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# AN OVERVIEW ON OPERATIONAL SEMANTICS IN MEMBRANE COMPUTING

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The aim of this paper is to give motivations for the development of operational semantics in membrane computing, and to survey existing proposals. In particular, the definitions are compared of three operational semantics available in the literature, namely a semantics proposed by Andrei, Ciobanu and Lucanu, another proposed by Busi, and another one proposed by the authors of the present paper. These definitions are different since they are given with different aims. However, we show that there is an operational correspondence among the three.

*Keywords*: P systems, Structural operational semantics, Labelled transition systems

## 1. Introduction

The definition of (a variant of) P systems [19] usually includes a precise description of a generic computation step. Such a description must be exhaustive and unambiguous in order to allow the reader to understand exactly how the system behaves in all cases, and to allow proofs of theorems to be reliable and technically sound. In some cases, however, a more formal description of the system behaviour based on mathematical means is preferable. This happens for instance when one is interested either in developing a software tool which executes the P systems (such as an interpreter or a simulator) or in developing analysis techniques for such systems. In these cases, it might be helpful having a definition of a *transition relation* on *states* (or *configurations*) of a P system given by using only set-theoretic ingredients and such that two states $s_1$ and $s_2$ are in the transition relation if and only if a P system in state $s_1$ can perform a computation step to state $s_2$.

In order to define a transition relation describing the behaviour of P systems the *structural operational semantics (SOS)* approach proposed by Plotkin in [21, 22] can be followed. Such an approach consists in a systematic methodology based on inference rules for defining a semantics in terms of a *labelled transition system*. The SOS approach offers several advantages (see also [1]): it makes easier defining the semantics of a systems in terms of the semantics of its components, it makes easier proving properties of the semantics of a system by induction, and offers the possibility to prove some results on the semantics (such as congruence results for behavioural equivalences) simply by ensuring that the given inference rules satisfy some requirements (or formats).

Operational semantics for P systems which follow the SOS approach have been defined in [3, 4, 12, 9, 10, 7, 8] with different aims. In [3, 4, 12] Andrei, Ciobanu and Lucanu proposed a semantics mainly aimed at developing a provably correct interpreter for P systems. The semantics in [9, 10] was defined by Busi to study properties of the behaviour of P systems such as divergence and causality of events. In [7, 8] the authors of the present paper have defined a compositional semantics aimed at studying behavioural equivalences for P systems. In [6] the latter semantics has also been used to define a notion of diagnosability for biological systems inspired by some notions from systems security. Another operational semantics for P systems has been defined by Freund and Verlan in [15]. This semantics does not follow the SOS approach and it is aimed at representing a unique modular framework for the mathematical description of the behaviour of different classes of P systems.

A different approach to the definition of operational semantics for P systems consists in translating such systems into formalisms whose semantics has already been defined. As examples of this approach we mention the translations of P systems into Petri Nets [23, 17, 16, 20], into event structures [13, 20, 2], into the Calculus of Looping Sequences [18], and into the process calculus Pi@ [25]. These translations allow analysis means of the considered target formalisms to be applied to P systems.

In this paper we consider and compare the three mentioned definitions of operational semantics for P systems that follow the SOS approach. The variants of P systems considered in the three definitions are not exactly the same. For the sake of the comparison we adapt them to describe the same class of P systems, namely transition P systems with cooperative rules and no further ingredients. We will show that the three semantics manage maximal parallelism and communication of objects between membranes in different ways. However, we will show that, as expected, there is an operational correspondence between the three semantics.

Two of the three considered semantics in their original definitions can deal with dissolution rules. Describing dissolution of membranes with SOS rules is not trivial and it would be interesting to compare the different approaches followed by these semantics. However, we omit such a comparison in this paper for reasons of space limitation.

## 2. Background Notions of Structural Operational Semantics

In this section we present some basic notions of structural operational semantics [1, 21, 22, 24] that are needed in the remainder of the paper. In particular, we recall the definition of *Labelled Transition System* (LTS) and we show how an LTS can be specified by means of inference rules. An LTS is a mathematical model describing something having a notion of *state* (or *configuration*) which may evolve by performing steps from one state to another. An LTS is formally defined as follows.

**Definition 1 (Labelled Transition System)** *A* Labelled Transition System (LTS) *is a triple* $(S, L, \rightarrow)$ *where* $S$ *is the set of* states *(or* configurations*) ranged over by* $s, s_0, s_1, \ldots$*,* $L$ *is a set of* labels *ranged over by* $l, l_0, l_1, \ldots$ *and* $\rightarrow \subseteq S \times L \times S$ *is the* labelled transition relation*. We write* $s_0 \xrightarrow{l} s_1$ *when* $(s_0, l, s_1) \in \rightarrow$*.*

In an LTS, the nature of the elements of $S$ usually depends on what the LTS describes. The transition relation, instead, represents the steps that can be performed by the system from one state to another one. In fact, $s_0 \xrightarrow{l} s_1$ means that a system in state $s_0$ can change its state to $s_1$ in one step. Transitions of an LTS are enriched with labels containing information on the event that caused them. Such information is often exploited to define the semantics in a *compositional way*, namely in a way in which the semantics of a system, given as an LTS, is constructed from the semantics of its components, also given as LTSs. An LTS in which $L = \emptyset$ is called *Transition System (TS)*. Given an LTS, we write $s_0 \stackrel{l}{\not\rightarrow}$ if there exists no state $s_1$ such that $s_0 \xrightarrow{l} s_1$, and we write $s_0 \not\rightarrow$ if $s_0 \stackrel{l}{\not\rightarrow}$ holds for every label $l$. Moreover, we write $s_0 \rightarrow^+ s$ if there exist $s_1, \ldots, s_n \in S$ such that $s_0 \xrightarrow{l_0} s_1 \xrightarrow{l_1} \ldots \xrightarrow{l_{n-1}} s_n \xrightarrow{l_n} s$ for some labels $l_0, \ldots, l_n$. Finally, we write $s_0 \rightarrow^* s$ if either $s_0 = s$, or $s_0 \rightarrow^+ s$.

