A Tour of Mathematica

Numerical Calculations

In[1]:=
```
5 + 7
```

Out[1]= 12

In[2]:=
```
3 ^ 100
```

Out[2]= 515 377 520 732 011 331 036 461 129 765 621 272 702 107 522 001

In[3]:=
```
N[%]
```

Out[3]= 5.15378×10^{47}

In[4]:=
```
N[ Sqrt[10], 40 ]
```

Out[4]= 3.162277660168379331998893544432718533720

In[5]:=
```
(3 + 4 I) ^ 10
```

Out[5]= $-9\,653\,287 + 1\,476\,984\, i$

In[6]:=
```
BesselJ[0, 10.5]
```

Out[6]= -0.236648

In[7]:=
```
N[ Zeta[ 1/2 + 13 I ], 70 ]
```

Out[7]= 0.4430047825053681891978974413328491262590327026475644014329460002982735 -
0.6554830983211689430513696491913355062167589194500585032932666697173853 i

In[8]:=
```
Zeta[ 2 ]
```

Out[8]= $\frac{\pi^2}{6}$

In[9]:=
```
Integrate[ x^3/(1+x^3), x ]
```

Out[9]= $x - \frac{\text{ArcTan}\left[\frac{-1+2x}{\sqrt{3}}\right]}{\sqrt{3}} - \frac{1}{3}\text{Log}[1+x] + \frac{1}{6}\text{Log}\left[1-x+x^2\right]$

In[10]:=
```
NIntegrate[ Sin[Sin[x]], {x, 0, Pi} ]
```

Out[10]= 1.78649

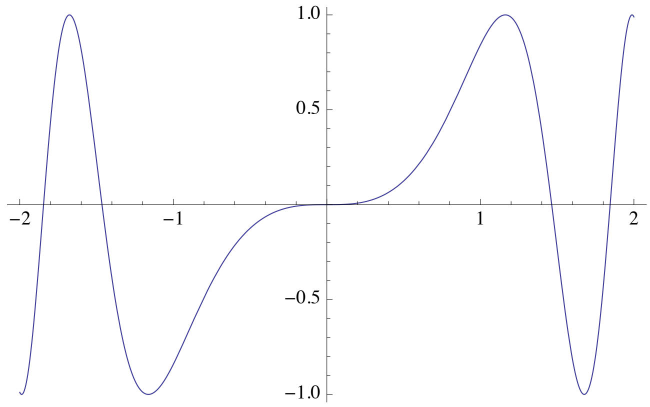
In[11]:=
```
FactorInteger[ 20654065386 ]
```

Out[11]= $\begin{pmatrix} 2 & 1 \\ 3 & 2 \\ 43 & 1 \\ 26\,684\,839 & 1 \end{pmatrix}$

