Towards an Axiomatic Semantics of P Systems

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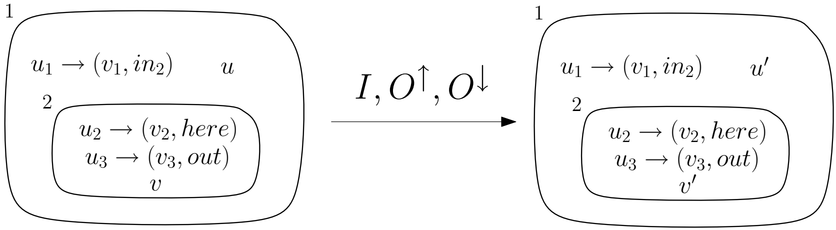
Introduction

We have defined a compositional operational semantics of P Systems as a labeled transition system [TCS, in press].

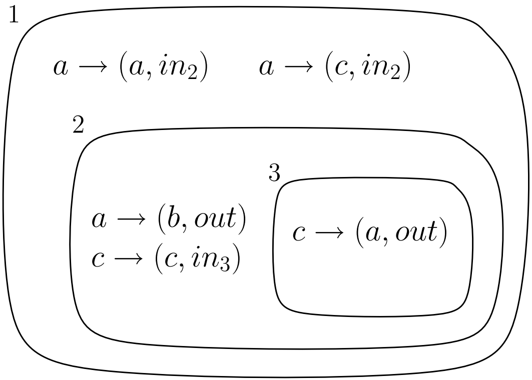
This semantics allows us to observe the behaviour of a membrane in terms of objects sent to and received from inner or external membranes.



where, I are object received (as an input), $O^{\uparrow}$ are the objects sent the external membrane and $O^{\downarrow}$ are the objects sent to inner membranes.

An Example (1)

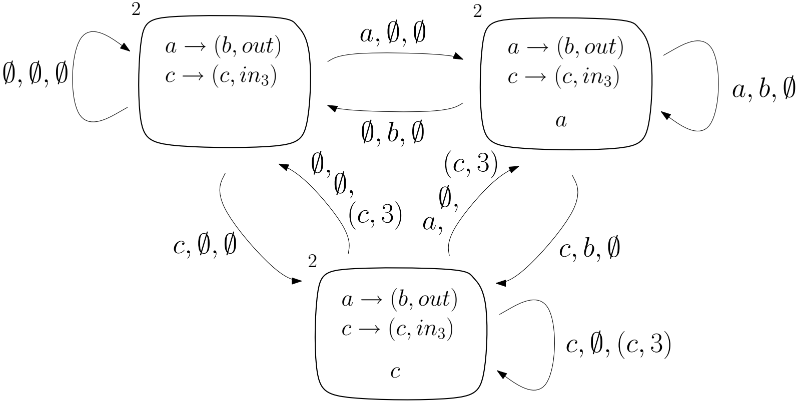
Let us consider the P System



and show the semantics of membrane number 2 in isolation. . .

An Example (2)

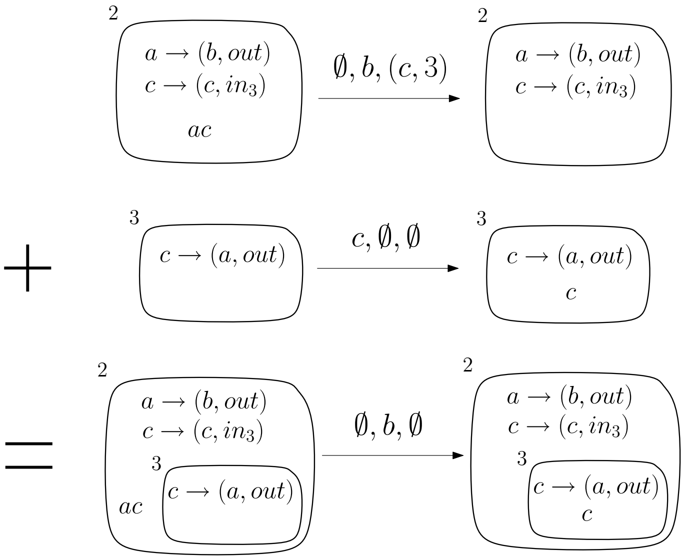
This is *a portion* of the semantics of membrane number 2:



Actually, the complete semantics has infinite states.

