Formal Methods for XML: Algorithms & Complexity

S. Margherita di Pula
August 2004

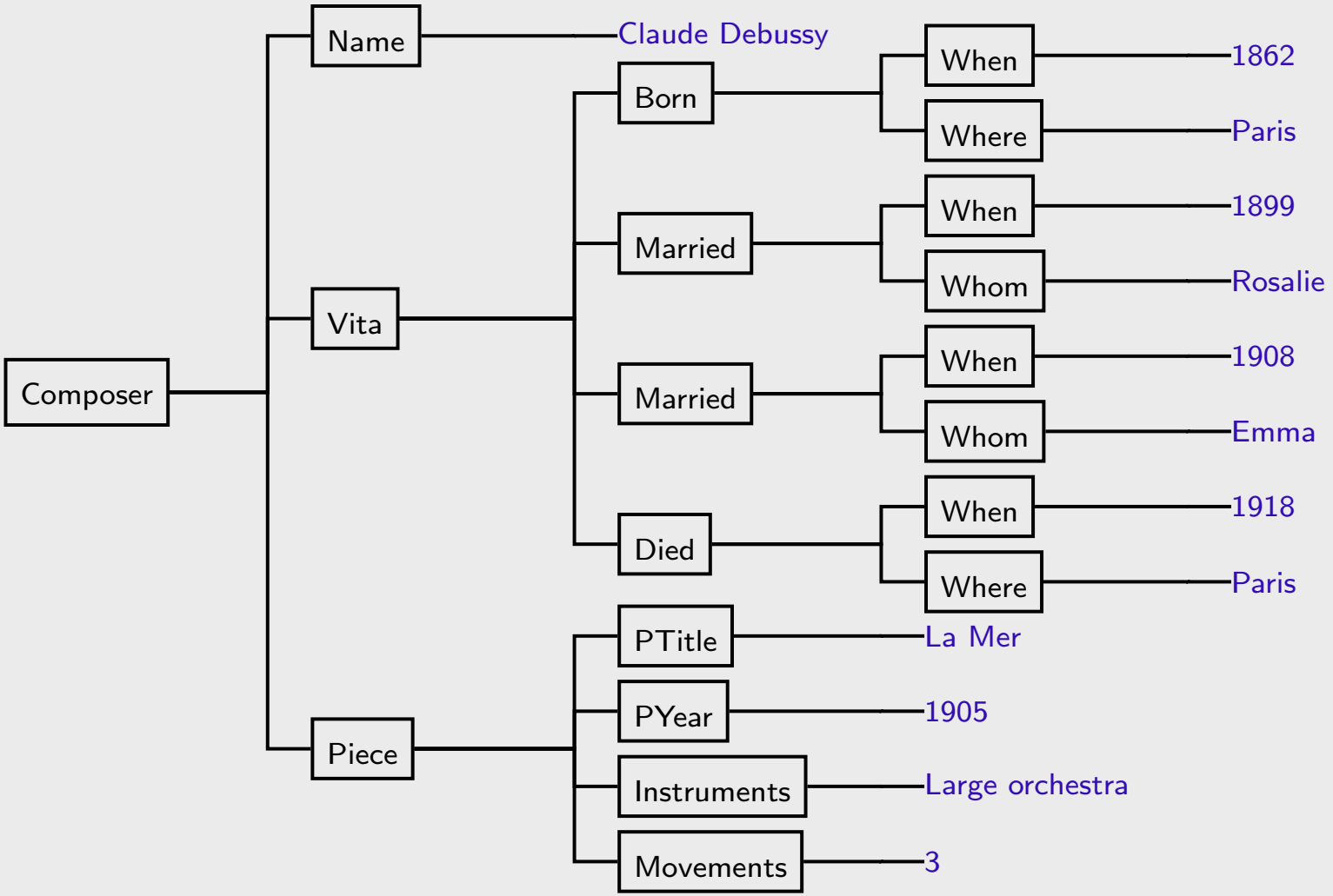
Thomas Schwentick

XML

Example Document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Example Tree



XML Processing

Four important kinds of XML processing

Validation

Check whether an XML document is of a given type

Navigation

Select a set of positions in an XML document

Querying

Extract information from an XML document

Transformation

Construct a new XML document from a given one

XML Processing

Four important kinds of XML processing ..... and their languages

Validation — `DTD, XML Schema`

Check whether an XML document is of a given type

Navigation — `XPath`

Select a set of positions in an XML document

Querying — `XQuery`

Extract information from an XML document

Transformation — `XSLT`

Construct a new XML document from a given one

Validation: DTD

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
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⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Validation: DTD

DTD

DTDs describe types of XML documents

Example document

⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩
    ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
    ⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
    ⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
    ⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
    ...
  ⟨/Piece⟩
  ...
⟨/Composer⟩
...

Validation: DTD

DTD

DTDs describe types of XML documents

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instrun
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Example

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
        Instruments, Movements)>
    ]>
```

Navigation: XPath

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
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⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Navigation: XPath

XPath

XPath expressions select sets of nodes of XML documents by specifying navigational patterns

Example doc

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Navigation: XPath

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⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
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⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Example query

```
//Vita/Died/*
```

Navigation: XPath

XPath

XPath expressions select sets of nodes of XML documents by specifying navigational patterns

Example doc

```
⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩
    ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
    ⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
    ⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
    ⟨Died⟩ ⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
    ...
  ⟨/Piece⟩
  ...
⟨/Composer⟩
...
```

Example query

```
//Vita/Died/*
```

Querying: XQuery

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Querying: XQuery

XQuery

XQuery is a full-fledged XML query language

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Querying: XQuery

XQuery

XQuery is a full-fledged XML query language

Example document

```
⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩
    ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
    ⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
    ⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
    ⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
    ...
  ⟨/Piece⟩
  ...
⟨/Composer⟩
...
```

Example query

```
for $x in doc('composers.xml')/Composer
where $x/Vita/Died/Where = 'Paris'
return $x/Name
```

Querying: XQuery

XQuery

XQuery is a full-fledged XML query language

Result

⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Name⟩ Eric Satie ⟨/Name⟩
⟨Name⟩ Hector Berlioz ⟨/Name⟩
⟨Name⟩ Camille Saint-Saëns ⟨/Name⟩
⟨Name⟩ Frédéric Chopin ⟨/Name⟩
⟨Name⟩ Maurice Ravel ⟨/Name⟩
⟨Name⟩ Jim Morrison ⟨/Name⟩
⟨Name⟩ César Franck ⟨/Name⟩
⟨Name⟩ Gabriel Fauré ⟨/Name⟩
⟨Name⟩ George Bizet ⟨/Name⟩
...

ere⟩ Paris ⟨/Where⟩⟨/Born⟩
n⟩ Rosalie⟨/Whom⟩⟨/Married⟩
n⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟩ Paris ⟨/Where⟩ ⟨/Died⟩

⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Example query

```
for $x in doc('composers.xml')/Composer
where $x/Vita/Died/Where = 'Paris'
return $x/Name
```

Transformation: XSLT

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Transformation: XSLT

XSLT

XSLT transforms documents by means of templates

Example documen

```
⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩
    ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
    ⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
    ⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
    ⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
    ⟨Movements⟩ 3 ⟨/Movements⟩
    ...
  ⟨/Piece⟩
  ...
⟨/Composer⟩
...
```

Transformation: XSLT

XSLT

XSLT transforms documents by means of templates

Example documen

```
⟨Composer⟩
  ⟨Name⟩ Claude Debussy ⟨/Name⟩
  ⟨Vita⟩
    ⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
    ⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
    ⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
    ⟨Died⟩⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
  ⟨/Vita⟩
  ⟨Piece⟩
    ⟨PTitle⟩ La Mer ⟨/PTitle⟩
    ⟨PYear⟩ 1905 ⟨/PYear⟩
    ⟨Instruments⟩ Large orchest
    ⟨Movements⟩ 3 ⟨/Movemer
    ...
  ⟨/Piece⟩
  ...
⟨/Composer⟩
...
```

Example

