PUMPING LEMMA

Se L è regolare
allora $\exists\, n \in \mathbb{N}$ tale
che tutte le stringhe
$|w|$, $|w| \geq n$ possono
essere divise in 3 parti

$$w = xyz$$

tali che:

- $y \neq \varepsilon$
- $|xy| \leq n$
- $xy^iz \in L$ per ogni $i \in \mathbb{N}$

L regolare $\Longrightarrow$ | **tutte** le stringhe più lunghe di n .....

$$\not\Longleftarrow$$

$$A \Longrightarrow B \qquad \cancel{B \Longrightarrow A}$$

$$\equiv$$

$$\neg B \Longrightarrow \neg A$$

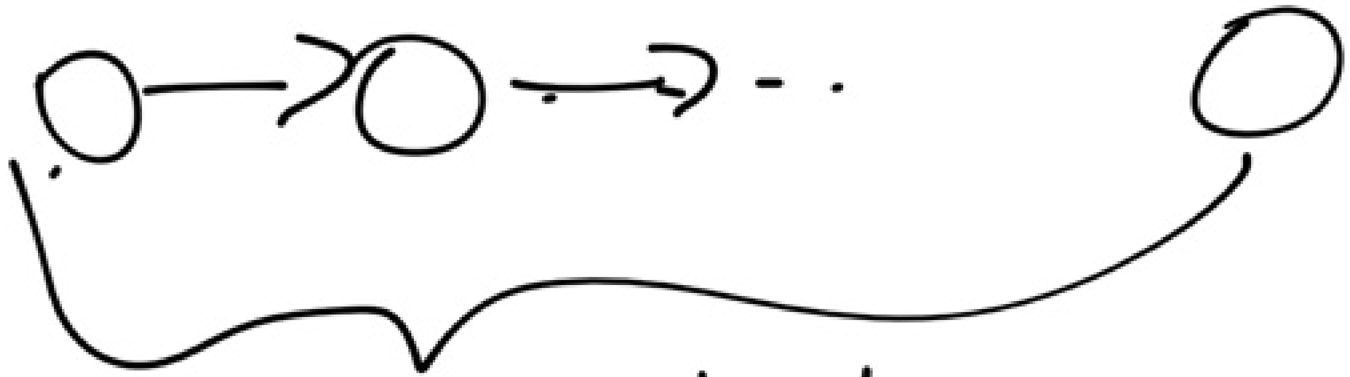
L regolare $\Rightarrow \exists n.$

$\forall w.\ |w| \geq n \quad w = xyz$

- $y \neq \varepsilon$
- $|xy| \leq n$
- $xy^i z \in L \quad i = 0, 1, 2 \ldots$

