Teo ricorsione

ambito formale per le definizioni ricorsive.

Le equazioni ricorsive hanno un

SIGNIFICATO

principio di induzione

$$f(x) = \begin{cases} 0 & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$$f : \mathbb{N} \to \mathbb{N}$$

$$f(x) = \begin{cases} \varnothing & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$\vdots$

3. $f(3) = 2 + f(2) = 6$

2. $f(2) = 2 + f(1) = 4$

1. $f(1) = 2 + f(0) = 2$

$\varnothing$ $\quad f(0) = \varnothing$

$$P(0) \wedge \left(\forall n.\, P(n) \Rightarrow P(n+1)\right) \Rightarrow \forall n.\, P(n)$$

$$f(x) = 2 \cdot x$$

$$f(x) = \begin{cases} 0 & \text{se } x = 0 \\ 2 + f(x-1) & \text{se } x > 0 \end{cases}$$

$$\forall n \in \mathbb{N}.\ f(n) = 2n$$

Caso base $n \doteq 0$

$$f(0) = 2 \cdot 0$$

Caso induttivo

$$\left(\forall n \in \mathbb{N}.\ f(n) = 2 \cdot n \Longrightarrow f(n+1) = 2 \cdot (n+1) \right)$$

Caso base

$$f(0) = 2 \cdot 0$$

$$f(0)$$

$$= \{\text{def. } f \ 1^\circ \text{ caso}\}$$

$$0$$

$$= \{\text{calcolo}\}$$

$$2 \cdot 0$$

Caso induttivo

$\forall n \in \mathbb{N}$

$$f(n) = 2\cdot n \Rightarrow f(n+1) = 2\cdot(n+1)$$

$$f(n+1)$$

$$= \{\text{def } f,\ 2^\circ \text{ caso}\}$$

$$2 + f(n)$$

$$= \{\text{ip. induttiva}\}$$

$$2 + 2\cdot n$$

$$= \{\text{calcolo}\}$$

$$2\cdot(n+1)$$

$$f(n) = \begin{cases} 1 & \text{se } n = 0 \\ n \cdot f(n-1) & \text{se } n > 0 \end{cases}$$

$$f : \mathbb{N} \to \mathbb{N}$$

$$\forall n \in \mathbb{N}$$

$$f(n) = n!$$

Caso base

$$f(0) = 0!$$

Caso induttivo

$$f(n) = n! \Longrightarrow f(n+1) = (n+1)!$$

Caso base

$$f(0) = 0!$$

$$f(0)$$

$$= \{\text{def } f,\ 1^\circ \text{ caso}\}$$

$$1$$

$$= \{\text{def. } 0! = 1\}$$

$$0!$$

Caso induttivo

$f(n) = n! \Rightarrow f(n+1) = (n+1)!$

$f(n+1)$

$= \{\text{def } f,\ 2^{o}\text{ caso}\}$

$(n+1) \cdot f(n)$

$= \{\text{ip. induttiva}\}$

$(n+1) \cdot (n!)$

$= \{\text{calcolo},\ n! = n \cdot (n-1) \cdots 1\}$

$(n+1)!$

$f(n) = \begin{cases} 1 & \text{se } n = 0 \\ n \cdot f(n-1) & \text{se } n > 0 \end{cases}$

$$f(n) = 4n + 2$$

$$f : \mathbb{N} \to \mathbb{N}$$

$$f(0) = 4 \cdot 0 + 2 = 2$$

Sapendo calcolare le funzioni su $n-1$

cioè
$$f(n-1) = 4 \cdot (n-1) + 2$$

Sappiamo calcolare il valore di $f(n)$ sulla base di $f(n-1)$

$$f(n) = \ldots\ldots \; f(n-1) \; \ldots\ldots$$
$$= \ldots\ldots \; 4 \cdot (n-1) + 2 \ldots$$

$$f(n) = \ldots\ldots\ 4\cdot(n-1)+2\ \ldots\ldots$$

$$= \ldots\ldots\ 4n-4+2\ \ldots\ldots$$

$$= \ldots\ldots\ \underline{\underline{4n-2}}\ \ldots\ldots$$

$$= 4+4n-2$$

$$= 4n+2$$

$$f(n) = \begin{cases} 2 & \text{se } n = \emptyset \\ f(n-1)+4 & \text{se } n > 0 \end{cases}$$