```
⟨xsl:template match="Composer[Vita//Where='Paris']"⟩
  ⟨ParisComposer⟩
    ⟨xsl:copy-of select="Name" /⟩
    ⟨xsl:copy-of select="Vita/Born" /⟩
  ⟨/ParisComposer⟩
⟨/xsl:template⟩
```

Transformation: XSLT

Result

⟨ParisComposer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Born⟩
⟨When⟩ August 22, 1862 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Born⟩
⟨/ParisComposer⟩
⟨ParisComposer⟩
⟨Name⟩ Frédéric Chopin ⟨/Name⟩
⟨Born⟩
⟨When⟩ March 1, 1810 ⟨/When⟩
⟨Where⟩ Želazowa ⟨/Where⟩
⟨/Born⟩
⟨/ParisComposer⟩
⟨ParisComposer⟩
⟨Name⟩ Camille Saint-Saëns ⟨/Name⟩
⟨Born⟩
⟨When⟩ October 9, 1835 ⟨/When⟩
⟨Where⟩ Paris ⟨/Where⟩
⟨/Born⟩
⟨/ParisComposer⟩

XSLT

XSLT transforms documents by means of templates

umer

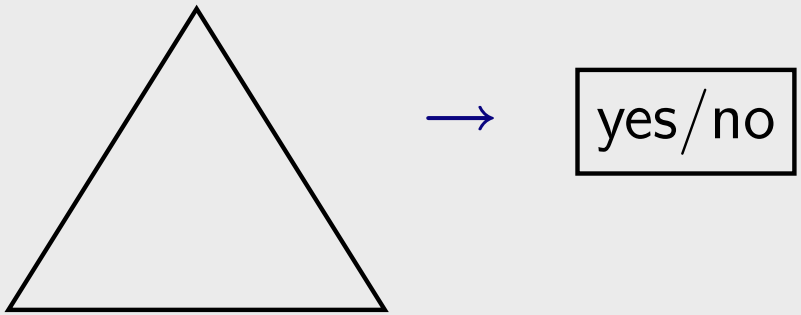
en⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩

Example

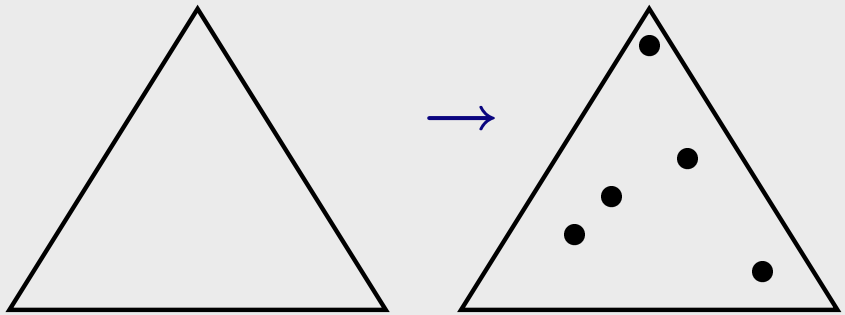
ate match=”Composer[Vita//Where=’Paris’]”⟩
omposer⟩
copy-of select=“Name” /⟩
copy-of select=“Vita/Born” /⟩
Composer⟩
late⟩

A Schematic View

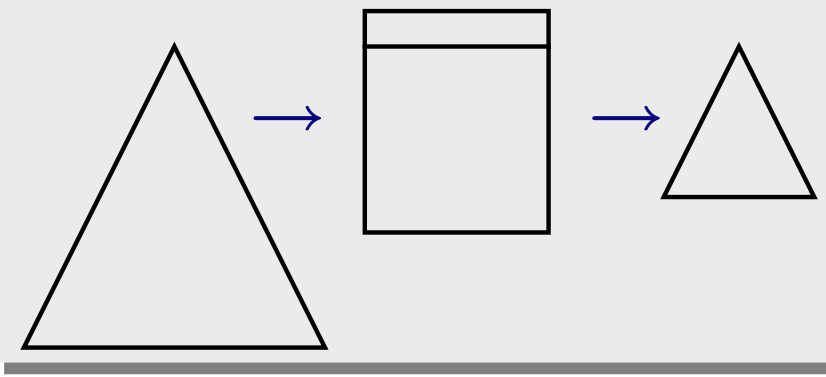
DTD/ XML Schema



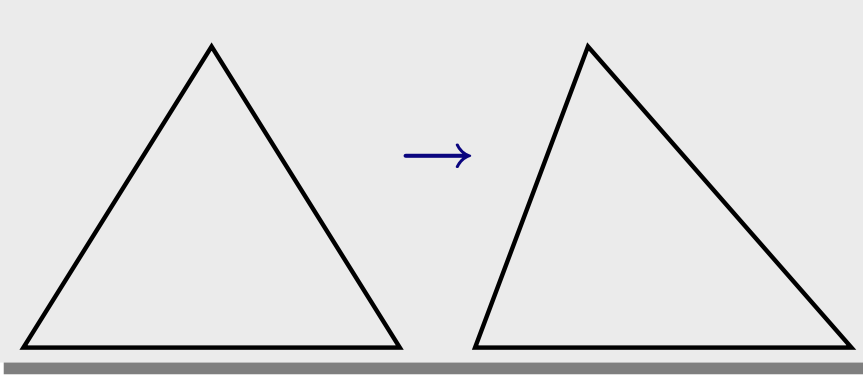
XPath



XQuery



XSLT



Focus of this Talk

Topics

- Expressive power of XML languages
- Complexity of algorithmic tasks related to XML processing
- Tradeoff between expressiveness and complexity

Goals of this Research

- Understand expressive power and complexity of XML languages
- Identify interesting fragments with good tradeoff

Algorithmic Tasks

Algorithmic Tasks

Evaluation

Evaluation (Combined)

I: Tree t, Query q

O: $q(t)$

Evaluation (Data(q))

I: Tree t

O: $q(t)$

Incremental Eval. (q)

I: Tree t,
Changes of t

O: $q(t)$

Static Analysis

Satisfiability

I: Query q

Q: Is $q(t) \neq \emptyset$
for some t?

Containment

I: Queries q_1, q_2

Q: Is always
$q_1(t) \subseteq q_2(t)$?

Equivalence

I: Queries q_1, q_2

Q: Is always
$q_1(t) = q_2(t)$?

Type Checking

I: Types d_1, d_2,
Transformation T

Q: Does $t \models d_1$ imply
$T(t) \models d_2$?

Type Inference

Expressive power

Question: How do we measure expressive power?

Remarks

- Classes of logical formulas are a good yardstick

$\longrightarrow$ They provide methods to prove that a query can not be expressed

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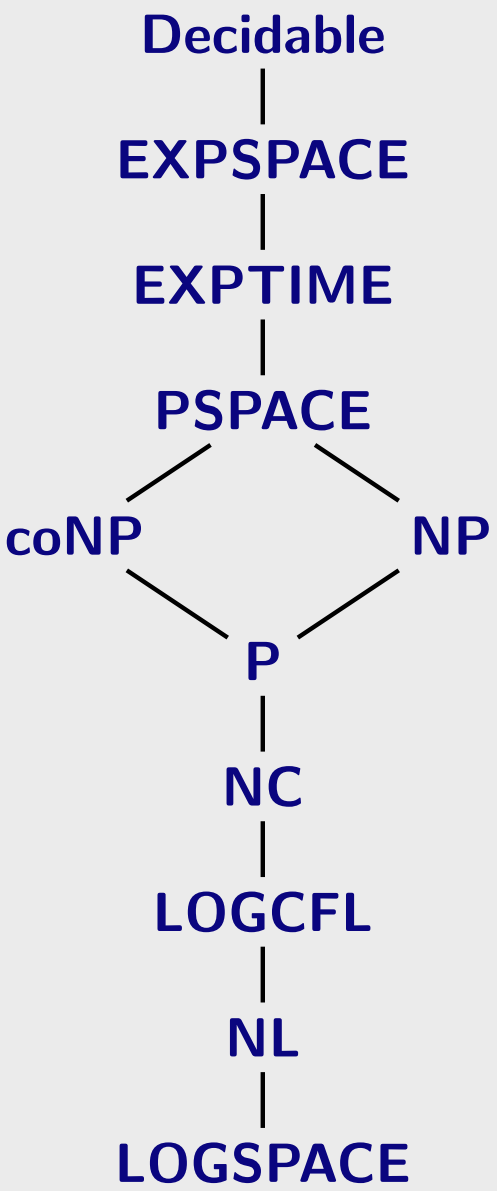
Recall Relational Databases

- Core of SQL $\equiv$ First-order Logic
- Most frequently asked queries $\equiv$ Conjunctive queries

Contents

Background: Complexity Classes

Overview of Complexity Classes



- EXPSPACE ..... Equivalence of reg. expressions with squaring
- EXPTIME ..... 2-Player Corridor Tiling
- PSPACE ..... Quantified Boolean Formulas
- coNP, NP ..... Satisfiability of prop. formulas
- P ..... Boolean circuit evaluation
- NC ..... *Efficiently parallelizable problems*
- LOGCFL ..... Acyclic conjunctive queries
- NL ..... Reachability in directed graphs
- LOGSPACE ..... Reachability in directed forests

XML, Trees and Automata

Question: Why is XML appealing for Theory people?

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Years ago...

- Theoretical Computer Science for Database Theorists: Logics, Complexity, Algorithms,...
- Database Theory for Theoretical Computer Scientists:

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After the advent of XML

Many connections between

Formal Languages & Automata Theory

and

XML & Database Theory