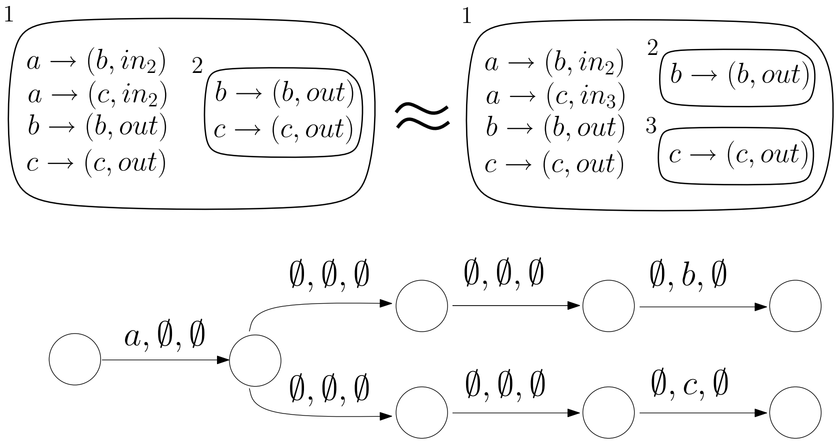
Compositionality

The semantics is compositional...



Behavioral Equivalences

The semantics allows us to define behavioral equivalences...



...that are congruences!

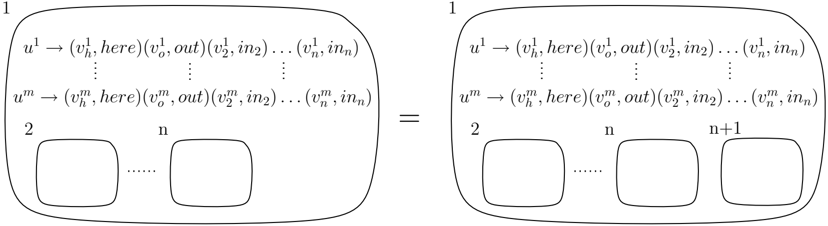
Axiomatization (1)

We would like to define an axiomatization of some behavioral equivalence.

Axioms are reversible syntactic transformations preserving the equivalence.

$$u \longrightarrow (v_h, here)(v_o, out)\delta \quad = \quad u \longrightarrow (v_h v_o, out)\delta$$

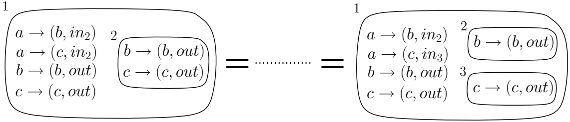
$$\{\, u \longrightarrow (v_h, here)(v_o, out)\delta \,\}$$
$$=$$
$$\{\, u \longrightarrow (v_h, here)(v_o, out)\delta \;\;,\;\; uu \longrightarrow (v_h v_h, here)(v_o v_o, out)\delta \,\}$$



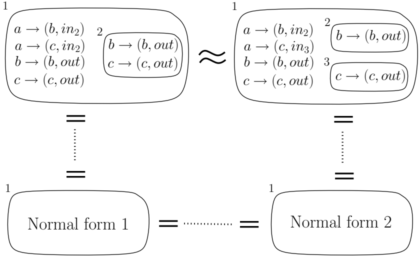
Axiomatization (2)

We would like the axioms to be sound and complete with respect to some behavioural equivalence $\approx$.

- This would allow us to forget about the semantics and to use axioms (syntactic transformations) to prove equivalence of two membranes



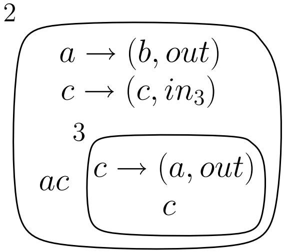
- Proving completeness is easier if we have a notion of **normal form**

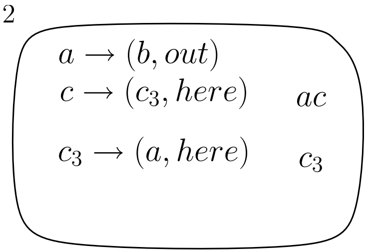


Towards a Normal Form

Idea: The normal form could be a *flat* membrane containing a *minimal* set of rules and multiset of objects.

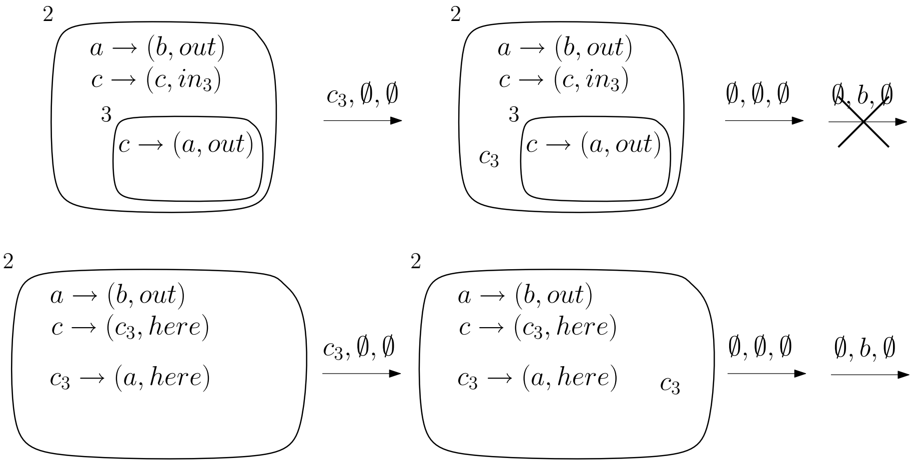
Flattening Technique: Given two membranes, one containing the other, the inner membrane is removed, its objects and rules are added to the ones of the containing membrane after suitable ridenomination.