$n \cdot 1$

n stati



m stati

prendo $|s| \geq m$

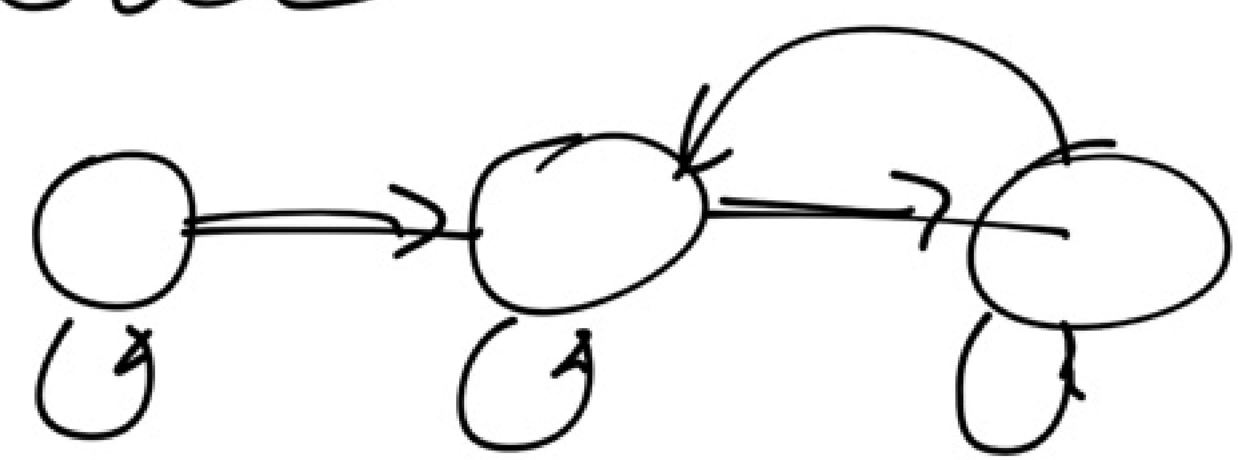
almeno

m archi passo almeno

2 volte nello stesso

stato

3 simboli



Principio delle buche dei piccioni:

Se voi volette mettere $n+1$ piccioni in n buche dovete mettere almeno 2 piccioni nelle stessa buca

n

$|s| \geq n$ per riconoscerla dovete passare almeno due volte nello stesso stato

$$s_0 \to s_1 \to s_2 \cdots s_i \to \cdots s_j \to s_m$$

$$s_i = s_j \qquad x y^i z$$

$$s_3 = xyyyz$$

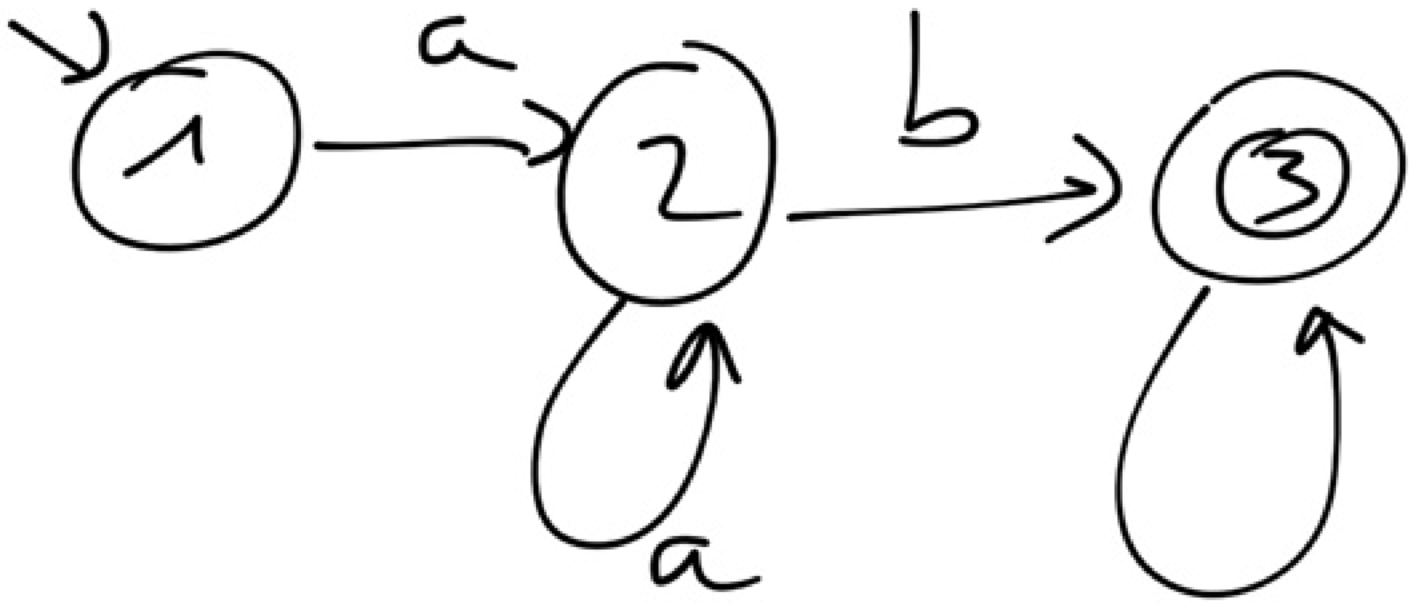
$$s = xy$$

$$\underbrace{s_1 \to s_2 \to \cdots \to}_{x} s_i \underbrace{\to \cdots \to s_m}_{z}$$