XML, Trees and Automata

Question: Why trees?

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A Natural Answer

- Trees reflect the hierarchical structure of XML
- Underlying data model of XML is tree based

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$\Rightarrow$ Sometimes extensions are needed

- E.g., directed graphs instead of trees

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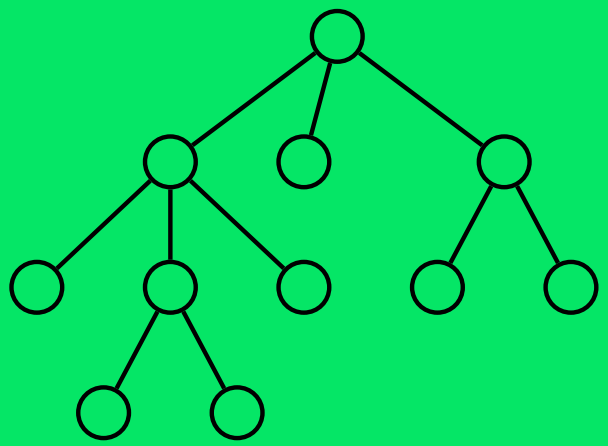

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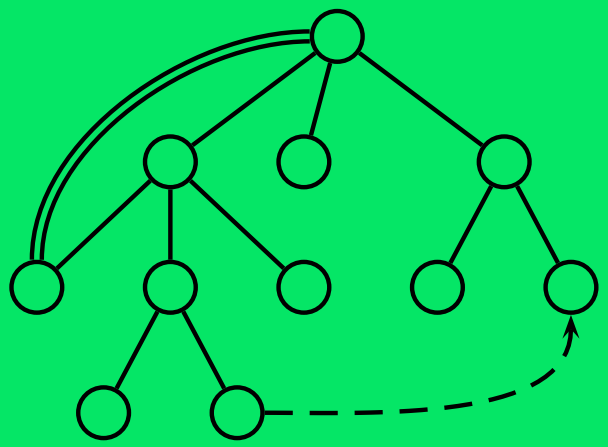

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Nevertheless

In this tutorial we will concentrate on the tree view at XML

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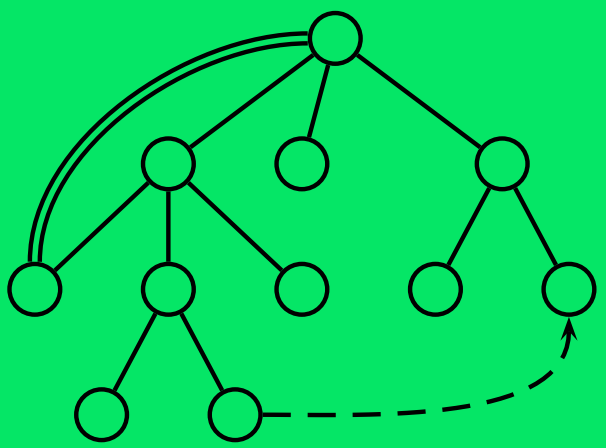

XML, Trees and Automata

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Ingredients of XML

Concepts from formal languages are obviously present around XML:

- Labelled trees
- `DTD`: context-free grammars
- `DTD`: regular expressions
- `XPath`: regular path expressions

We will see

Automata turn out to be useful as:

- a means to define robust classes with clear semantics
- a tool for proofs
- an algorithmic tool for static analysis
- a tool for query evaluation

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From Strings to Trees

A String

abcab

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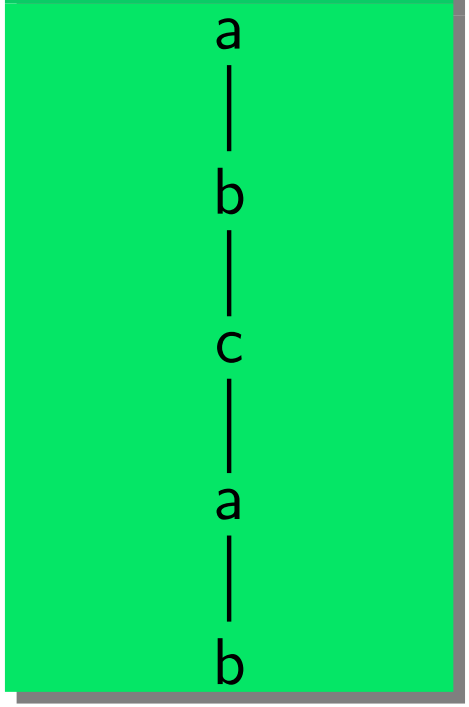

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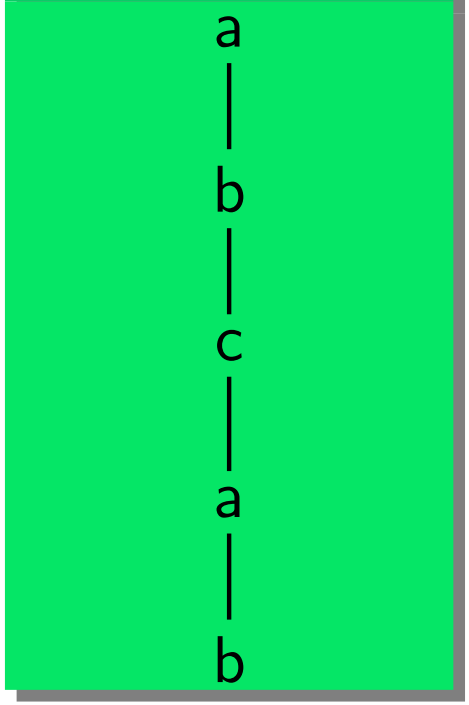

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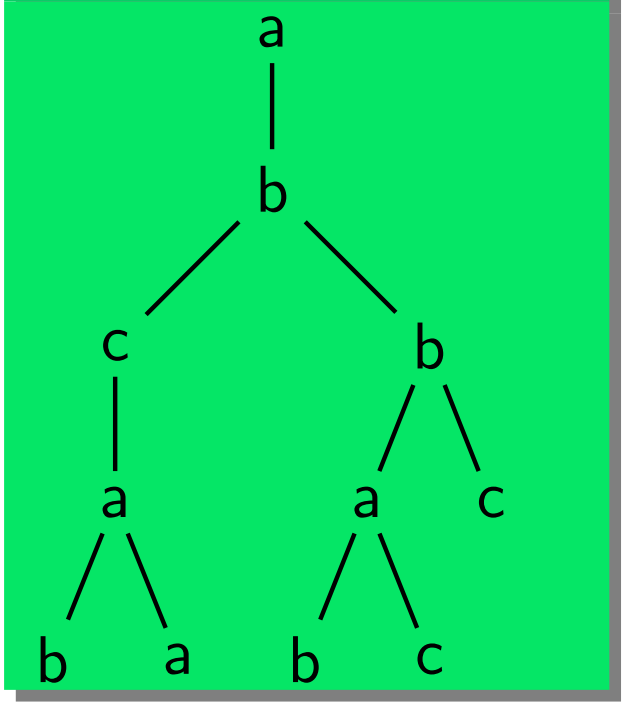

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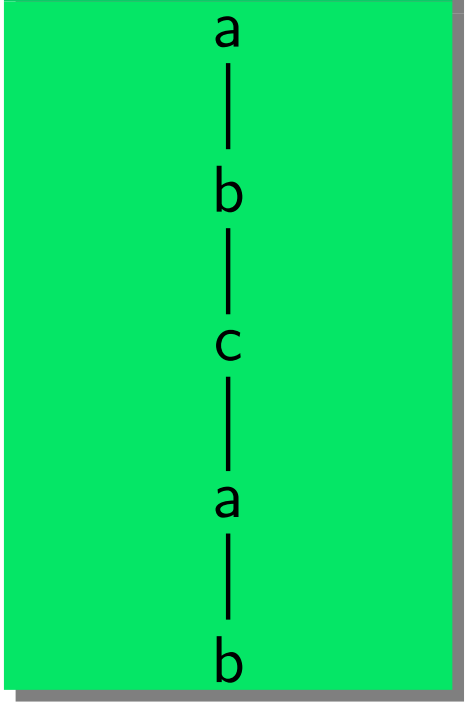

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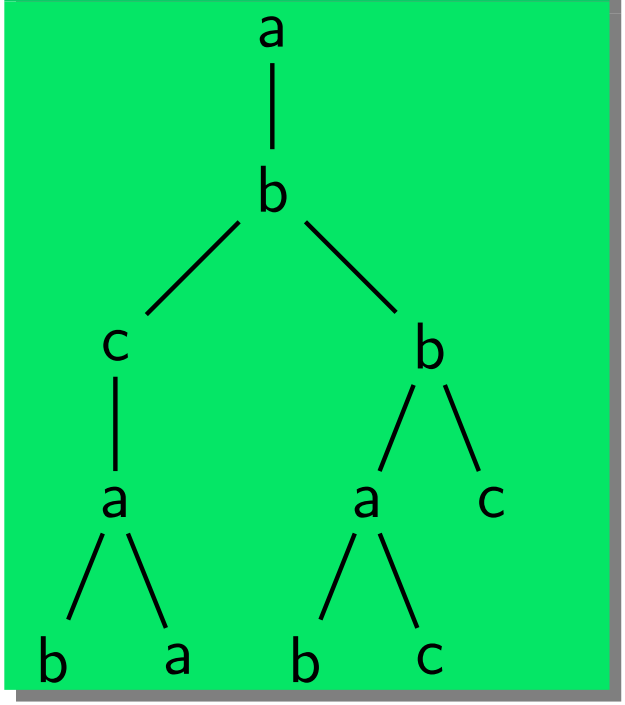
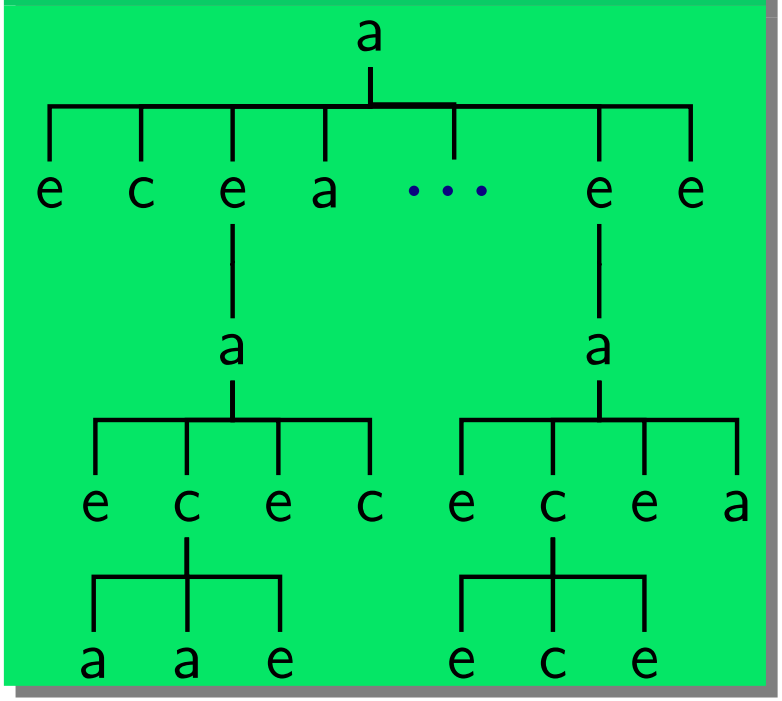


An Unranked Tree



From Strings to Trees (cont.)

XML and Trees

- XML trees are **unranked**:

  the number of children of a node is not restricted

- Automata have first been considered on **ranked** trees, where each symbol has a fixed number of children (rank)
- Most important ideas were already developed for ranked trees

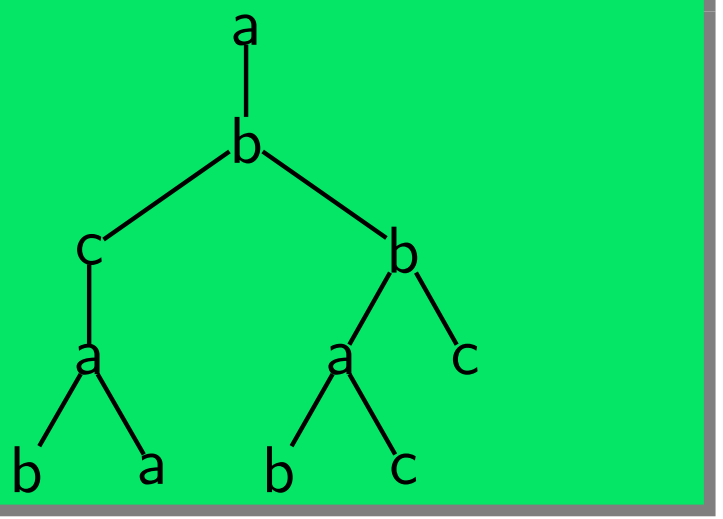
$\longrightarrow$ Let us take a look at this first

Trees as Terms

Remark

Sometimes trees are viewed as terms

Example



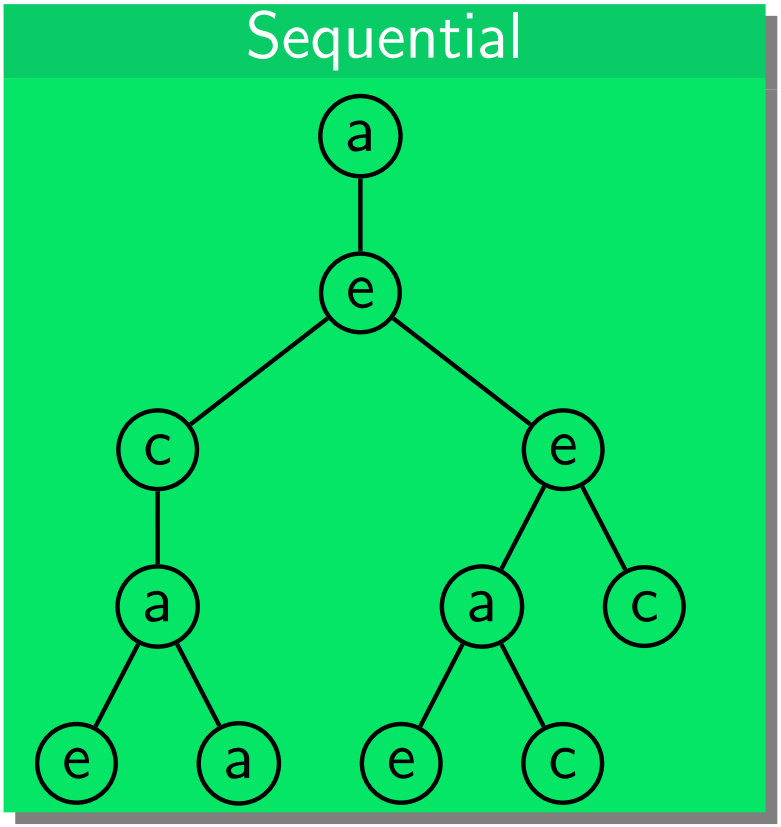
Example Tree as Term

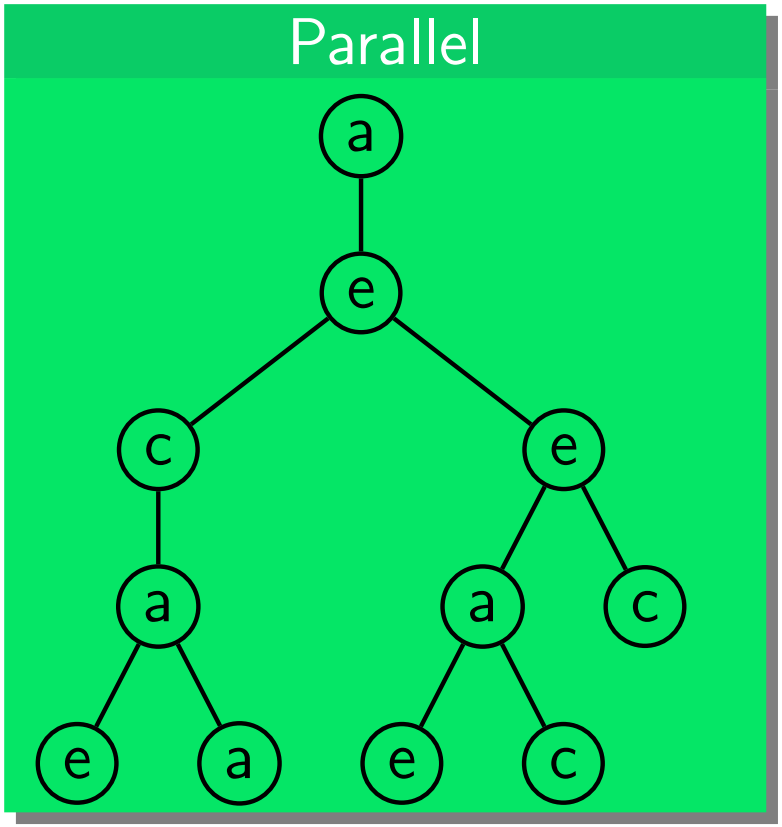
$$a^1(b^2(c^1(a^2(b,a)),b^2(a^2(b,c),c)))$$

From String Automata to Tree Automata

Question

How do automata generalize to trees?





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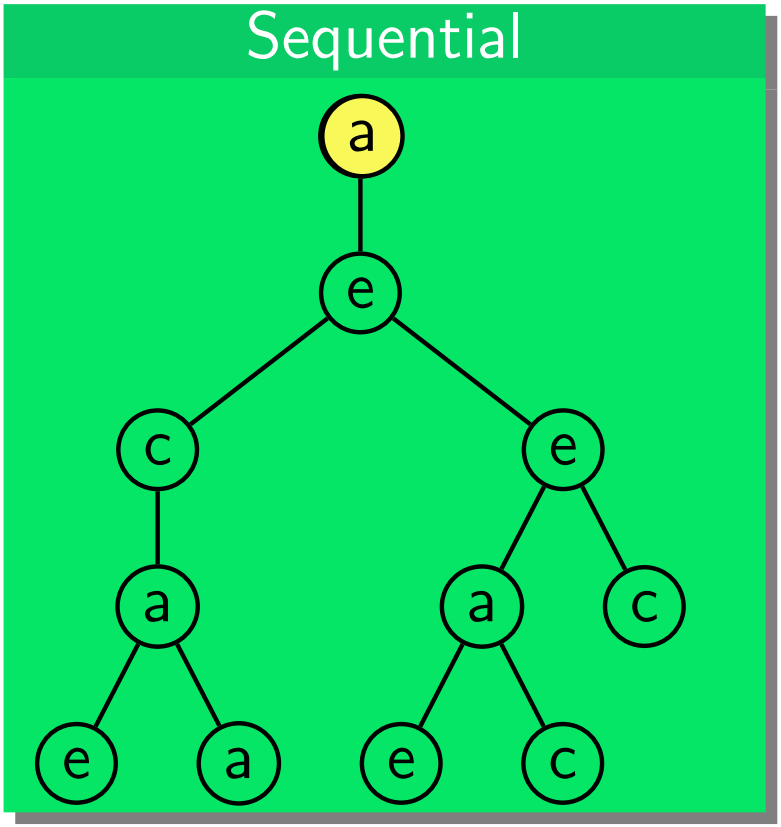

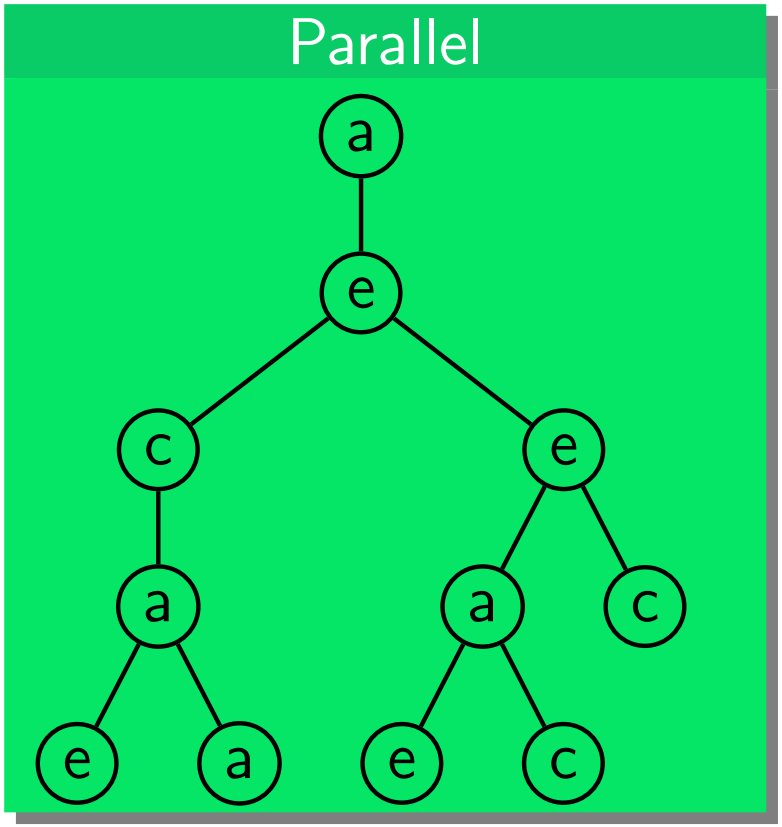

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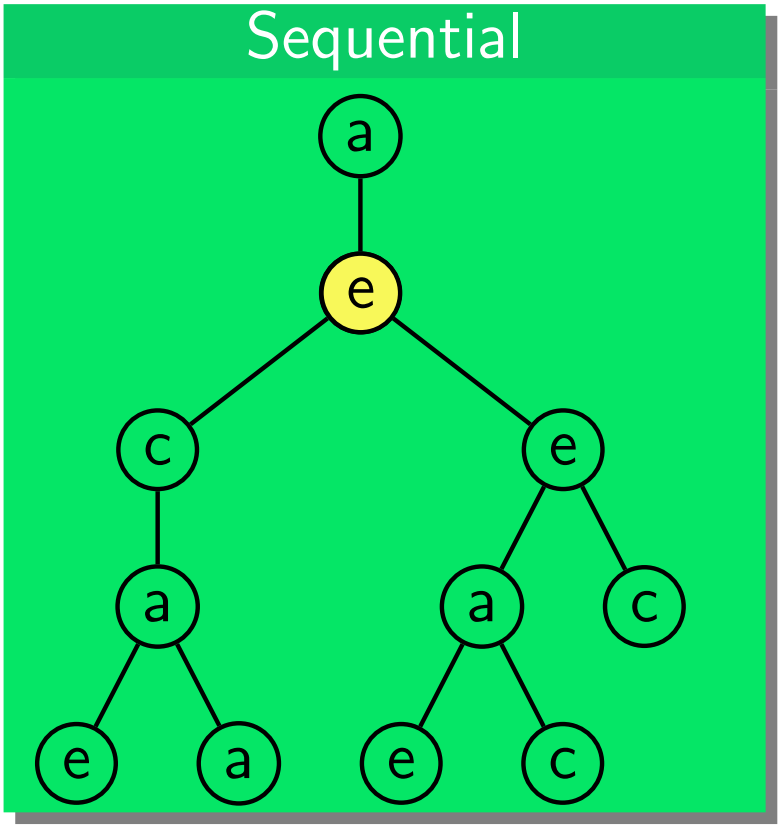

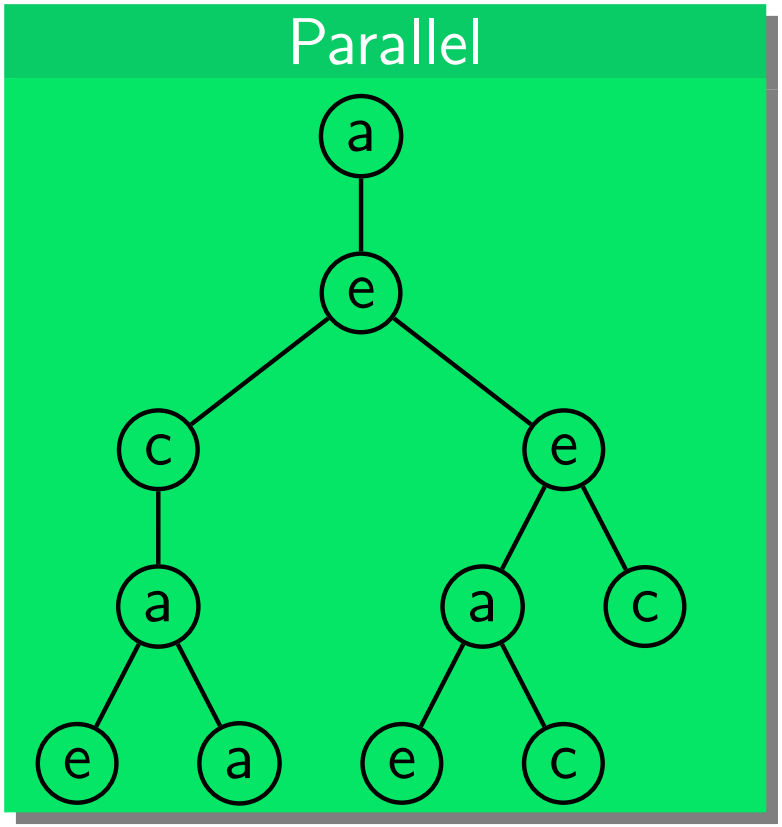

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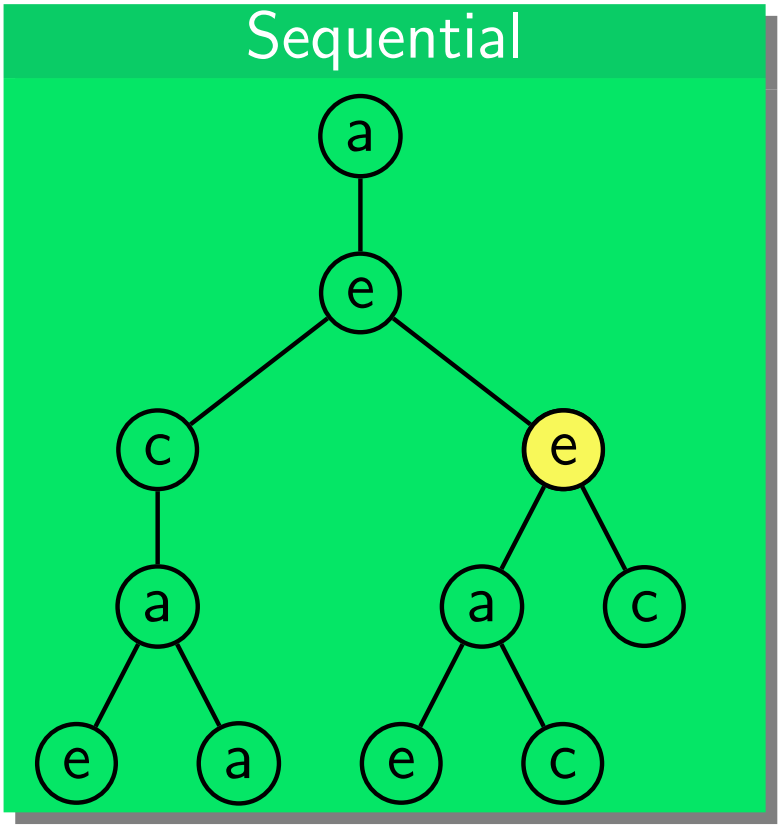

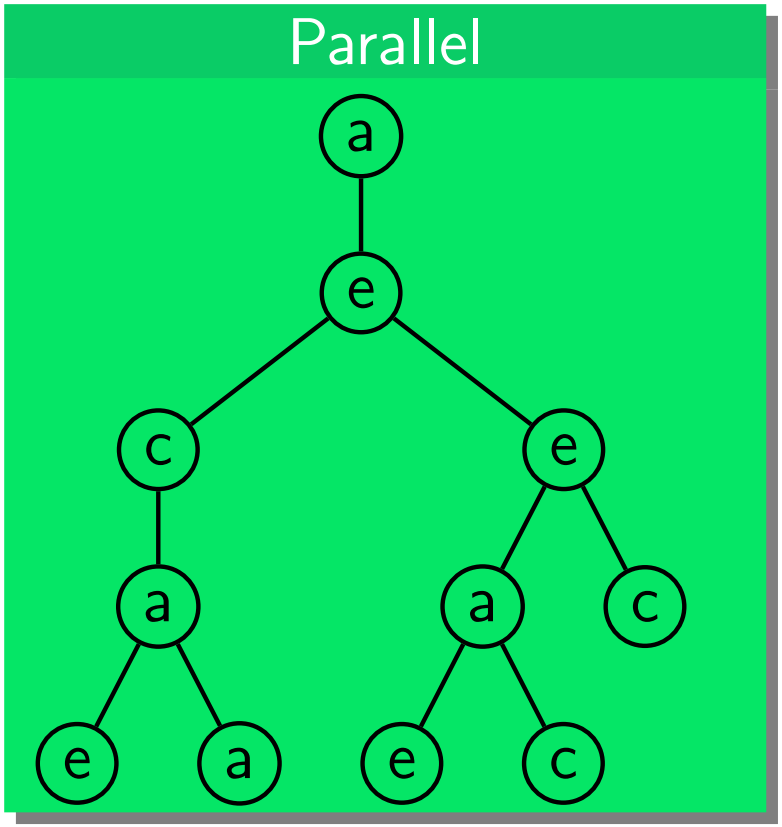

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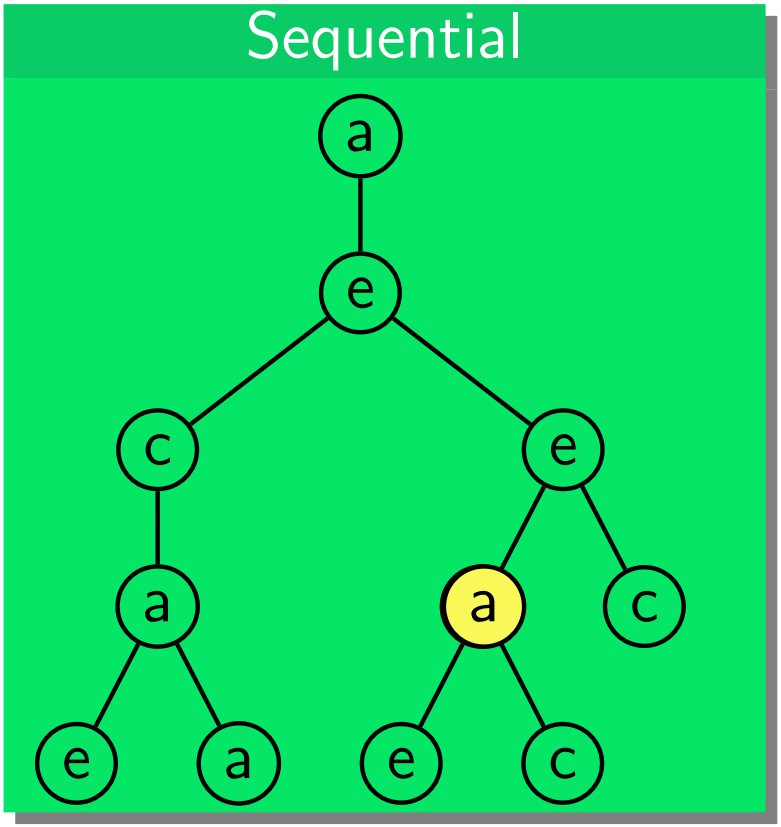

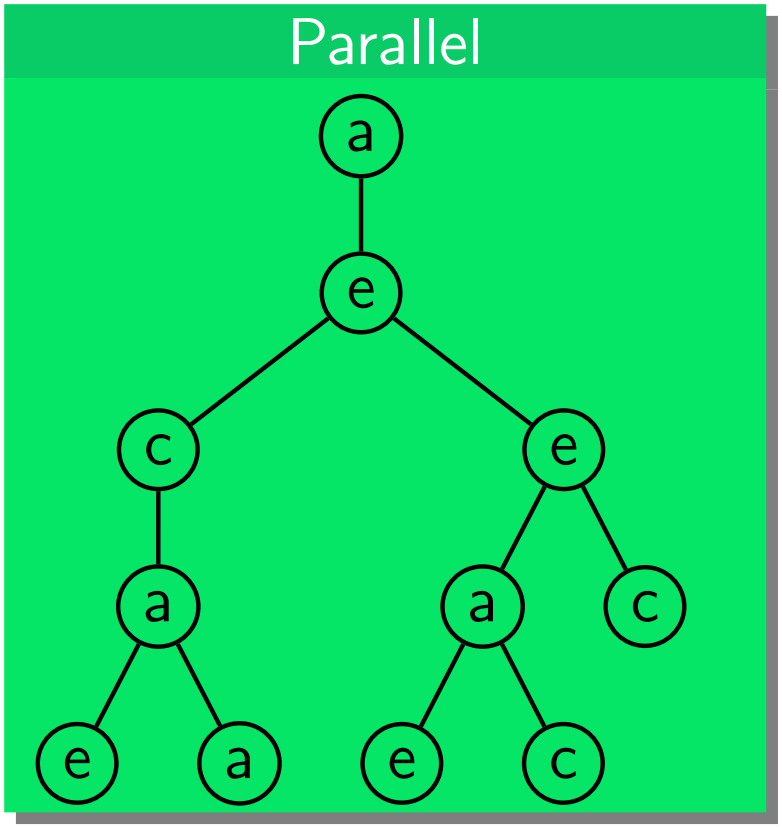

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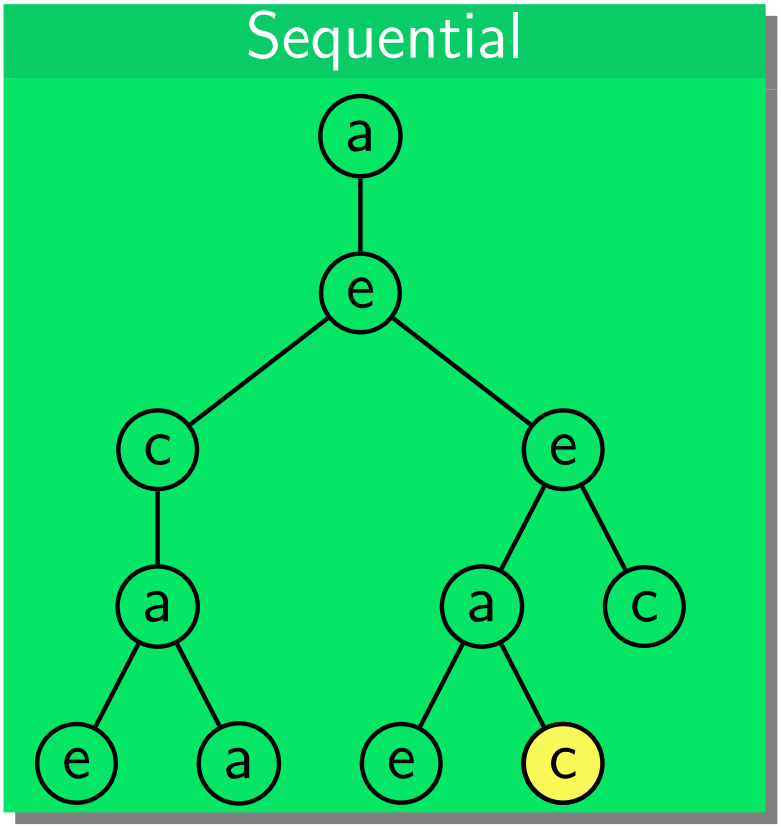

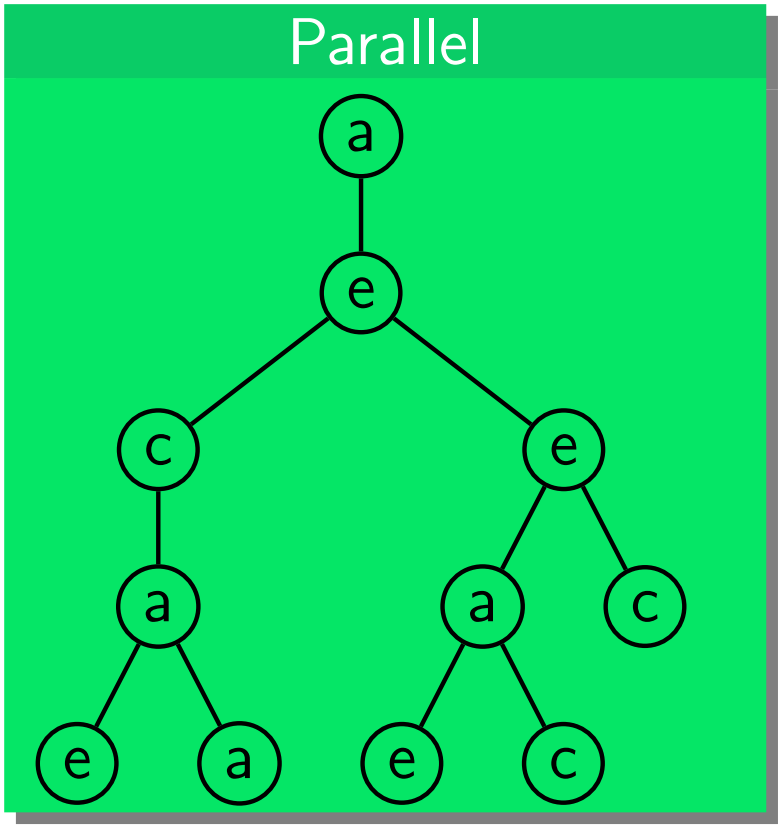

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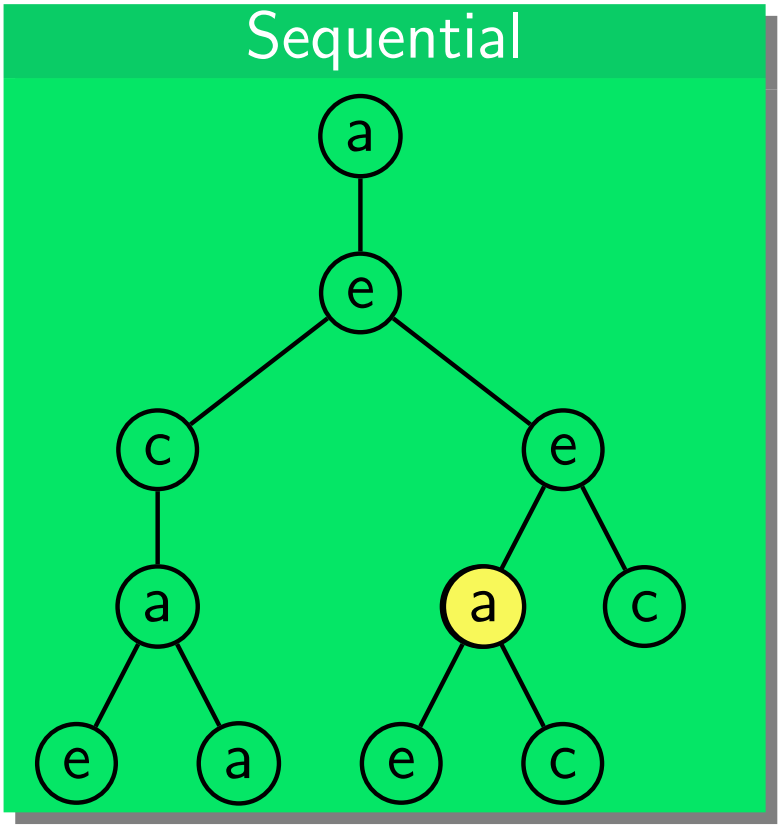

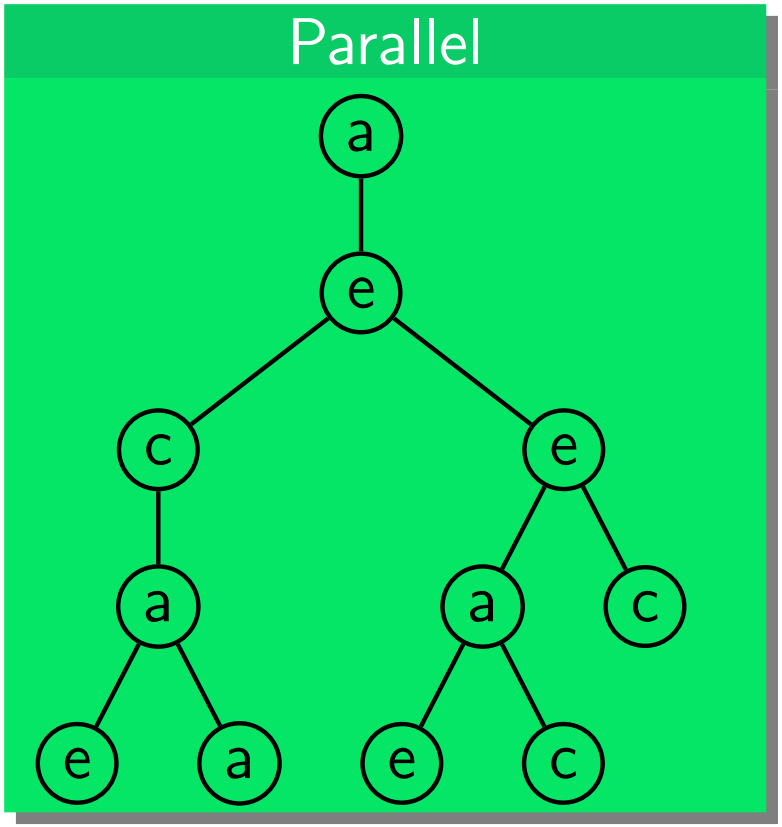