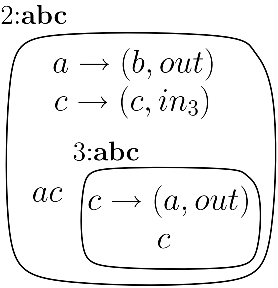
Flattening (1)

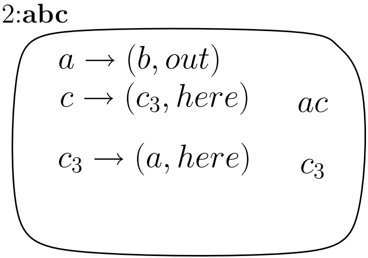
Problem n.1: The flat membrane has not the same behaviour as the original one. A rule added to membrane 2 could be applied to objects entering the membrane.



Flattening (2)

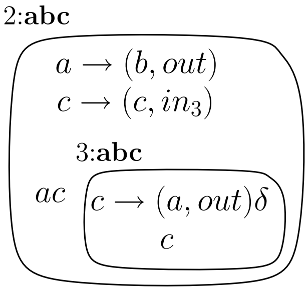
Possible solution: Introduce in the model a concept of *interface* of a membrane that specifies which objects are allowed to enter the membrane.

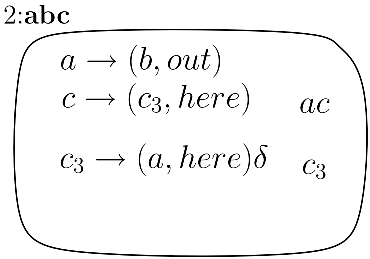




Flattening (3)

Problem n.2: If the inner membrane can be dissolved, dissolution must be simulated.

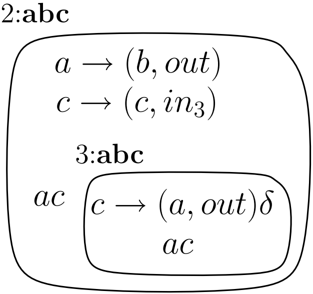


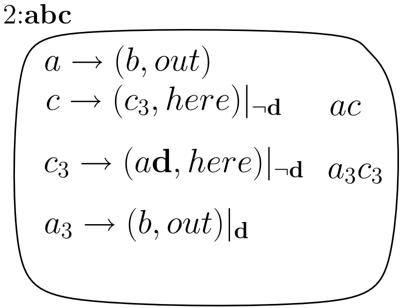


(This is wrong... It would dissolve membrane 2!!!)

Flattening (4)

Possible solution: Allow promoters and inhibitors in rules and replace δ with a special object **d**.

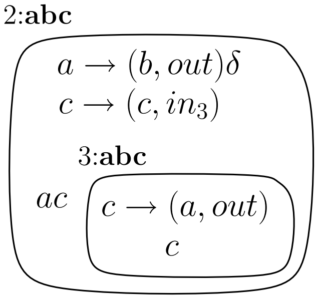


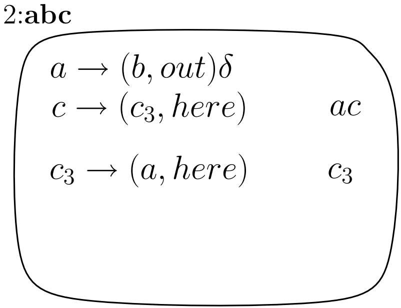


The rules simulating those in 3 are inhibited by **d**, and a copy of those in 2, after a suitable ridenomination, are promoted by **d**.

Flattening (5)

Problem n.3: If the containing membrane can be dissolved, one may have problems with rules of membranes containing the membrane which is dissolved.

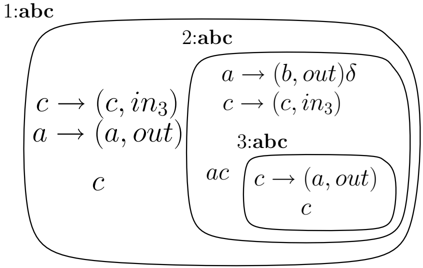




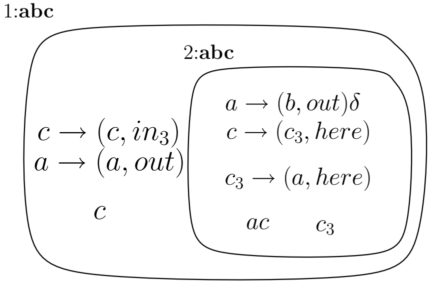
This is wrong, in fact...