$$\begin{tikzcd} s_i \arrow[loop below, "y"] \end{tikzcd}$$

$$s_1 = xz \qquad s_2 = xyyz$$

$$L = \{ a^n b^m \mid n, m > 0 \}$$



$M = 3$

$$\underbrace{a}_{x}\underbrace{a}_{y}\underbrace{b}_{z}$$

$xz = ab$

$xyz = aab$

$xy^2z = aaab$

$xy^3z = aaaab$

$\vdots$

$$\underbrace{a}_{x}\underbrace{b}_{y}\underbrace{b}_{z}$$

$$xz = ab$$
$$xyz = abb$$
$$xy^2z = abbb$$
$$xy^3z = abbbb$$
$$\vdots$$

aaab

abbb

$L = \{ a^k b^k \mid k > 0 \}$

$y \neq \varepsilon$

$|xy| \leq n$

$\forall n$ prendo la stringa $|a^n b^n| > n$

$\overbrace{a \ldots a}^{x,\,y}\,\overbrace{b \ldots b}^{z}$

$\underbrace{a \ldots a}_{n}\;\underbrace{b \ldots b}_{n}$

s

$y = \underbrace{a \ldots a}_{m}$

$|xy| \leq n$

$|x| = s \quad |y| = m$

$m + s \leq n$

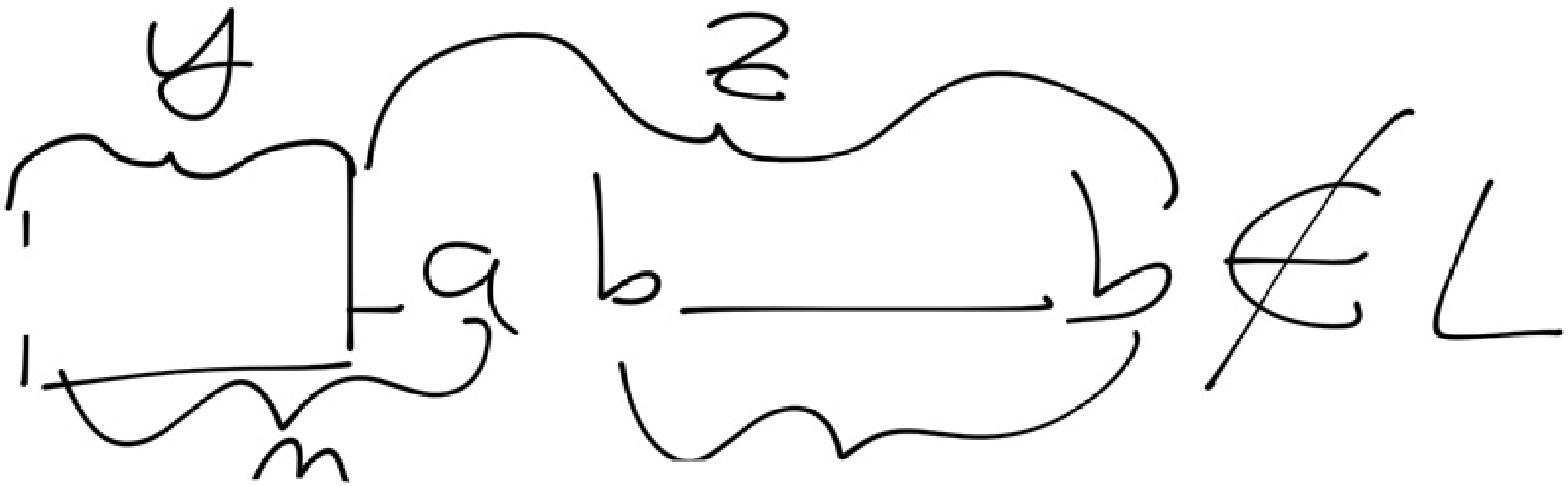
Qualunque na n

$$\overbrace{a\,-}^{x}\,\overbrace{-\,-}^{y}\,\overbrace{a\;b\,-\,-\,-\,b}^{z}$$

$$\underbrace{a\,-\,-\,-\,a}_{n}\;\underbrace{b\,-\,-\,-\,b}_{n}$$