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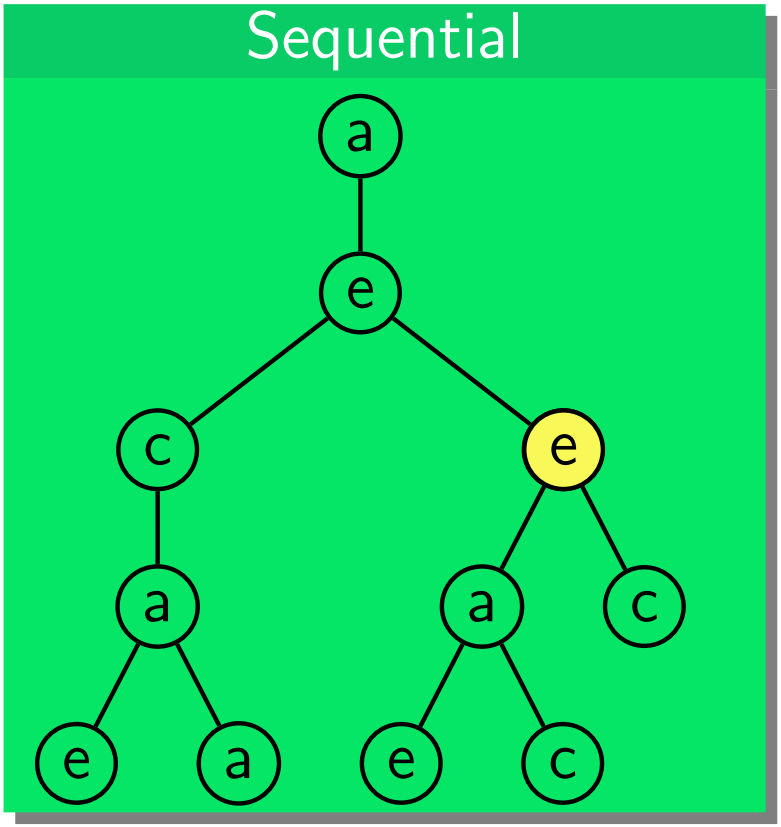

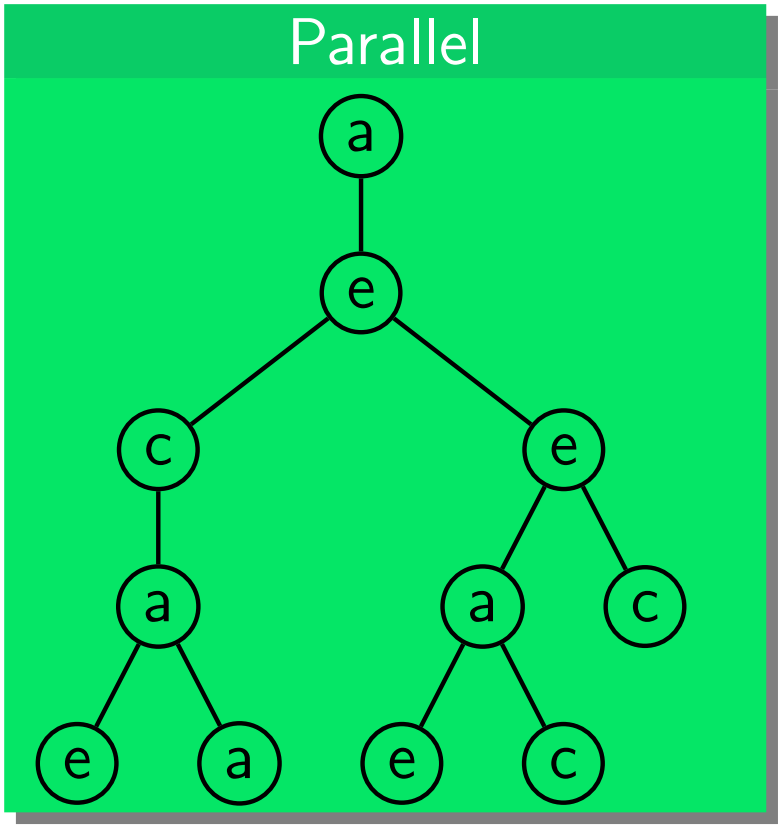

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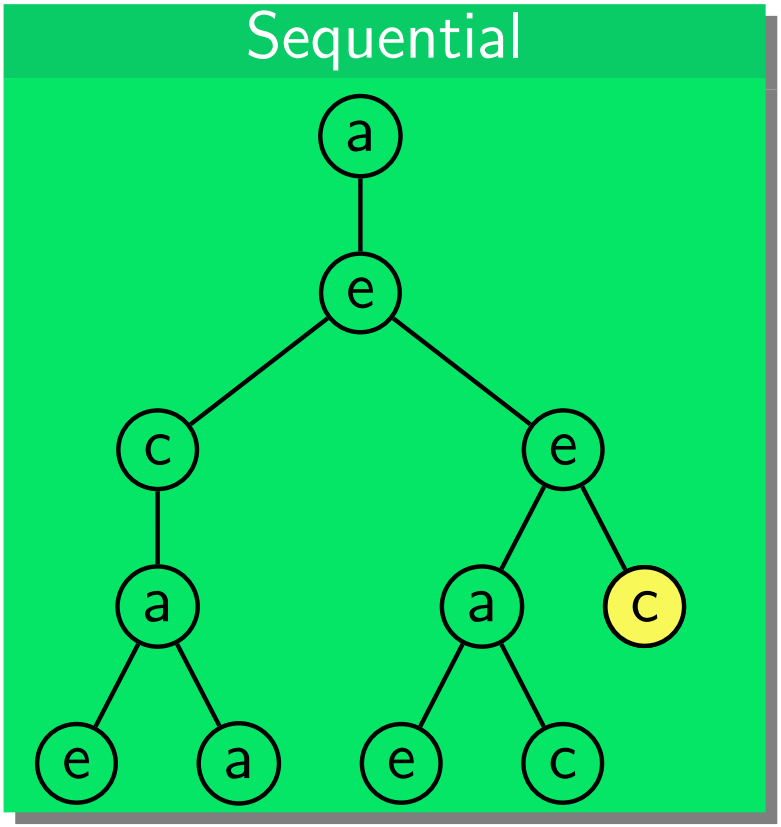

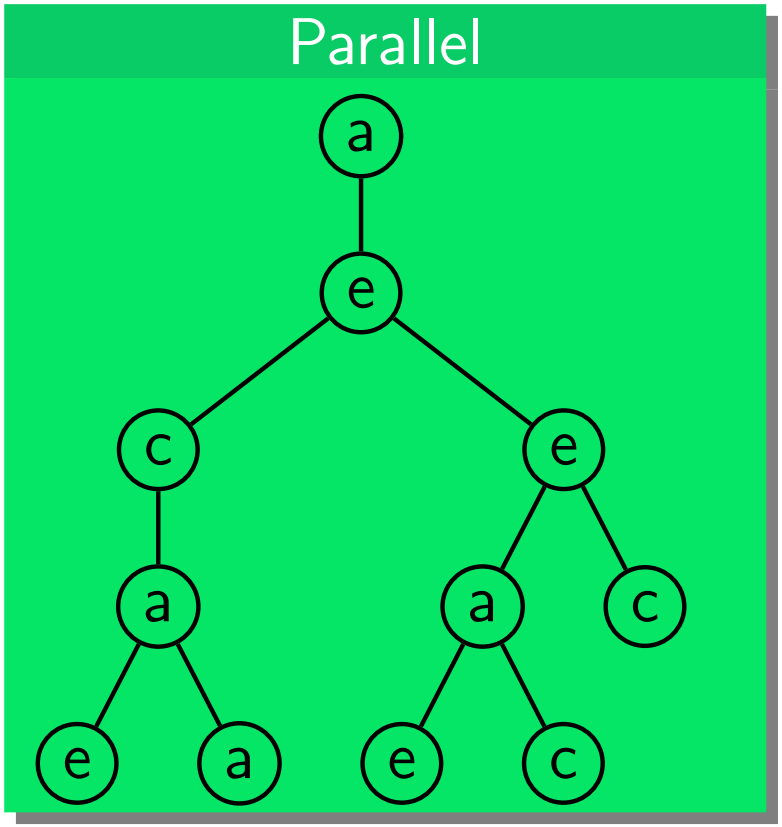

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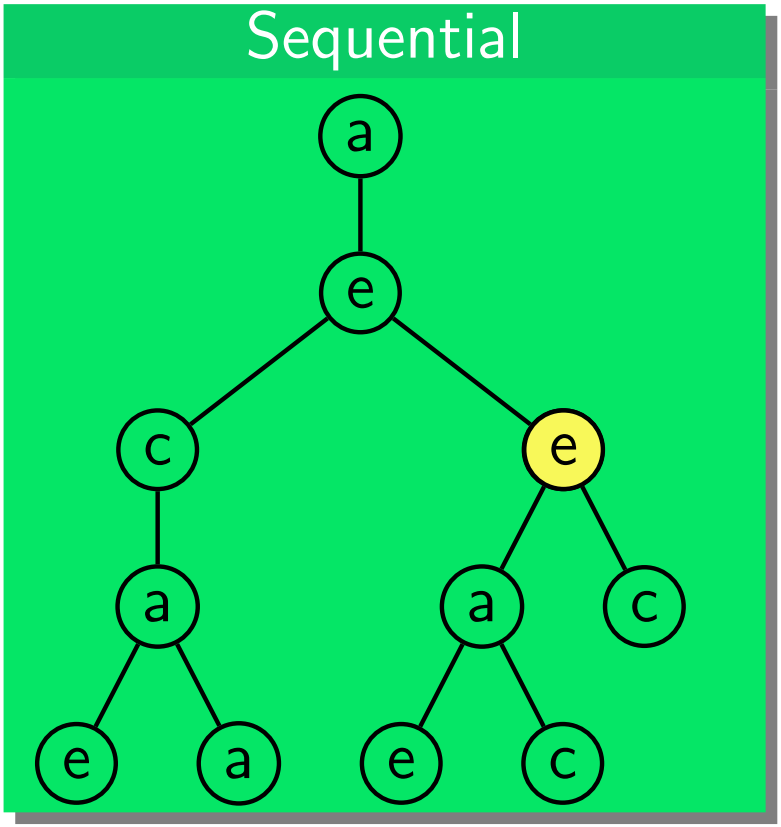

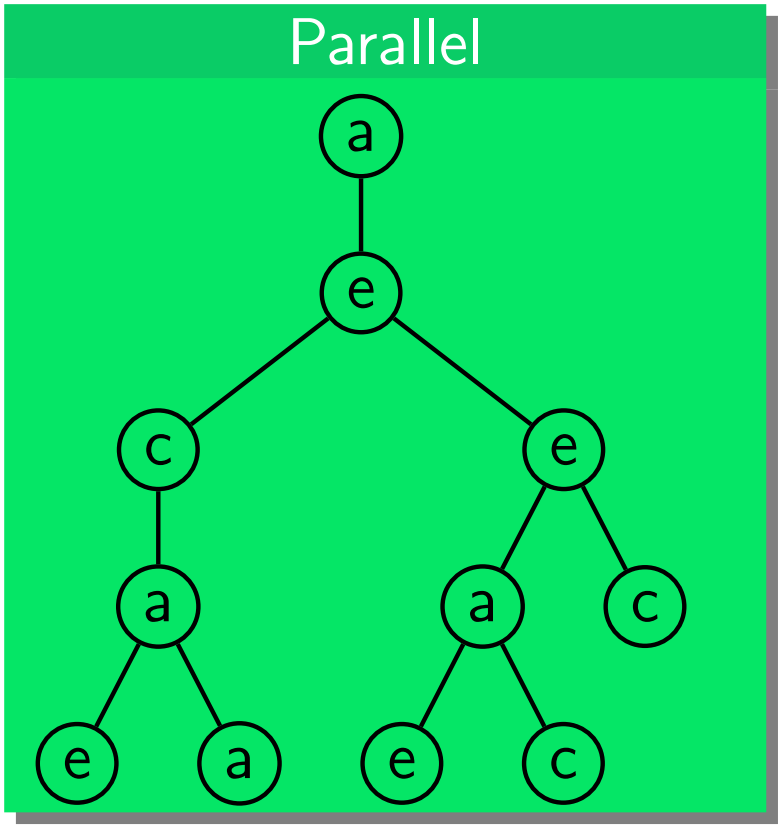

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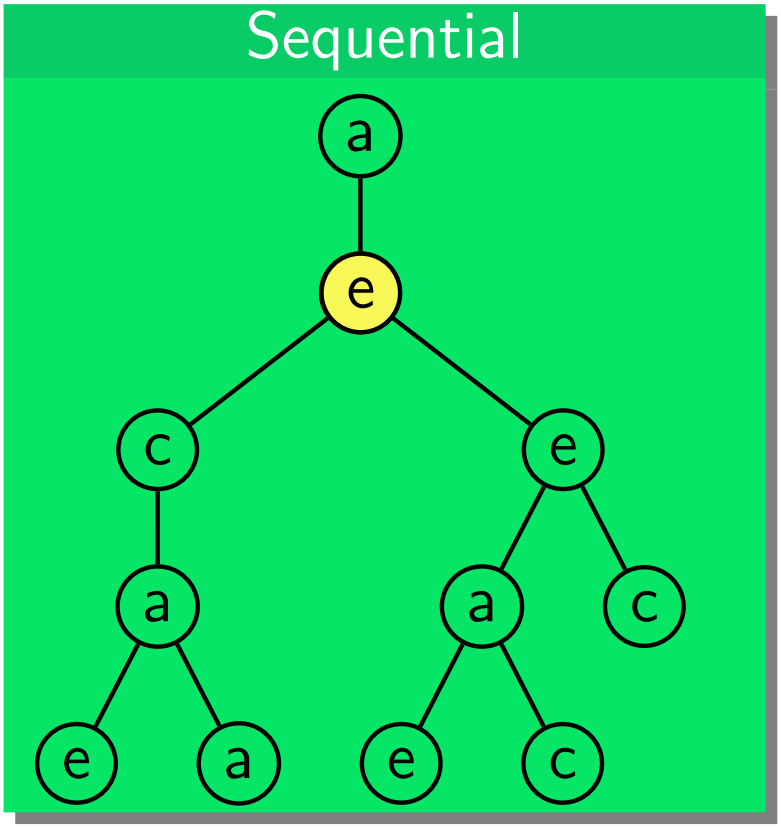

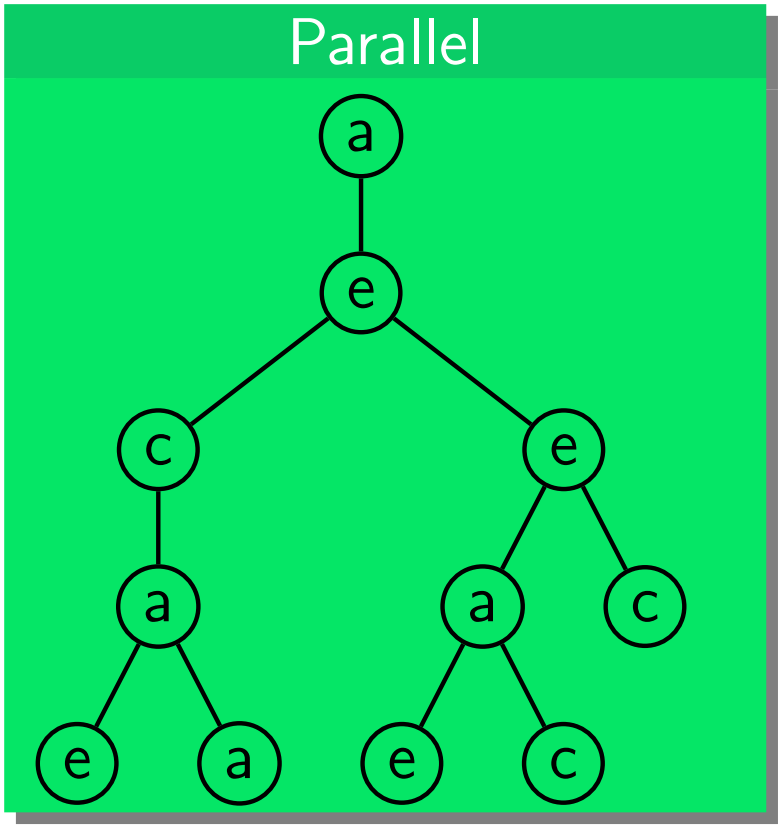

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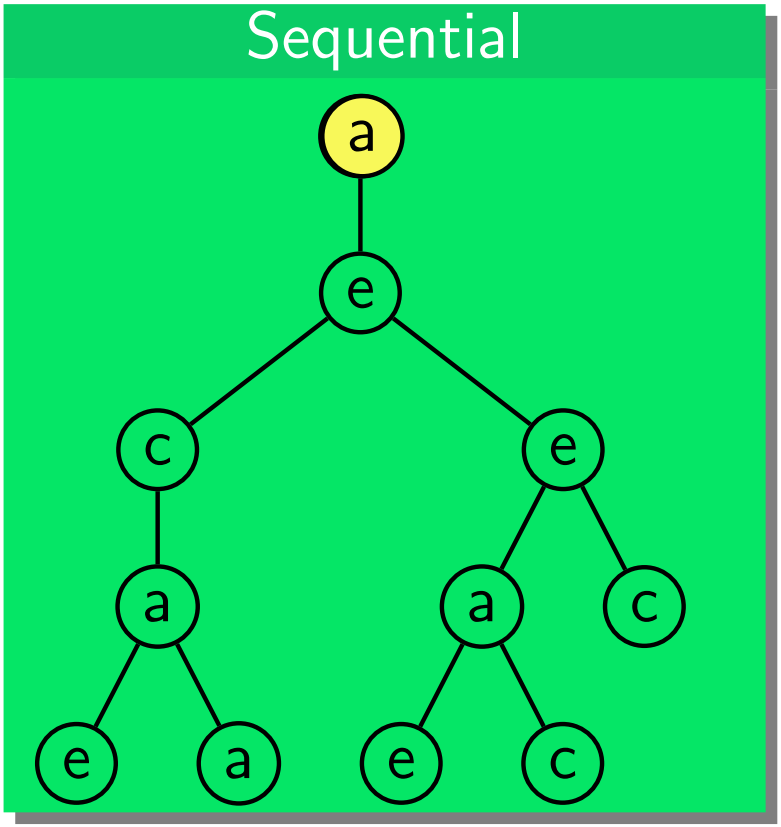

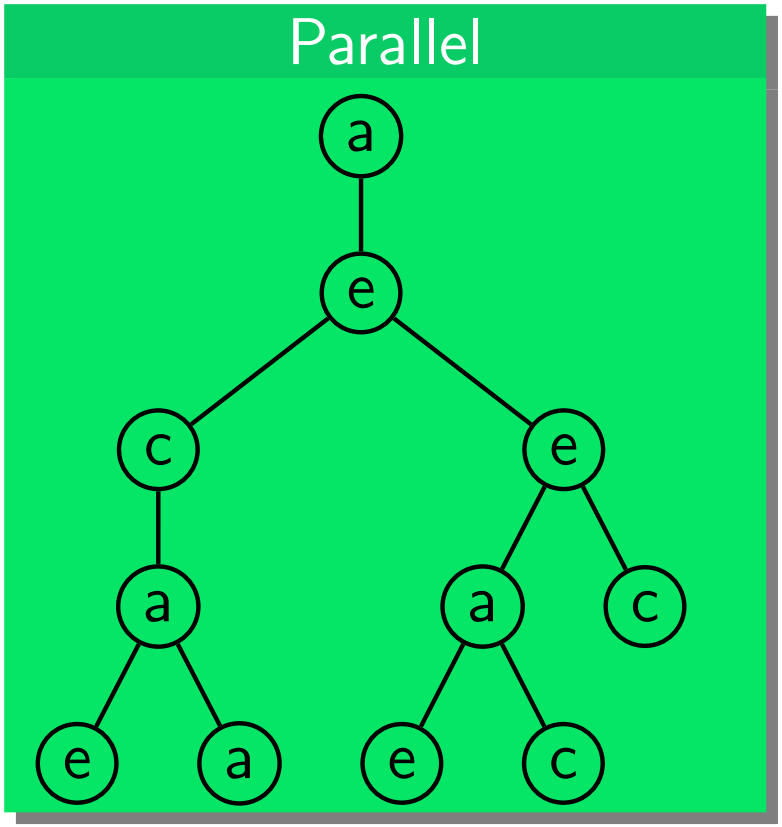

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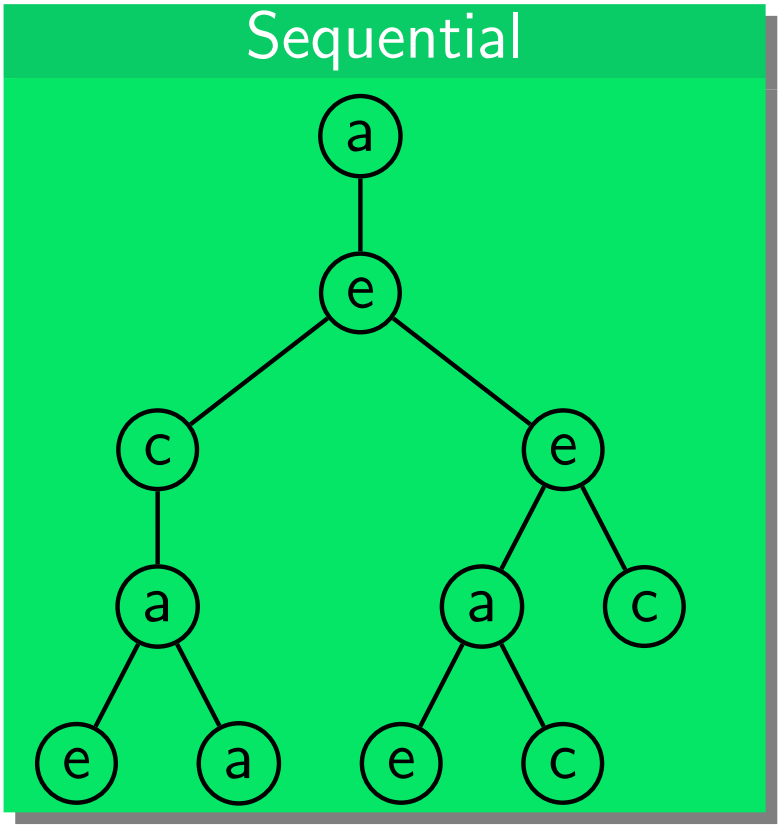

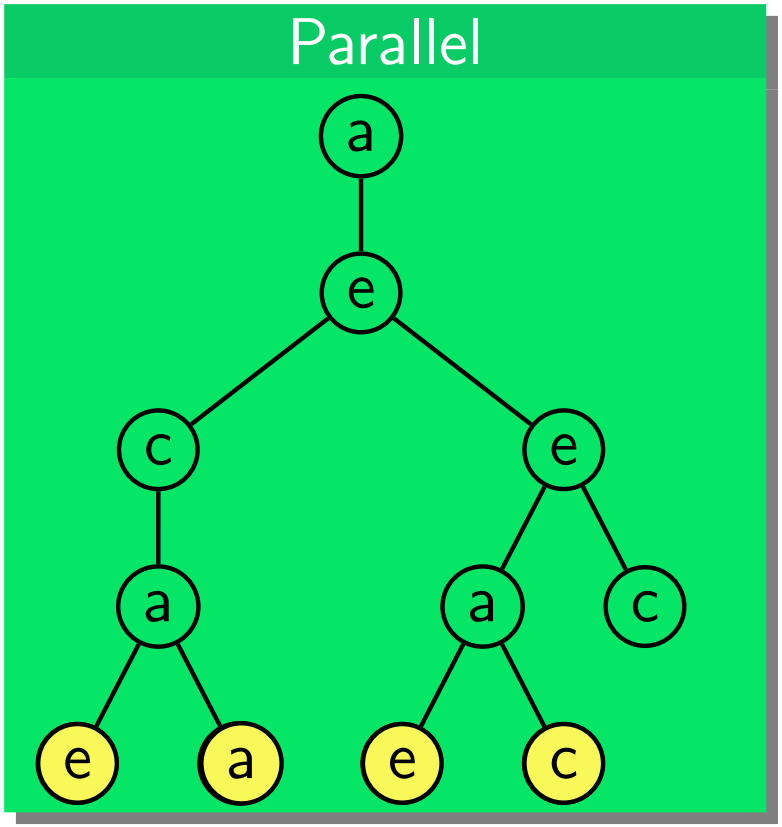

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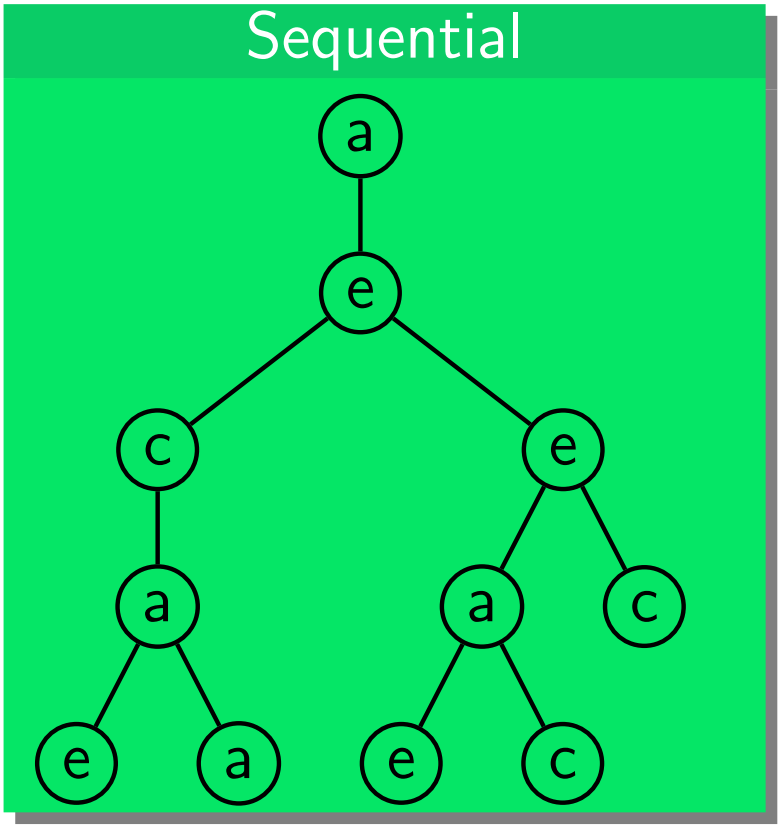

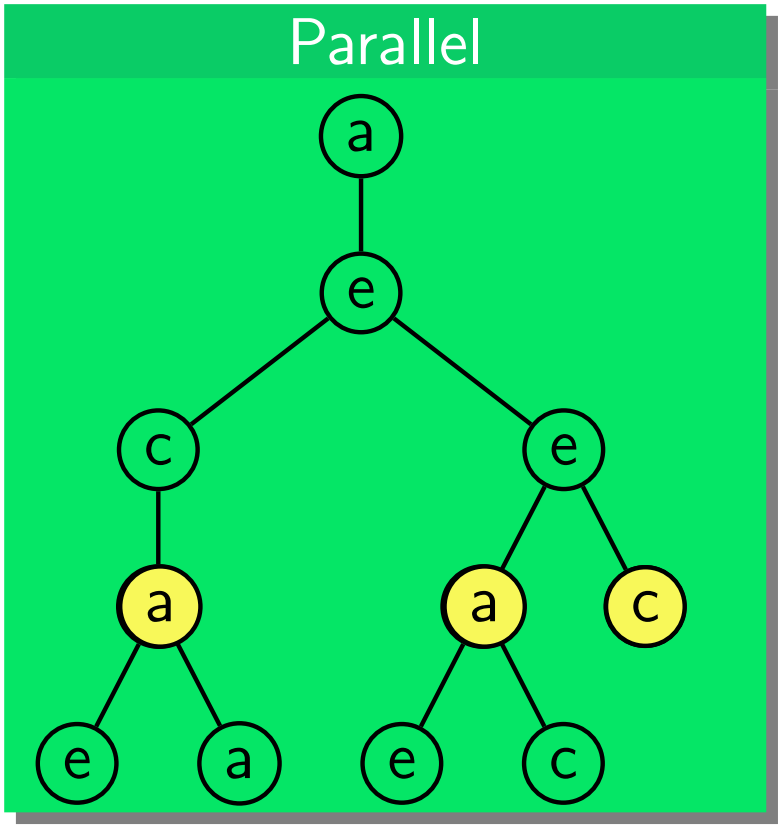

From String Automata to Tree Automata

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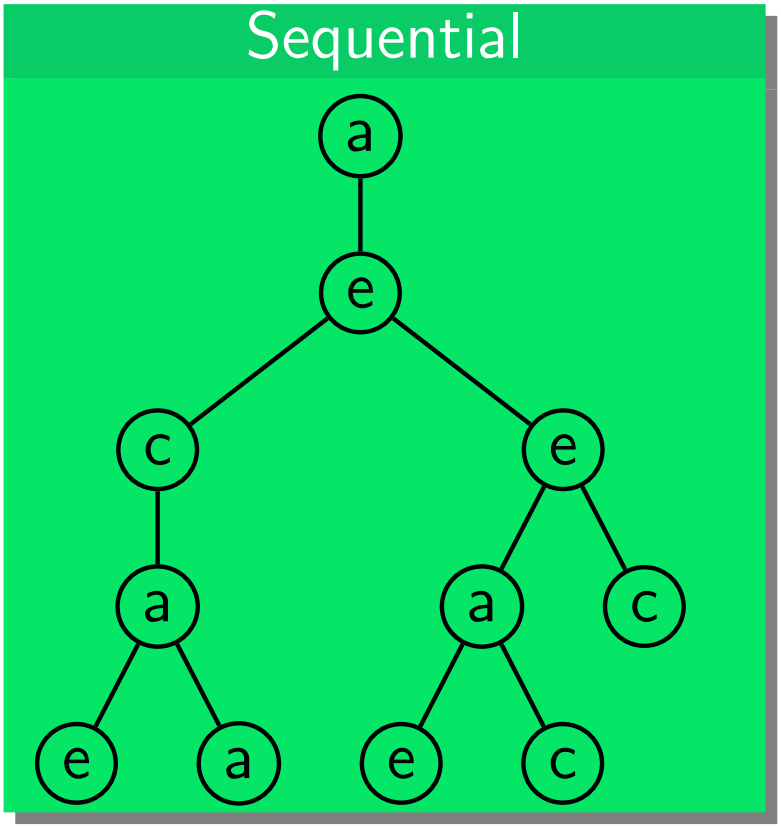

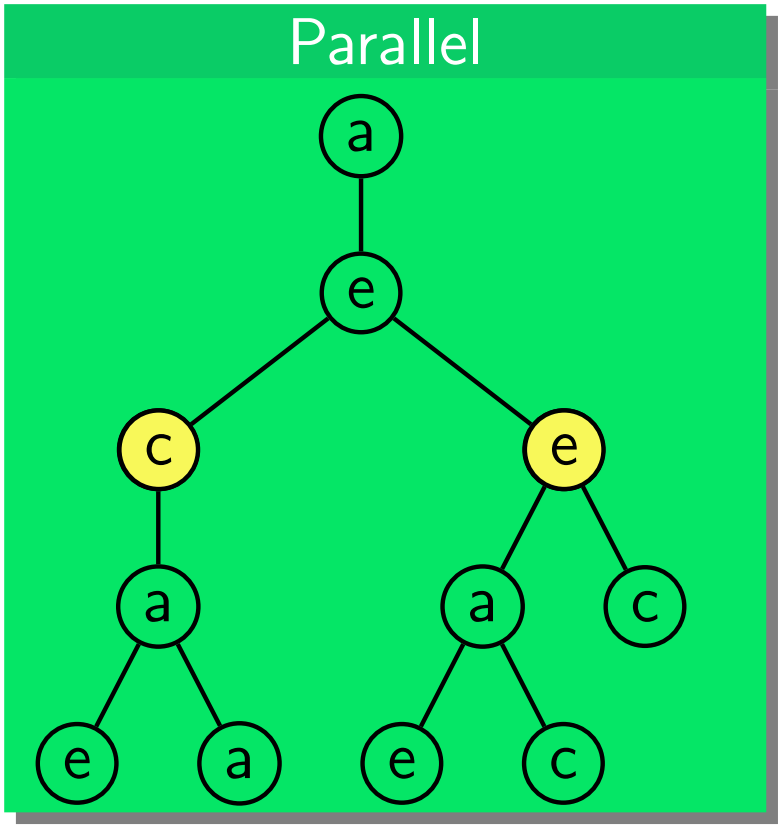

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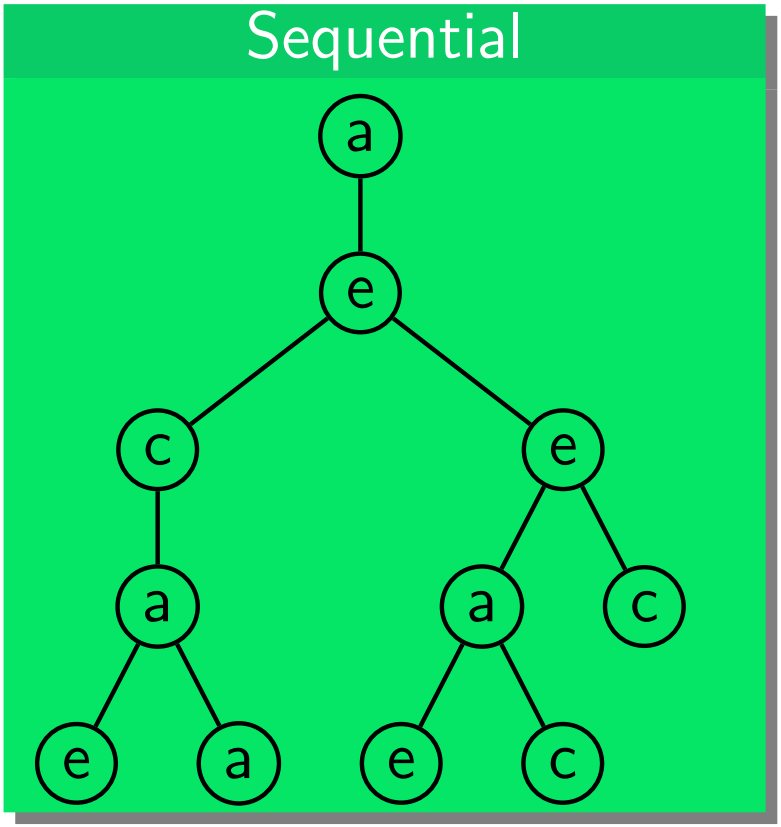

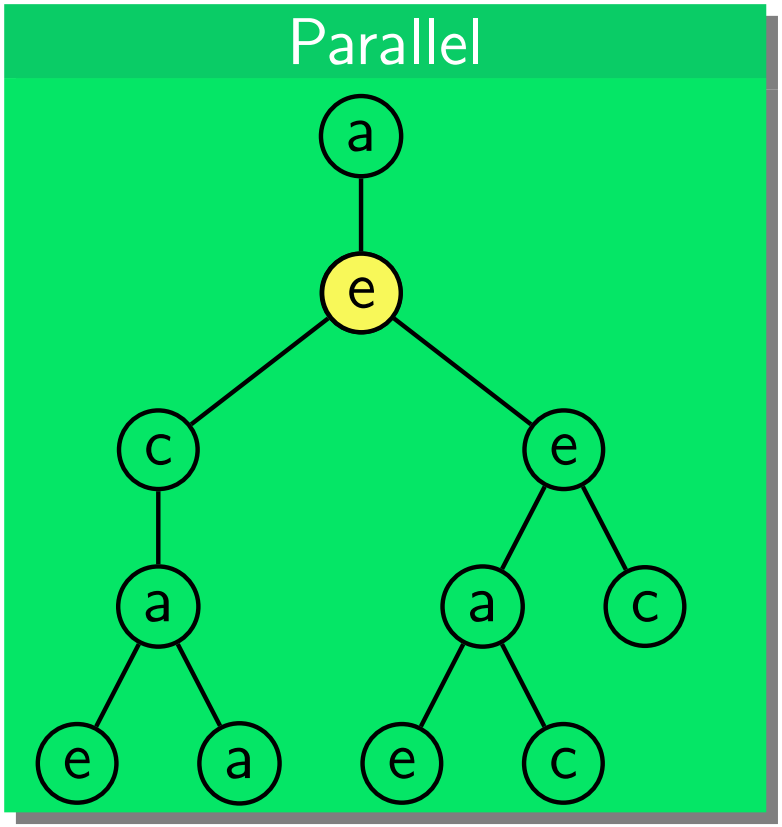

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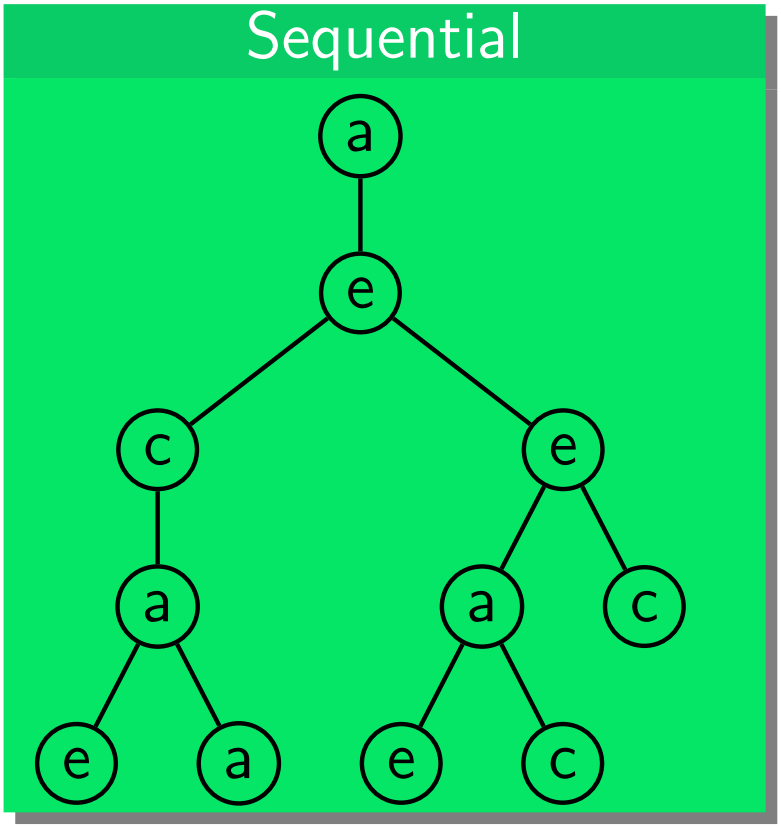
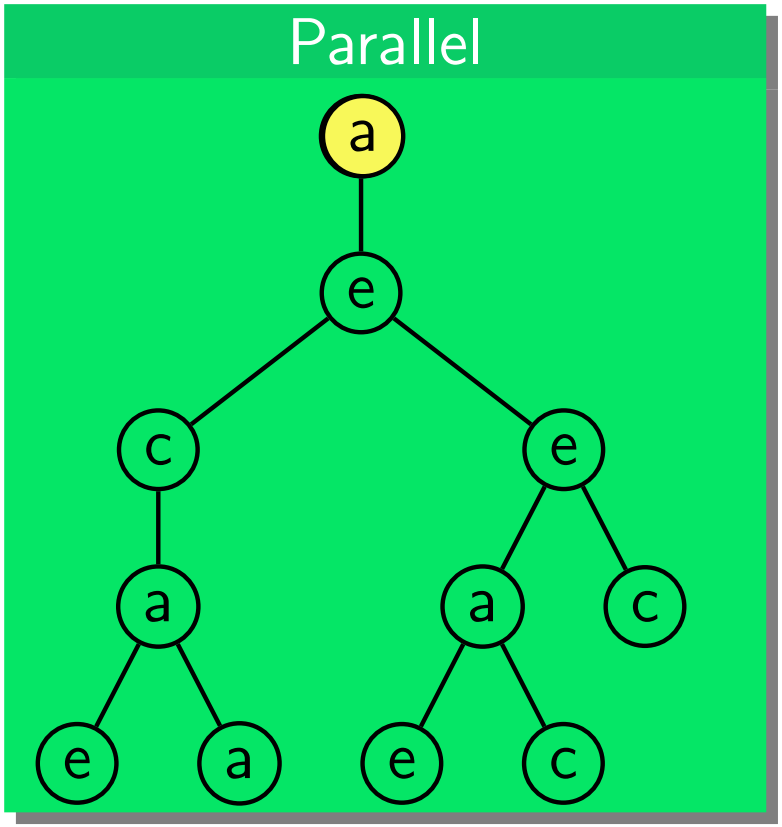
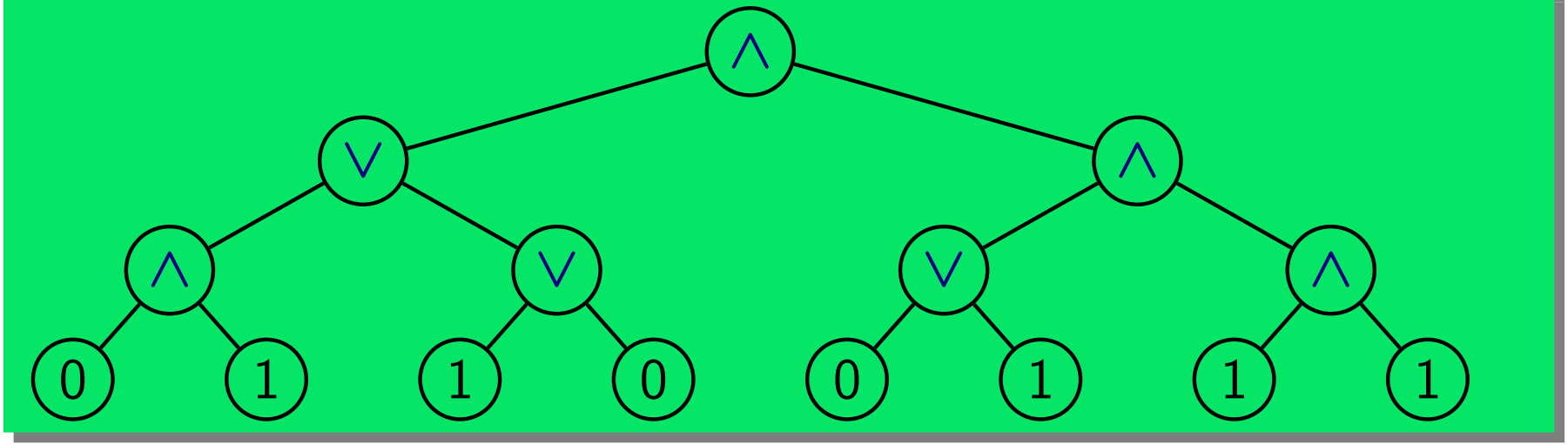




Bottom-Up Automata

Example: Tree-structured Boolean Circuits



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Accepting at the root: q_1

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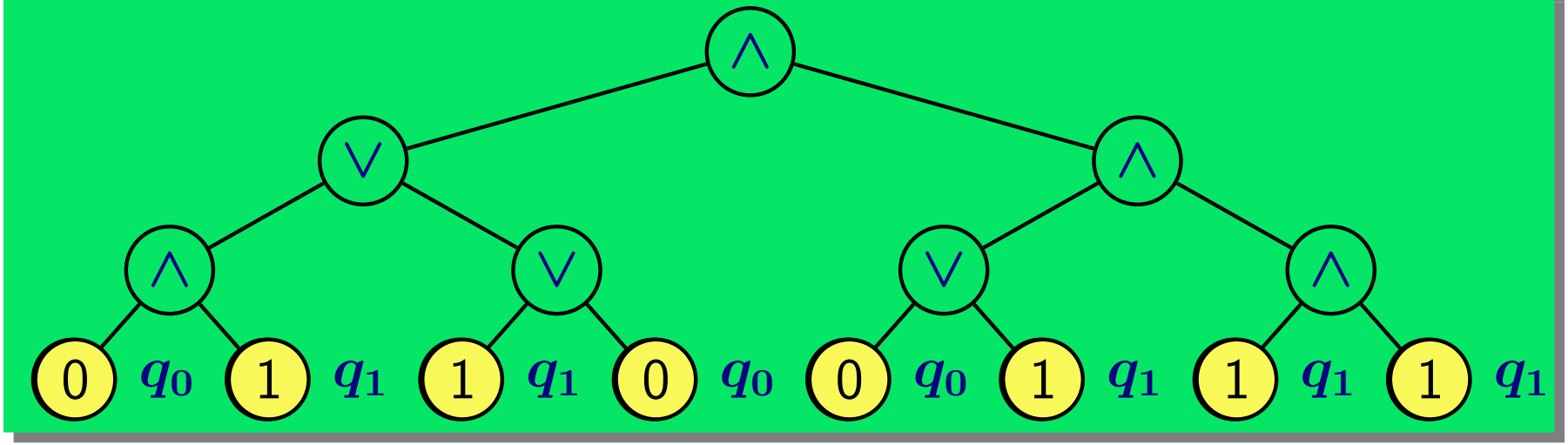

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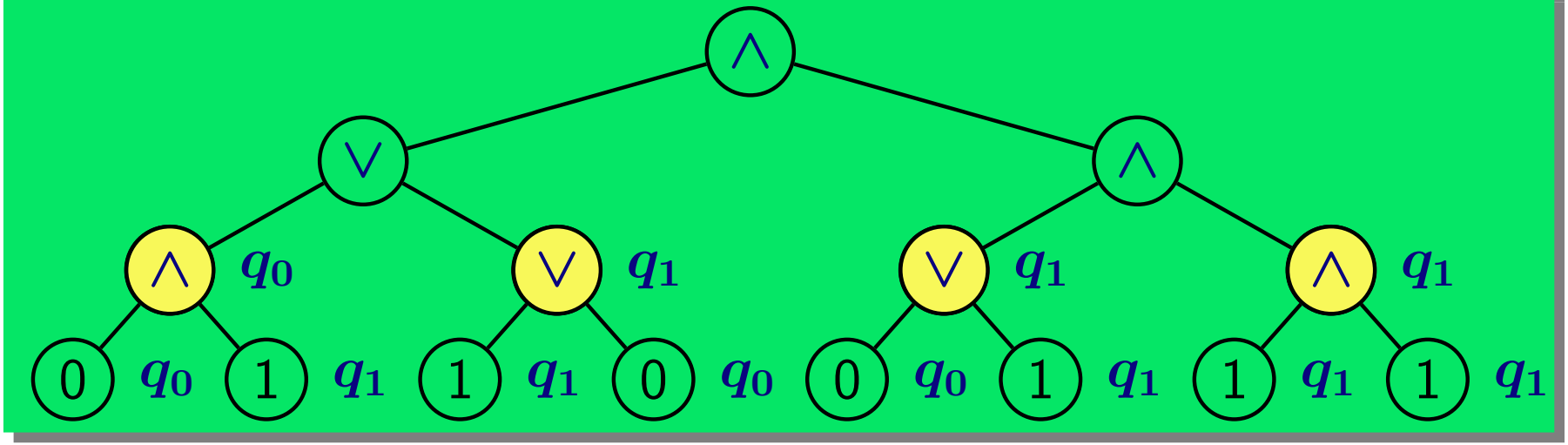

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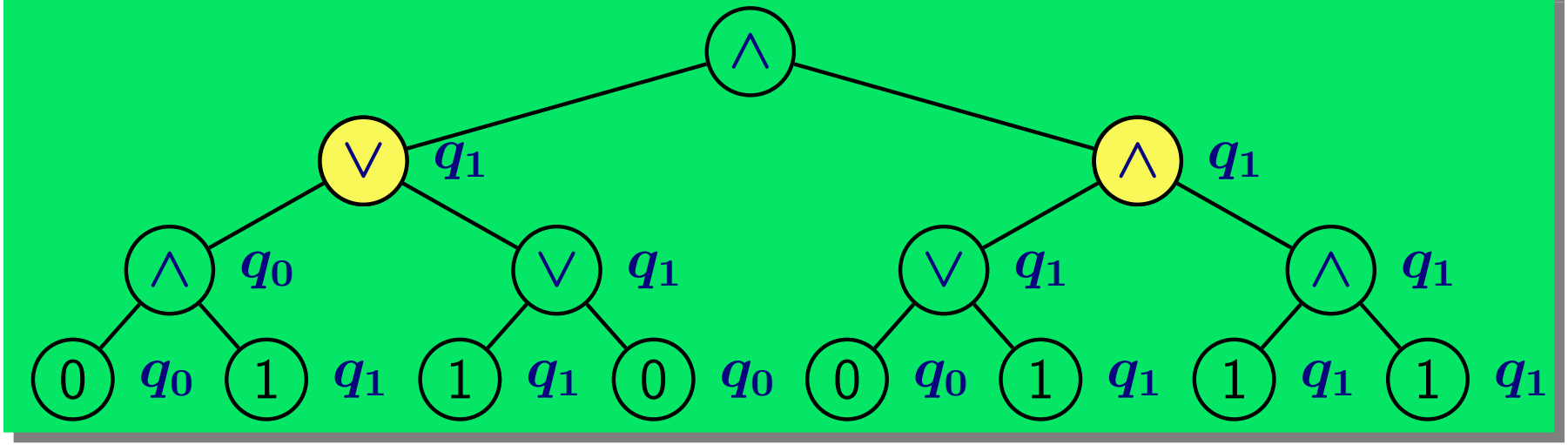

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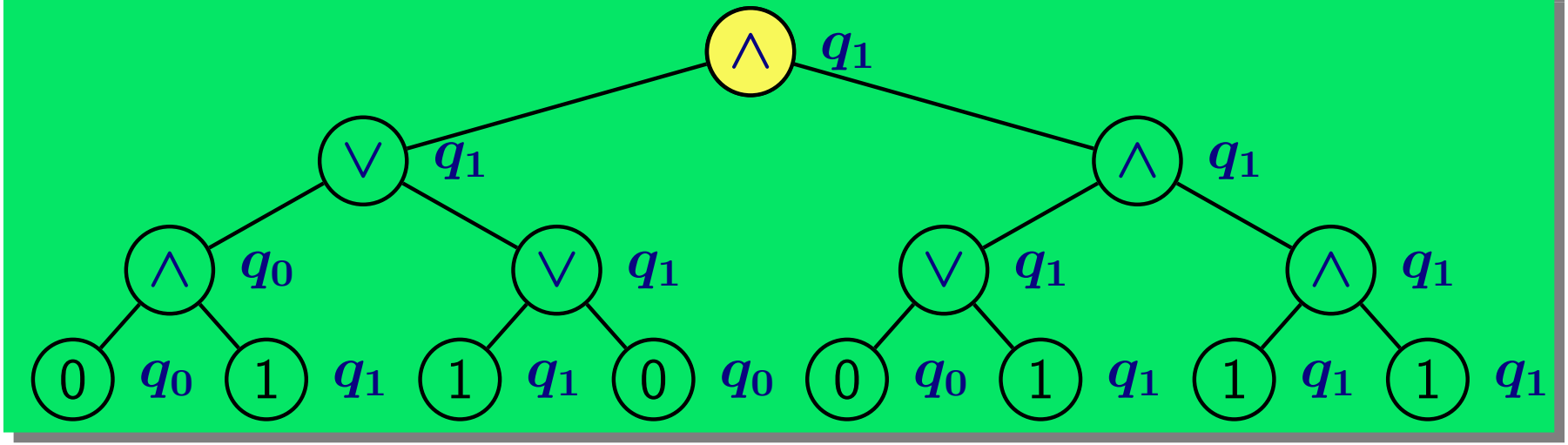

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Non-det. Top-Down Automata

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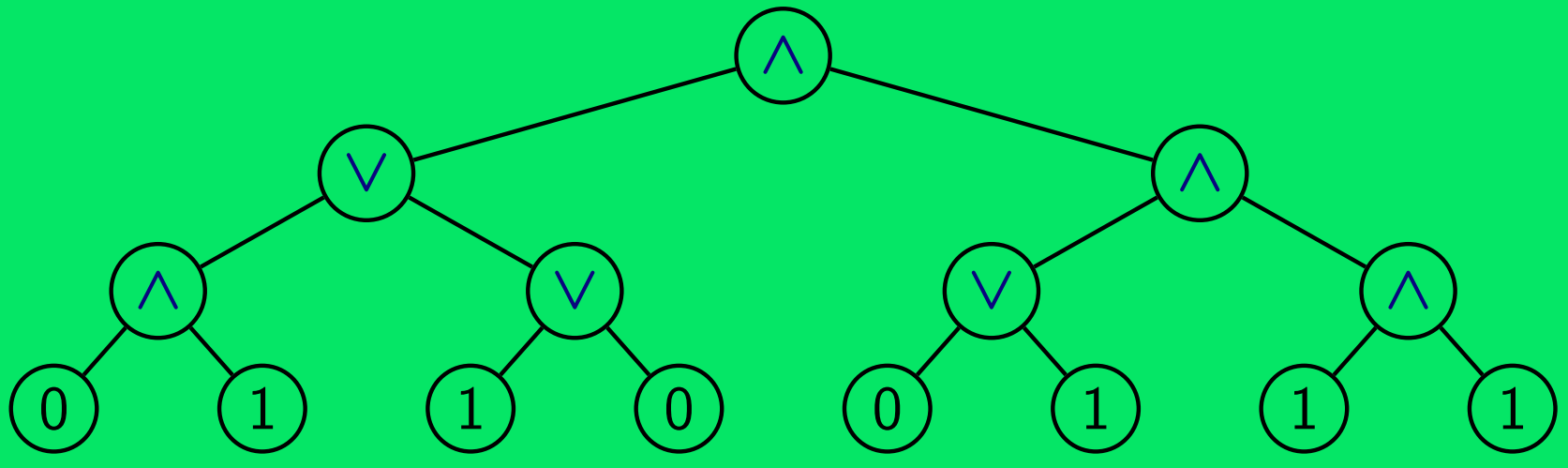

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Guess the correct values starting from the root

Check at the leaves

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Accepting if all leaves end in acc

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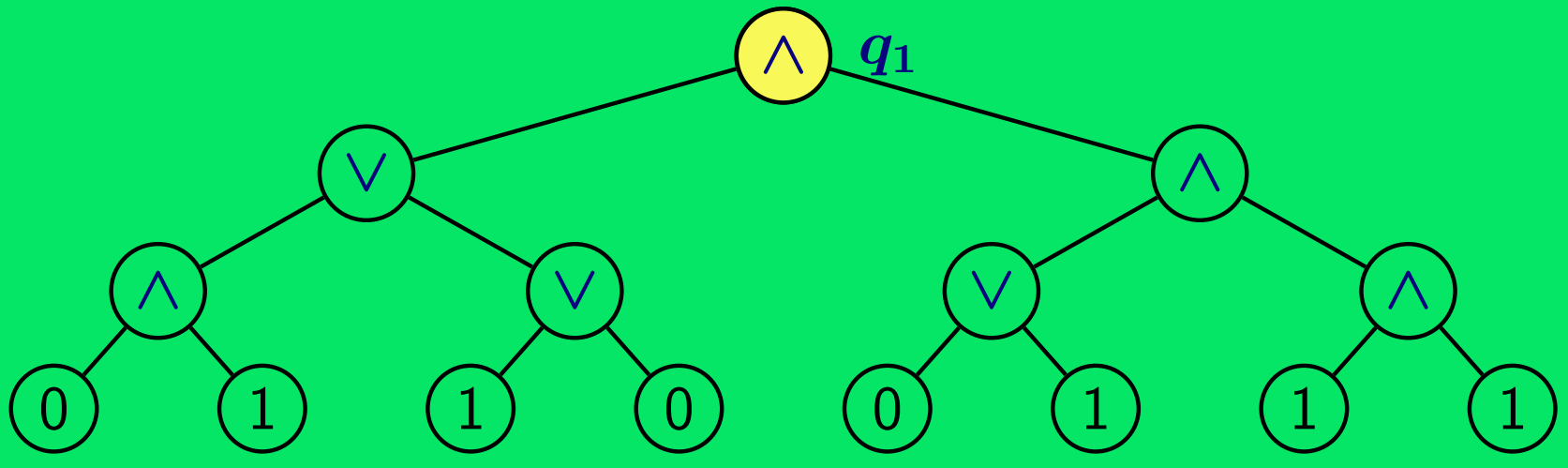

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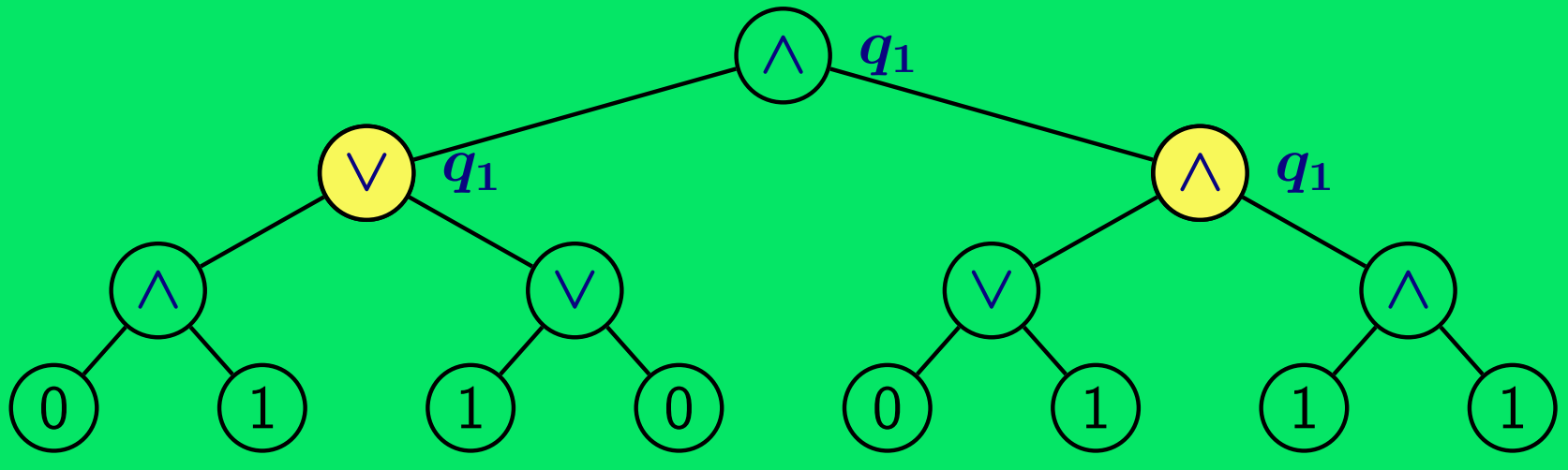

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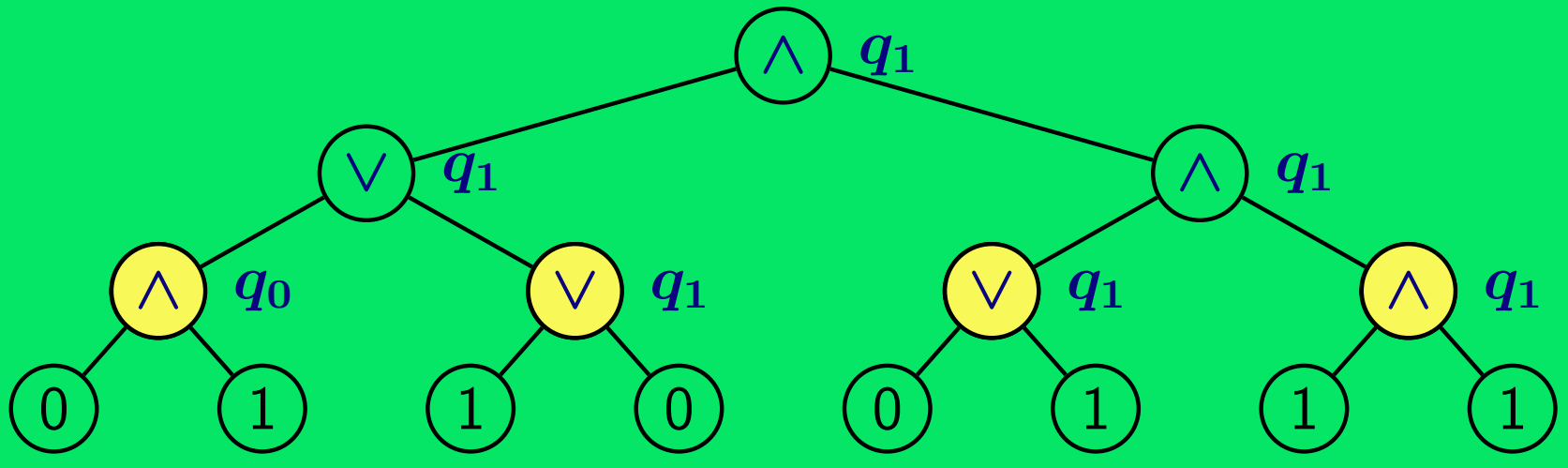

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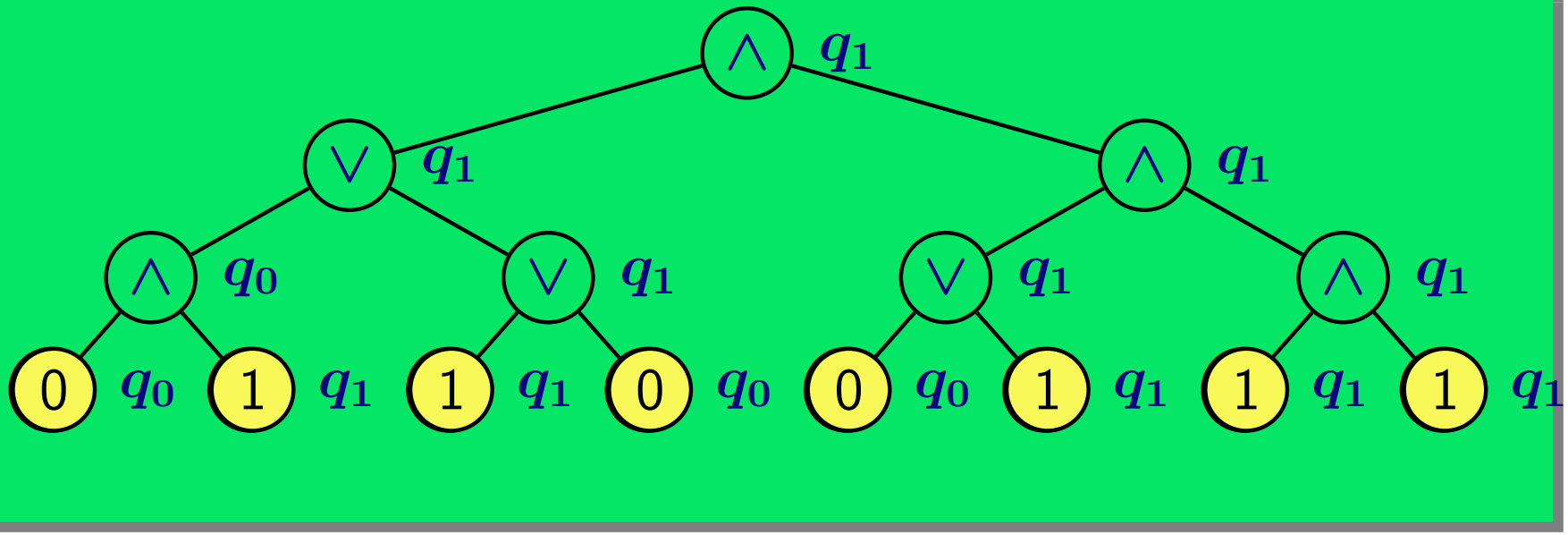

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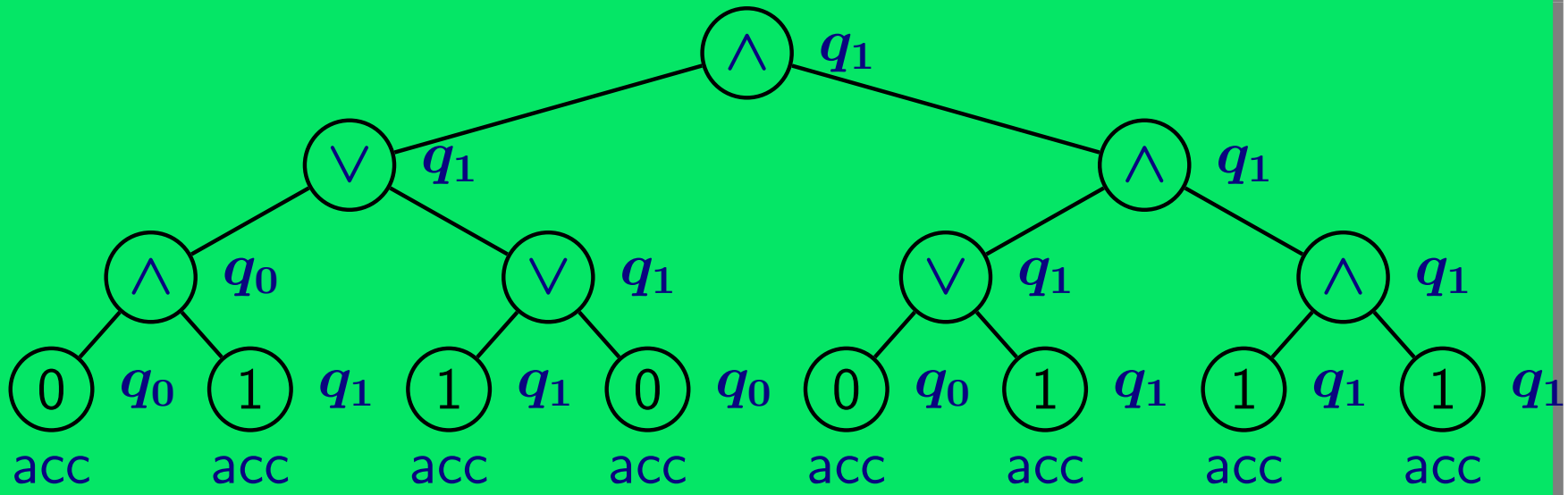

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Regular Tree Languages

Definition

A bottom-up automaton is **deterministic** if
for each a and $p \neq q$: $\delta(a, p) \cap \delta(a, q) = \emptyset$

Theorem

The following are equivalent for a tree language L:

(a) L is accepted by a nondeterministic bottom-up automaton

(b) L is accepted by a deterministic bottom-up automaton

(c) L is accepted by a nondeterministic top-down automaton

Proof idea

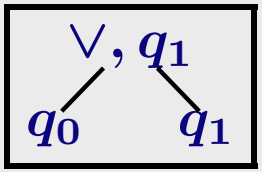
(a) $\Longrightarrow$ (b): Powerset construction