An LTS whose states are closed terms of a term algebra can be specified by means of a set of inference rules. According to [5] an inference rule for the specification of an LTS (a *transition rule*) is a logical rule having the form

$$\frac{t_1 \xrightarrow{l_1} t'_1 \quad \cdots \quad t_k \xrightarrow{l_k} t'_k \qquad t_{k+1} \stackrel{l_{k+1}}{\not\longrightarrow} \quad \cdots \quad t_n \stackrel{l_n}{\not\rightarrow}}{t \xrightarrow{l} t'}$$

where $t_1, t'_1, \ldots$ are (open) terms over the signature, namely terms that may contain state variables, $t_i \xrightarrow{l_i} t'_i$, for $1 \leq i \leq k$, and $t_j \stackrel{l_j}{\not\rightarrow}$ for $k+1 \leq j \leq n$, are the *premises* and $t \xrightarrow{l} t'$ is the *conclusion*. A transition rule without premises is called an *axiom*. Whenever we apply to a transition rule a substitution mapping variables to closed terms we obtain a closed transition rule, which states that whenever the premises are transitions of the LTS, then also the conclusion is a transition of the LTS.

Transition rules can be extended by allowing unary transition relations of the form $t\ P$, called *predicates* [11], among the premises (possibly negated) and as the conclusion. Side conditions can be associated with a transition rule with the effect of imposing that the conclusion of the rule is a transition of the LTS whenever both the premises and the side conditions are satisfied. Side conditions are often placed in the upper part of a transition rule, together with the premises. In what follows we shall freely use side conditions by assuming that they could be substitued by the use of some predicates, of more rules, and of more complex rules.

## 3. Operational Semantics in Membrane Computing

In this section we compare the operational semantics proposed in [4, 7, 9], adapted here to describe P systems defined as follows.

**Definition 2.** *A* P system $\Pi$ *is given by* $\Pi = (V, \mu, w_1, \ldots, w_N, R_1, \ldots, R_N)$ *where:*

- $V$ *is an* alphabet *whose elements are called* objects*;*
- $\mu \subset \mathbb{N} \times \mathbb{N}$ *is a* membrane structure*, such that* $(i, j) \in \mu$ *denotes that the membrane labelled by* $j$ *is contained in the membrane labelled by* $i$*;*
- $w_i$ *with* $1 \leq i \leq N$ *are strings from* $V^*$ *representing multisets over* $V$ *associated with the membranes* $1, 2, \ldots, N$ *of* $\mu$*;*
- $R_i$ *with* $1 \leq i \leq N$ *are finite sets of* evolution rules *associated with the membranes* $1, 2, \ldots, N$ *of* $\mu$*.*

An evolution rule is a pair $(u, v)$, denoted $u \to v$, where $u \in V^*$ and $v \in (V^* \times Tar)^*$, with $Tar = \{here, out\} \cup \{in_l \mid l \in \mathbb{N}\}$. Given a set of membrane labels $\{l_1, \ldots, l_n\} \subset \mathbb{N}$, we will always assume that multisets in $(V^* \times Tar)^*$ have the following form, in which objects with the same target are all grouped together:

$$(v_h, here)(v_o, out)(v_1, in_{l_1}) \ldots (v_n, in_{l_n}) .$$

Consequently, we will always assume that evolution rules have the following form:

$$u \to (v_h, here)(v_o, out)(v_1, in_{l_1}) \ldots (v_n, in_{l_n})$$

where $u$ is the multiset of objects consumed by the rule, $v_h$ is the multiset of objects produced by the rule which remain in the same membrane, and $v_o, v_1, \ldots, v_n$ are the multisets of objects produced by the rule and sent to the outer membrane and to the child membranes $l_1, \ldots, l_n$, respectively.

We will always represent multisets as strings, hence we denote multiset union as concatenation of strings. Moreover, given a multiset $w \in V^* \cup (V^* \times Tar)^*$ such that $w = v(v_h, here)(v_o, out)(v_1, in_{l_1}) \ldots (v_n, in_{l_n})$, we define $\mathbf{obj}(w), \mathbf{here}(w), \mathbf{out}(w)$ and $\mathbf{in}_l(w)$ as follows:

$$\mathbf{obj}(w) = v \qquad \mathbf{here}(v) = v_h \qquad \mathbf{out}(w) = v_o \qquad \mathbf{in}_{l_i}(v) = v_i .$$

Given a membrane structure $\mu$ (usually assumed from the context) and a membrane label $l$, we define $Skin$, $father(l)$ and $children(l)$ as follows:

$$Skin = l \text{ iff } \nexists l'.(l', l) \in \mu \qquad father(l) = l' \text{ iff } (l', l) \in \mu \qquad children(l) = \{l' \mid (l, l') \in \mu\}$$

Finally, we assume P system $\Pi_{toy} = (\{abcd\}, \{(1, 2), (1, 3)\}, aabb, \emptyset, \emptyset, R_1, R_2, R_3)$, with $R_1 = \{ab \to (b, here)(c, in_2), ab \to (b, out)(d, in_3)\}$, $R_2 = \{c \to (a, out)\}$ and $R_3 = \{d \to (a, out)\}$.

In subsections 3.1, 3.2 and 3.3 we recall the definitions of the semantics proposed in [9],[4] and [7], respectively. For reasons of space limitations we give only a brief description (by exploiting the toy example $\Pi_{toy}$) of the inference rules of the three semantics. Details can be found in the referenced papers. In subsection 3.4 we give some results of operational correspondence among the three considered semantics.

### 3.1. *The semantics proposed by Busi*

The semantics proposed by Busi [9] exploits a notion of *partial configuration* in which for each membrane of a P system $\Pi$ we have two multisets of objects, namely *active* and *frozen* objects.