- $y \neq \varepsilon$
- $|xy| \leq n$
- $xy^i z \in L$

$xz \notin L$



$$A \Rightarrow B \not\equiv B \Rightarrow A$$

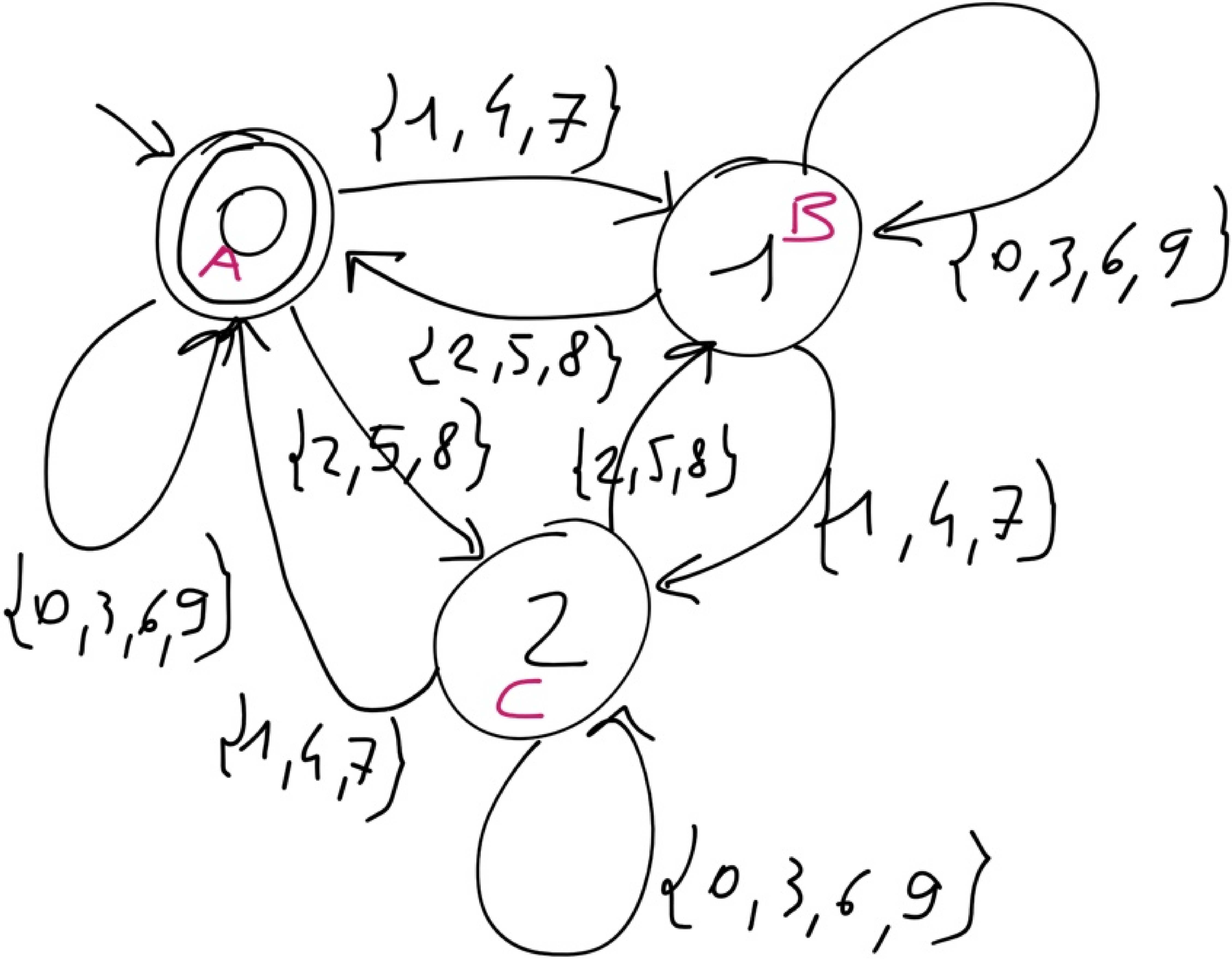
$$\equiv \neg B \Rightarrow \neg A$$

$\Sigma = \{0, 1, \dots, 9\}$

$L = \{\alpha \mid \alpha \in \Sigma^* \text{ e } \alpha \text{ è multiplo di } 3\}$

$L = \{3, 12, 6, 18, 111, \dots\}$

$$\begin{array}{l|}
3\,6\,1\,1\,1 \\
\hline
\quad 1\,2\,1\,0
\end{array}$$



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3	2	1	1	4	8	2		← strings
∅	∅	2	∅	1	2	1	∅	← stato

8	5	4	2	1		← strings
∅	2	1	2	1	2	← stato

Monē mc.

$$L = \{a^n b^m c^k \mid n, m, k > 0 \text{ e } m = n + k\}$$

$$\Sigma = \{a, b, c\}$$

$$\Updownarrow$$

$$L = \{a^n b^n b^m c^m \mid n, m > 0\}$$

$$S \to AB$$
$$A \to ab \mid aAb$$
$$B \to bc \mid bBc$$

$$L = \{ a^n b^m c^k \mid n, k > 0 \text{ e } m = n + k \}$$

$$L = \{ a^n b^m \mid n, m > 0 \text{ e } n \neq m \}$$