(a) $\Longleftrightarrow$ (c): Just the same thing, viewed in two different ways

Automata as Tiling Systems

Observation

- $(q_0, q_1) \in \delta(\vee, q_1)$ can be interpreted as an allowed pattern:



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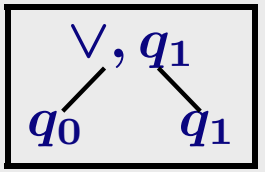

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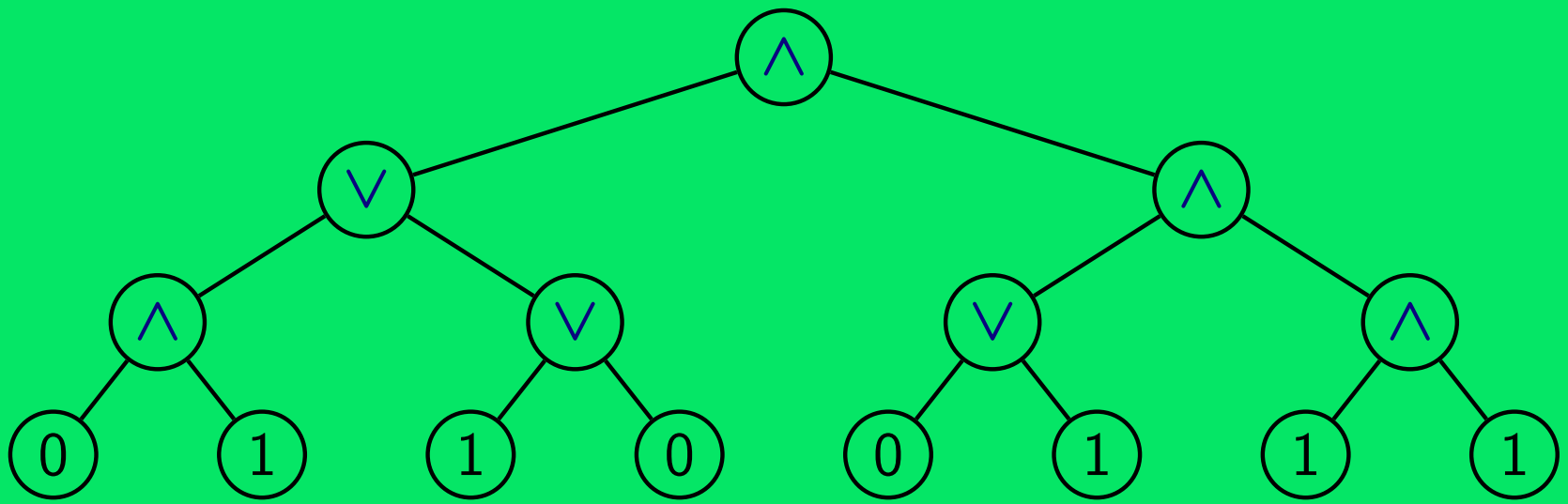

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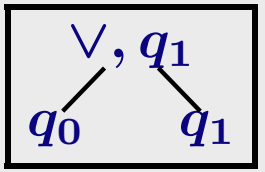

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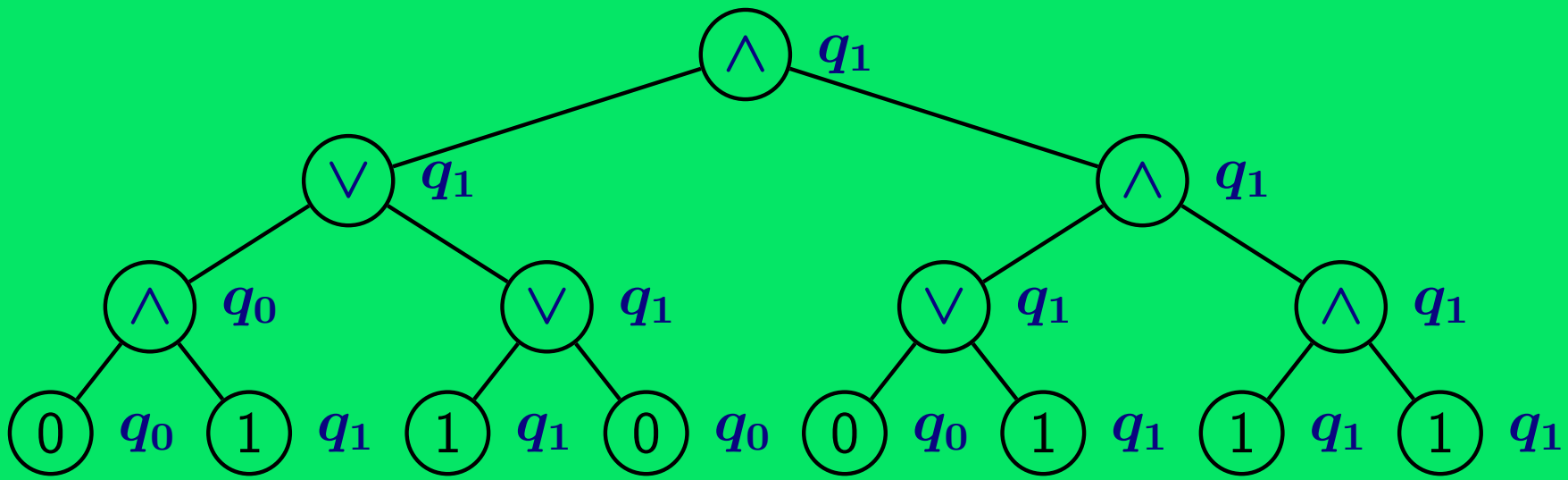


Regular tree languages and logic

Definition: (MSO logic)

- **Formulas** talk about
  - edges of the tree (E)
  - node labels ($Q_0, Q_1, Q_\wedge, Q_\vee$)
  - the root of the tree (root)
- **First-order-variables** represent nodes
- **Monadic second-order** (MSO) variables represent sets of nodes

Example: Boolean Circuits

$$\begin{aligned}\text{Boolean circuit true} \equiv\ & \exists X\ X(\text{root}) \wedge\ \forall x \\ & (Q_0(x) \rightarrow \neg X(x)) \wedge \\ & ((Q_\wedge(x) \wedge X(x)) \rightarrow (\forall y[E(x,y) \rightarrow X(y)])) \wedge \\ & ((Q_\vee(x) \wedge X(x)) \rightarrow (\exists y[E(x,y) \wedge X(y)]))\end{aligned}$$

Theorem [Doner 70; Thatcher, Wright 68]

MSO $\equiv$ Regular Tree Languages

Regular tree languages and logic (cont.)

Theorem

MSO $\equiv$ Regular Tree Languages

Proof idea

Automata $\Rightarrow$ **MSO**:

Formula expresses that there exists a correct tiling

MSO $\Rightarrow$ **Automata**: more involved

Basic idea:

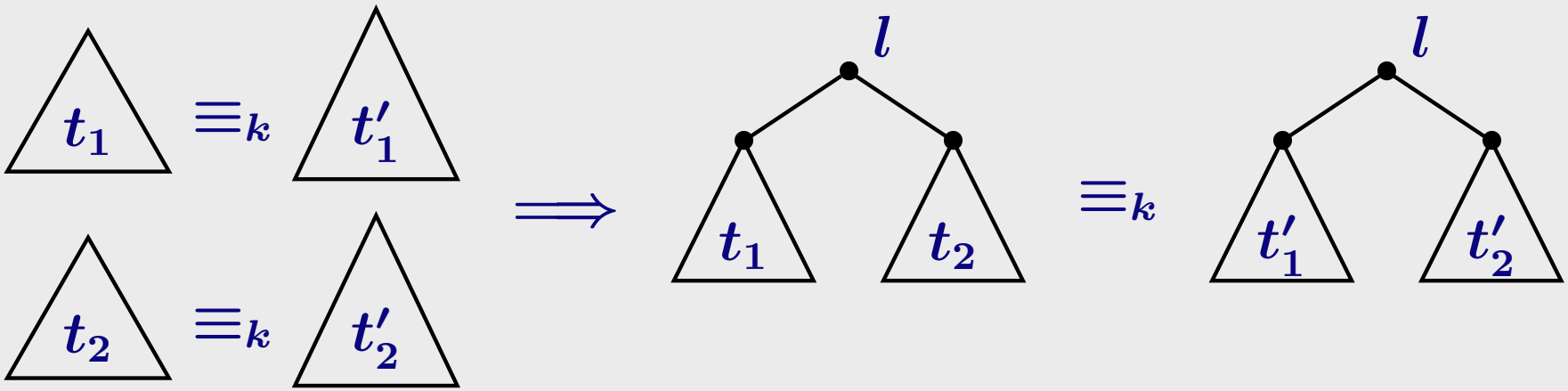
Automaton computes for each node v the set of formulas which hold in the subtree rooted at v

Regular tree languages and logic (cont.)

Formula $\Rightarrow$ automaton

- Let φ be an MSO-formula, $k :=$ quantifier-depth of φ
- **k-type** of a tree $t :=$ (essentially)
  set of MSO-formulas ψ of quantifier-depth $\leq k$ which hold in t
- **$t_1 \equiv_k t_2$**: k-type$(t_1) = k$-type(t_2)
- Automaton computes k-type of tree and concludes whether φ holds

Crucial fact



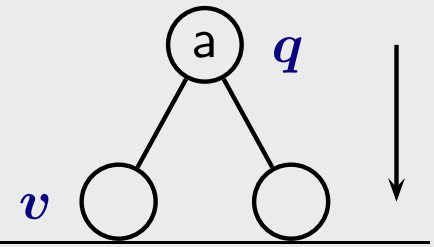
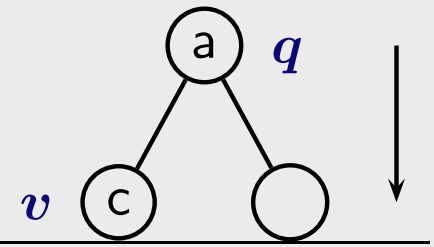
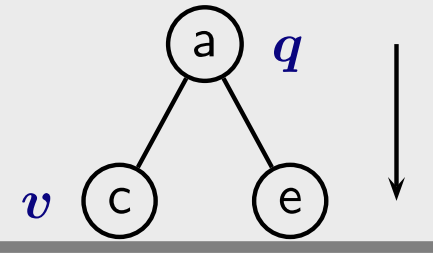
Det. Top-Down Automata

Question

What is the right notion for deterministic top-down automata?

3 Possibilities

State at a node v might depend on

state and symbol of parent	
state and symbol of parent and symbol of v	
state and symbol of parent and symbols at v and its sibling	

Det. Top-Down Automata: Acceptance

Question

What is a good acceptance mechanism for deterministic top-down automata?

Several possibilitites

(1) At all leaves states have to be accepting

(2) There is a leave with an accepting state

(2) is problematic for complement and intersection

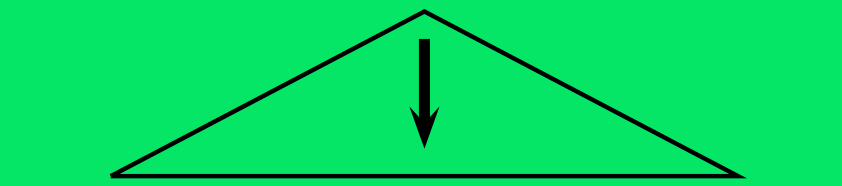
(1) is problematic for complement and union

Det. Top-Down Automata: Acceptance (cont.)

Definition: (Root-to-frontier automata with regular acceptance condition)

- Tree automata $\mathcal{A}$ are equipped with an additional regular string language L over $Q \times \Sigma$
- $\mathcal{A}$ accepts t if the (state,symbol)-string at the leaves (from left to right) is in L [Jurvanen, Potthoff, Thomas 93]

Illustration



A robust class

- The resulting class is closed under Boolean operations
- Good algorithmic properties
- Does not capture all regular tree languages

Summary