**Definition 3 (Configuration)** *A* partial configuration *of a P system* $\Pi$ *is a tuple* $((w_1, \bar{w}_1), \dots, (w_N, \bar{w}_N))$*, where* $w_i \in V^*$ *and* $\bar{w}_i \in V^*$ *with* $1 \leq i \leq N$. *A* configuration *of a P system* $\Pi$ *is a partial configuration in which* $\bar{w}_1 = \dots = \bar{w}_N = \emptyset$. *The set of all partial configurations of a P system* $\Pi$ *is denoted by* $PConf_\Pi$*, with* $Conf_\Pi$ *denoting the subset of all (non-partial) configurations.*

We shall denote a partial configuration $((w_1, \bar{w}_1), \dots, (w_n, \bar{w}_N))$ with $\times_{i=1}^{N}(w_i, \bar{w}_i)$. Moreover, we will assume $\gamma, \gamma', \gamma_1, \gamma_2, \dots$ to range over partial configurations. Finally, the function *heated*, defined as $heated\left(\times_{i=1}^{N}(w_i, \bar{w}_i)\right) = \times_{i=1}^{N}(w_i\bar{w}_i, \emptyset)$, will be used to transform partial configurations into non-partial ones.

**Definition 4 (Semantics)** *Let* $\mapsto \subset PConf_\Pi \times PConf_\Pi$ *be the least transition relation satisfying the following rule:*

$$\frac{\begin{array}{c} u \to v \in R_k \qquad u \subseteq w_k \qquad w'_k = (w_k \setminus u) \cup \mathbf{here}(v) \\ k \neq 1 \textit{ implies } w'_{father(k)} = w_{father(k)}\mathbf{out}(v) \\ \forall i \in children(k). w'_i = w_i \mathbf{in}_i(v) \end{array}}{\times_{i=1}^{N}(w_i, \bar{w}_i) \mapsto \times_{i=1}^{N}(w'_i, \bar{w}'_i)}$$

*The semantics of a P system* $\Pi$ *is given by the least transition relation* $\Mapsto \subset Conf_\Pi \times Conf_\Pi$ *such that* $\gamma_1 \Mapsto \gamma_2$ *holds if and only if* $\gamma_1 \mapsto^+ \gamma', \gamma' \not\mapsto$ *and* $\gamma_2 = heated(\gamma')$.

Let us consider the toy example of P system $\Pi_{toy}$. Its initial configuration is $((aabb, \emptyset), (\emptyset, \emptyset), (\emptyset, \emptyset))$. By using the definition of $\mapsto$ we can obtain the following derivation describing one application of each rule in the skin membrane:

$$((aabb, \emptyset), (\emptyset, \emptyset), (\emptyset, \emptyset)) \mapsto ((ab, b), (\emptyset, c), (\emptyset, \emptyset)) \mapsto ((\emptyset, b), (\emptyset, c), (\emptyset, d)) \not\mapsto \ .$$

Such a derivation allows us to derive the following maximally parallel step:

$$((aabb, \emptyset), (\emptyset, \emptyset), (\emptyset, \emptyset)) \Mapsto ((b, \emptyset), (c, \emptyset), (d, \emptyset)) \ .$$

### 3.2. *The semantics proposed by Andrei, Ciobanu and Lucanu*

Configurations of this semantics (see [4]) are called *membranes*.

**Definition 5 (Membranes)** Membranes $M$ *and* sibling membranes $M_+$ *are given by the following grammar:*

$$M \ ::= \ \langle\, l \mid w \rangle \ \big| \ \langle\, l \mid w\,;\, M_+ \rangle \qquad\qquad M_+ \ ::= \ M \ \big| \ M, M_+$$

*where* $l \in \mathbb{N}$ *is a membrane label and* $w \in V^* \cup (V^* \times Tar)^*$. *We denote with* $\mathcal{M}(\Pi)$ *and* $\mathcal{M}^+(\Pi)$ *the sets of membranes and sibling membranes for a P system* $\Pi$ *(which provides the alphabet and the membrane labels).*

We shall assume $NULL$ to represent an empty set of sibling membranes such that $\langle\, l \mid w\,;\, NULL\rangle \equiv \langle\, l \mid w\rangle$. We define $\mathcal{M}^*(\Pi) = \mathcal{M}^+(\Pi) \cup \{NULL\}$. Moreover, given $M = \langle\, l \mid w\,;\, M_*\rangle$ with $M_* \in \mathcal{M}^*(\Pi)$, we define $L(M) = l$ and $\mathbf{w}(M) = w$.

The set of membranes $\mathcal{M}(\Pi)$ can be partitioned into two subsets: the first consisting of membranes in which all multisets $w$ contain only objects from the alphabet $V$, and the second consisting of membranes in which there is a multiset $w$ containing elements from $(V^* \times Tar)$. Membranes of the first subset, denoted $\mathcal{C}(\Pi)$, represent *configurations* of the P system $\Pi$, namely states that can be reached by $\Pi$ by performing maximally parallel steps. Membranes of the second subset, denoted $\mathcal{C}^\#(\Pi)$, represent *intermediate configurations*, namely states that can be reached in the middle of the execution of a maximally parallel step.

This semantics exploits some irreducibility notions defined as follows.

The *l-irreducibility* and the *mpr-irreducibility* properties are defined as follows: a multiset $w \in V^* \cup (V^* \times Tar)^*$ is $l$-irreducible iff there are no rules in $R_l$ applicable to $\mathbf{obj}(w)$; $NULL$ is mpr-irreducible; a set of sibling membranes $M_1, \ldots, M_n$ is mpr-irreducible iff each $M_i$, with $1 \leq i \leq n$ is mpr-irreducible; a membrane $\langle\, l \mid w\,;\, M_*\rangle$ is mpr-irreducible iff $w$ is $l$-irreducible and $M_*$ is mpr-irreducible.