$$L = \{ a^n b^m \mid n, m > 0 \text{ e } n > m \}$$

$$\cup \{ a^n b^m \mid n, m > 0 \text{ e } m > n \}$$

$$S \rightarrow S_1 \mid S_2$$

$$S_1 \rightarrow aab \mid aS_1 \mid aS_1 b$$

$$S_2 \rightarrow abb \mid S_2 b \mid aS_2 b$$

date due grammatiche

$G_1 = (\Lambda, V_1, S_1, P_1)$

e $G_2 = (\Lambda, V_2, S_2, P_2)$

con $V_1 \cap V_2 = \emptyset$

definire una grammatica G tale che

$L_G = L_{G_1} \cup L_{G_2}$

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$$G = \langle \Sigma, V_1 \cup V_2 \cup \{S\}, S, P_1 \cup P_2 \cup \{S \to S_1 \mid S_2\} \rangle$$

con $S \notin V_1 \cup V_2$

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data

$$G_1 = \langle \Lambda, V_1, S_1, P_1 \rangle$$

definire

G che genera il

linguaggio

$$L = \left\{ \alpha_1 \alpha_2 \ldots \alpha_n \;\middle|\; \begin{array}{l} n > 0 \text{ e} \\ \alpha_i \in L_{G_1} \end{array} \right\}$$