Regular tree languages

- Regular tree languages are a robust class
- Characterized by
  - parallel tree automata
  - MSO logic
  - several other models
- They are the natural analog of regular string languages
- Deterministic top-down automata with regular acceptance conditions define a weaker but nevertheless robust class

Contents

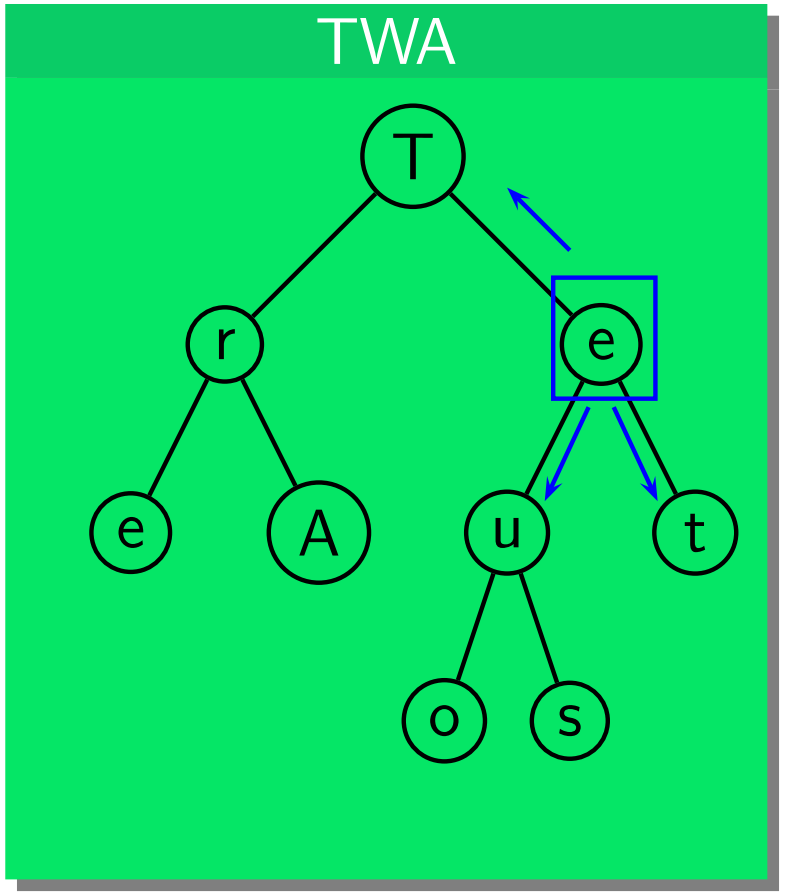
Tree-Walk Automata

Definition: (Tree-walk automata)

Depending on

- current state
- symbol of current node
- position of current node wrt its siblings

the automaton moves to a neighbor and takes a new state



Question

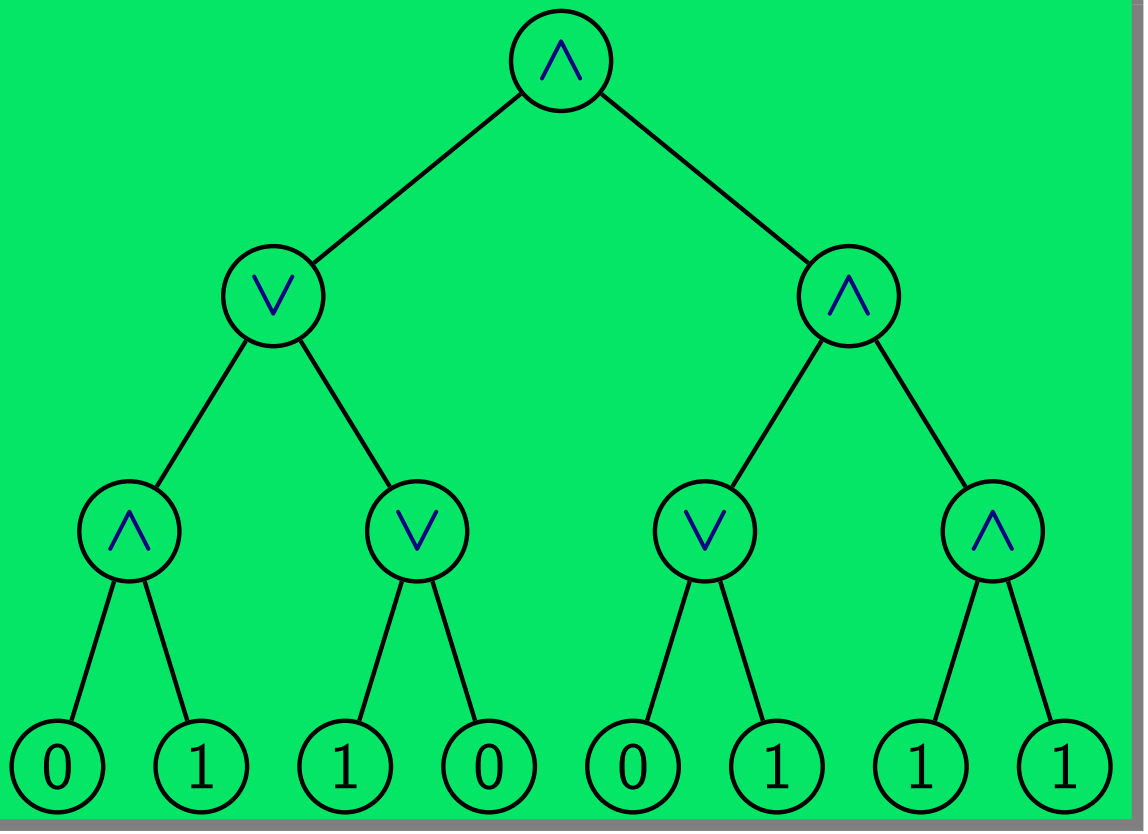
What is the expressive power of tree-walk automata?

Tree-Walk Automata (cont.)

Fact

- Tree-walk automata can evaluate Boolean circuit trees
- 5 states

Example



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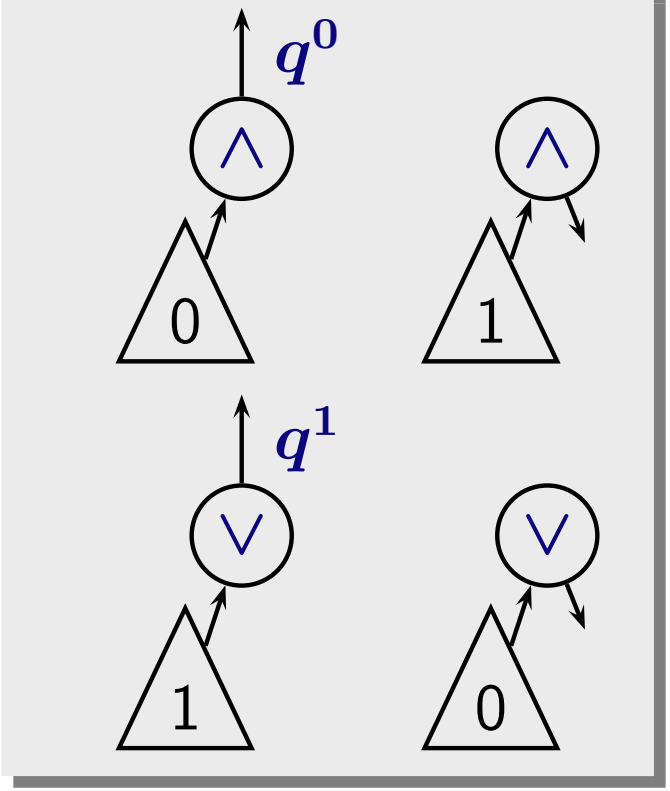

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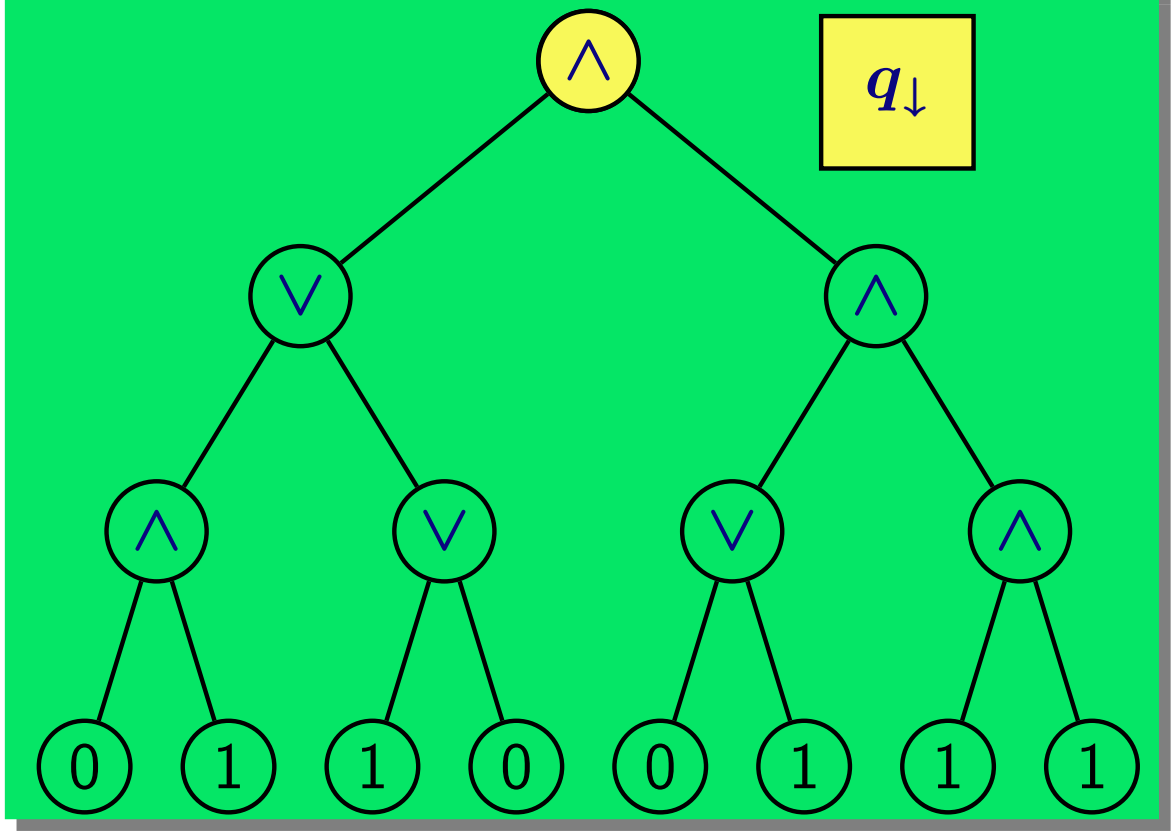

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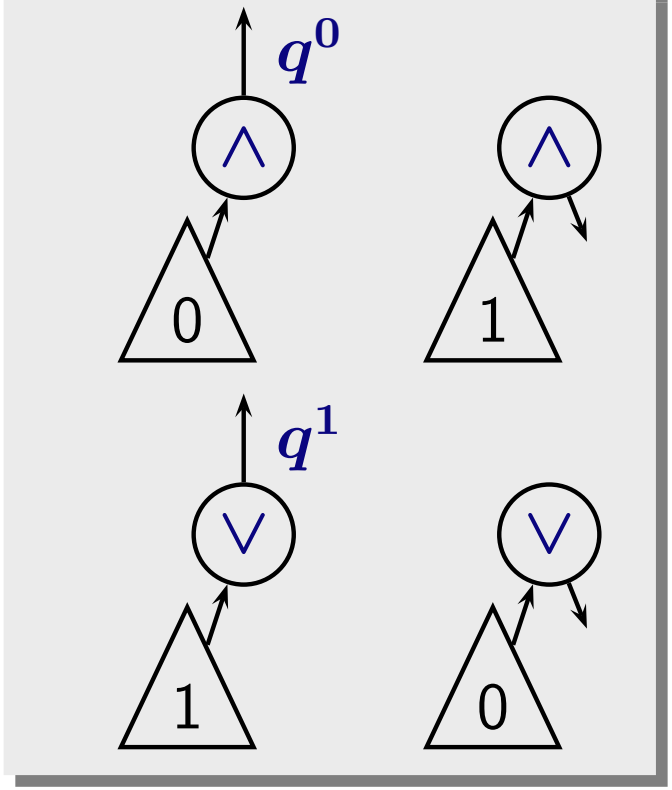

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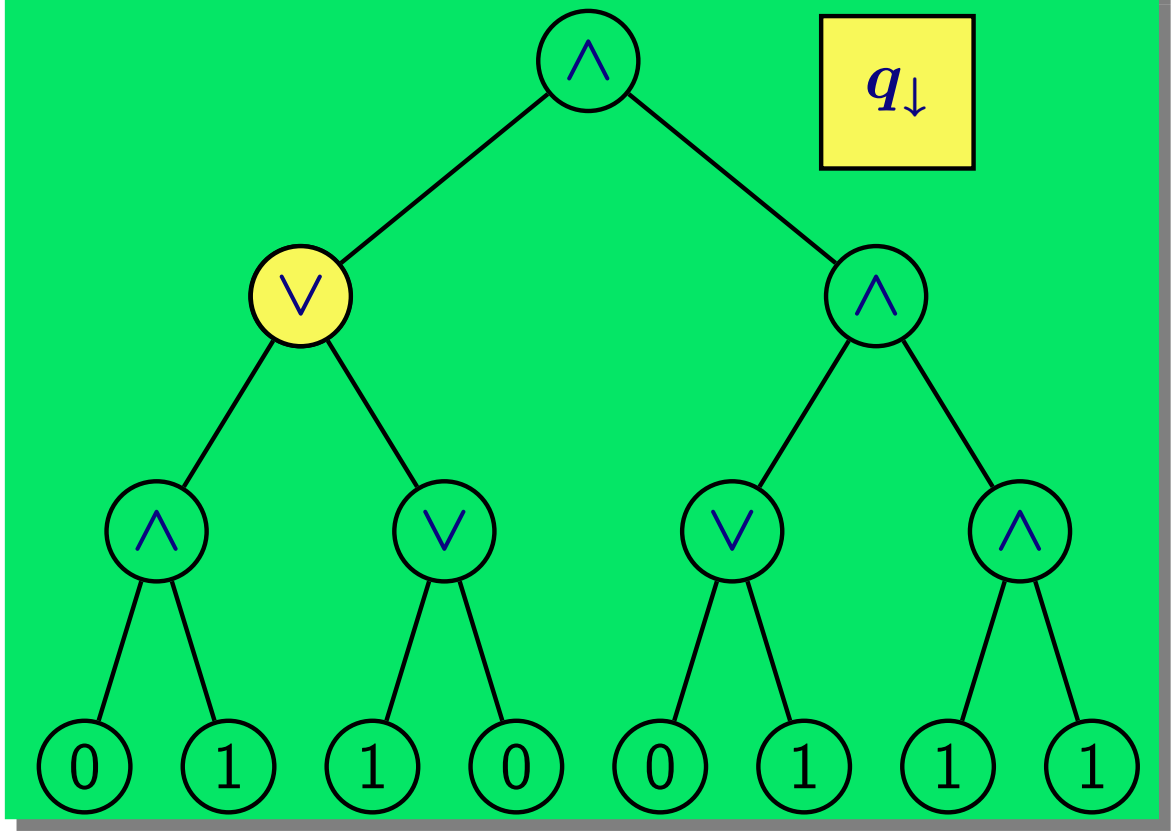

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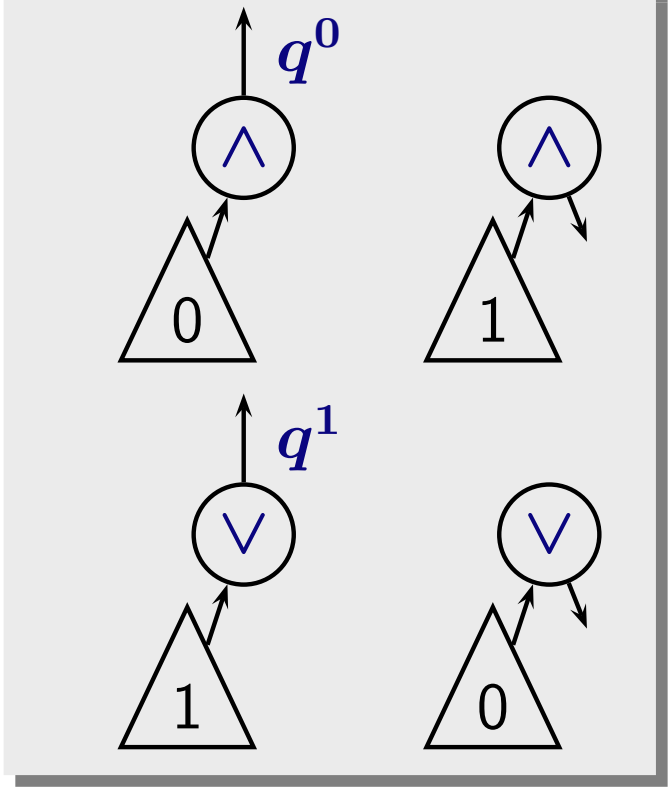

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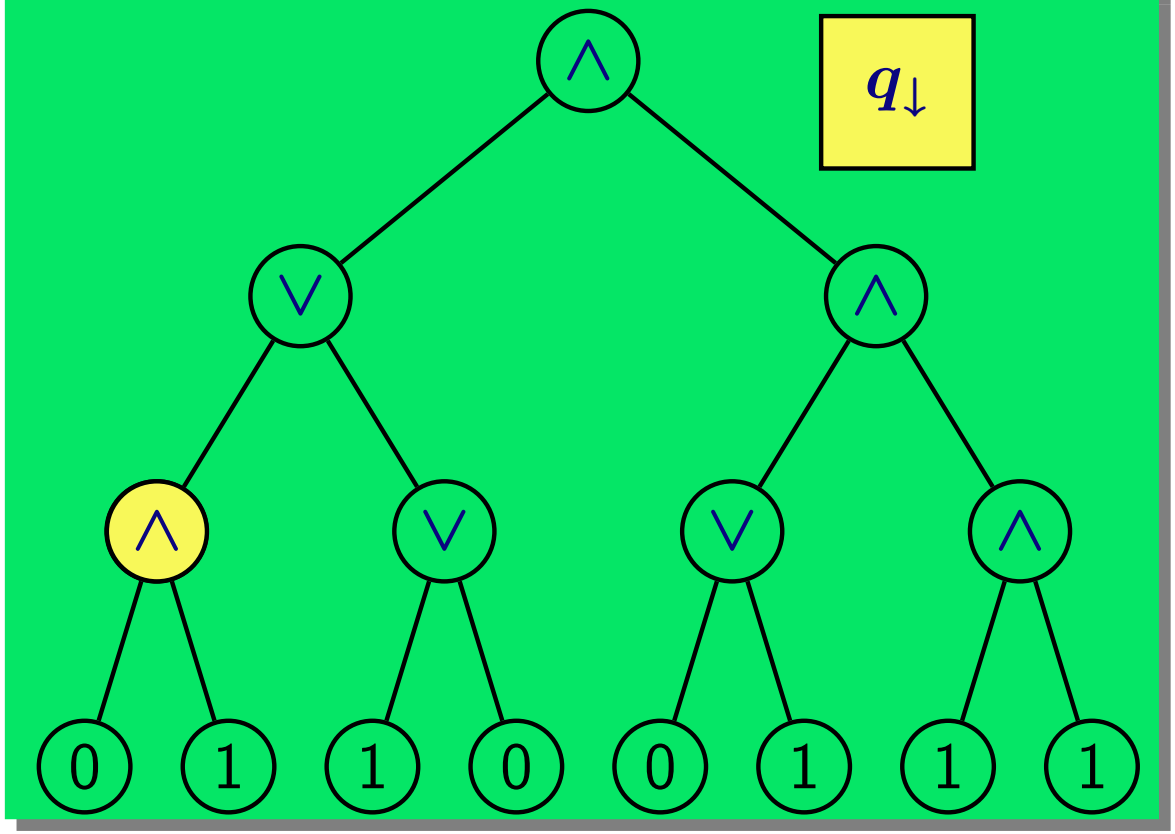

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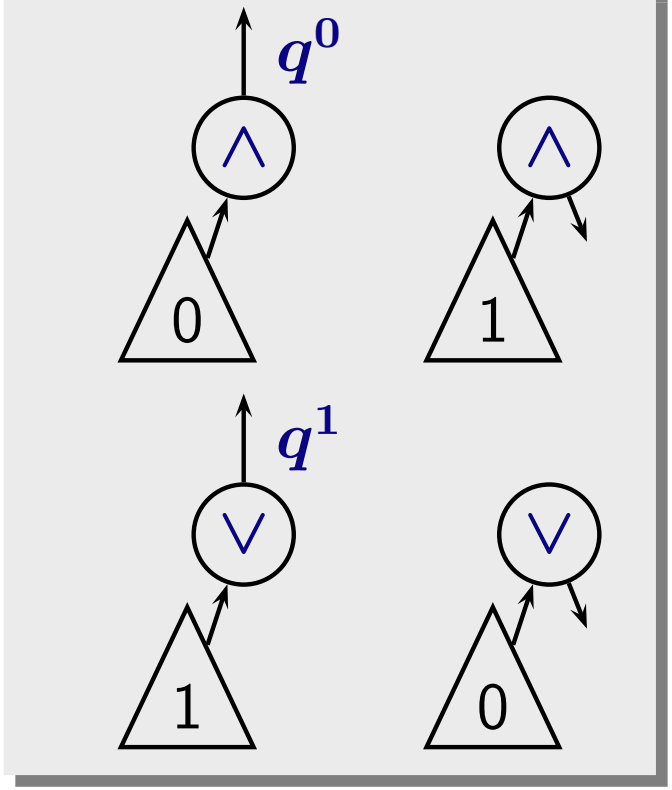

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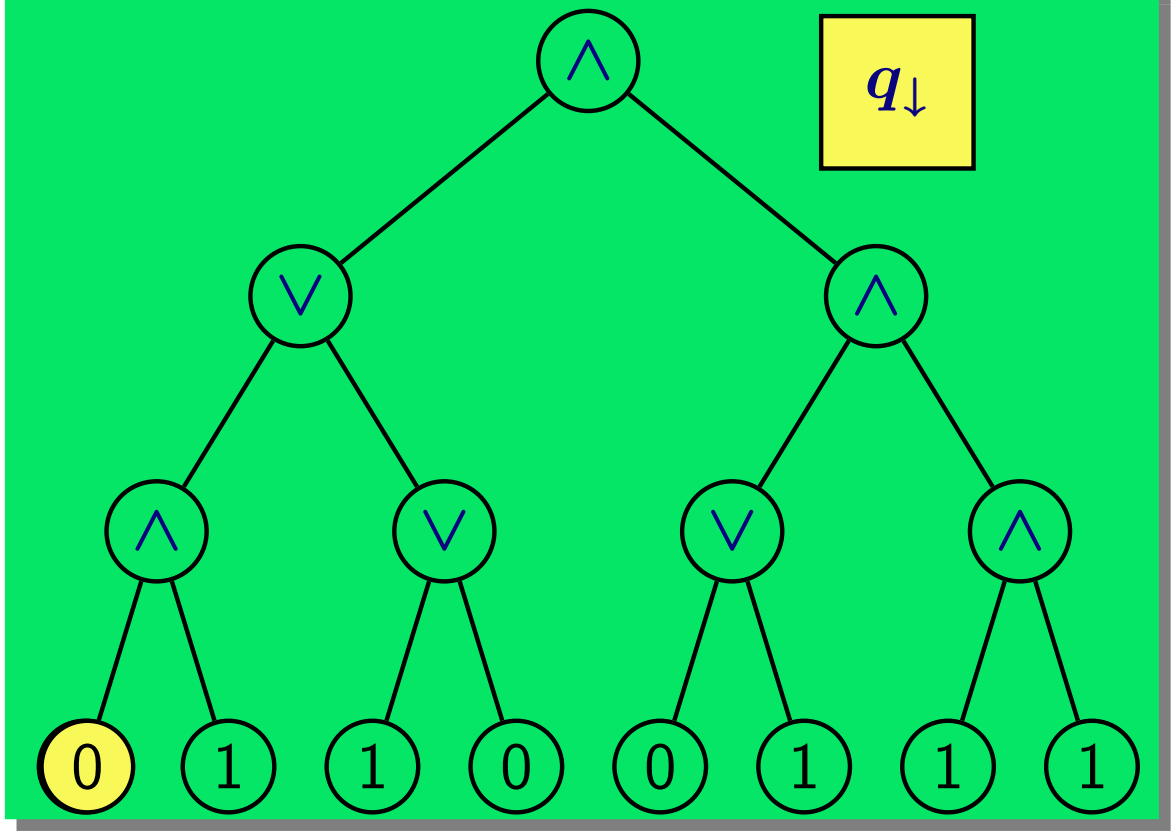

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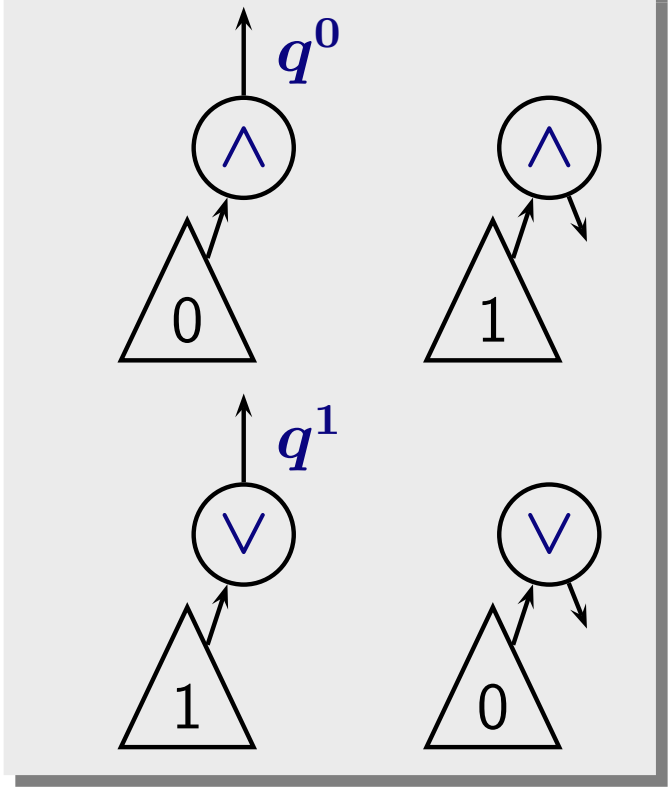

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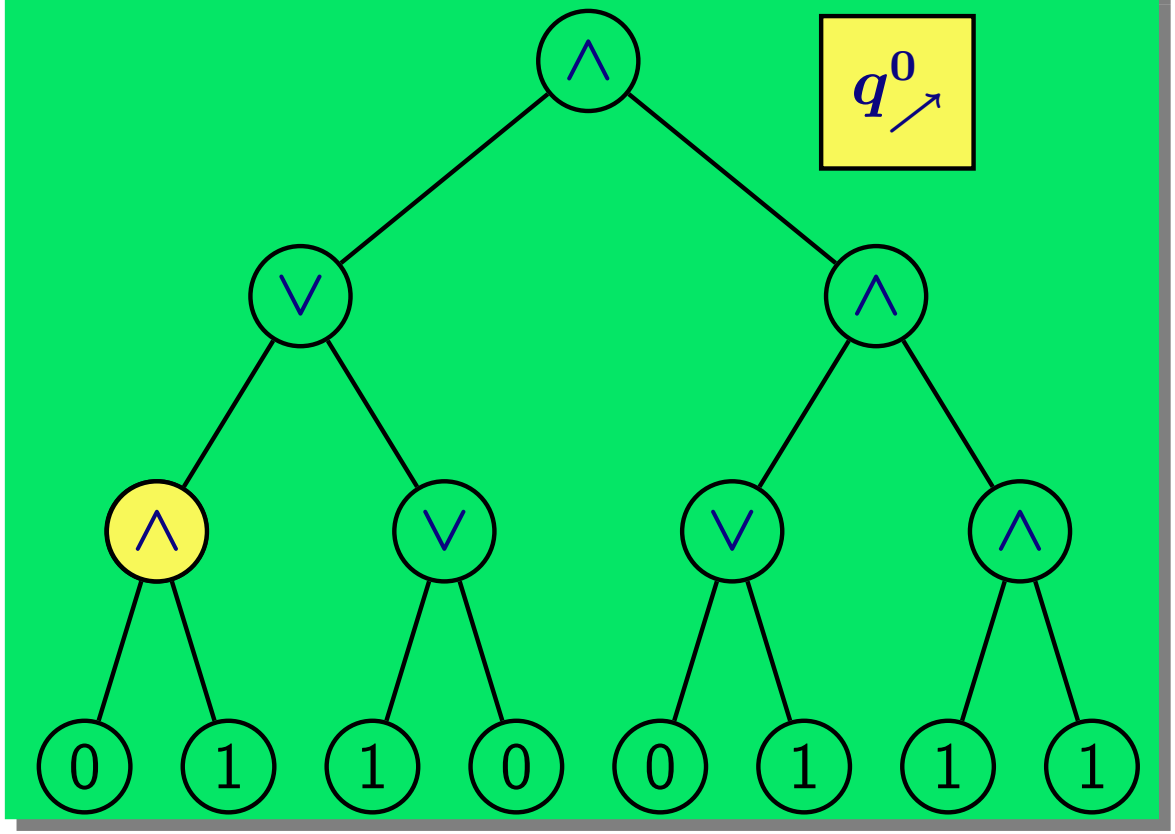

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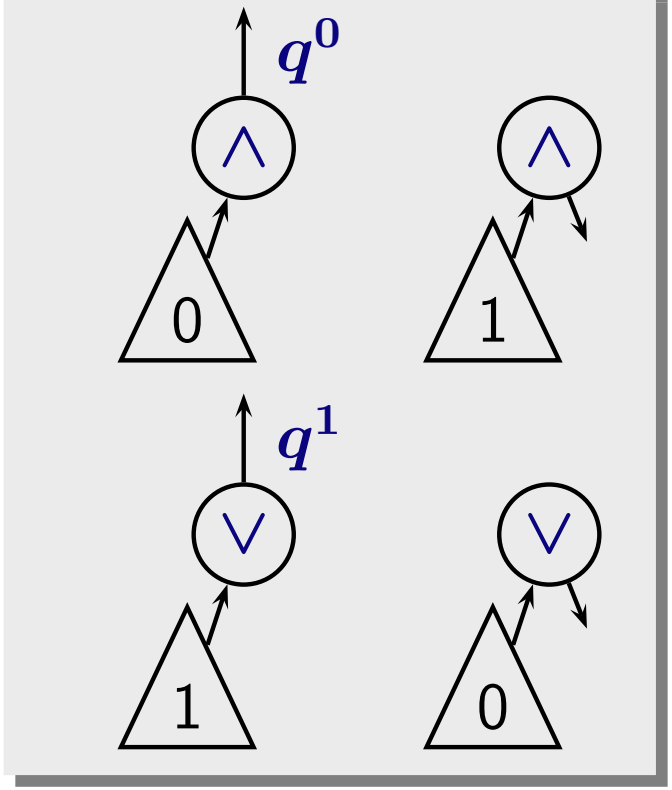

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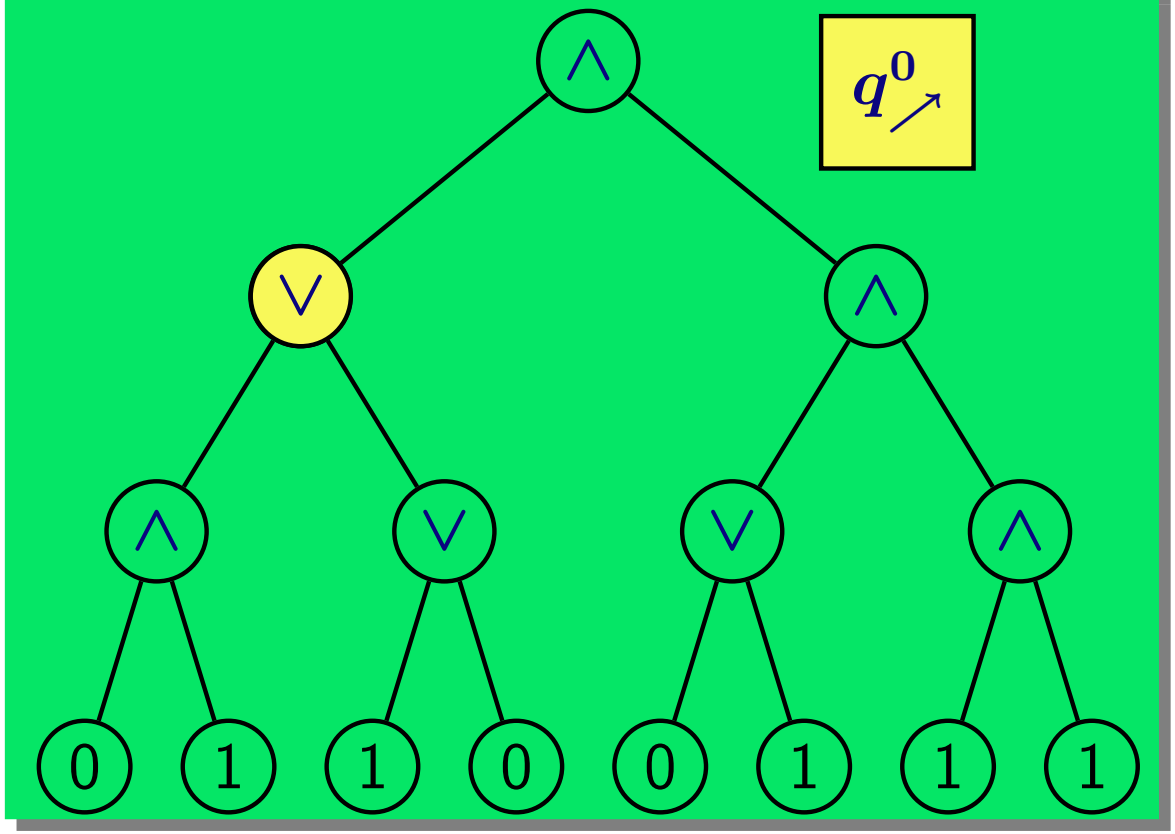

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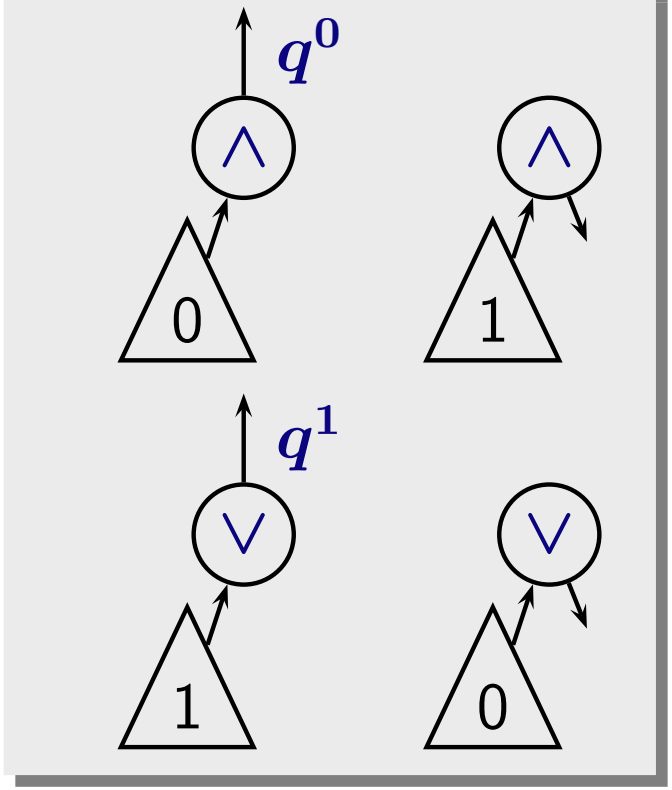

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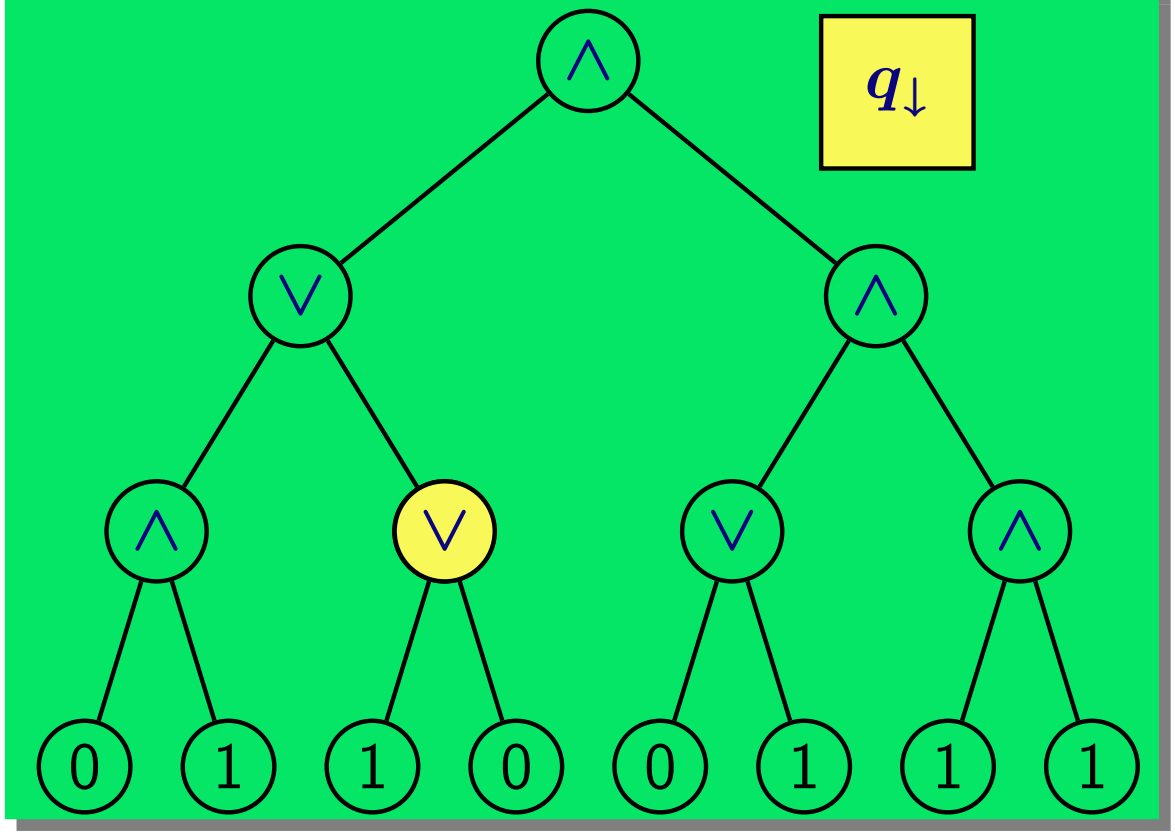

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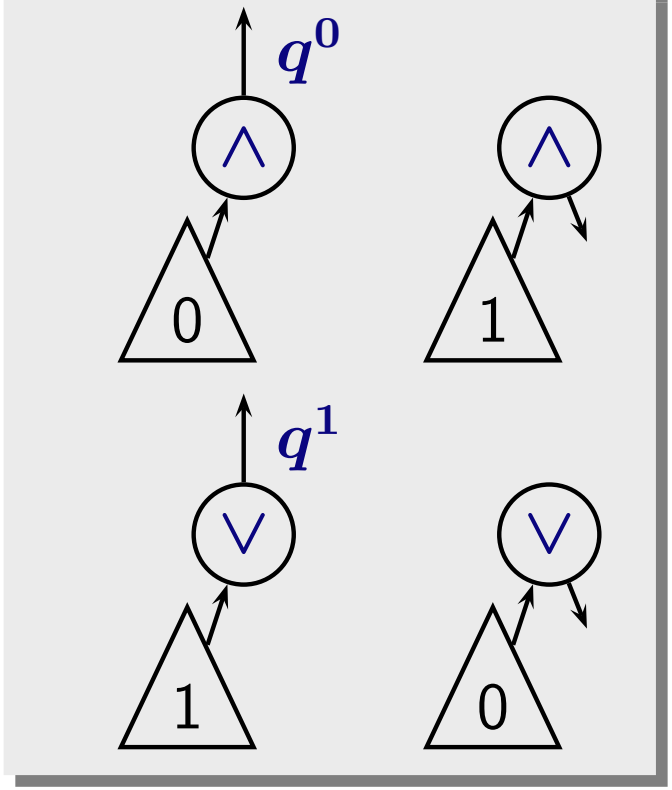

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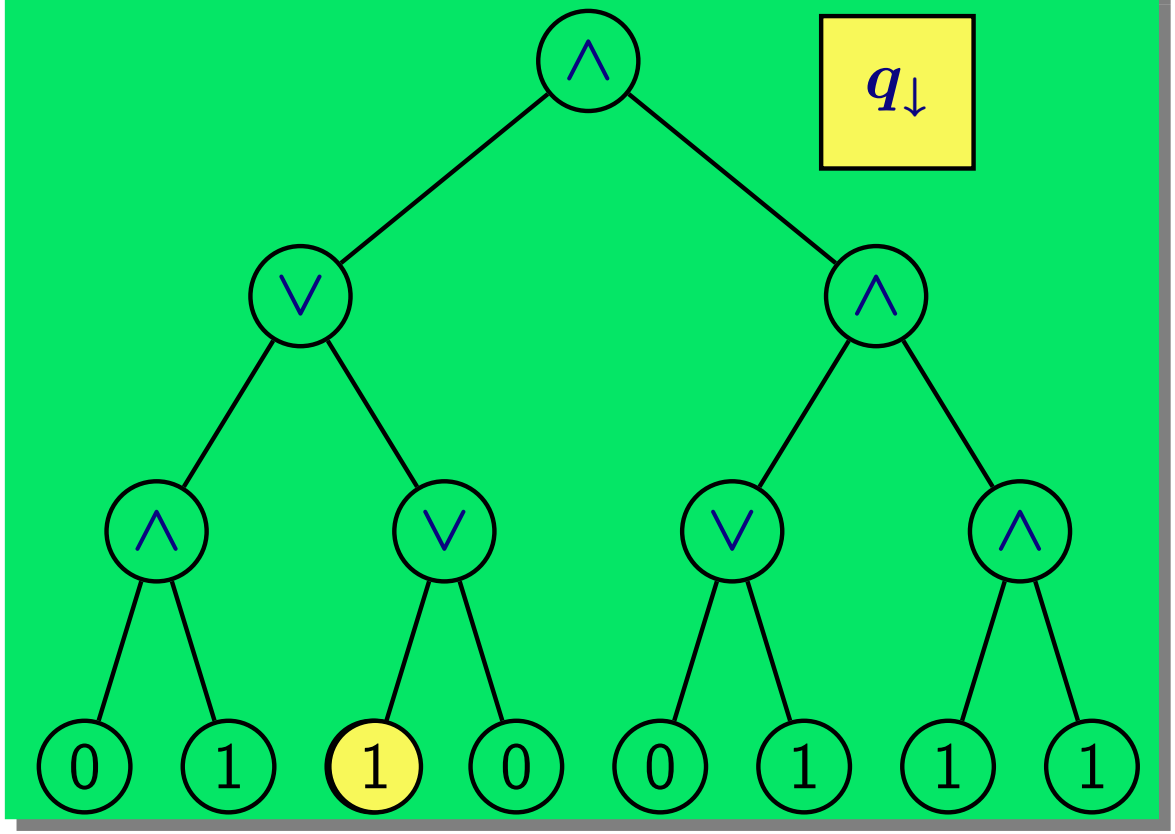

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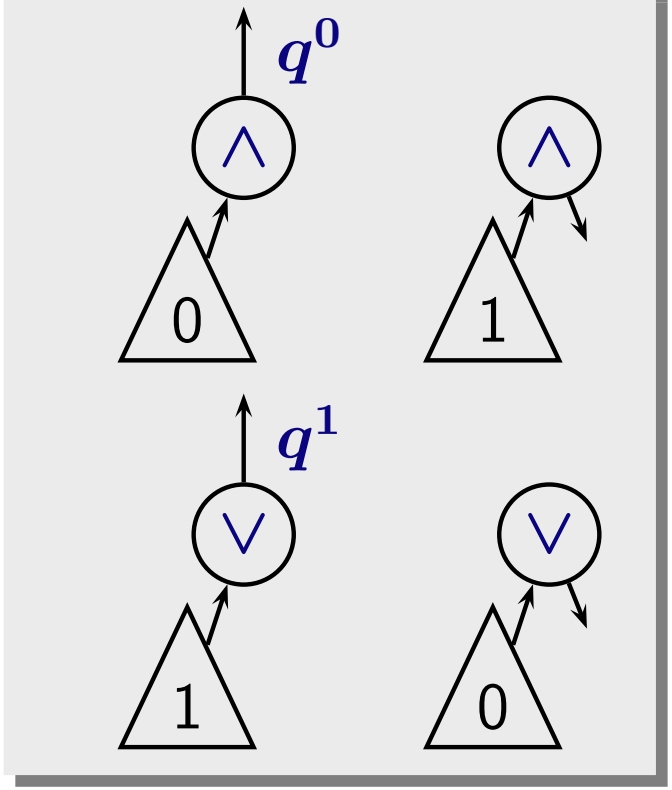

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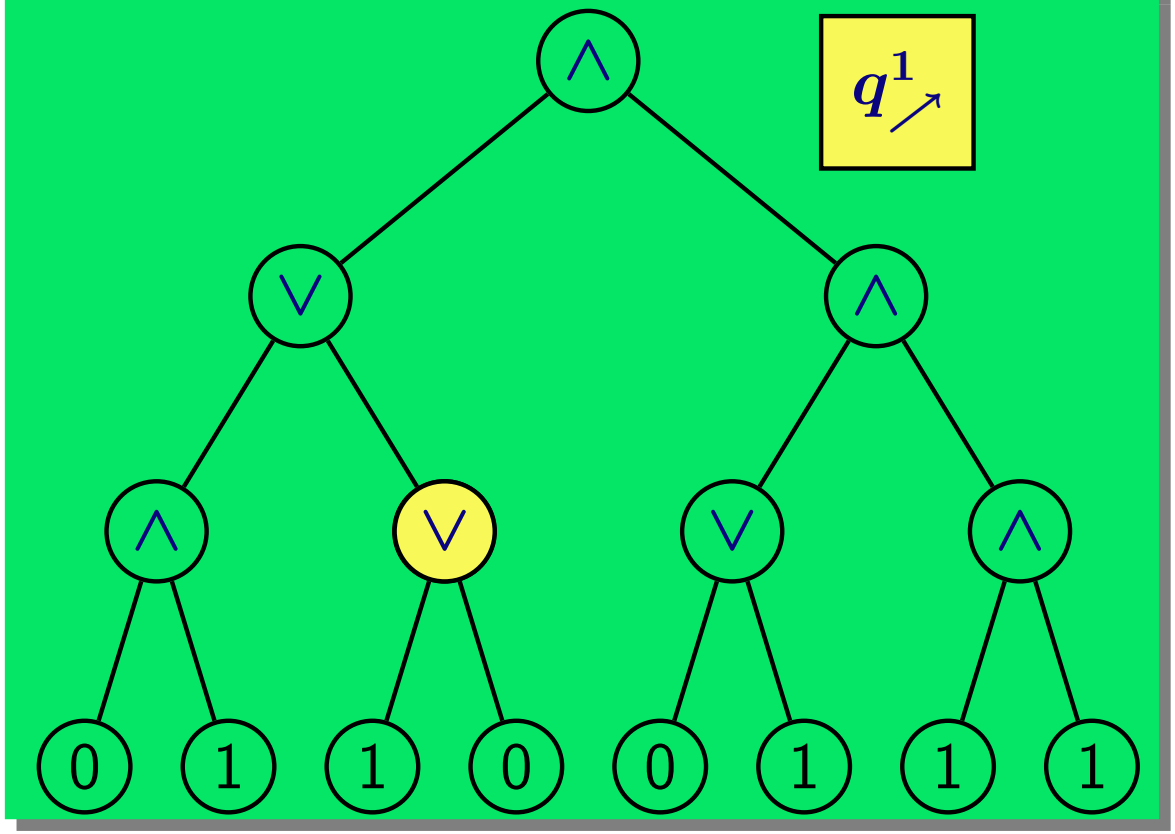

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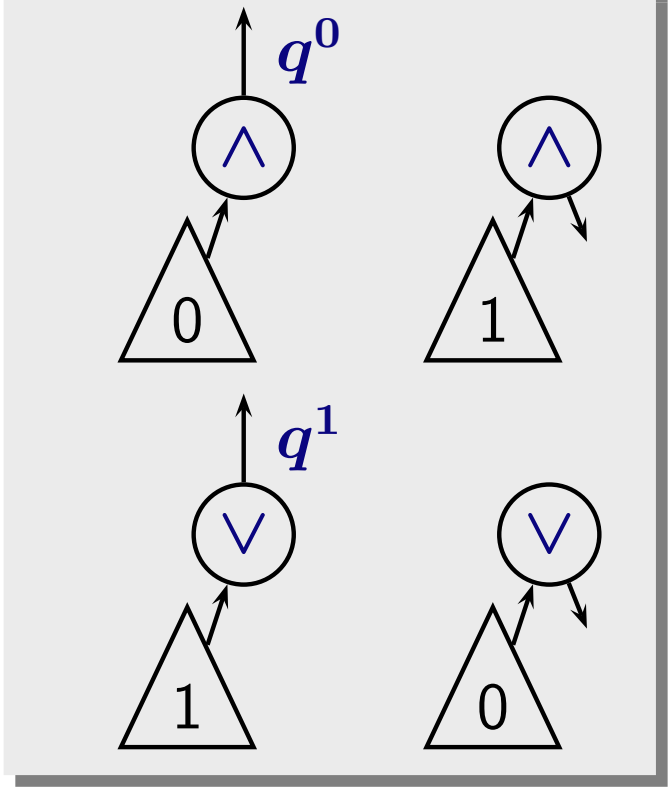

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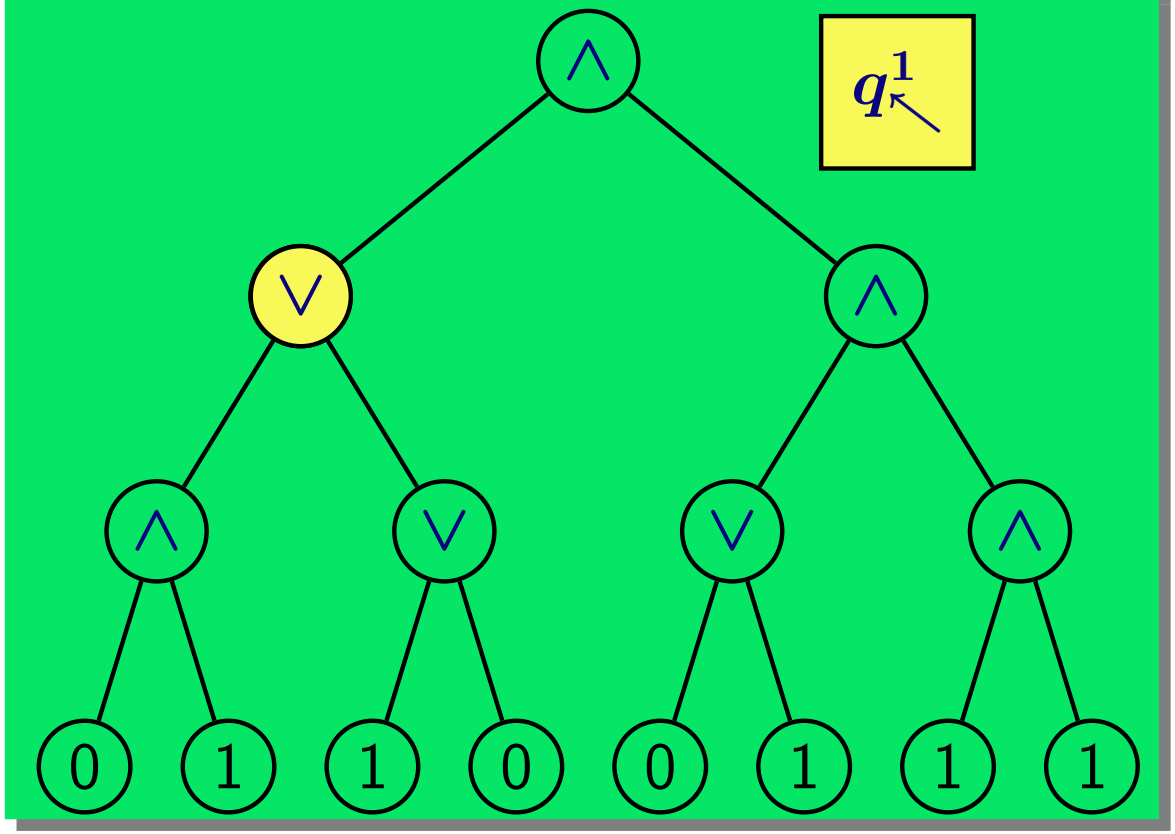

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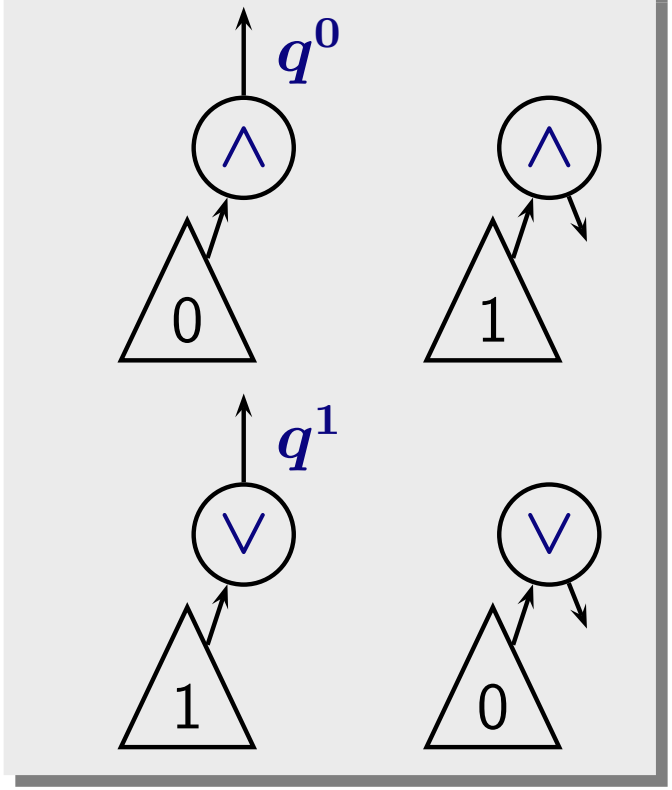

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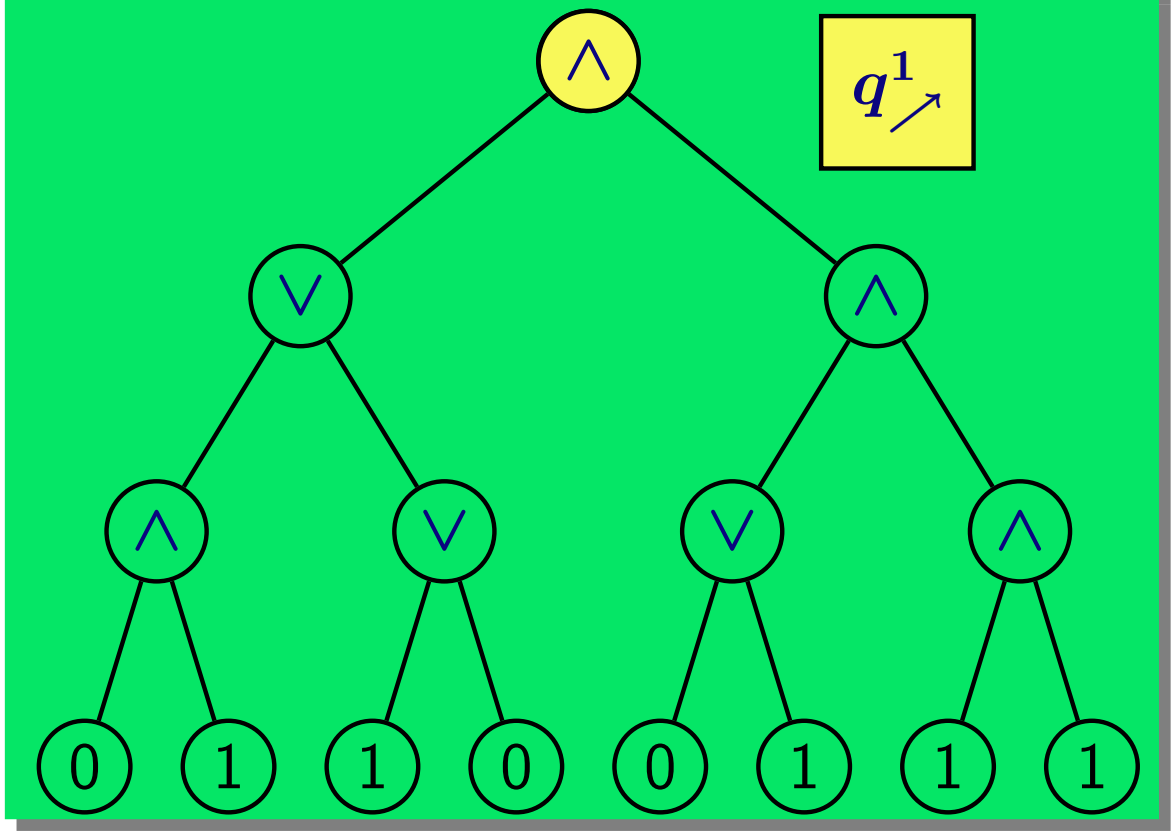

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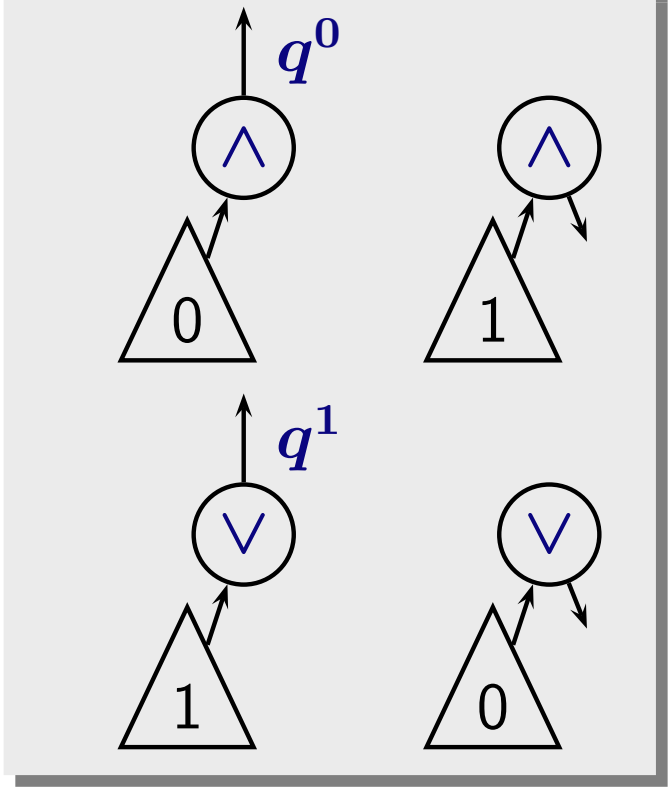

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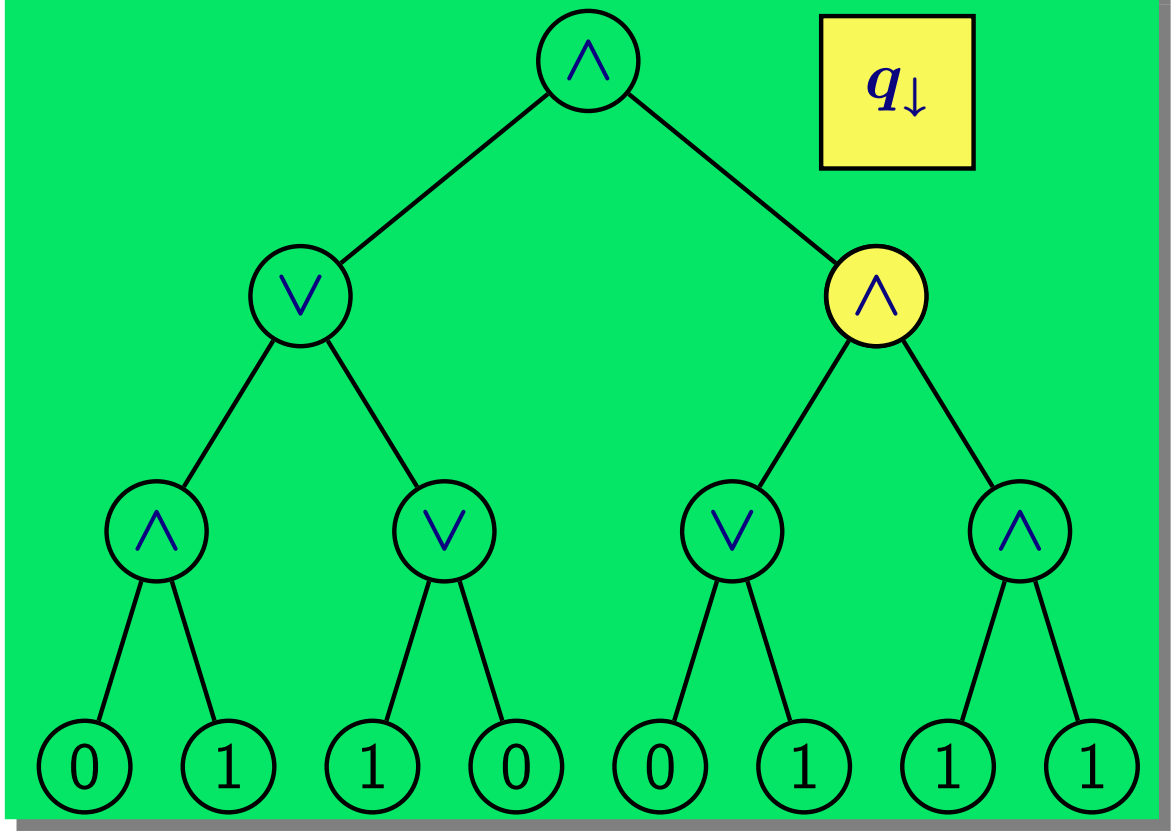

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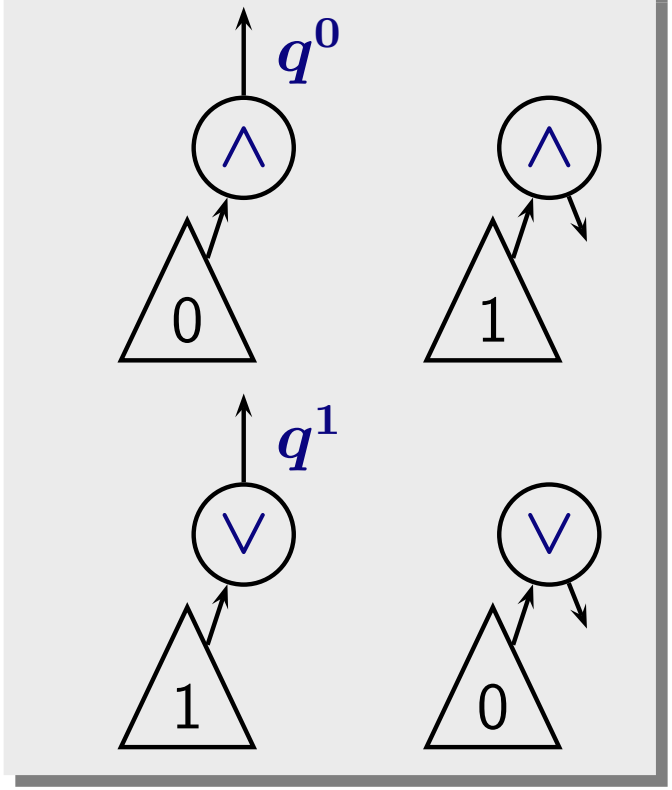

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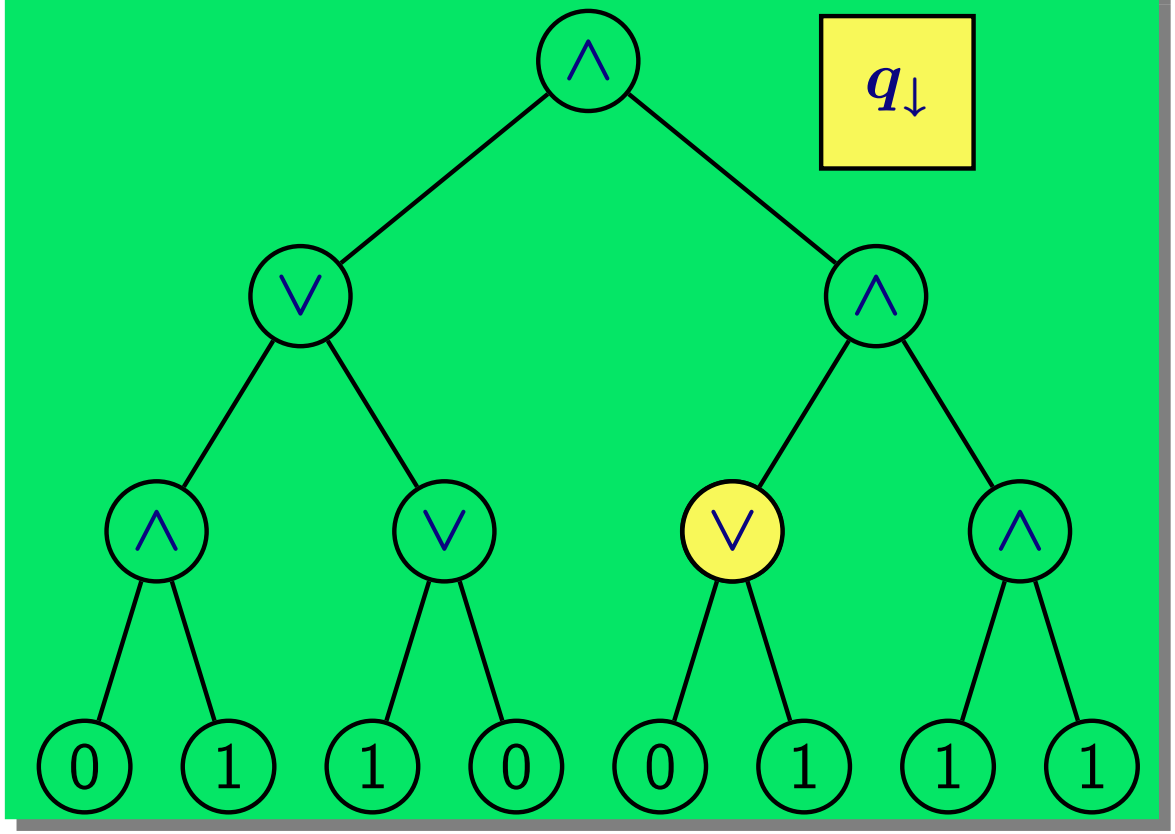

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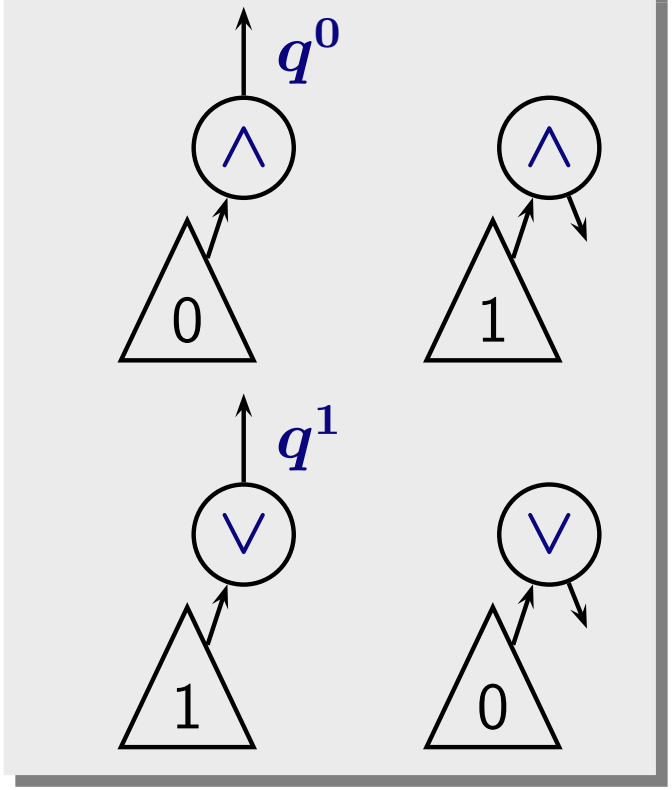