Graphics

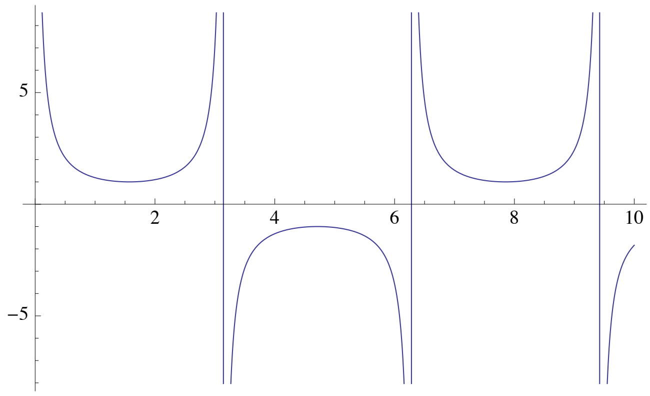
In[12]:= `Plot[Sin[x^3], {x, -2, 2}]`

Out[12]=



In[13]:= `Plot[1/Sin[x], {x, 0, 10}]`

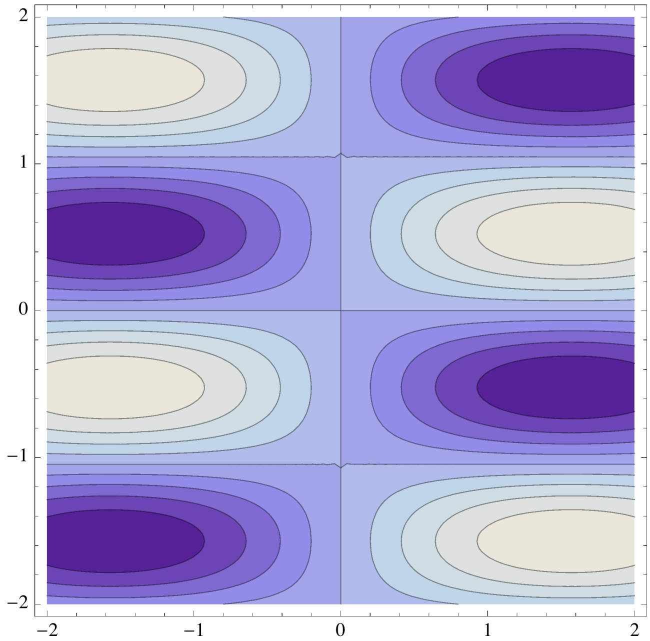
Out[13]=



Three-Dimensional Plots

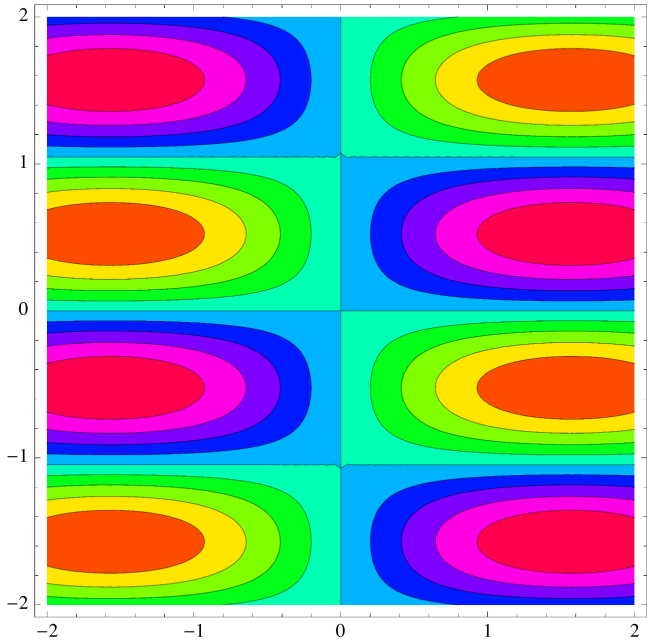
In[14]:=
```
ContourPlot[ Sin[x] Sin[3y], {x, -2, 2}, {y, -2, 2} ]
```

Out[14]=



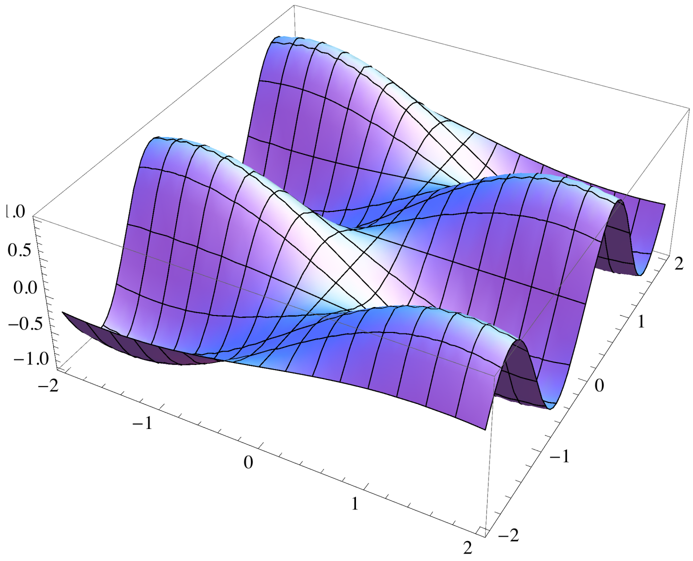
In[15]:=
```
ContourPlot[ Sin[x] Sin[3y], {x, -2, 2}, {y, -2, 2},ColorFunction->Hue ]
```