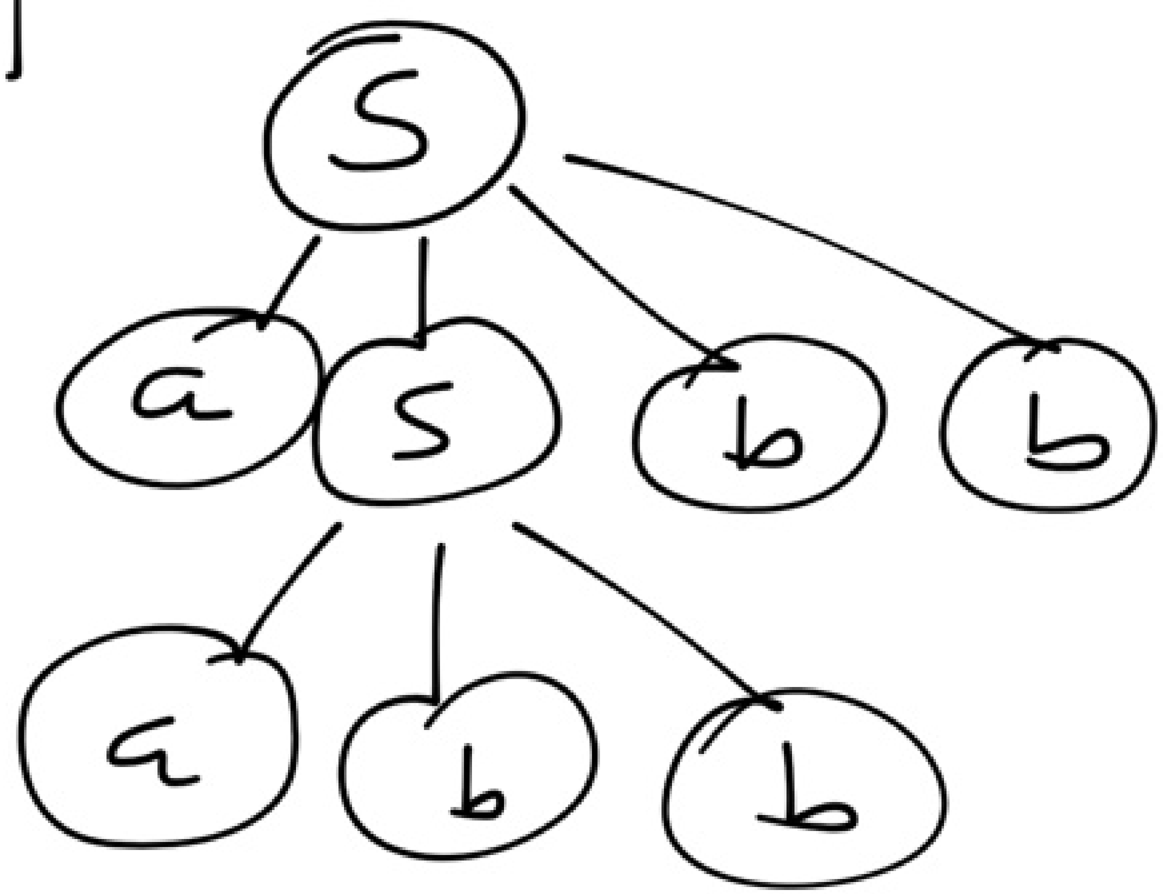
$$G = \{ \Lambda,\ V_1 \cup \{S\},\ S,$$
$$P_1 \cup \{ S \to S_1 \mid S_1 S \} \}$$