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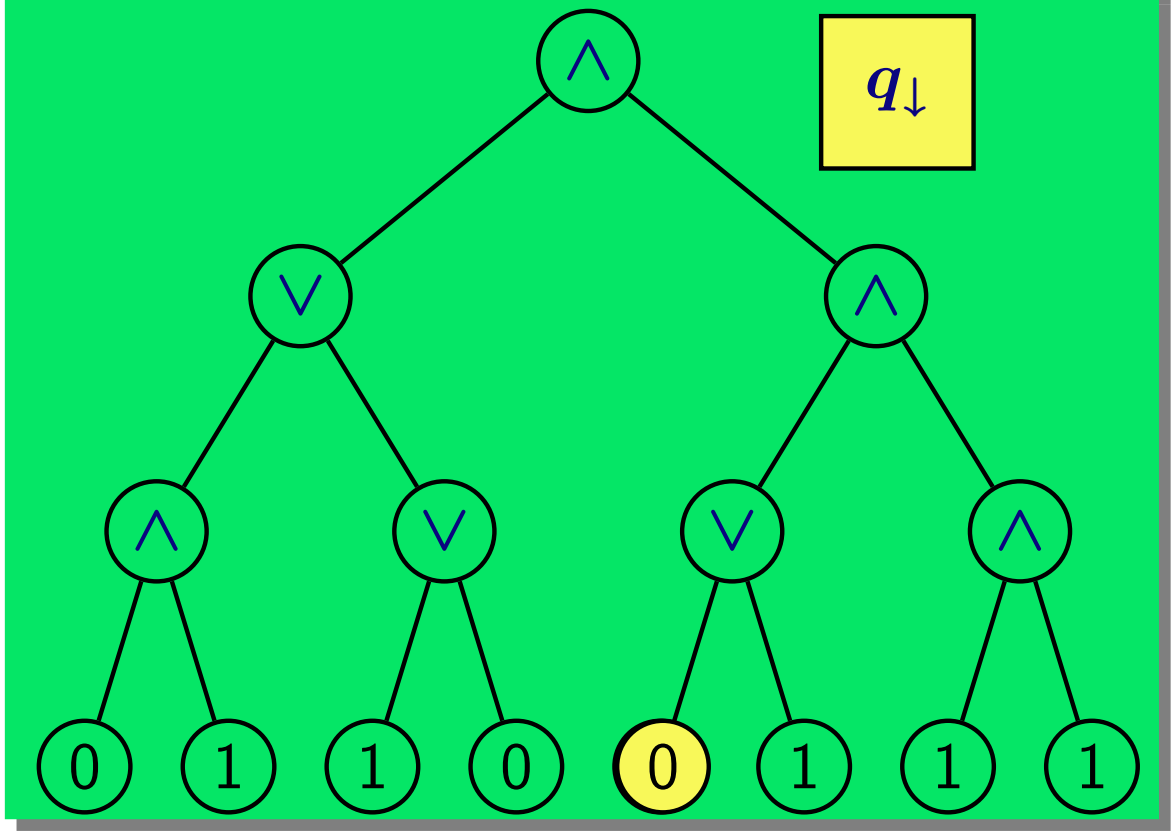

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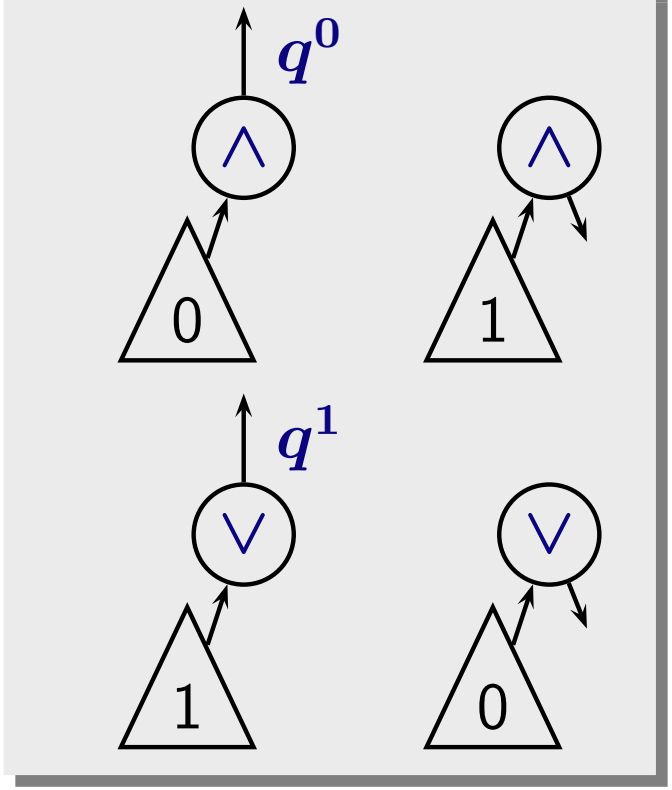

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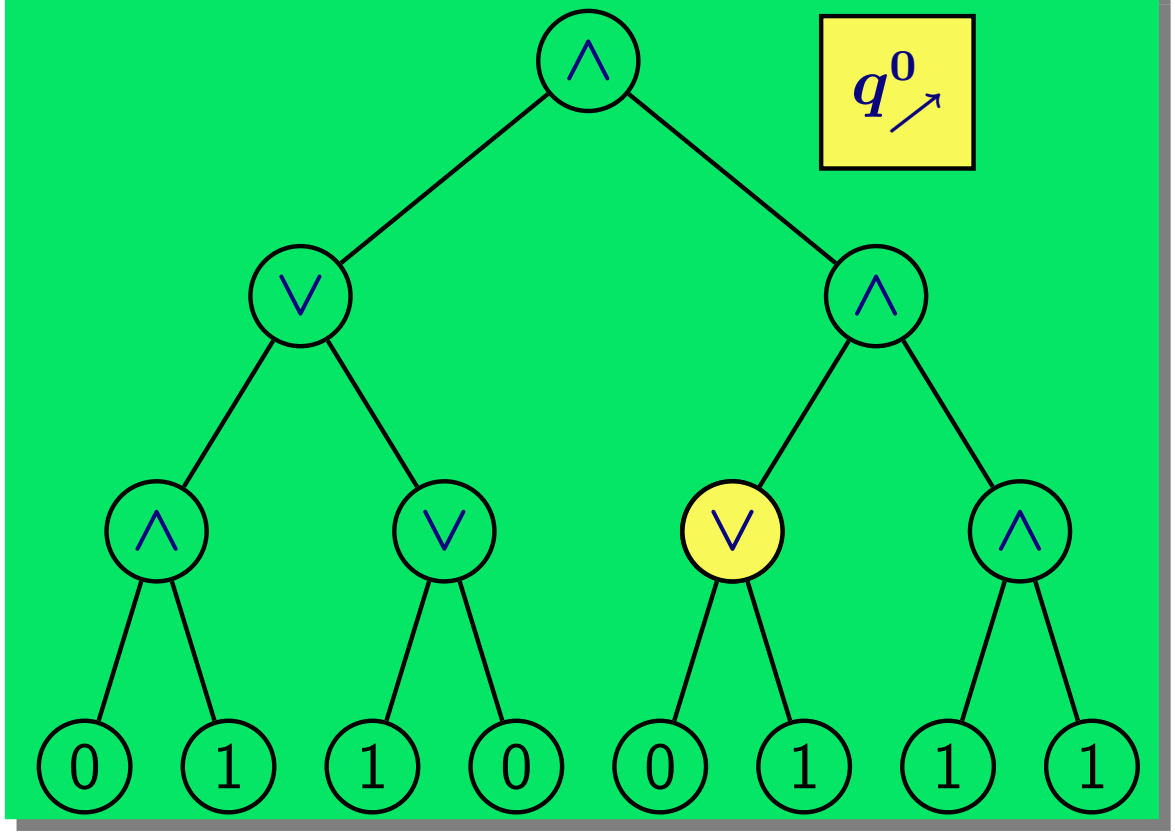

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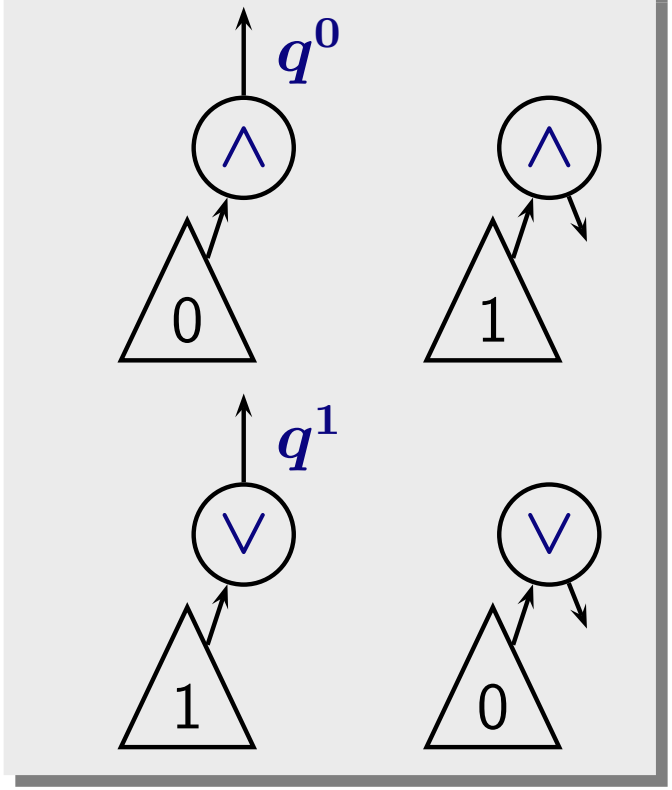

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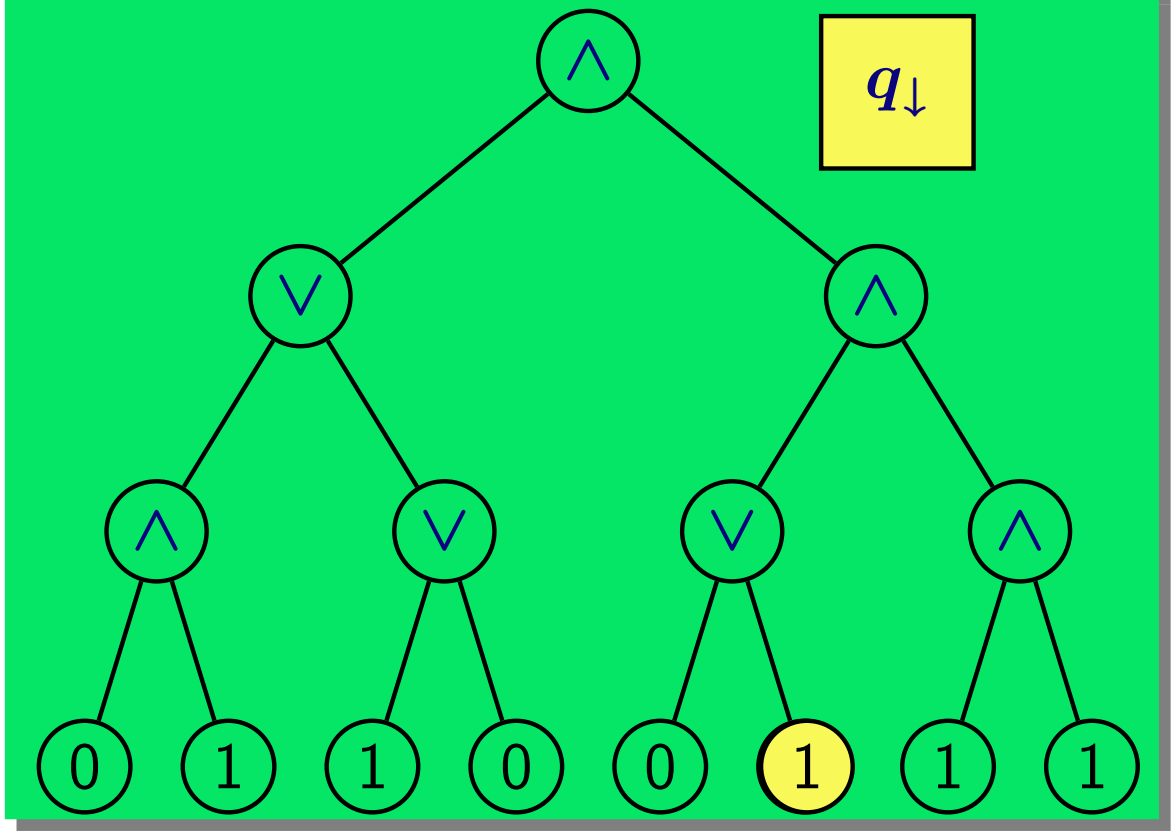

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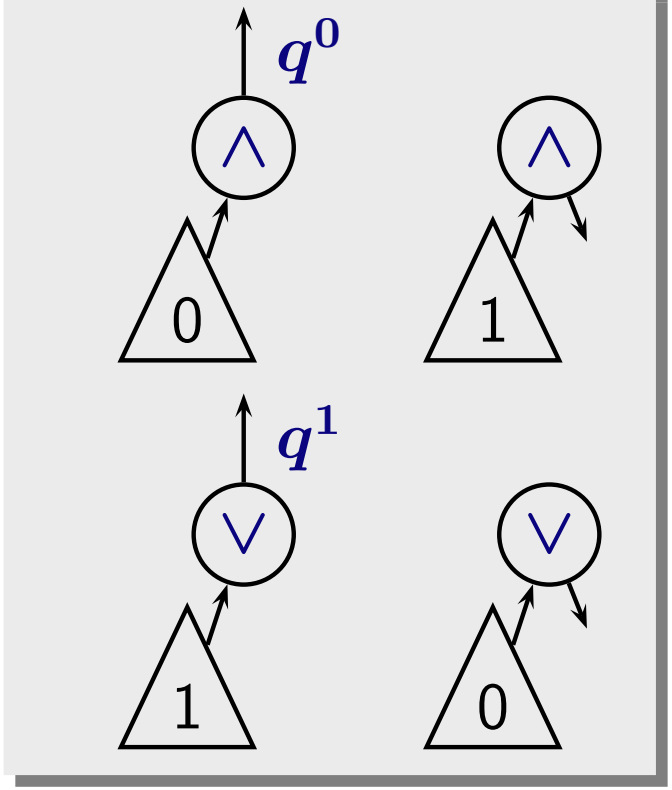

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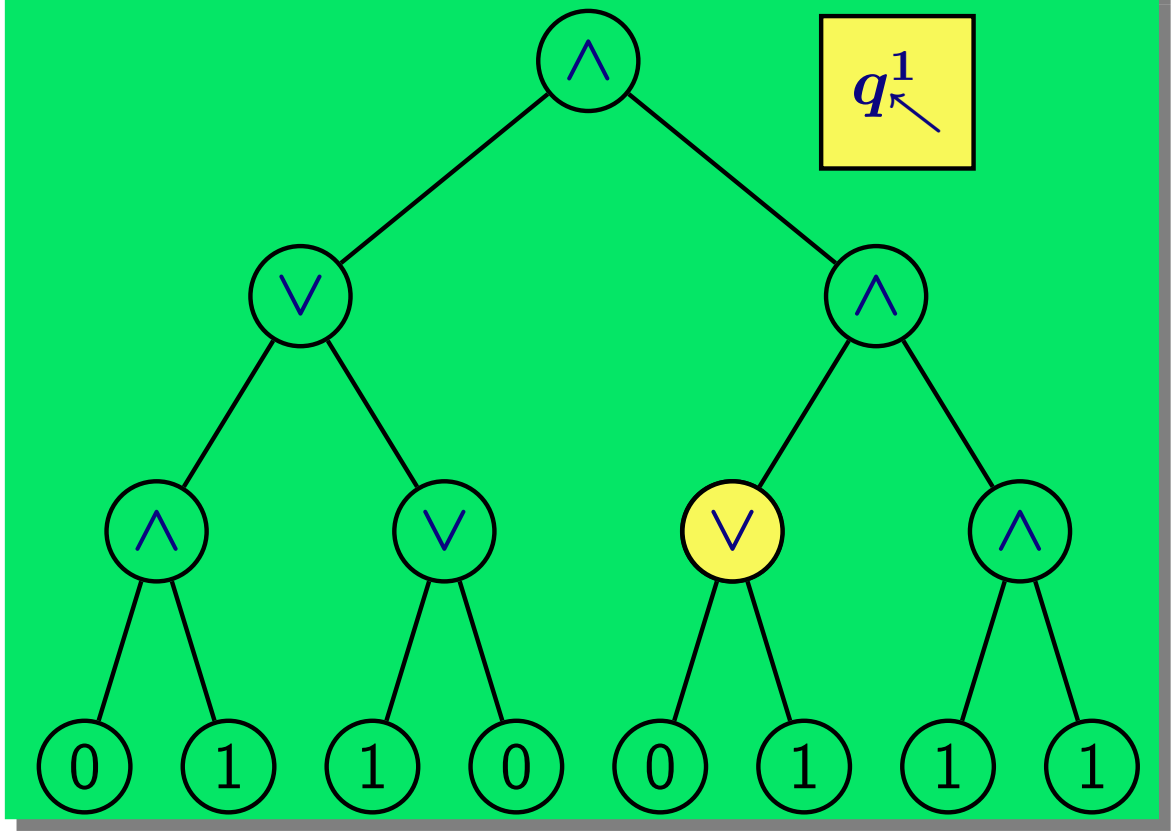

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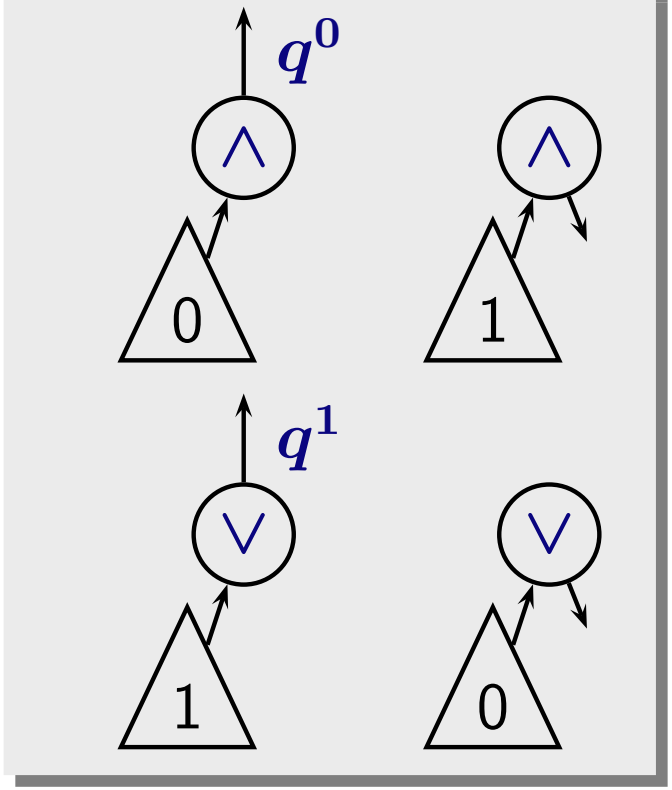

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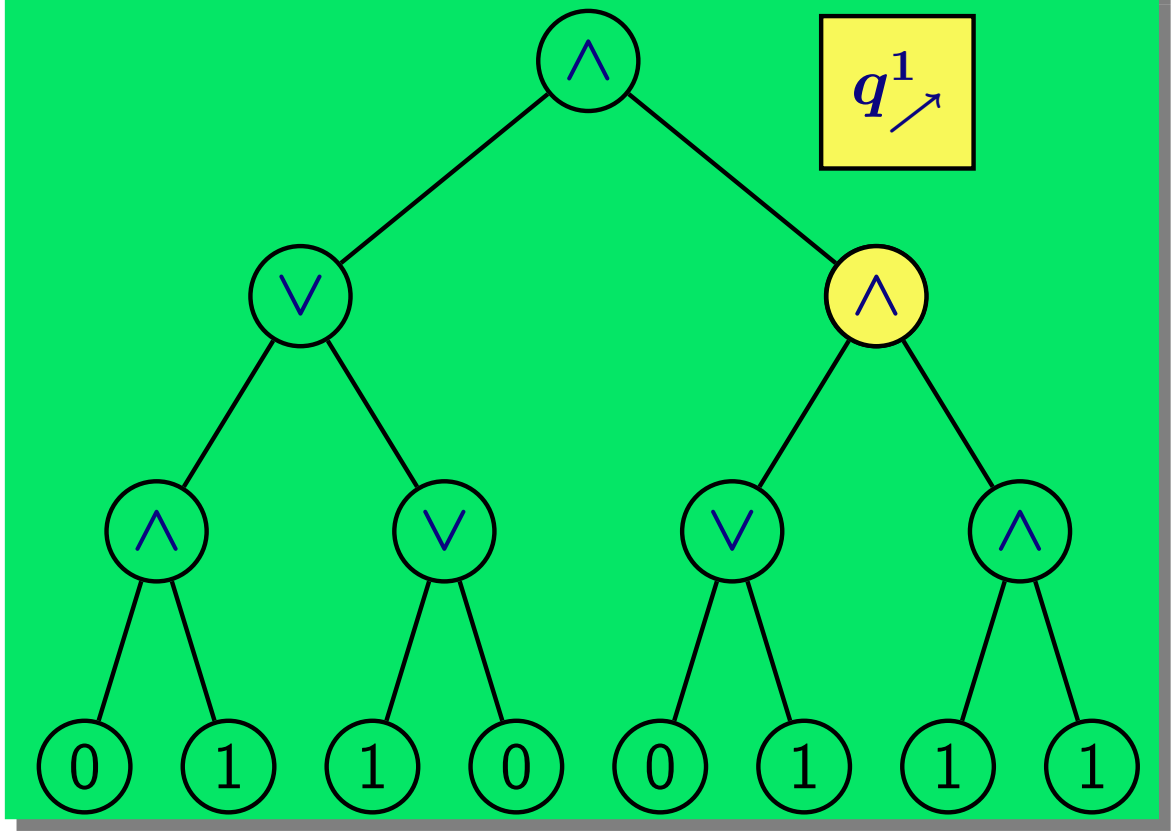

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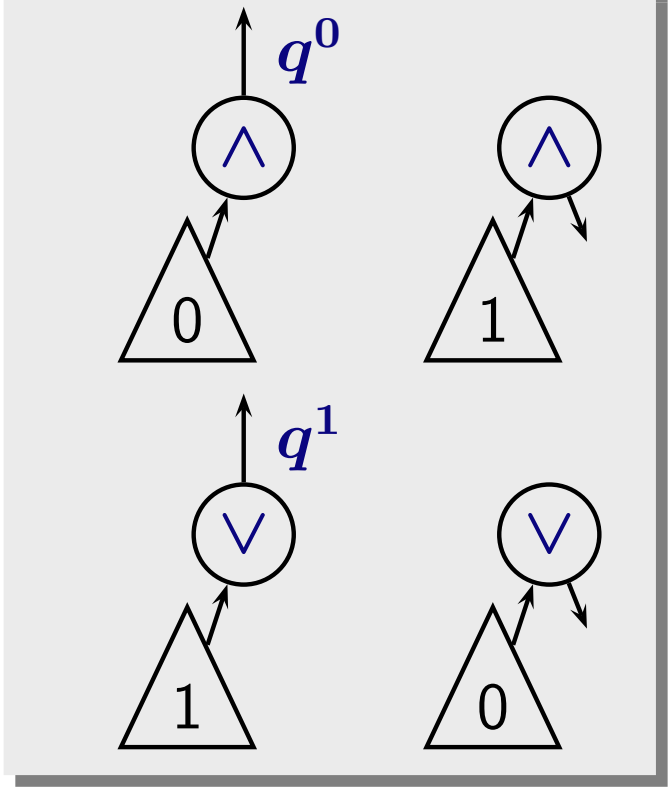

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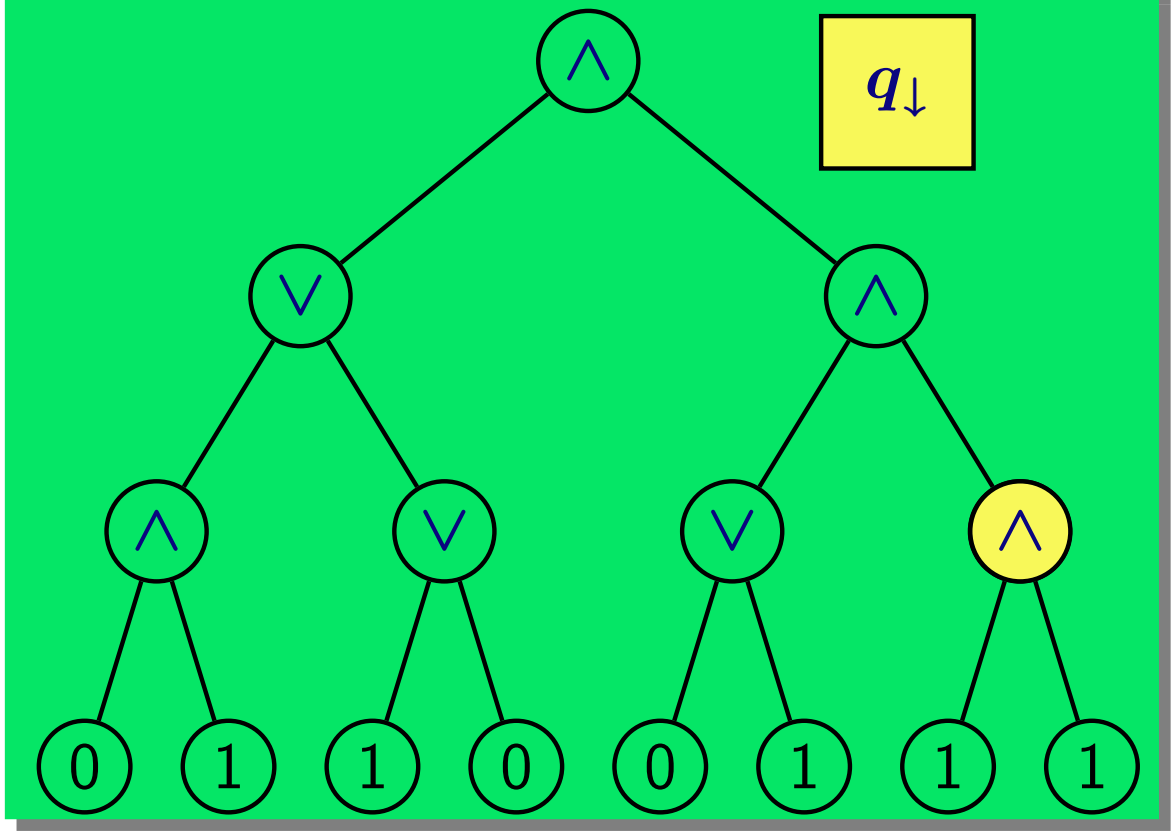

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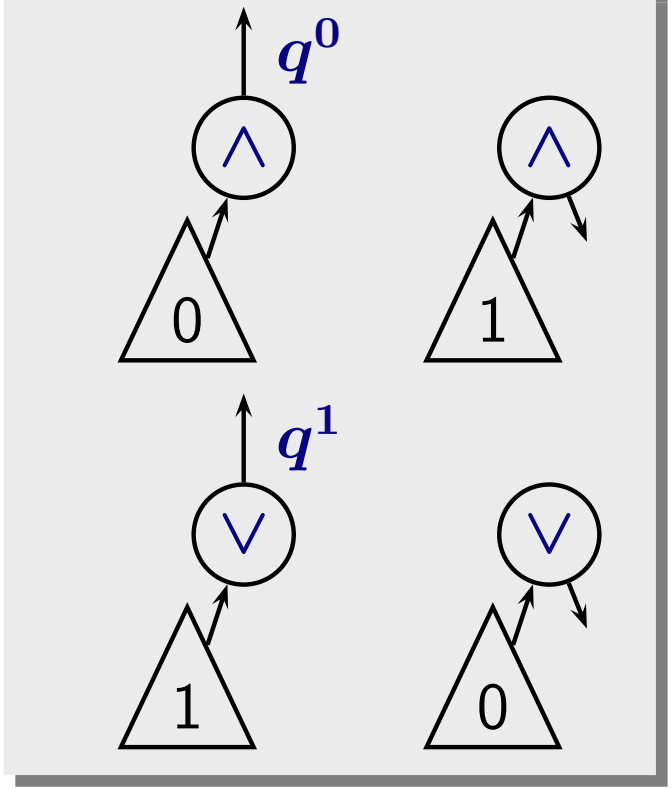

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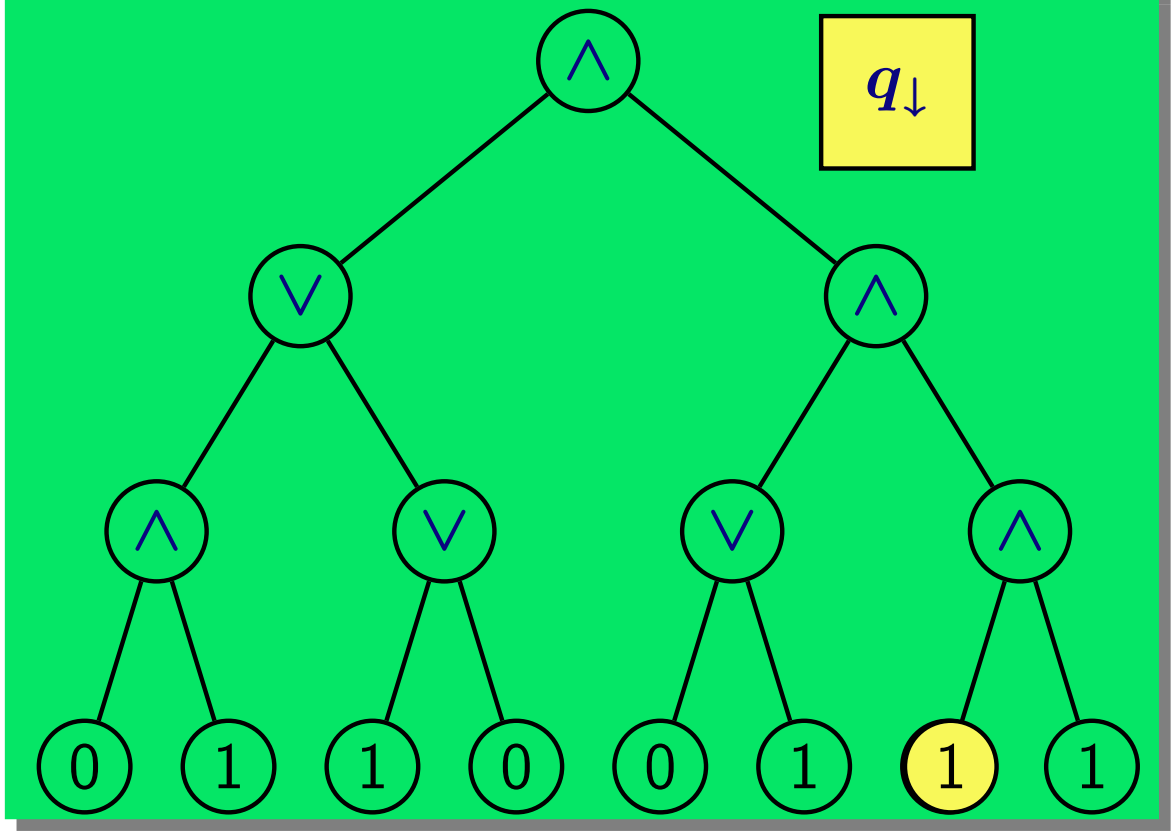

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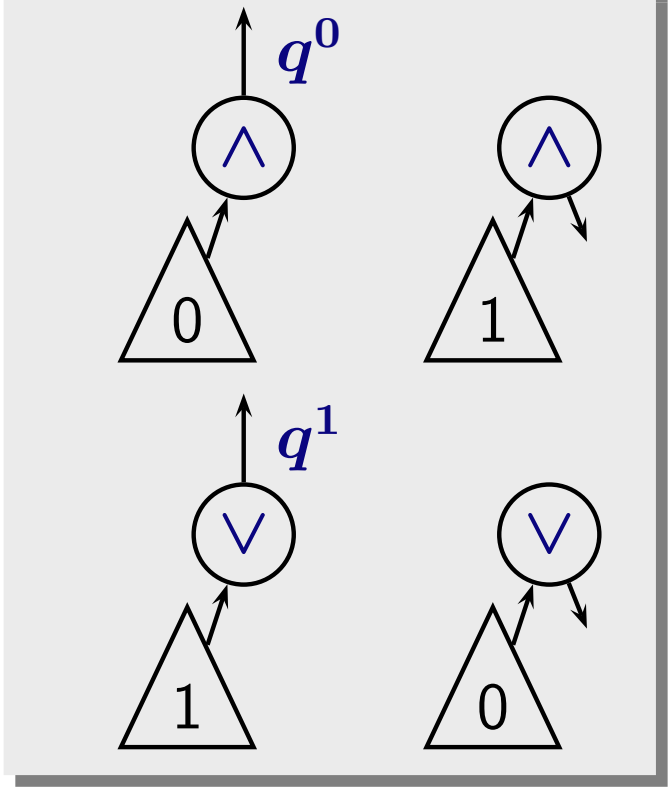

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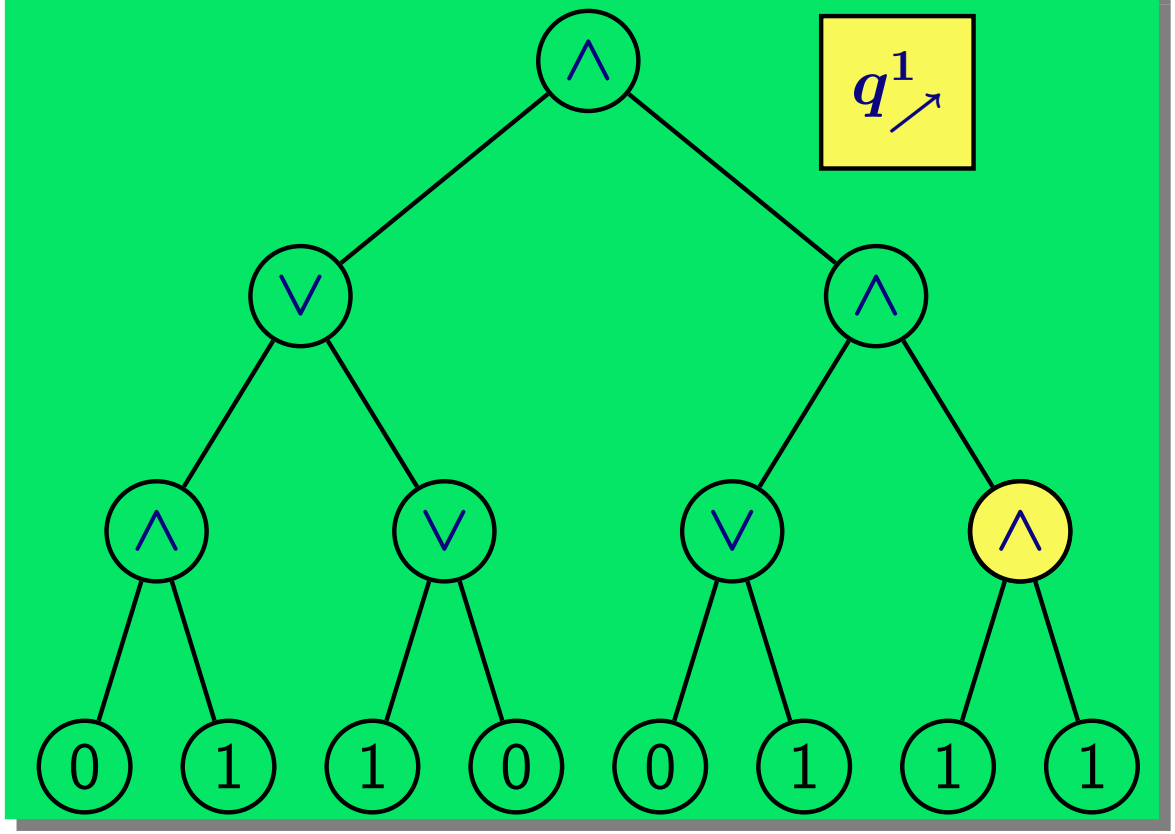

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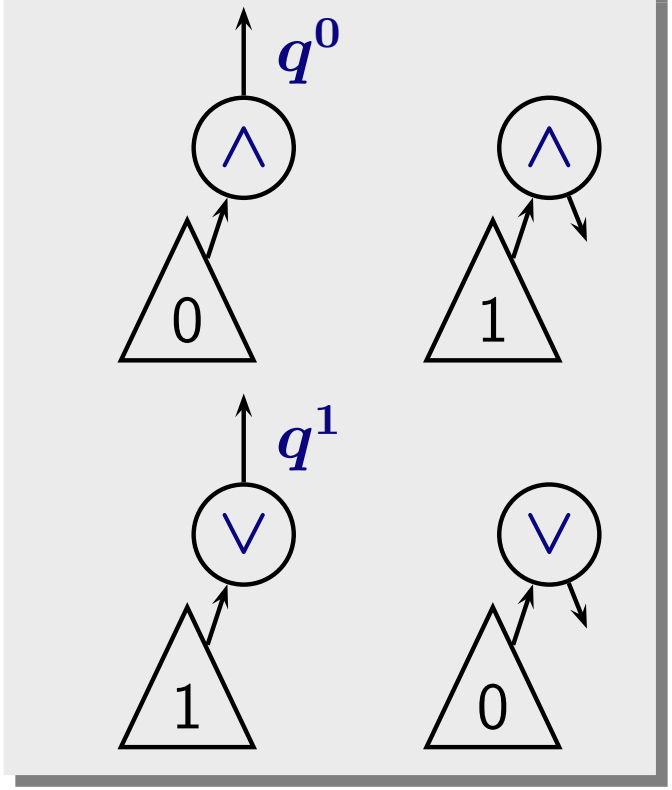

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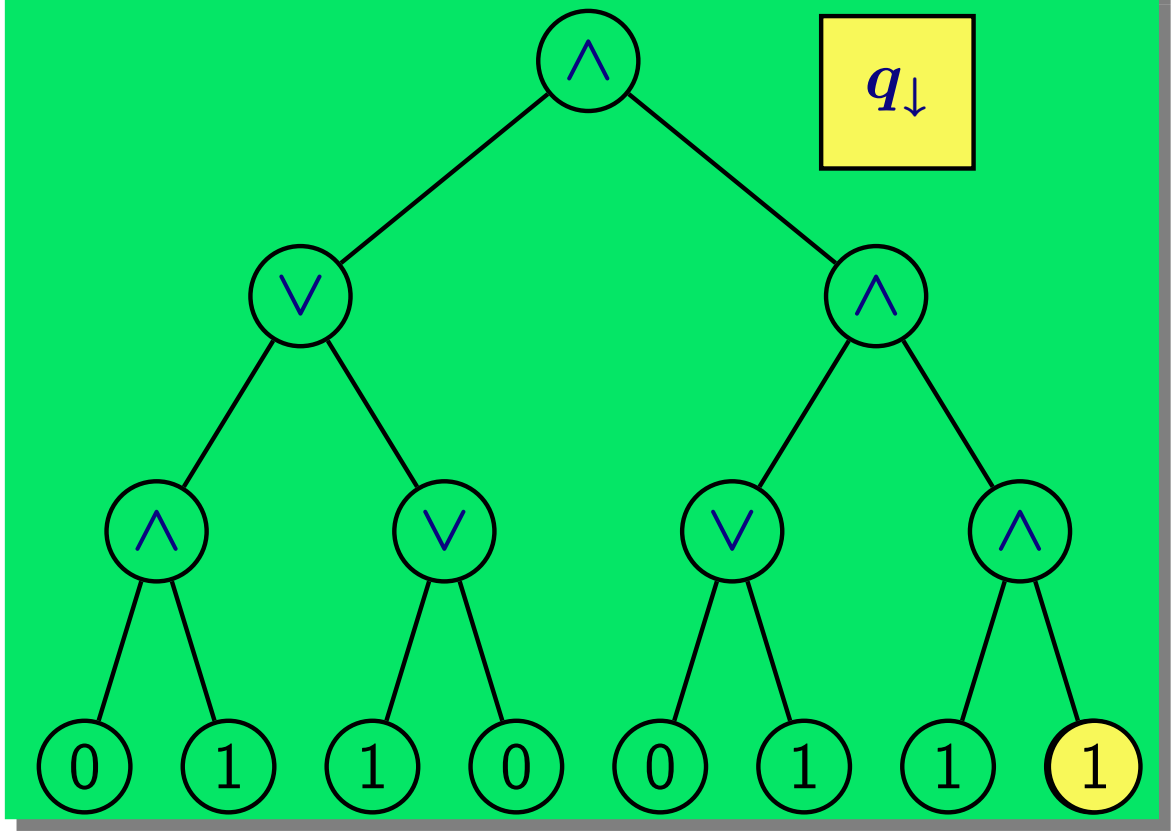

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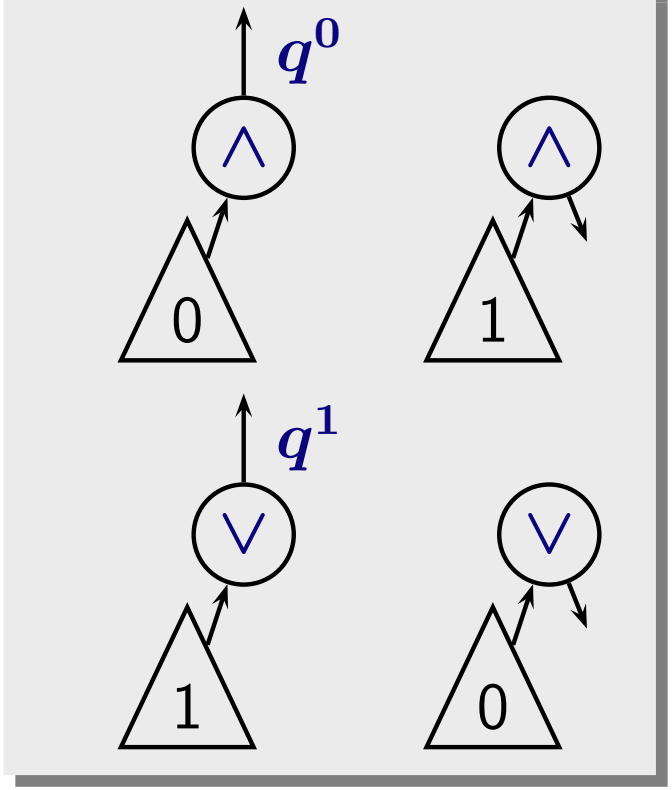
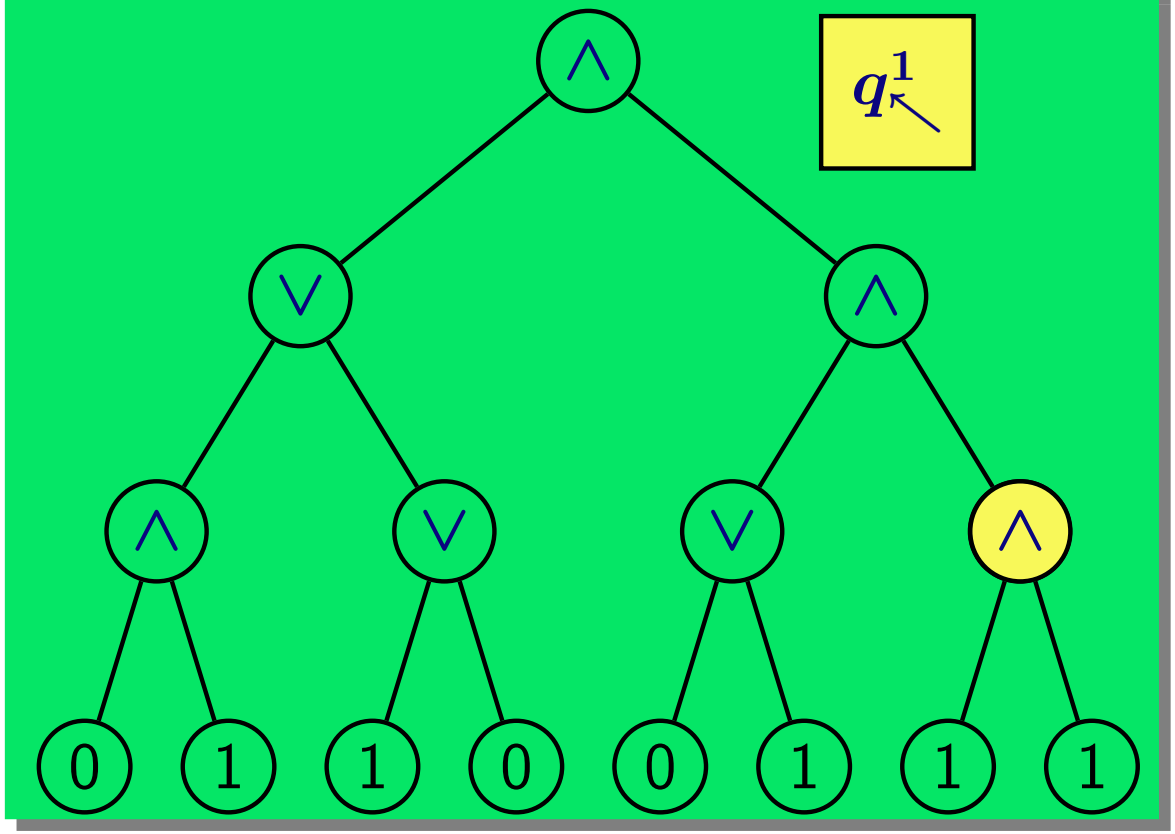


Tree-Walk Automata (cont.)

Fact

- Tree-walk automata can evaluate Boolean circuit trees
- 5 states

Example



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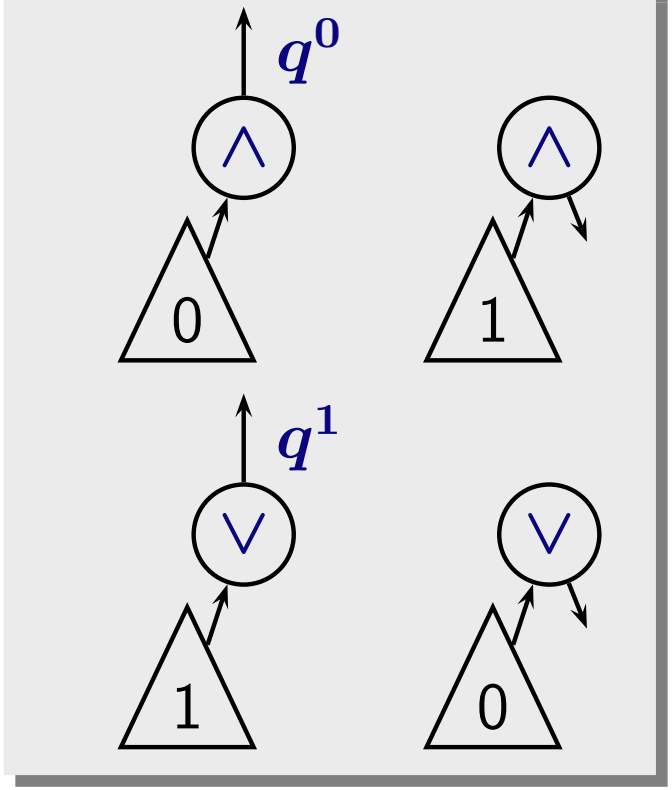

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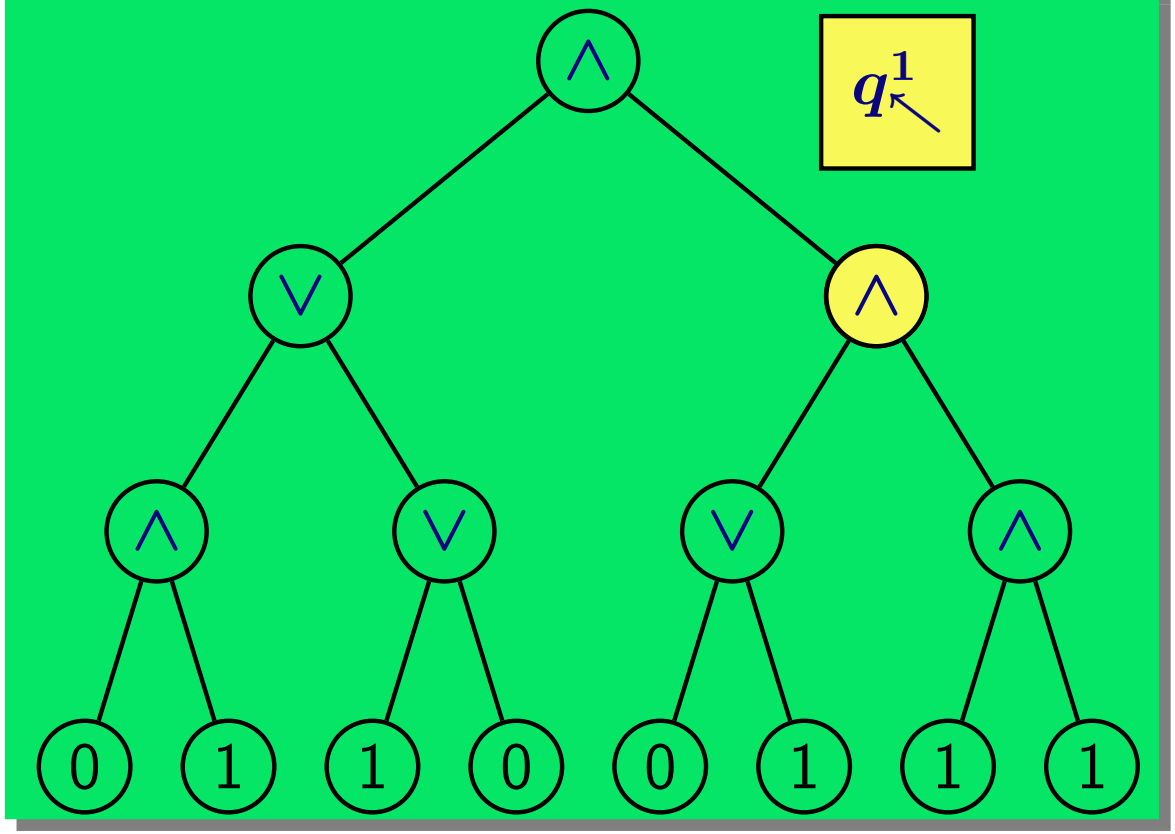

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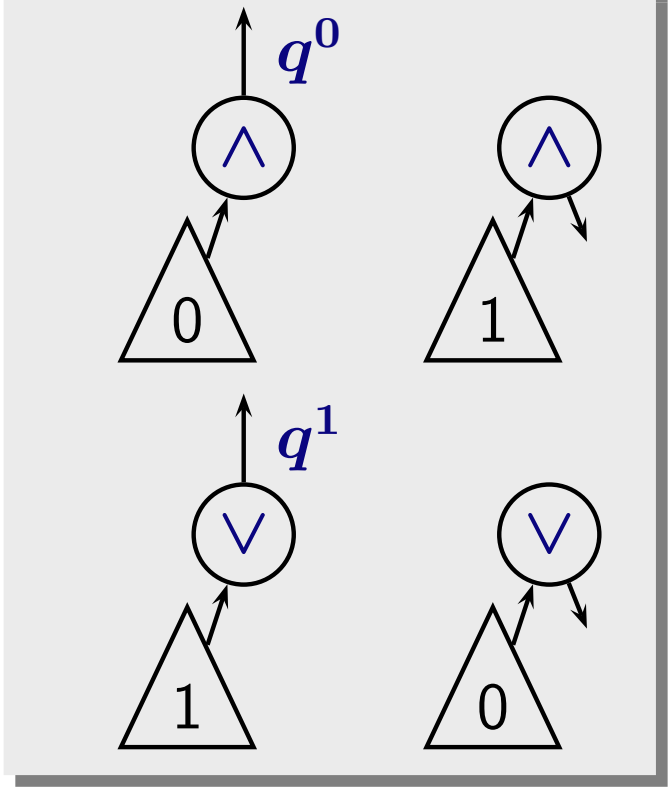

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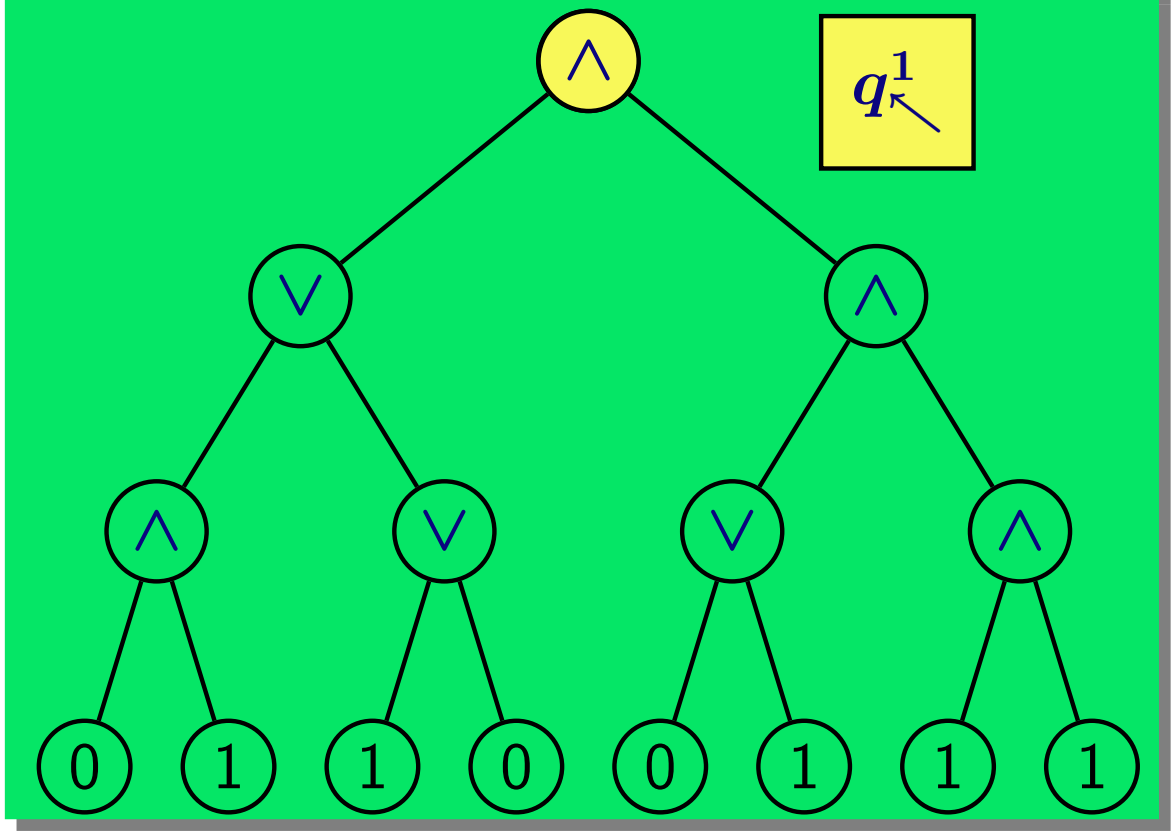

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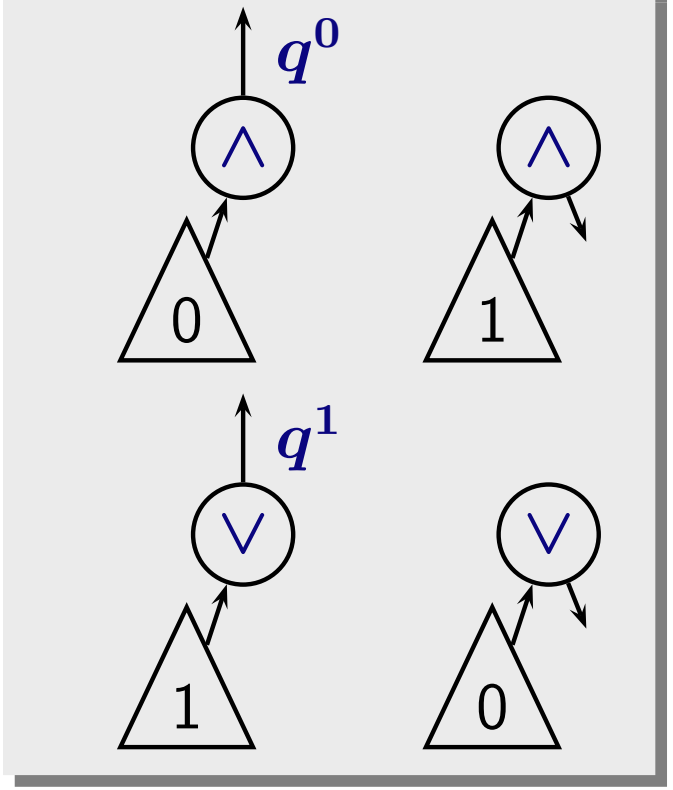


A Recent Result and an Even More Recent Result

Theorem [Bojanczyk, Colcombet 04]

Deterministic TWAs are weaker than nondeterministic TWAs

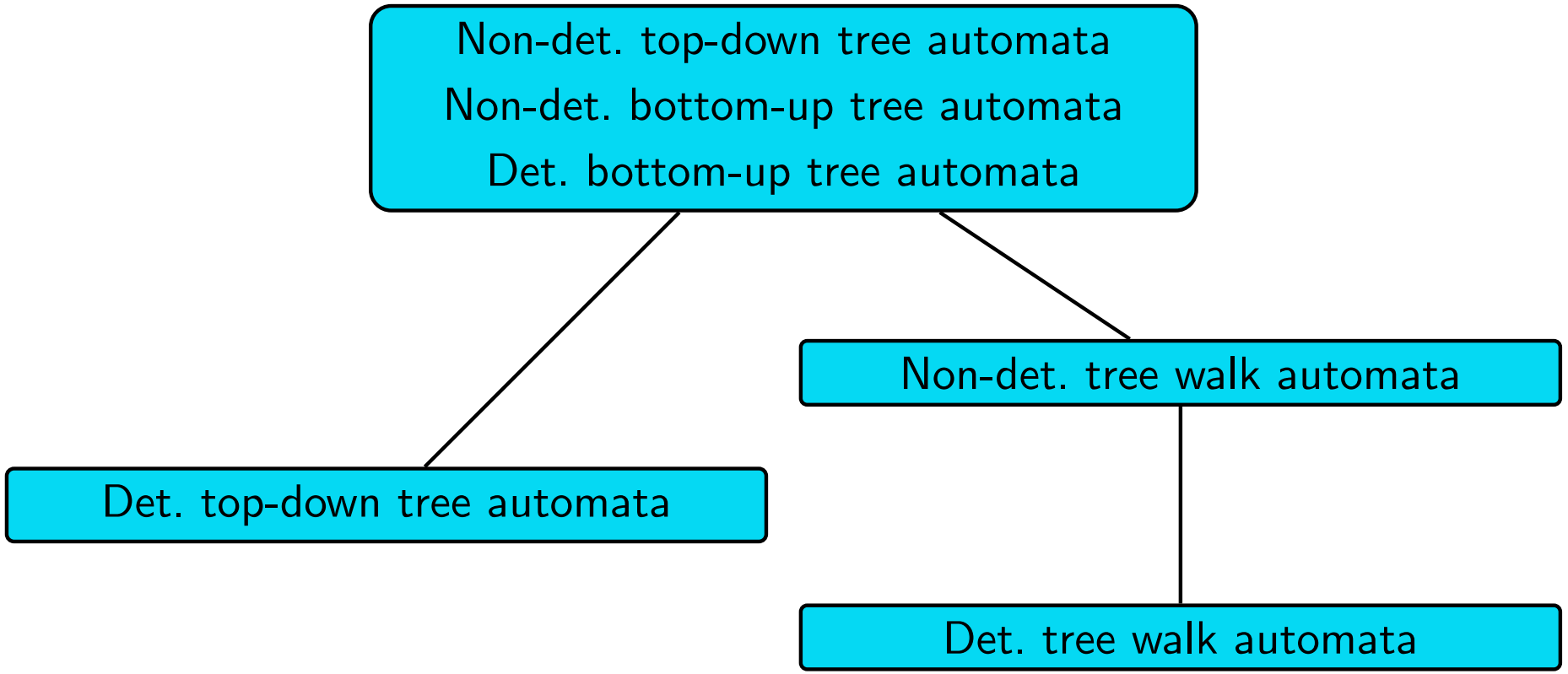
Corollary

Deterministic TWAs do not capture all regular tree languages

Theorem [Bojanczyk, Colcombet 04]

Nondeterministic TWAs do not capture all regular tree languages

Overview of Models



Contents

Decision Problems

Algorithmic problems

- We consider the following algorithmic problems
- All of them will be useful in the XML context

Membership test for $\mathcal{A}$

Given: Tree t

Question: Is $t \in L(\mathcal{A})$?

Membership test (combined)

Given: Automaton $\mathcal{A}$, tree t

Question: Is $t \in L(\mathcal{A})$?

Non-emptiness

Given: Automaton $\mathcal{A}$

Question: Is $L(\mathcal{A}) \neq \emptyset$?

Containment

Given: Automata $\mathcal{A}_1, \mathcal{A}_2$

Question: Is $L(\mathcal{A}_1) \subseteq L(\mathcal{A}_2)$?

Equivalence

Given: Automata $\mathcal{A}_1, \mathcal{A}_2$

Question: Is $L(\mathcal{A}_1) = L(\mathcal{A}_2)$?

Membership Test

Facts