Out[15]=



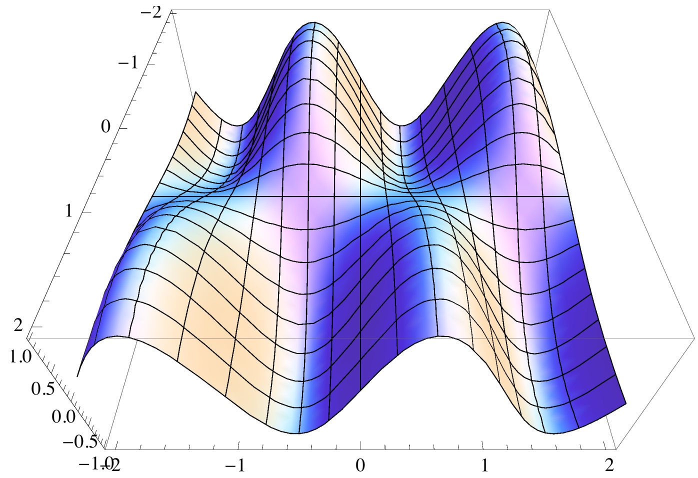
In[16]:= **Plot3D[Sin[x] Sin[3y], {x, -2, 2}, {y, -2, 2}]**

Out[16]=



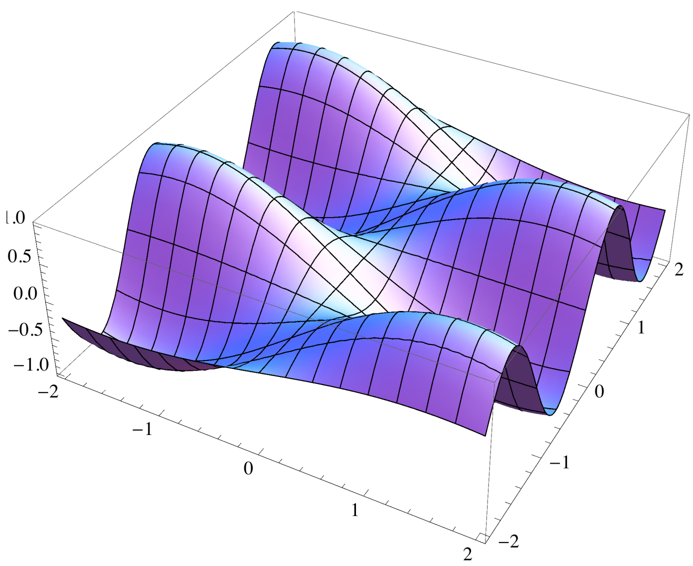
In[17]:= **Show[%, ViewPoint -> {1, 0, 1}]**

Out[17]=



In[18]:= **Plot3D[Sin[x] Sin[3 y], {x, -2, 2}, {y, -2, 2}, PlotPoints \[Rule] 40, Lighting \[Rule] Automat.**

Out[18]=



Algebraic Formulae

In[19]:=
```
(x + y)^2 + 9 (2 + x) (x + y)
```

Out[19]= $9\,(2+x)\,(x+y)+(x+y)^2$

In[20]:=
```
Expand[ % ]
```

Out[20]= $18\,x+10\,x^2+18\,y+11\,x\,y+y^2$

In[21]:=
```
% ^ 3
```

Out[21]= $\left(18\,x+10\,x^2+18\,y+11\,x\,y+y^2\right)^3$

In[22]:=
```
Expand[ % ]
```

Out[22]= $5832\,x^3+9720\,x^4+5400\,x^5+1000\,x^6+17\,496\,x^2\,y+30\,132\,x^3\,y+17\,280\,x^4\,y+3300\,x^5\,y+17\,496\,x\,y^2+32\,076\,x^2\,y^2+19\,494\,x^3\,y^2+3930\,x^4\,y^2+5832\,y^3+12\,636\,x\,y^3+8802\,x^2\,y^3+1991\,x^3\,y^3+972\,y^4+1242\,x\,y^4+393\,x^2\,y^4+54\,y^5+33\,x\,y^5+y^6$

In[23]:=
```
Factor[ % ]
```

Out[23]= $(x+y)^3\,(18+10\,x+y)^3$

In[24]:=
```
Series[ Exp[-x] Sin[2x], {x, 0, 6} ]
```

Out[24]= $2\,x-2\,x^2-\frac{x^3}{3}+x^4-\frac{19\,x^5}{60}-\frac{11\,x^6}{180}+O[x]^7$

Solving Equations

In[25]:=
```
x^4 - 7 x^3 + 3 a x^2 == 0
```

Out[25]= $3\,a\,x^2-7\,x^3+x^4==0$

In[26]:=
```
Solve[%, x ]
```

Out[26]= $\begin{pmatrix} x\to 0 \\ x\to 0 \\ x\to \frac{1}{2}\left(7-\sqrt{49-12\,a}\right) \\ x\to \frac{1}{2}\left(7+\sqrt{49-12\,a}\right) \end{pmatrix}$