The *tar-irreducicibility* property is defined as follows: $NULL$ is tar-irreducible; a set of sibling membranes $M_1, \ldots, M_n$ is tar-irreducible iff each $M_i$, with $1 \leq i \leq n$ is tar-irreducible; a skin membrane $\langle\, Skin \mid w\,;\, M_*\rangle$ is tar-irreducible iff $w = \mathbf{obj}(w)$, for all $M \in M_*$ it holds $\mathbf{w}(M) = \mathbf{obj}(\mathbf{w}(M))$ and $M_*$ is tar-irreducible; a membrane $\langle\, l \mid w\,;\, M_*\rangle$ with $l \neq Skin$ is tar-irreducible iff $w = \mathbf{obj}(w)\mathbf{out}(w)$, for all $M \in M_*$ it holds $\mathbf{w}(M) = \mathbf{obj}(\mathbf{w}(M))$ and $M_*$ is tar-irreducible.

**Definition 6 (Semantics)** *Let* $\stackrel{mpr}{\Longrightarrow} \subset \mathcal{C}(\Pi) \times \mathcal{M}(\Pi)$ *and* $\stackrel{tar}{\Longrightarrow} \subset \mathcal{C}^\#(\Pi) \times \mathcal{C}(\Pi)$ *be the least transition relations given by the inference rules in Fig. 1 and Fig. 2, respectively. The* semantics *of a P system* $\Pi$ *is given by the transition relation* $\Rightarrow \subset \mathcal{C}(\Pi) \times \mathcal{C}(\Pi)$ *such that* $M \Rightarrow M'$ *holds if and only if*

- *either there exists* $M'' \in \mathcal{C}^\#(\Pi)$ *such that* $M \stackrel{mpr}{\Longrightarrow} M'' \stackrel{tar}{\Longrightarrow} M'$,
- *or* $M \stackrel{mpr}{\Longrightarrow} M'$ *and* $M'$ *is tar-irreducible.*

Let us consider again the P system $\Pi_{toy}$. In this case the initial configuration is membrane $M_{toy} = \langle\, 1 \mid aabb\,;\, \langle\, 2 \mid \emptyset\rangle, \langle\, 3 \mid \emptyset\rangle\rangle$. The maximally parallel step of $\Pi_{toy}$ described with the previous semantics is described now by the following transitions:

$$M_{toy} \stackrel{mpr}{\Longrightarrow} \langle\, 1 \mid (b, here)(b, out)(c, in_2)(d, in_3)\,;\, \langle\, 2 \mid \emptyset\rangle, \langle\, 3 \mid \emptyset\rangle\rangle \stackrel{tar}{\Longrightarrow} \langle\, 1 \mid b\,;\, \langle\, 2 \mid c\rangle, \langle\, 3 \mid d\rangle\rangle$$

that implies $M_{toy} \Rightarrow \langle\, 1 \mid b\,;\, \langle\, 2 \mid c\rangle, \langle\, 3 \mid d\rangle\rangle$.

$$\frac{z \text{ is } l\text{-irreducible} \qquad \forall i \in [1,n].u_i \to v_i \in R_l}{u_1 \dots u_n z \overset{mpr}{\Longrightarrow}_l v_1 \dots v_n z} \quad (r1)$$

$$\frac{w \overset{mpr}{\Longrightarrow}_l w' \qquad M \in \mathcal{M}^*(\Pi) \qquad M \text{ is mpr-irreducible}}{\langle\, l \mid w\,;\, M \rangle \overset{mpr}{\Longrightarrow} \langle\, l \mid w'\,;\, M \rangle} \quad (r2)$$

$$\frac{M \overset{mpr}{\Longrightarrow} M' \qquad w \text{ is } l\text{-irreducible}}{\langle\, l \mid w\,;\, M \rangle \overset{mpr}{\Longrightarrow} \langle\, l \mid w\,;\, M' \rangle} \quad (r3) \qquad \frac{w \overset{mpr}{\Longrightarrow}_l w' \qquad M \overset{mpr}{\Longrightarrow} M'}{\langle\, l \mid w\,;\, M \rangle \overset{mpr}{\Longrightarrow} \langle\, l \mid w'\,;\, M' \rangle} \quad (r4)$$

$$\frac{M_1 \overset{mpr}{\Longrightarrow} M_1' \qquad M_2 \overset{mpr}{\Longrightarrow} M_2'}{M_1, M_2 \overset{mpr}{\Longrightarrow} M_1', M_2'} \quad (r5) \qquad \frac{M_1 \overset{mpr}{\Longrightarrow} M_1' \qquad M_2 \text{ is mpr-irreducible}}{M_1, M_2 \overset{mpr}{\Longrightarrow} M_1', M_2} \quad (r6)$$

Fig. 1. Inference rules of the semantics proposed by Andrei, Ciobanu and Lucanu. The rule dual to (r6) is omitted.

$$\frac{\mathbf{out}(w)\mathbf{here}(w) \neq \emptyset \qquad w' = \begin{cases} \mathbf{obj}(w)\mathbf{here}(w) & \text{if } l = Skin \\ \mathbf{obj}(w)\mathbf{here}(w)(\mathbf{out}(w), out) & \text{otherwise} \end{cases}}{\langle\, l \mid w \rangle \overset{tar}{\Longrightarrow} \langle\, l \mid w' \rangle} \quad (r7)$$

$$\frac{\begin{array}{c} M_1, \dots, M_n \text{ is tar-irreducible} \\ \mathbf{out}(w) \neq \emptyset \vee \exists i \in [1,n].\mathbf{in}_{L(M_i)}(w)\mathbf{out}(\mathbf{w}(M_i)) \neq \emptyset \\ \forall j \in [1,n].\mathbf{w}(M_j') = \mathbf{obj}(\mathbf{w}(M_j))\mathbf{in}_{L(M_j)}(w) \\ w' = \begin{cases} \mathbf{obj}(w)\mathbf{here}(w)\mathbf{out}(\mathbf{w}(M_1)\dots\mathbf{w}(M_n)) & \text{if } l = Skin \\ \mathbf{obj}(w)\mathbf{here}(w)(\mathbf{out}(w), out)\mathbf{out}(\mathbf{w}(M_1)\dots\mathbf{w}(M_n)) & \text{otherwise} \end{cases} \end{array}}{\langle\, l \mid w\,;\, M_1, \dots, M_n \rangle \overset{tar}{\Longrightarrow} \langle\, l \mid w'\,;\, M_1', \dots, M_n' \rangle} \quad (r8)$$