$\forall n \in \mathbb{N}$

$$f(n) = 4 \cdot n + 2$$

Caso base

$$f(0) = 4 \cdot 0 + 2$$

Caso induttivo

$$f(n) = 4 \cdot n + 2 \implies$$

$$f(n+1) = 4 \cdot (n+1) + 2$$

<u>Caso induttivo</u>

$$f(n-1) = 4\cdot(n-1)+2$$

$$\Longrightarrow$$

$$f(n) = 4\cdot n+2$$

$\underline{n>0}$

Caso base

$$f(\varnothing) = 4 \cdot \varnothing + 2$$

$$f(0)$$

$$= \{\text{def. } f\}$$

$$2$$

$$= \{\text{calcolo}\}$$

$$4 \cdot \varnothing + 2$$

<u>Caso induttivo</u>

$f(n) = 4 \cdot n + 2 \Rightarrow f(n+1) = 4 \cdot (n+1) + 2$

$f(n+1)$

$= \{ \text{def } f, 2^{o} \text{ caso} \}$

$f(n) + 4$

$= \{ \text{ip. induttiva} \}$

$4 \cdot n + 2 + 4$

$= \{ \text{calcolo} \}$

$4 \cdot (n+1) + 2$

sapendo calcolare la funzioni su n so calcolarla su $n+1$?

$$f(n+1) = \dots\; f(n) \;\dots$$

$$f(n+1) = \dots\; 4 \cdot n + 2 \;\dots$$

$$f(n+1) = 4n+2+4 = 4(n+1)+2$$

$$f : \mathbb{N} \to \mathbb{N}$$

$$g : (\underline{\mathbb{N} \times \mathbb{N}}) \to \underline{\mathbb{N}}$$

$$g(n, m) = 2n + m + 1$$

$$g(n,m) = \begin{cases} m+1 & \text{se } n = 0 \\ 2 + g(n-1, m) & \text{se } n > 0 \end{cases}$$

n, m

$|$

$n-1, m$

$\vdots$

$0, m$

Induzione ben fondata

Relazione di precedenza

$x \prec y$

$(S, <)$ si dice ben fondato sse non esistono catene infinite decrescenti secondo la relazione di precedenza $<$