In[27]:=
```
Solve[{
    a x + b y == 0,
    x - y     == c
}, {x, y} ]
```

Out[27]= $\left(x\to \frac{b\,c}{a+b}\quad y\to -\frac{a\,c}{a+b}\right)$

In[28]:=
```
Solve[{
    x^3 + y^3 == 1,
    x + y     == 2
}, {x, y} ]
```

Out[28]= $\begin{pmatrix} x\to \frac{1}{6}\left(6-i\sqrt{6}\right) & y\to \frac{1}{6}\left(6+i\sqrt{6}\right) \\ x\to \frac{1}{6}\left(6+i\sqrt{6}\right) & y\to \frac{1}{6}\left(6-i\sqrt{6}\right) \end{pmatrix}$

There are some equations, however, where it is mathematically impossible to get closed forms for all the solutions. Mathematica gets the solutions it can, then leaves a symbolic representation of the ones that cannot be found.

In[29]:=
```
Solve[ 1 + 8 x^3 + x^5 - 2 x^6 + 4 x^7 == 0, x ]
```

Out[29]=

$$\begin{pmatrix} x \to \frac{1}{4}\left(1 - i\sqrt{3}\right) \\ x \to \frac{1}{4}\left(1 + i\sqrt{3}\right) \\ x \to \text{Root}\left[1 + 2\,\#1 + \#1^5\,\&,\,1\right] \\ x \to \text{Root}\left[1 + 2\,\#1 + \#1^5\,\&,\,2\right] \\ x \to \text{Root}\left[1 + 2\,\#1 + \#1^5\,\&,\,3\right] \\ x \to \text{Root}\left[1 + 2\,\#1 + \#1^5\,\&,\,4\right] \\ x \to \text{Root}\left[1 + 2\,\#1 + \#1^5\,\&,\,5\right] \end{pmatrix}$$

You can use Mathematica to get a numerical approximation to all the solutions.

In[30]:=
```
N[ % ]
```

Out[30]=

$$\begin{pmatrix} x \to 0.25 - 0.433013\,i \\ x \to 0.25 + 0.433013\,i \\ x \to -0.486389 \\ x \to -0.701874 - 0.879697\,i \\ x \to -0.701874 + 0.879697\,i \\ x \to 0.945068 - 0.854518\,i \\ x \to 0.945068 + 0.854518\,i \end{pmatrix}$$

Lists

This makes a list of the first twenty factorials.

In[31]:=
```
Table[ n!, {n, 1, 20} ]
```

Out[31]=
```
{1, 2, 6, 24, 120, 720, 5040, 40 320, 362 880, 3 628 800, 39 916 800, 479 001 600,
 6 227 020 800, 87 178 291 200, 1 307 674 368 000, 20 922 789 888 000, 355 687 428 096 000,
 6 402 373 705 728 000, 121 645 100 408 832 000, 2 432 902 008 176 640 000}
```

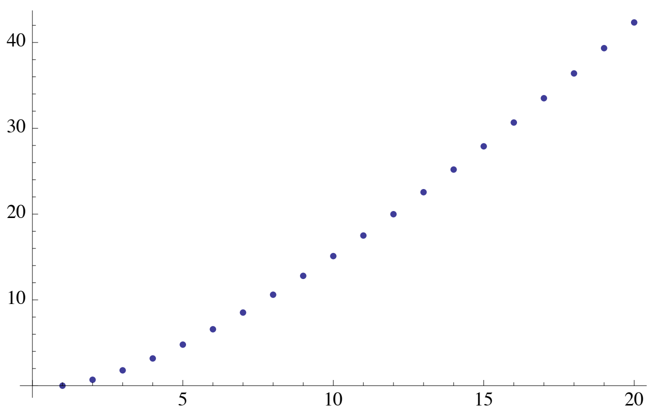
In[32]:=
```
N[ Log[ % ] ]
```

Out[32]=
```
{0., 0.693147, 1.79176, 3.17805, 4.78749, 6.57925, 8.52516, 10.6046, 12.8018, 15.1044,
 17.5023, 19.9872, 22.5522, 25.1912, 27.8993, 30.6719, 33.5051, 36.3954, 39.3399, 42.3356}
```

Here is a plot of the entries in the list.