$$\frac{\begin{array}{c} M_1, \dots, M_n \overset{tar}{\Longrightarrow} M_1', \dots, M_n' \\ \forall j \in [1,n].\mathbf{w}(M_j'') = \mathbf{obj}(\mathbf{w}(M_j'))\mathbf{in}_{L(M_j)}(w) \\ w' = \begin{cases} \mathbf{obj}(w)\mathbf{here}(w)\mathbf{out}(\mathbf{w}(M_1')\dots\mathbf{w}(M_n')) & \text{if } l = Skin \\ slobjw\mathbf{here}(w)(\mathbf{out}(w), out)\mathbf{out}(\mathbf{w}(M_1')\dots\mathbf{w}(M_n')) & \text{otherwise} \end{cases} \end{array}}{\langle\, l \mid w\,;\, M_1, \dots, M_n \rangle \overset{tar}{\Longrightarrow} \langle\, l \mid w'\,;\, M_1'', \dots, M_n'' \rangle} \quad (r9)$$

$$\frac{M_1 \overset{tar}{\Longrightarrow} M_1' \quad M_2 \overset{tar}{\Longrightarrow} M_2'}{M_1, M_2 \overset{tar}{\Longrightarrow} M_1', M_2'} \quad (r10) \qquad \frac{M_1 \overset{tar}{\Longrightarrow} M_1' \quad M_2 \text{ is tar-irreducible}}{M_1, M_2 \overset{tar}{\Longrightarrow} M_1', M_2} \quad (r11)$$

Fig. 2. Further inference rules of the semantics proposed by Andrei, Ciobanu and Lucanu. The rule dual to (r11) is omitted.

### 3.3. *The semantics proposed by Barbuti et al.*

Configurations in this semantics (see [7]) are given by terms of an algebra, called *P algebra*. The abstract syntax of the P algebra is defined as follows. For the sake of legibility, in this subsection we write $u \to v_h v_o \{v_{l_i}\}$ for the generic evolution rule $u \to (v_h, here)(v_o, out)(v_1, in_{l_1}) \dots (v_n, in_{l_n})$.

**Definition 7 (P algebra)** *The abstract syntax of* membrane contents $c$*,* membranes $m$*, and* membrane systems $ms$ *is given by the following grammar, where* $l$ *ranges over* $\mathbb{N}$ *and* $a$ *over* $V$*:*

$$c \ ::= \ (\emptyset,\emptyset) \ \big| \ (u \to v_h v_o\{v_{l_i}\},\emptyset) \ \big| \ (\emptyset, a) \ \big| \ c \cup c$$
$$m \ ::= \ [_l \, c\,]_l \qquad\qquad ms \ ::= \ ms \mid ms \ \big| \ \mu(m, ms) \ \big| \ F(m)$$

A membrane content $c$ represents a pair $(R, u)$, where $R$ is a set of evolution rules and $u$ is a multiset of objects. A membrane content is obtained trough the union operation $\_ \cup \_$ from constants representing single evolution rules and constants representing single objects, and can be plugged into a membrane $l$ by means of the operation $[_l \, \_\,]_l$. Membrane systems $ms$ have the following meaning: $ms_1 \mid ms_2$ represents the juxtaposition of $ms_1$ and $ms_2$, $\mu(m, ms)$ represents the containment of $ms$ in $m$, and $F(m)$ represents a *flat membrane*, namely it states that $m$ does not contain any child membrane. Juxtaposition is used to group sibling membranes, namely membranes all having the same parent in a membrane structure. This operation allows containment $\mu$ to be defined as a binary operator on a single membrane (the parent) and a juxtaposition of membranes (all the children).

In what follows we will often write $[\![_l \, c \,]\!]_l$ for $F([_l \, c\,]_l)$. We shall also often write $(R, u)$, where $R = \{r_1, \ldots, r_n\}$ is a set of rules and $u = o_1 \ldots o_m$ a multiset of objects, rather than $(r_1, \varnothing) \cup \ldots \cup (r_n, \varnothing) \cup (\varnothing, o_1) \cup \ldots \cup (\varnothing, o_m)$. Moreover, we shall often omit parentheses around membrane contents.

The semantics of the P algebra is given as an LTS. Labels of the LTS can be of the following forms:

- $(u, v, v', I, O^{\uparrow}, O^{\downarrow})$, describing a computation step of a membrane content $c$, where: $u, v, v'$ are multisets of objects used to handle maximal parallelism in a compositional way, $I$ is the multiset of objects received from the parent and the child membranes, $O^{\uparrow}$ is the multiset of objects sent to the parent membrane, and $O^{\downarrow}$ is a set of pairs $(l_i, v_{l_i})$ describing the multiset of objects sent to each child membrane $l_i$. The role of $u, v, v'$ can be detailed as follows: $u$ and $v$ are the objects that are consumed by evolution rules, where $u$ originate by transition rules encoding moves by membrane contents consisting of evolution rules and $v$ originate by transition rules encoding moves by membrane contents consisting of objects, and $v'$ are the objects that are not consumed by evolution rules. When operators $[_l \, \_\,]_l$ is applied to a membrane content, we check that, coherently, $u = v$.
- $(\mathcal{I}^{\downarrow}, I^{\uparrow}, O^{\uparrow}, O^{\downarrow})$, describing a computation step of a membrane $m$, where: $\mathcal{I}^{\downarrow}$ is a set containing only the pair $(l, I)$ where $l$ is the label of $m$ and $I$ is the multiset of objects received by $m$ from the parent membrane, $I^{\uparrow}$ is the multisets of objects received by $m$ from its child membranes, and $O^{\uparrow}$ and $O^{\downarrow}$ are as before.
- $(\mathcal{I}^{\downarrow}, O^{\uparrow})$, describing a computation step of a membrane system $ms$, where $O^{\uparrow}$ is as before, whereas $\mathcal{I}^{\downarrow}$ may contain more than one pair $(l, I)$.