Flattening (6)

...there exist a context in which they behave differently.



Here $b \rightarrow (c, in_3)$ can be eventually applied, leading to an output of a after a few steps.



Here $b \rightarrow (c, in_3)$ cannot be applied.

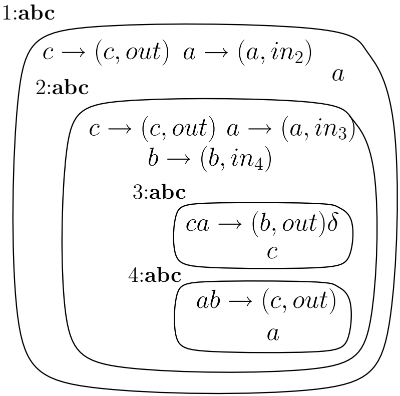
Flattening (7)

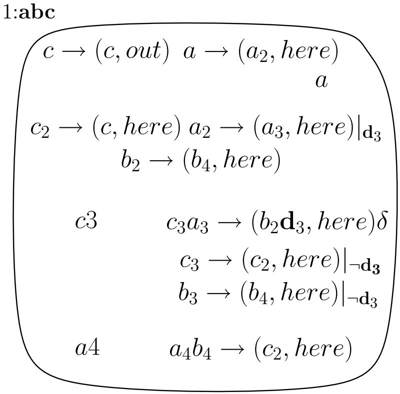
Possible solution: Avoid flattening compositions of membranes in which the external one can be dissolved.

As a consequence, the normal form of membrane systems contained in a membrane that can be dissolved will not be flat.

Normal Form (1)

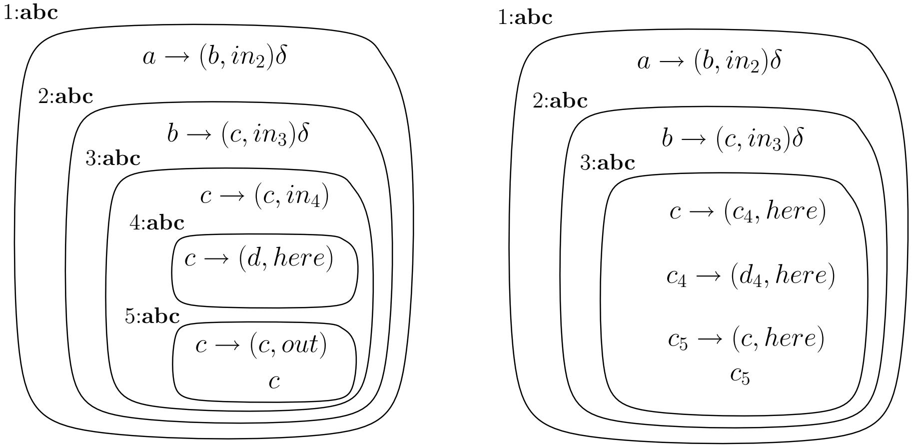
If the external membrane of a membrane structure cannot be dissolved its normal form is a single flat membrane that cannot be dissolved.





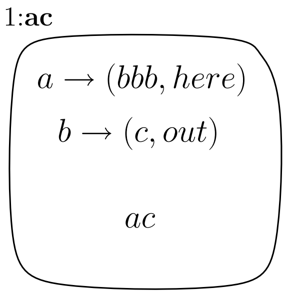
Normal Form (2)

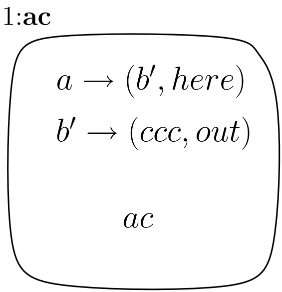
If the external membrane of a membrane structure can be dissolved its normal form is a structure consisting only of membranes that can be dissolved, but for innermost membranes that might be non–dissolvable.



Normal Form (3)

In order to reach a normal form, we would need also to transform rules and objects.





Conclusions

We belive that, given two systems in normal form, their equivalence could be checked as follows:

- if they are both flat, they should contain the same rules and objects, up to a suitable ridenomination;
- if they are both non flat, they should have the same membrane structure of equivalent membranes.

Open problems:

- defining flattening by means of axioms;
- defining rules and objects transformations into normal forms;
- decidability of the behavioural equivalences
  - step–by–step equivalence is not language equivalence
  - restrictions under which decidability holds