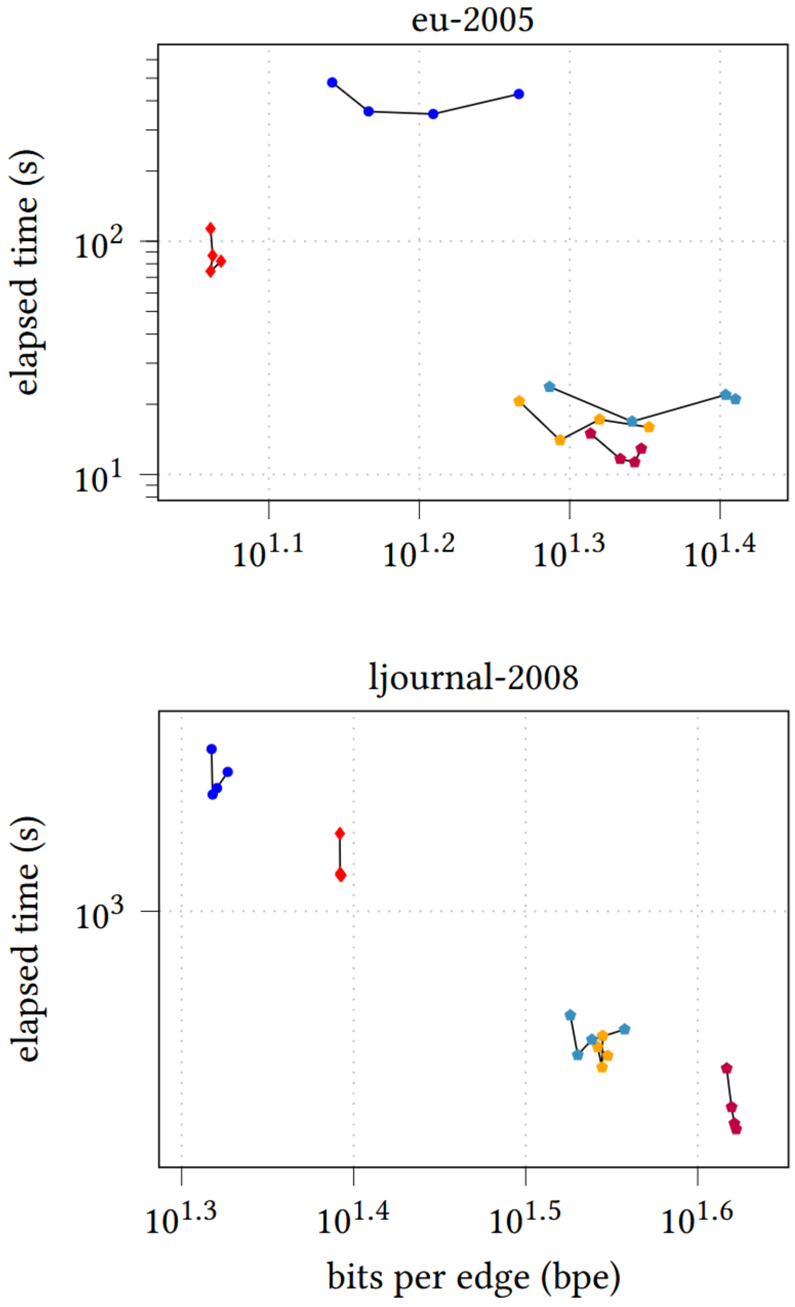


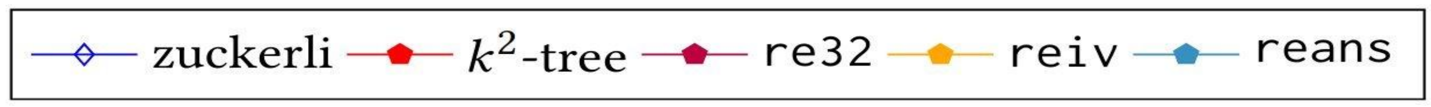
L1d/L3 cache access patterns

on the Raspberry Pi

5 smallest graphs. Up to ≤8 threads.

- Non-monotonic behaviour observed when scaling from 4 to 8 threads (notably for Zuckerli).
- Single-threaded k^2-tree outperforms all Zuckerli configurations on Raspberry Pi
- Less pronounced convex trend on Intel® Xeon® suggests different resource management.





Conclusion

- Appropriate compressed representations enable handling large datasets on resource-constrained devices.

- Careful selection of compressed representation can reduce energy usage by one or two orders of magnitude across different environments.

- The k^2-tree is nearly as fast as grammar-based compressors and almost as space-efficient as Zuckerli, with minimal degradation in compression ratio as threads increase.

- ✓ Optimal thread counts may differ for minimising energy vs. optimizing runtime.

- Increased L1 and L3 cache operations affect cycles per instruction

Future work

Investigate additional lossless compression formats for matrices and vectors.

Expand applications beyond PageRank.

In-depth studies on optimising energy-time tradeoffs. ⇒ Insights for software engineers to reduce carbon footprints and enhance battery life.

Study energy-efficient implementations of major compressed data structures (e.g., FM-index, Rank and select structures, Suffix arrays, Succinct tree topologies, …)

ML optimisation: explore combinations of lossless and state-of-the-art lossy compression strategies.

Guidelines for software engineers

Utilize computationally-friendly compressed formats: Efficiently handle large datasets on resource-constrained devices using domain-specific compression.

Employ k2-trees for matrix-vector dominated applications: k2-trees offer a simple, robust, and energy-efficient choice for applications like PageRank.

Prioritize cache efficiency: Implement cache-oblivious solutions to improve performance and reduce energy consumption by streamlining data access.

Investigate time-energy relationships: Explore the interplay between thread counts, runtime, and energy usage to develop multi-criteria algorithmic frameworks for performance and sustainability.

Optimise for space-time trade-offs and thread count: Balance performance and energy by optimizing thread allocation based on hardware.

Transparency and reproducibility

The entire codebase to reproduce the experiments is made available at gitlab: https://gitlab.com/ftosoni/green-lossless-spmv

Datasets from https://sparse.tamu.edu/LAW

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- 3 research groups
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Questions & Answers

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