$$\frac{I \in V^* \qquad n \in \mathbb{N}}{(u \to v_h v_o\{v_{l_i}\}, \emptyset) \xrightarrow[u^n,\emptyset,\emptyset]{I,v_o^n,\{(l_i,v_{l_i}^n)\}} (u \to v_h v_o\{v_{l_i}\}, I v_h^n)} \;(mc1_n)$$

$$\frac{I \in V^*}{(\emptyset, a) \xrightarrow[\emptyset,a,\emptyset]{I,\emptyset,\emptyset} (\emptyset, I)} \;(mc2) \qquad \frac{I \in V^*}{(\emptyset, a) \xrightarrow[\emptyset,\emptyset,a]{I,\emptyset,\emptyset} (\emptyset, Ia)} \;(mc3) \qquad \frac{I \in V^*}{(\emptyset, \emptyset) \xrightarrow[\emptyset,\emptyset,\emptyset]{I,\emptyset,\emptyset} (\emptyset, I)} \;(mc4)$$

$$\frac{x_1 \xrightarrow[u_1,v_1,v_1']{I_1,O_1^{\uparrow},O_1^{\downarrow}} y_1 \qquad x_2 \xrightarrow[u_2,v_2,v_2']{I_2,O_2^{\uparrow},O_2^{\downarrow}} y_2}{x_1 \cup x_2 \xrightarrow[u_1u_2,v_1v_2,v_1'v_2']{I_1I_2,O_1^{\uparrow}O_2^{\uparrow},O_1^{\downarrow}\cup_{\mathbb{N}}O_2^{\downarrow}} y_1 \cup y_2} \;(u1) \qquad \frac{x \xrightarrow[u,u,v']{I,O^{\uparrow},O^{\downarrow}} y}{[_l\, x\,]_l \xrightarrow{\emptyset,I,O^{\uparrow},O^{\downarrow}} [_l\, y\,]_l} \;(m1)$$

$$\frac{x \xrightarrow[u,u,v']{I_1I_2,O^{\uparrow},O^{\downarrow}} y}{[_l\, x\,]_l \xrightarrow{\{(l,I_1)\},I_2,O^{\uparrow},O^{\downarrow}} [_l\, y\,]_l} \;(m2) \qquad \frac{x \xrightarrow{\mathcal{I}^{\downarrow},\emptyset,O^{\uparrow},\emptyset} y}{F(x) \xrightarrow{\mathcal{I}^{\downarrow},O^{\uparrow}} F(y)} \;(f1) \qquad \frac{x_1 \xrightarrow{\mathcal{I}_1,O_1^{\uparrow}} y_1 \quad x_2 \xrightarrow{\mathcal{I}_2,O_2^{\uparrow}} y_2}{x_1|x_2 \xrightarrow{\mathcal{I}_1\mathcal{I}_2,O_1^{\uparrow}O_2^{\uparrow}} y_1|y_2} \;(j1)$$

$$\frac{x_1 \xrightarrow{\mathcal{I}_1^{\downarrow},I_1^{\uparrow},O_1^{\uparrow},O_1^{\downarrow}} y_1 \qquad x_2 \xrightarrow{\mathcal{I}_2,O_2^{\uparrow}} y_2 \qquad O_1^{\downarrow} \doteq \mathcal{I}_2 \qquad O_2^{\uparrow} = I_1^{\uparrow}}{\mu(x_1, x_2) \xrightarrow{\mathcal{I}_1^{\downarrow},O_1^{\uparrow}} \mu(y_1, y_2)} \;(h1)$$

Fig. 3. Rules of the semantics proposed by Barbuti, Maggiolo-Schettini, Milazzo and Tini.

**Definition 8 (Semantics)** *The* semantics *of a P system* $\Pi$ *is given by the LTS having P algebra terms as states and the least transition relation obtained by the inference rules in Fig. 3 as labelled transition relation.*

Labels of this semantics are richer than those of the semantics by Busi and by Andrei, Ciobanu and Lucanu: they contain information that is needed to define the semantics in a compositional way with just one transition relation. The transition rules of this semantics respect the so called *de Simone format* [14].

Let us consider again the P system $\Pi_{toy}$. It corresponds to the following term of the P algebra: $ms_{toy} = \mu\big([_1\, R_1, aabb\,]_1\,,\, \llbracket_2\, R_2, \emptyset\,\rrbracket_2 \mid \llbracket_3\, R_3, \emptyset\,\rrbracket_3\big)$. The computation step of $\Pi_{toy}$ described with the two other semantics correspond in this case to a transition from $ms_{toy}$ that can be derived compositionally. A transition for each membrane in $ms_{toy}$ can be derived by using inference rules for individual rules and objects, and for operations $\cup$, $[_l\, \_\,]_l$ and $F(\_)$. Such transitions are as follows:

$$[_1\, R_1, aabb\,]_1 \xrightarrow{\{(1,\emptyset)\},\emptyset,b,\{(2,c),(3,d)\}} [_1\, R_1, b\,]_1$$

$$\llbracket_2\, R_2, \emptyset\,\rrbracket_2 \xrightarrow{\{(2,c)\},\emptyset} \llbracket_2\, R_2, c\,\rrbracket_2 \qquad\qquad \llbracket_3\, R_3, \emptyset\,\rrbracket_3 \xrightarrow{\{(3,d)\},\emptyset} \llbracket_3\, R_3, d\,\rrbracket_3$$

The latter two transitions can be used as premises of inference rule (j1) to derive the following transition:

$$\llbracket_2\, R_2, \emptyset\,\rrbracket_2 \mid \llbracket_3\, R_3, \emptyset\,\rrbracket_3 \xrightarrow{\{(2,c),(3,d)\},\emptyset} \llbracket_2\, R_2, c\,\rrbracket_2 \mid \llbracket_3\, R_3, d\,\rrbracket_3$$