In[33]:=
```
lp=ListPlot[ % ]
```

Out[33]=



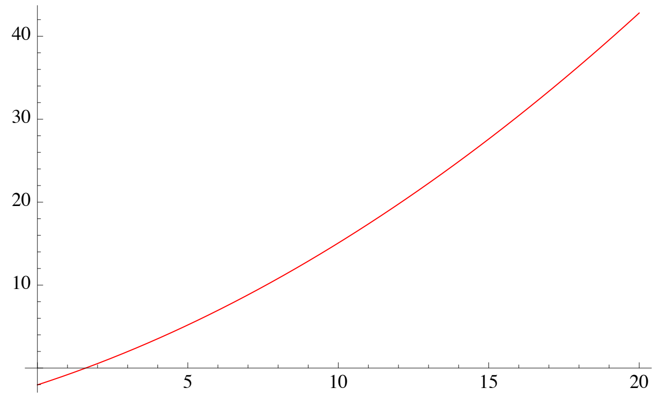
Fit finds least-squares fits to data.
This finds the quadratic formula which gives the best fit to the list of numbers.

In[34]:=
```
pol=Fit[ %%, {1, x, x^2}, x ]
```

Out[34]= $-2.02963 + 1.17902\,x + 0.0531166\,x^2$

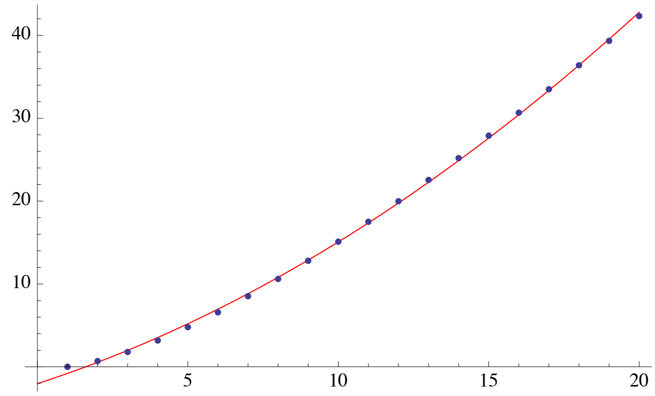
In[35]:=
```
pl = Plot[pol, {x, 0, 20}, PlotStyle → Red]
```

Out[35]=



In[36]:=
```
Show[pl, lp]
```

Out[36]=



▪ Vectors

Mathematica uses lists to represent vectors.
Here is the dot product of two three-dimensional vectors.

In[37]:=
```
{x, y, z} . {a, b, c}
```

Out[37]= a x + b y + c z

You can also do purely symbolic operations with lists.
Permutations gives all possible permutations of a list.

In[38]:=
```
Permutations[{a, b, c}]
```

Out[38]=

$$\begin{pmatrix} a & b & c \\ a & c & b \\ b & a & c \\ b & c & a \\ c & a & b \\ c & b & a \end{pmatrix}$$

\[FilledSmallSquare] Matrices

In[39]:= `m = Table[ 1 / (i + j + 1), {i, 3}, {j, 3} ]`

Out[39]= $\begin{pmatrix} \frac{1}{3} & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} & \frac{1}{6} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} \end{pmatrix}$

In[40]:= `Inverse[ m ]`

Out[40]= $\begin{pmatrix} 300 & -900 & 630 \\ -900 & 2880 & -2100 \\ 630 & -2100 & 1575 \end{pmatrix}$

In[41]:= `% . m`

Out[41]= $\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

In[42]:= `m - x IdentityMatrix[3]`

Out[42]= $\begin{pmatrix} \frac{1}{3} - x & \frac{1}{4} & \frac{1}{5} \\ \frac{1}{4} & \frac{1}{5} - x & \frac{1}{6} \\ \frac{1}{5} & \frac{1}{6} & \frac{1}{7} - x \end{pmatrix}$

In[43]:= `Det[ % ]`

Out[43]= $\frac{1}{378\,000} - \frac{317\,x}{25\,200} + \frac{71\,x^2}{105} - x^3$

In[44]:= `N[ Solve[ % == 0, x ] ]`

Out[44]= $\begin{pmatrix} x \to 0.657051 - 3.46945 \times 10^{-18}\, i \\ x \to 0.000212737 - 1.38778 \times 10^{-16}\, i \\ x \to 0.0189263 + 1.38778 \times 10^{-16}\, i \end{pmatrix}$

In[45]:= `Chop[%]`

Out[45]= $\begin{pmatrix} x \to 0.657051 \\ x \to 0.000212737 \\ x \to 0.0189263 \end{pmatrix}$