dove $S \notin V_1$

$$L = \{ a^n b^{2n} \mid n > 0 \}$$

$$S \rightarrow abb \mid aSbb$$

aabbbb

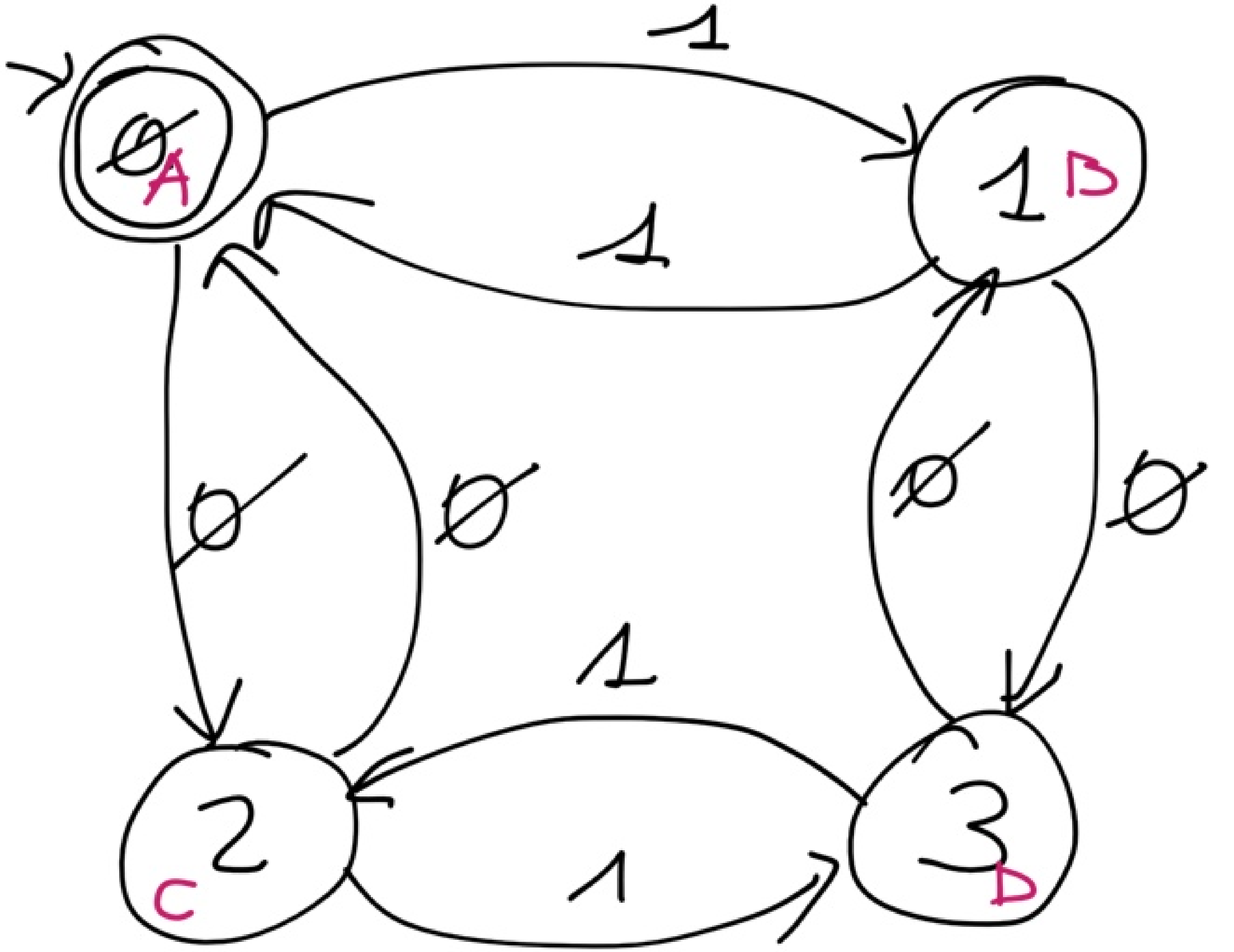


$\Lambda = \{0, 1\}$ $\qquad \varepsilon \in L$

L contiene un numero pari di occorrenze sia di 0 che di 1

$010100 \in L$

$\underbrace{0\underline{1}0\underline{11}}_{3} \notin L$



$A \rightarrow 1B \mid 0C \mid \varepsilon$