This transition can now be used to derive the following final transition:

$$ms_{toy} \xrightarrow{\{(1,\emptyset)\},b} \mu\big([_1\, R_1, b\,]_1\,,\, \llbracket_2\, R_2, c\,\rrbracket_2 \mid \llbracket_3\, R_3, d\,\rrbracket_3\big)$$

### 3.4. *Relating the different approaches*

We can define an encoding $(\!|\_|\!)$ of membranes in $\mathcal{M}(\Pi)$ into (partial) configurations in $PConf(\Pi)$ as follows:

$$(\!|\langle\, l \mid w\rangle|\!) = \times_{i=1}^{N}(w_i, \bar{w}_i) \text{ where } \begin{cases} w_l = \mathbf{obj}(w) \qquad \bar{w}_l = \mathbf{here}(w) \\ \forall i \neq l. w_i = \emptyset \wedge \bar{w}_i = \mathbf{in}_i(w) \\ \bar{w}_{father(l)} = \mathbf{out}(w) \text{ if } l \neq Skin \end{cases}$$

$$(\!|\langle\, l \mid w\,;\, M_1, \ldots, M_n\rangle|\!) = (\!|\langle\, l \mid w\rangle|\!) \oplus (\!|M_1|\!) \oplus \ldots \oplus (\!|M_n|\!)$$

where $\times_{i=1}^{N}(w_i, \bar{w}_i) \oplus \times_{i=1}^{N}(w'_i, \bar{w}'_i) = \times_{i=1}^{N}(w_i w'_i, \bar{w}'_i \bar{w}'_i)$.

Note that this encoding is not an injective function. For example, let us consider the following two intermediate configurations reachable by the toy P system $\Pi_{toy}$: $M_1 = \langle\, 1 \mid \emptyset\,;\, \langle\, 2 \mid (a, out)\rangle, \langle\, 3 \mid \emptyset\rangle\rangle$ and $M_2 = \langle\, 1 \mid \emptyset\,;\, \langle\, 2 \mid \emptyset\rangle, \langle\, 3 \mid (a, out)\rangle\rangle$. It holds $(\!|M_1|\!) = (\!|M_2|\!) = \big((\emptyset, a), (\emptyset, \emptyset), (\emptyset, \emptyset)\big)$.

By using this encoding we can prove some results of operational correspondence between the first two considered semantics. Let us start with some auxiliary results.

**Lemma 9.** *The following properties hold:*

- $M \stackrel{mpr}{\Longrightarrow} M'$ *implies* $(\!|M|\!) \mapsto^{+} (\!|M'|\!)$, *with* $M \in \mathcal{C}(\Pi)$ *and* $M' \in \mathcal{M}(\Pi)$;
- $M$ *is mpr-irreducible if and only if* $(\!|M|\!) \not\mapsto$, *with* $M \in \mathcal{C}(\Pi)$;
- $(\!|M|\!) \mapsto^{+} \gamma$ *and* $\gamma \not\mapsto$ *imply that there exists an* $M' \in \mathcal{M}(\Pi)$ *s.t.* $\gamma = (\!|M'|\!)$ *and* $M \stackrel{mpr}{\Longrightarrow} M'$, *with* $M \in \mathcal{C}(\Pi)$;
- $M \stackrel{tar}{\Longrightarrow} M'$ *if and only if* $(\!|M'|\!) = heated((\!|M|\!))$, *with* $M \in \mathcal{C}^{\#}(\Pi)$ *and* $M' \in \mathcal{C}(\Pi)$.

These auxiliary results allow the following theorem to be proved.

**Theorem 10.** *The following properties of operational correspondence hold:*

- $M \Rightarrow M'$ *implies* $(\!|M|\!) \Mapsto (\!|M'|\!)$;
- $(\!|M|\!) \Mapsto \gamma$ *implies that there exists some* $M'$ *s.t.* $M \Rightarrow M'$ *and* $\gamma = (\!|M'|\!)$.

Now, we can define also an encoding $\{\!|\_|\!\}$ of P algebra membrane systems (corresponding to some P system $\Pi$) into membranes in $\mathcal{M}(\Pi)$ as follows:

$$\{\!|[\![_l\, R, w\,]\!]_l|\!\} = \langle\, l \mid w\rangle \qquad \{\!|ms_1 \mid ms_2|\!\} = \{\!|ms_1|\!\}, \{\!|ms_2|\!\}$$
$$\{\!|\mu(m, ms)|\!\} = \langle\, l \mid w\,;\, \{\!|ms|\!\}\rangle \text{ if } \{\!|F(m)|\!\} = \langle\, l \mid w\rangle$$

Note that this time the encoding is an invertible function. The new encoding allows us to give a result on the correspondence of the latter two semantics.

**Theorem 11.** *The following properties of operational correspondence hold:*

- $ms \xrightarrow{\emptyset, w} ms'$ *implies* $\{\!|ms|\!\} \Rightarrow \{\!|ms'|\!\}$;
- $\{\!|ms|\!\} \Rightarrow \{\!|ms'|\!\}$ *implies that there exists exactly one* $w$ *s.t.* $ms \xrightarrow{\emptyset, w} ms'$.

## 4. Discussion and Conclusions

The definition of an operational semantics for P systems allows the behaviour of of systems to be described with set-theoretic means such as LTSs. There might be different reasons for defining an operational semantics, which may lead to different definitions. Such different definitions should be seen as different viewpoints of the same thing: the behaviour of P systems.

In this paper we have given a survey on operational semantics for P systems. Moreover, we have considered three proposals of semantics that follow the SOS approach, we have recalled their definitions restricted to a common class of P systems, and we have given some results of operational correspondence among the three.

The first semantics we have considered, proposed by Busi in [9], is the simplest one. It handles maximal parallelism with an exhaustive sequence of applications of evolution rules, and it has been used in [9, 10] to study properties of the behaviour of P systems such as divergence and causality of events.

The second semantics, proposed by Andrei, Ciobanu and Lucanu in [4], handles maximal parallelism and communication of objects in two subsequent substeps. Notions of irreducibility of configurations are exploited to assess whether each of the two substeps is enabled in a configuration. This way of defining the semantics was suitable for the development of a provably correct interpreter for P systems based on rewriting logic, that was one of the main aims of the authors.