$$((\mathbb{N}\times\mathbb{N}), \prec) \qquad (3,4) \not\prec (4,5)$$

$$(\forall\, n, m, n', m' \in \mathbb{N}.$$

$$(n, m) \prec (n', m')$$

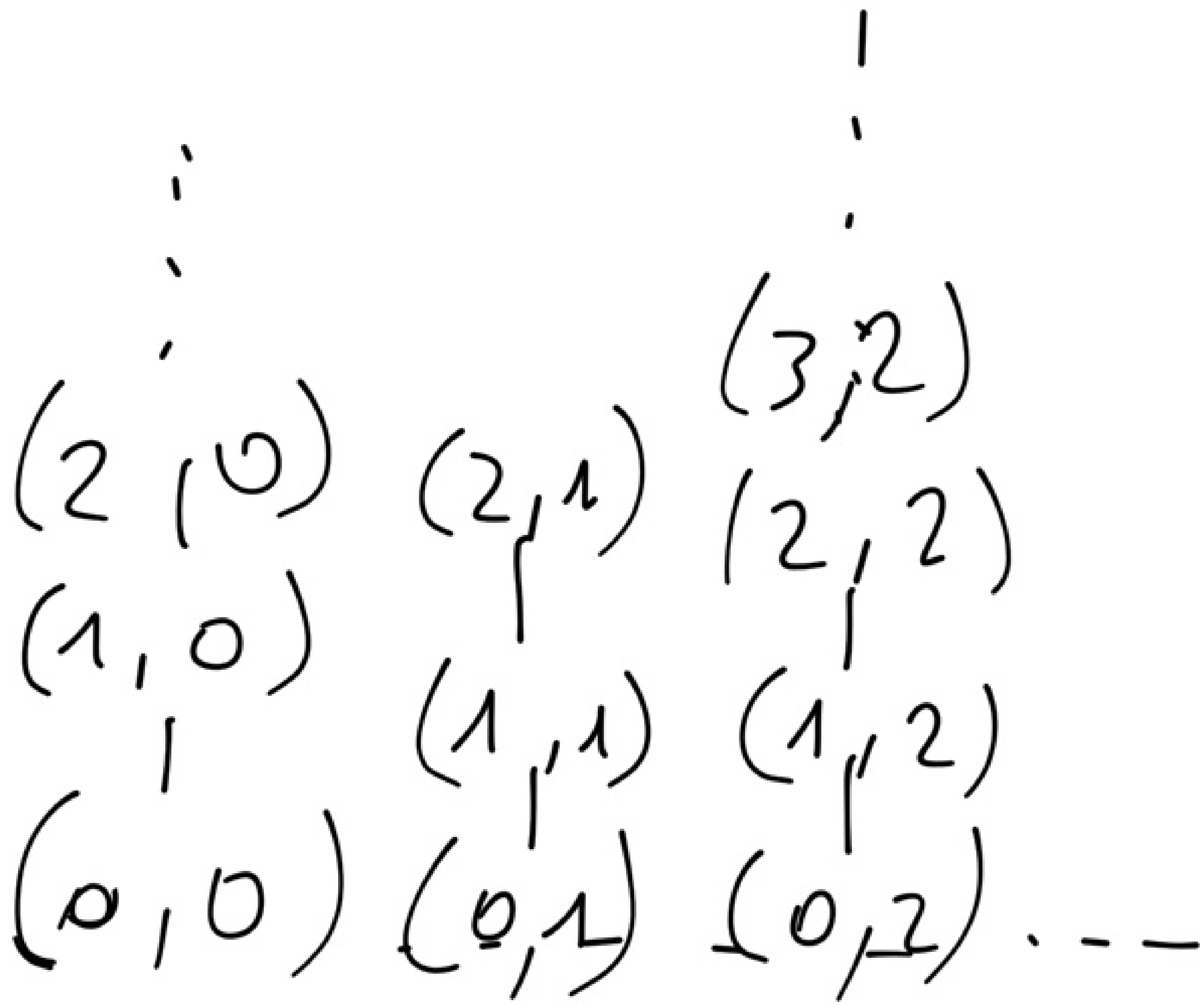
$$\equiv$$

$$n' = n+1 \wedge m' = m\,)$$

$$(3,4) \prec (4,4) \quad \Big| \quad (3,4) \not\prec (5,4)$$

$$\begin{tikzcd}
(m', m') \arrow[d, dash] & (4, 4) \arrow[d, dash] \\
(m, m) & (3, 4)
\end{tikzcd}$$

$(\mathbb{N} \times \mathbb{N}, \prec)$



$(\mathbb{N} \times \mathbb{N}, <)$

$$\begin{array}{c} \vdots \\ (2, m) \\ | \\ (1, m) \\ | \\ (0, m) \end{array}$$

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Induzione ben fondata

P proprietà su $(S, \prec)$

-

- Dimostro P sui minimali di $(S, \prec)$

- Supponendo P vera su x dimostro P su y tale che $x \prec y$

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$$g(\underline{n,m}) = \begin{cases} m+1 & \text{se } n = 0 \\ 2 + g(\underline{n-1,m}) & \text{se } n > 0 \end{cases}$$

Si dimostra per induzione ben fondata che

$$\forall n,m \in \mathbb{N}.\ g(n,m) = 2n + m + 1$$

$$\forall n,m \quad g(n,m) = 2n + m + 1$$

$$(\mathbb{N} \times \mathbb{N}, \prec)$$

$$\Big(\forall n, m, n', m' \in \mathbb{N}.$$

$$(n,m) \prec (n',m') \equiv$$

$$n' = n+1 \wedge m' = m \Big)$$

$$n = n' - 1 \wedge m = m'$$

Caso base

$$g(\emptyset, m) = 2 \cdot \emptyset + m + 1$$

Caso induttivo

$$g(n, m) = 2 \cdot n + m + 1$$

$$\Longrightarrow$$

$$g(n+1, m) = 2 \cdot (n+1) + m + 1$$

$g(0,m) = 2\cdot\emptyset + m + 1$

$g(0,m)$

$= \{\text{def } g,\ 1^\circ \text{ caso}\}$

$m+1$

$= \{\text{calcolo}\}$

$2\cdot\emptyset + m + 1$

Caso induttivo

$g(n,m) = 2n + m + 1 \Rightarrow g(n+1, m) = 2\cdot(n+1) + m + 1$

ip. ind.

$g(n+1, m)$

$= \{\text{def } g, \ 2^\circ \text{ caso}\}$

$2 + g(n,m)$

$= \{\text{ip. induttiva}\}$

$2 + 2\cdot n + m + 1$

$= \{\text{calcolo}\}$

$2\cdot(n+1) + m + 1$

$$g(n,m) = 2\cdot n + m + 1$$

caso base

$$g(n,m) = \begin{cases} \\ \dots\, g(n-1,m) \end{cases}$$

caso ricorsivo

Sapendo calcolare $g(n-1,m)$ che deve essere uguale a $2\cdot(n-1)+m+1$ so calcolarla su (n,m)?

$$\underline{g(n,m)} = \ldots\ldots g(n-1,m) \ldots\ldots$$

$$e \quad = \ldots\ldots\ldots 2\cdot(n-1)+m+1 \ldots$$

$$= \ldots\ldots 2n-2+m+1$$

$$= \ldots\ldots 2n+m-1$$

$$= 2+2n+m-1$$

$$= 2n+m+1$$

$$g(n,m) = 2 \cdot n + m + 1$$

$$g(n,m) = \begin{cases} 2n+1 & \text{se } m=0 \\ 1 + g(n, m-1) & \text{se } m>0 \end{cases}$$

$$\underline{(n, m-1) < (n, m)}$$

$$g(n,m) = \ldots\ldots\; g(n, m-1) \ldots$$

$$= \ldots\ldots\; 2n + (m-1) + 1$$

$$= \ldots\ldots\; 2n + m$$

$$= 2n + m + 1$$