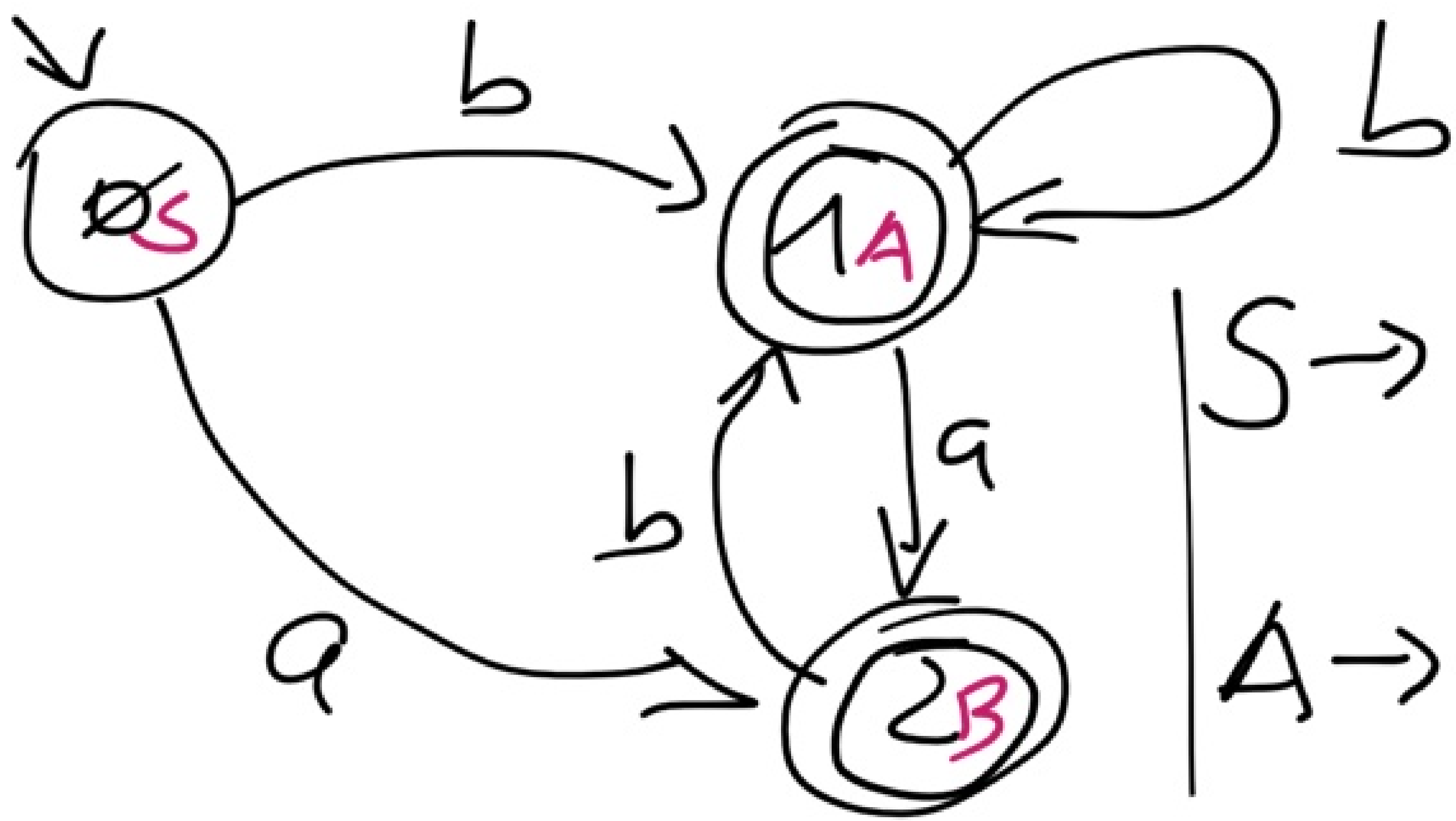
$D \rightarrow 1C \mid 0B$

$B \rightarrow 1A \mid 0D \mid 1$

$C \rightarrow 1D \mid 0A \mid 0$

$\Lambda = \{a, b\}$

$L = \Lambda^+ \setminus \{\alpha a a \beta \mid \alpha, \beta \in \Lambda^*\}$



$S \to bA \mid b \mid aB \mid a$

$A \to aB \mid a \mid bA \mid b$

$B \to bA \mid b.$