$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{B}$$

$$\left(\forall n, m \in \mathbb{N}.\ f(n, m) = \underbrace{(n = m+2)} \right)$$

$$\left(\forall n, m, n', m' \in \mathbb{N}.\ \langle n, m \rangle \prec \langle n', m' \rangle \equiv n' = n+1 \wedge m' = m+1 \right)$$

$$n \geq m \left\{ \begin{array}{c} (n, m) \\ | \\ (n-1, m-1) \\ \vdots \\ (n-m, \emptyset) \end{array} \right. \qquad \left. \begin{array}{c} (n, m) \\ | \\ (n-1, m-1) \\ \vdots \\ (\emptyset, m-n) \end{array} \right\} m \geq n$$

$$
f(n, m) = \begin{cases} \text{true} & \text{se } m = 0 \text{ e } n = 2 \\ \text{false} & \text{se } m = 0 \text{ e } n \neq 2 \\ \text{false} & \text{se } n = 0 \\ f(n-1, m-1) & \text{se } n > 0 \text{ e } m > 0 \end{cases}
$$

Casi base

$f(2,\emptyset) = 2 = \emptyset + 2$

$f(n,\emptyset) = n = \emptyset + 2 \qquad n \neq 2$

$f(0,m) = \emptyset = m + 2$

$f(2,0) = 2 = 0+2$

$f(2,0)$

$= \{\text{def. } f, 1^\circ \text{ caso}\}$

true

$= \{\text{calcolo}\}$

$2 = 0+2$

$f(n,0) = n = 0+2 \qquad n \neq 2$

$f(n,0)$

$= \{\text{def } f, 2^\circ \text{ caso}\}$

false $\longrightarrow$ $n = 2 \quad n \neq 2$

$= \{\text{calcolo}\}$

$n = 0+2 \qquad n \neq 2$

$f(0, m) = 0 = m+2$ 3° caso base

Caso induttivo

$$\underbrace{f(n, m) = (n = m+2)}_{\text{ip. induttiva}} \Longrightarrow f(n+1, m+1) = (n+1 = m+1+2)$$

$f(n+1, m+1)$

$\equiv$ {def f, $n+1>0$, $m+1>0$, 4° caso}

$f(n, m)$

$\equiv$ {ip. induttiva}

$n = m+2$

$\equiv$ {calcolo}

$n+1 = m+1+2$

Caso induttivo

$$\underbrace{f(n,m) = (n = m+2)}_{\text{ip. induttiva}} \Rightarrow f(n+1, m+1) = (n+1 = m+1+2)$$

$$f(n+1, m+1) = (n+1 = m+1+2)$$

$$\equiv \{\text{def. } f,\ m+1>0,\ n+1>0,\ 4^\circ \text{ caso}\}$$

$$f(n,m) = (n+\cancel{1} = m+\cancel{1}+2)$$

$$\equiv \{\text{calcolo}\}$$

$$f(n,m) = (n = m+2)$$

$$\equiv \{\text{ip. induttiva}\}$$

true

Caso induttivo

$$f(n-1,\ m-1) = (n-1 = m-1+2) \Longrightarrow$$

$$f(n,\ m) = (n = m+2)$$

$n > 0$

$m > 0$

$$f : \mathbb{N} \times \mathbb{N} \to \mathbb{N}$$

$$\left(\forall n, m \in \mathbb{N}.\ f(n,m) = \underline{4n}\right)$$

$$f(n,m) = \begin{cases} 0 & \text{se } n = 0 \text{ e } m = 0 \\ 4 + f(n-1, m+1) & \text{se } n > 0 \\ f(n, m-1) & \text{se } n = 0 \text{ e } m > 0 \end{cases}$$

$$\begin{array}{c} (n, m) \\ | \\ (n-1, m+1) \\ \vdots \\ (0, n+m) \\ | \\ (0, n+m-1) \\ | \\ (0, n+m-2) \\ \vdots \\ (0, 0) \end{array}$$

Caso induttivo

$$\underbrace{f(n,m) = 4n}_{\text{ip. ind.}} \Rightarrow f(n+1, m-1) = 4\cdot(n+1) \qquad m>0$$

$$f(n+1, m-1)$$

$$= \{\text{def } f,\ n+1>0,\ 2^\circ \text{ caso}\}$$

$$4 + f(n,m)$$

$$= \{\text{ip. induttiva}\}$$

$$4 + 4\cdot n$$

$$= \{\text{calcolo}\}$$

$$4(n+1)$$

Caso induttivo

$$\underbrace{f(0, n) = 4 \cdot 0}_{\text{ip. ind.}} \Rightarrow f(0, n+1) = 4 \cdot 0$$

$f(0, n+1)$

$= \{\text{def. } f, \ 3^\circ \text{ caso}\}$

$f(0, n)$

$= \{\text{ip. ind}\}$

$4 \cdot 0$

$= \{\text{calcolo}\}$

$4 \cdot 0$

let f x y = (x y) + 2 = 4 ;;

$$f : \underbrace{(\text{'a} \to \text{int})}_{\text{Tipo } x} \to \underbrace{\text{'a}}_{\text{Tipo } y} \to \underbrace{\text{bool}}_{\text{Tipo ris}} = \text{<fun>}$$

let f x y = (x y) = y ;;

$$f : \underbrace{(\text{'a} \to \text{'a})}_{x} \to \underbrace{\text{'a}}_{y} \to \underbrace{\text{bool}}_{\text{ris}} = \text{<fun>}$$

let f x y = (x y) y ;;

$$f : \underbrace{(\text{'a} \to \text{'a} \to \text{'b})}_{x} \to \underbrace{\text{'a}}_{y} \to \underbrace{\text{'b}}_{\text{ris}} = \text{<fun>}$$

```
let f x y z = if (x y) = y then z y else y ;;
```

$$f : \underbrace{(\text{'a} \to \text{'a})}_{x} \to \underbrace{\text{'a}}_{y} \to \underbrace{(\text{'a} \to \text{'a})}_{z} \to \underbrace{\text{'a}}_{res}$$

$f : int \to int \to int$

$$\text{let } f\ x\ y = \underbrace{x + y + 482}_{\text{espressione}} ;;$$

tipo x ē int
tipo y " "
tipo esp. " "

$$f : \underbrace{(\text{int} \to \text{int})} \to \underbrace{\text{int}}$$

let f x = $\underbrace{(x\ 25) + 487}_{\text{int}}$;;

$$\cancel{f\ x = \underbrace{(x\ 25)}_{(\text{int} \to \text{'a})} + x}$$

NO

```
let f x = (x 25) + (x 25);;

f : (int -> int) -> int

let f x = (x 25) = (x 25);;

f : (int -> 'a) -> bool
```