In[46]:= `Eigenvalues[ N[ m ] ]`

Out[46]= {0.657051, 0.0189263, 0.000212737}

In[47]:= `Eigenvalues[ {{a, b}, {-b, 2a}} ] // ExpandAll`

Out[47]= $\left\{\frac{3\,a}{2} - \frac{1}{2}\sqrt{a^2 - 4\,b^2},\ \frac{3\,a}{2} + \frac{1}{2}\sqrt{a^2 - 4\,b^2}\right\}$

In[48]:= `Eigenvalues[m]//Simplify`

Out[48]= $\{\text{Root}[-1 + 4755\,\#1 - 255\,600\,\#1^2 + 378\,000\,\#1^3\,\&,\,3],$
$\text{Root}[-1 + 4755\,\#1 - 255\,600\,\#1^2 + 378\,000\,\#1^3\,\&,\,2],$
$\text{Root}[-1 + 4755\,\#1 - 255\,600\,\#1^2 + 378\,000\,\#1^3\,\&,\,1]\}$

Making Definitions in Mathematica

This defines a value for the variable v.

In[49]:=
```
v = 1 + x
```
Out[49]= 1 + x

Now the value you have defined for v is used whenever v appears.

In[50]:=
```
5 + 2 v + 3 v^2
```
Out[50]= $5 + 2 (1 + x) + 3 (1 + x)^2$

You can actually define a value for any expression in Mathematica. This gives a value to w[2].

In[51]:=
```
w[2] = 1 + 2 a
```
Out[51]= 1 + 2 a

Whenever w[2] appears, it is now replaced by its value. Since you have not yet specified any value for w[1], it stays uncha

In[52]:=
```
w[1] + b w[2]
```
Out[52]= (1 + 2 a) b + w[1]

This defines a function f. The definition can be thought of as a rule for transforming expressions of the form f[anything].

In[53]:=
```
f[x_] := x^2
```

The occurrences of f in an expression like this are transformed according to the rule you have just given.

In[54]:=
```
f[3] + f[a+b]
```
Out[54]= $9 + (a + b)^2$

Here is the recursive rule for the factorial function.

In[55]:=
```
fac[n_] := n fac[n-1]
```

This gives a rule for the end condition of the factorial function.

In[56]:=
```
fac[1] = 1
```
Out[56]= 1

Here are the two rules you have defined for fac.

In[57]:=
```
?fac
```

```
Global`fac
```

```
fac[1] = 1

fac[n_] := n fac[n - 1]
```

Mathematica can now apply these rules to find values for factorials.

In[58]:=
```
fac[20]
```
Out[58]= 2 432 902 008 176 640 000

Funzioni che ricordano i valori calcolati

In[75]:=
```
Clear[fib]
fib[0]=0;
fib[1]=1;
fib[n_]:=fib[n-1]+fib[n-2];
```

In[82]:=
```
Timing[fib[33]]
```

Out[82]= {9.58823, 3 524 578}

In[84]:=
```
? fib
```

Global`fib

fib[0] = 0

fib[1] = 1

fib[n_] := fib[n - 1] + fib[n - 2]

In[85]:=
```
Clear[fib];
fib[0]=0;
fib[1]=1;
fib[n_]:= fib[n] = fib[n-1]+fib[n-2];
```

In[89]:=
```
Timing[fib[33]]
```

Out[89]= {0.000248, 3 524 578}

In[90]:=
```
? fib
```

Global`fib

fib[0] = 0

fib[1] = 1

fib[2] = 1

fib[3] = 2

fib[4] = 3

fib[5] = 5

fib[6] = 8

fib[7] = 13

fib[8] = 21

fib[9] = 34

fib[10] = 55

fib[11] = 89

fib[12] = 144

fib[13] = 233

fib[14] = 377

fib[15] = 610

fib[16] = 987

fib[17] = 1597

fib[18] = 2584

fib[19] = 4181

fib[20] = 6765

fib[21] = 10 946

fib[22] = 17 711

fib[23] = 28 657

fib[24] = 46 368

fib[25] = 75 025

fib[26] = 121 393

fib[27] = 196 418

fib[28] = 317 811

fib[29] = 514 229

fib[30] = 832 040

fib[31] = 1 346 269

fib[32] = 2 178 309

```
fib[33] = 3 524 578

fib[n_] := fib[n] = fib[n - 1] + fib[n - 2]

In[91]:= Timing[fib[100]]

Out[91]= {0.000473, 354 224 848 179 261 915 075}
```