The third semantics, proposed by the authors of the present paper in [7], is the most complex since it describes maximal parallel behaviour in a compositional way, by starting from the behaviour of individual rules and objects. Maximal parallelism is, by its nature, not a compositional notion. In fact, in the previous two semantics it is either sequentialized or handled by considering a membrane as a whole. Hence, in order to handle it compositionally it has been necessary to include a lot of information of the transition labels. However, having a compositional semantics allows meaningful behavioural equivalences to be defined, and used to reason on the behaviour of systems components (e.g. to find equivalent replacements for components, to prove properties on their behaviour, etc...).

In conclusion, the differences among the three considered semantics are motivated by their different aims. The existence of an operational correspondence among them shows that they agree in the interpretation of the behaviour of P systems.

## References

[1] L. Aceto, W.J. Fokkink and C. Verhoef, Structural operational semantics, in *Handbook of Process Algebra*, eds. J.A. Bergstra, A. Ponse and S.A. Smolka (Elsevier, 2001), pp. 197–292.

[2] O. Agrigoroaiei and G. Ciobanu, Rule-based and object-based event structures for membrane systems, *J. Log. Algeb. Program.* (2010), doi:10.1016/j.jlap.2010.03.010

[3] O. Andrei, G. Ciobanu and D. Lucanu, Structural operational semantics of P systems, in *Proc. 6th Workshop on Membrane Computing (WMC 2005)*, LNCS, Vol. 3850 (Springer, 2006), pp. 31–28.

[4] O. Andrei, G. Ciobanu and D. Lucanu, A rewriting logic framework for operational semantics of membrane systems, *Theoret. Comp. Sci.* **373** (2007) 163–181.

[5] B. Bloom, S. Istrail and A. Meyer, Bisimulation can't be traced, *J. Assoc. Comput. Mach* **43** (1995) 232–268.

[6] R. Barbuti, D.P. Gruska, A. Maggiolo-Schettini and P.Milazzo, A Notion of Biological Diagnosability Inspired by the Notion of Opacity in Systems Security, *Fund. Inform.* **102** (2010) 19–34.

[7] R. Barbuti, A. Maggiolo-Schettini, P. Milazzo and S. Tini, Compositional semantics and behavioral equivalences for P systems, *Theoret. Comput. Sci.* **395** (2008) 77–100.

[8] R. Barbuti, A. Maggiolo-Schettini, P. Milazzo and S. Tini, A P systems flat form preserving step-by-step behaviour, *Fund. Inform.* **87** (2008) 1–34.

[9] N. Busi, Using well–structured transition systems to decide divergence for catalytic P systems, *Theoret. Comput. Sci.* **372** (2007) 125–135.

[10] N. Busi, Causality in membrane systems, in *Proc. 8th Workshop on Membrane Computing (WMC 2007)*, LNCS, Vol. 4860 (Springer, 2007), pp. 160–171.

[11] J.C.M. Baeten and C. Verhoef, A congruence theorem for structured operational semantics with predicates. in *Proc. 4th Conference on Concurrency Theory (CONCUR 1993)*, LNCS, Vol. 715 (Springer, 1993), pp. 477–492.

[12] G. Ciobanu, Semantics of P systems, in *The Oxford Handbook of Membrane Computing*, eds. G. Păun, G. Rozenberg and A. Salomaa (Oxford University Press, 2009).

[13] G. Ciobanu and D. Lucanu, Events, causality, and concurrency in membrane computing, in *Proc. 8th Workshop on Membrane Computing (WMC 2007)*, LNCS, Vol. 4860 (Springer, 2007), pp. 209–227.

[14] R. de Simone, High level synchronization devices in Meije-SCCS, *Theoret. Comput. Sci.* **37** (1985) 245–267.

[15] R. Freund and S. Verlan, A formal framework for static (tissue) P Systems, in *Proc. 8th Workshop on Membrane Computing (WMC 2007)*, LNCS, Vol. 4860 (Springer, 2007), pp. 271–284.

[16] P. Frisco, P systesms, Petri nets, and program machines, in *Proc. 6th Workshop on Membrane Computing (WMC 2005)*, LNCS, Vol. 3850 (Springer, 2006), pp. 209–223.

[17] J.H.C.M. Kleijn, M. Koutny and G. Rozenberg, Towards a Petri net semantics for membrane systems, in *Proc. 6th Workshop on Membrane Computing (WMC 2005)*, LNCS, Vol. 3850 (Springer, 2006), pp. 292–309.

[18] P. Milazzo, *Qualitative and quantitative formal modelling of biological systems*, PhD Thesis, University of Pisa, 2007.

[19] G. Păun, *Membrane computing. An introduction* (Springer, 2002).

[20] M. Pinna and A. Saba, Dependencies and simultaneity in membrane systems, in *Proc. of Membrane Computing and Biologically Inspired Process Calculi (MeCBIC'09)*, Electronic Proc. in Theoretical Computer Science, Vol. 11 (2009), pp. 155-169.

[21] G. Plotkin, A structural approach to operational semantics, *Tech. Rep. DAIMI FN-19*, University of Aarhus, 1981.

[22] G. Plotkin, A structural approach to operational semantics, *J. Log. Algebr. Program.* **60–61** (2004) 17–139.

[23] Z. Qi, J. You and H. Mao, P systems and Petri nets, in *Proc. 4th Workshop on Membrane Computing (WMC 2003)*, LNCS, Vol. 2933 (Springer, 2004), pp. 286–303.

[24] R.J. van Glabbeek, Full abstraction in structural operational semantics, in *Proc. 3rd Int. Conference on Algebraic Methodology and Software Technology (AMAST'93)*, Workshops on Computing (Springer, 1993), pp. 77–84.

[25] C. Versari, *A Core Calculus for the Analysis and Implementation of Biologically Inspired Languages*, PhD Thesis, University of Bologna, 2009.