Time Bounds for the combined complexity of membership test for tree automata:

- Deterministic (parallel) tree automata: $O(|\mathcal{A}||t|)$
- Nondeterministic (parallel) tree automata: $O(|\mathcal{A}|^2|t|)$
  (Compute, for each node, the set of reachable states)
- Deterministic TWAs: $O(|\mathcal{A}|^2|t|)$
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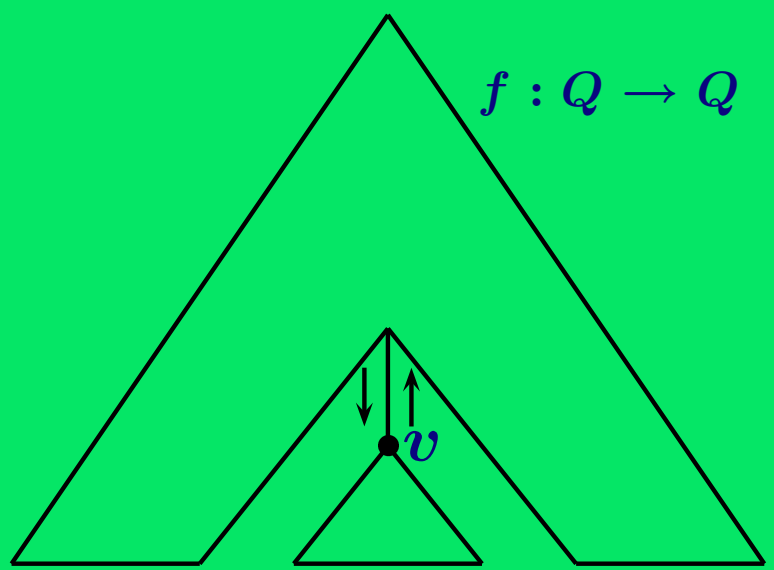
Behavior Function



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Membership Test (cont.)

Question: What is the structural complexity for the various models?

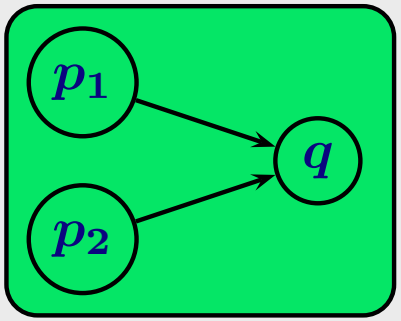
[Lohrey 01, Segoufin 03]

Model	Time Complexity	Structural Complexity
Det. top-down TA	$O(\|\mathcal{A}\|\|t\|)$	**LOGSPACE**
Det. bottom-up TA	$O(\|\mathcal{A}\|\|t\|)$	**LOGDCFL**
Nondet. bottom-up TA	$O(\|\mathcal{A}\|^2\|t\|)$	**LOGCFL**
Nondet. top-down TA	$O(\|\mathcal{A}\|^2\|t\|)$	**LOGCFL**
Det. TWA	$O(\|\mathcal{A}\|^2\|t\|)$	**LOGSPACE**
Nondet. TWA	$O(\|\mathcal{A}\|^3\|t\|)$	**NLOGSPACE**

Non-emptiness

Facts

- Non-emptiness for string automata corresponds to `Graph Reachability` (complete for **NLOGSPACE**)
- Non-emptiness for tree automata corresponds to `Path Systems` :



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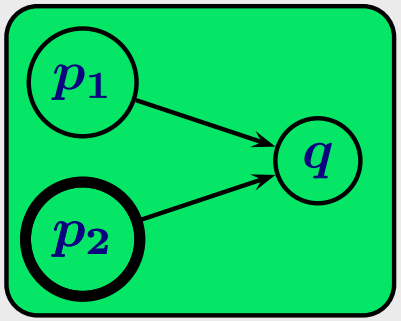

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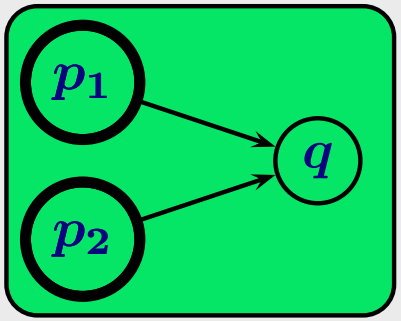

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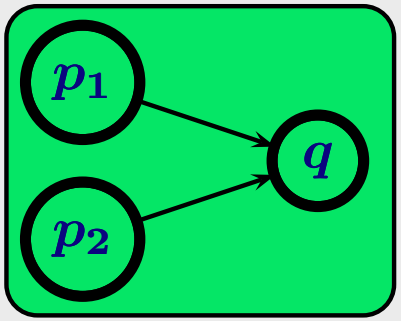

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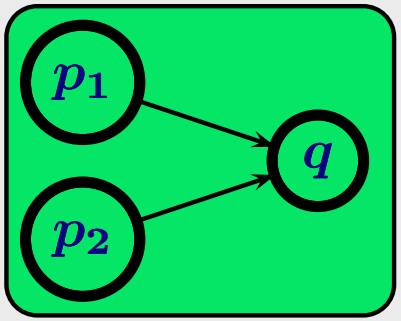


Result

- Non-emptiness for bottom-up tree automata can be checked in linear time
- It is complete for **PTIME**

Containment/Equivalence

Observations

- Of course:

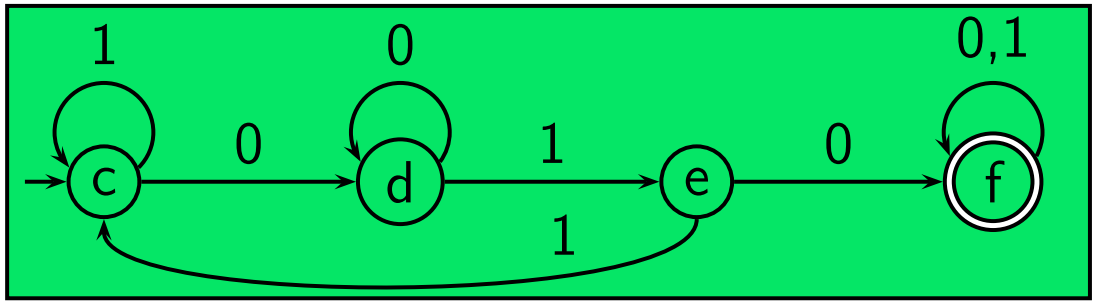
$$L(\mathcal{A}_1) = L(\mathcal{A}_2) \Longleftrightarrow [L(\mathcal{A}_1) \subseteq L(\mathcal{A}_2) \text{ and } L(\mathcal{A}_2) \subseteq L(\mathcal{A}_1)]$$

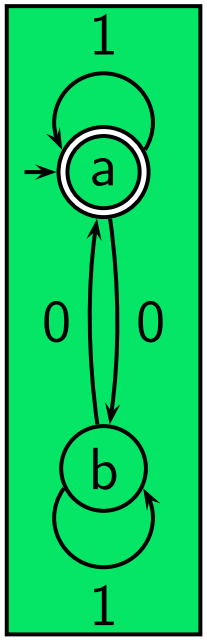
- Complexity of containment problem is very different for deterministic and non-deterministic automata
- Deterministic automata: construct product automaton

Reminder: Product automaton

Product of 2 string automata

- "even number of zeros"
- "contains substring 00"





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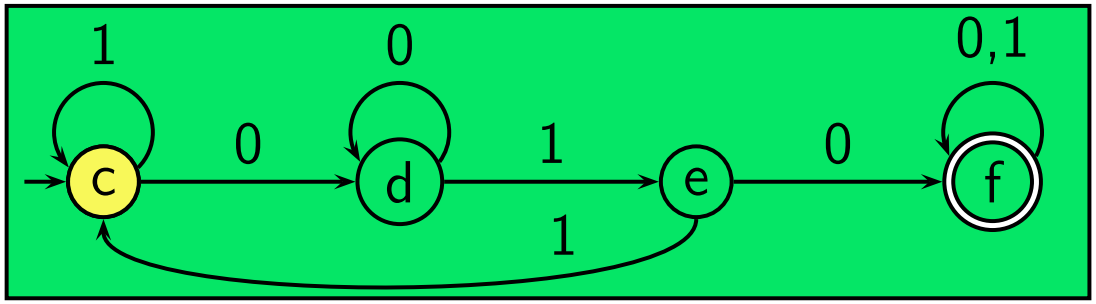

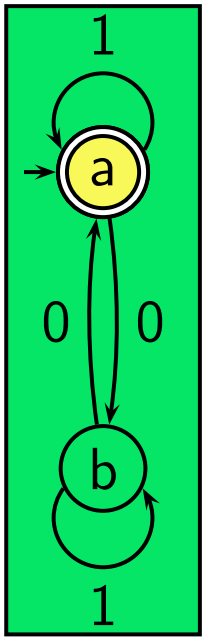

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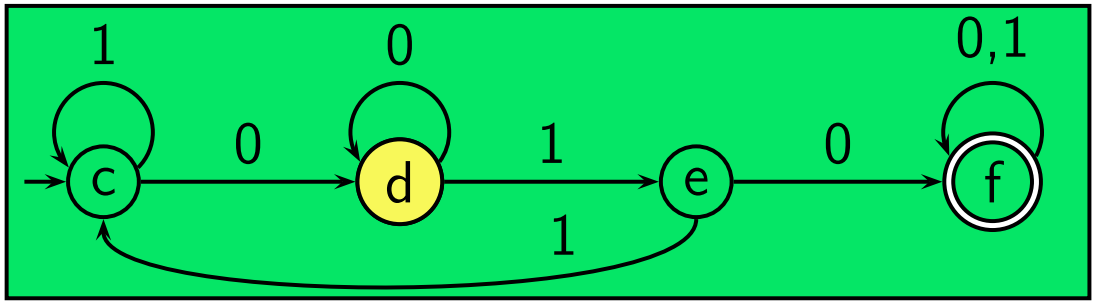

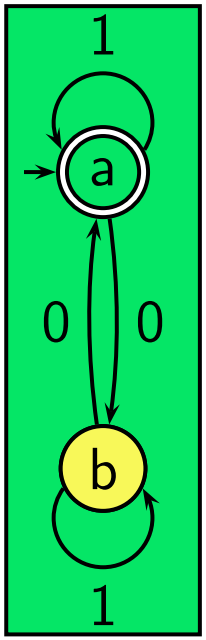

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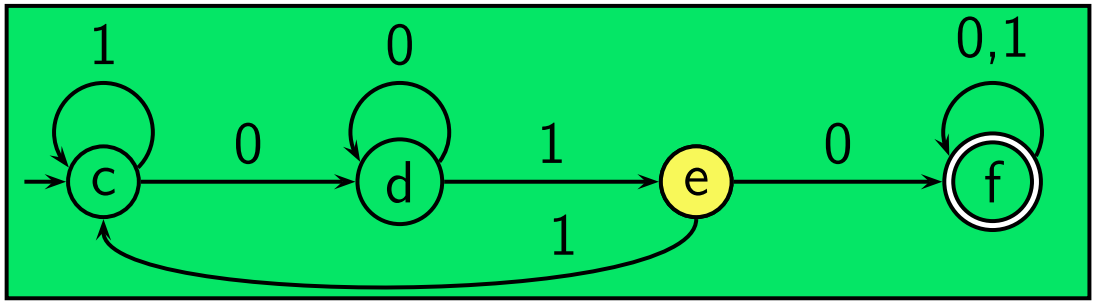

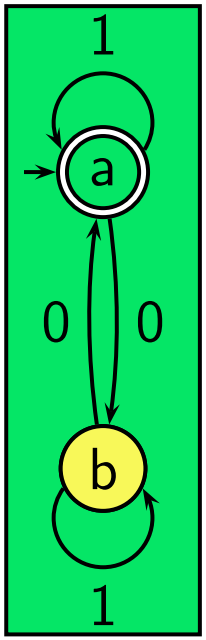

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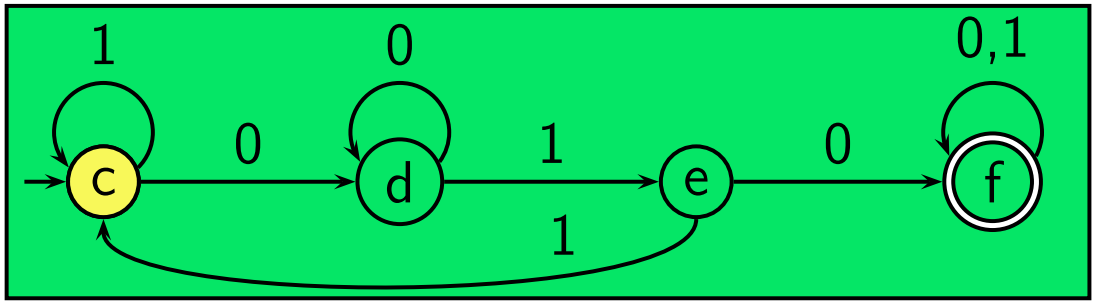

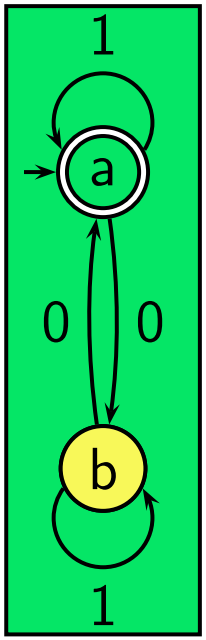

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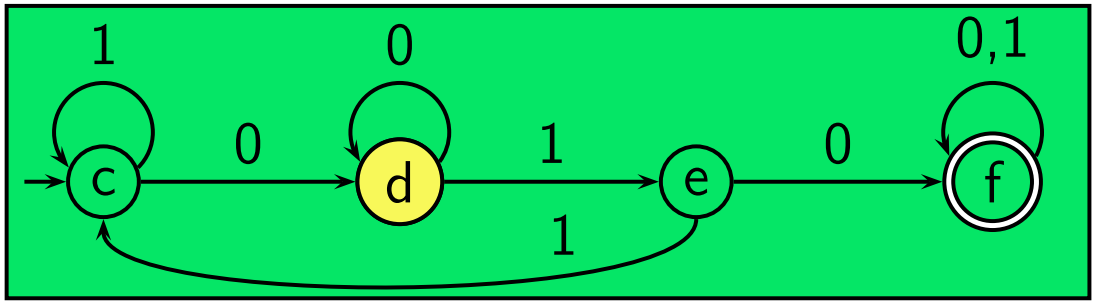

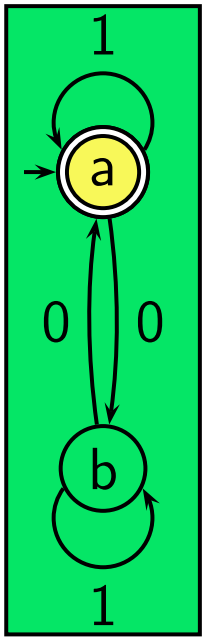

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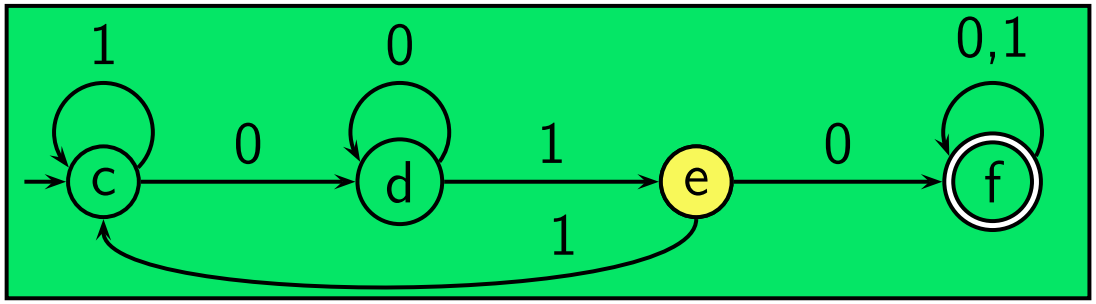

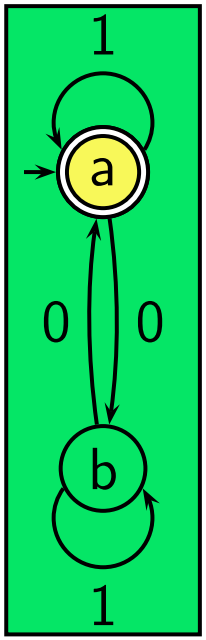

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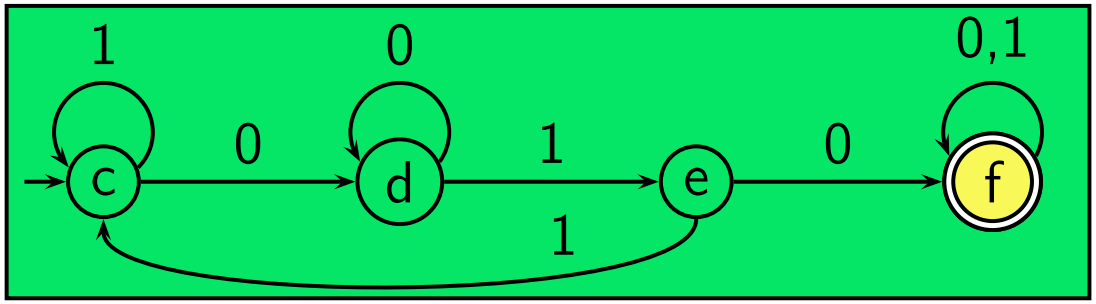

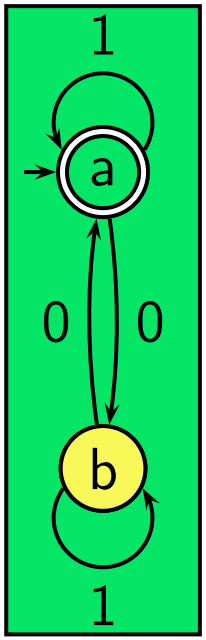

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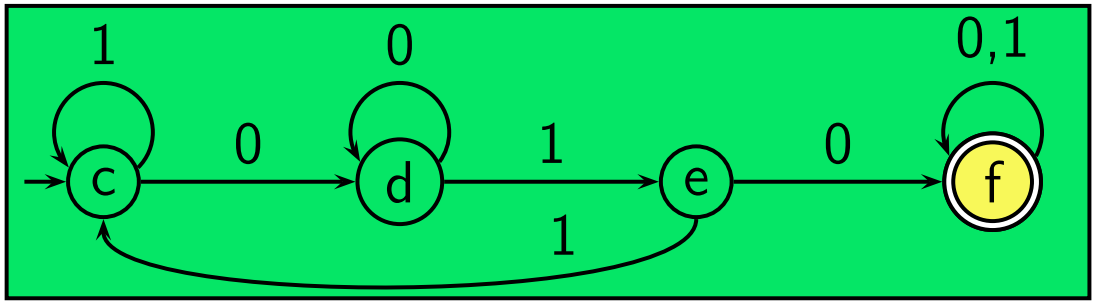

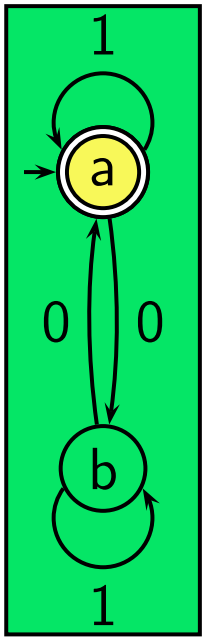

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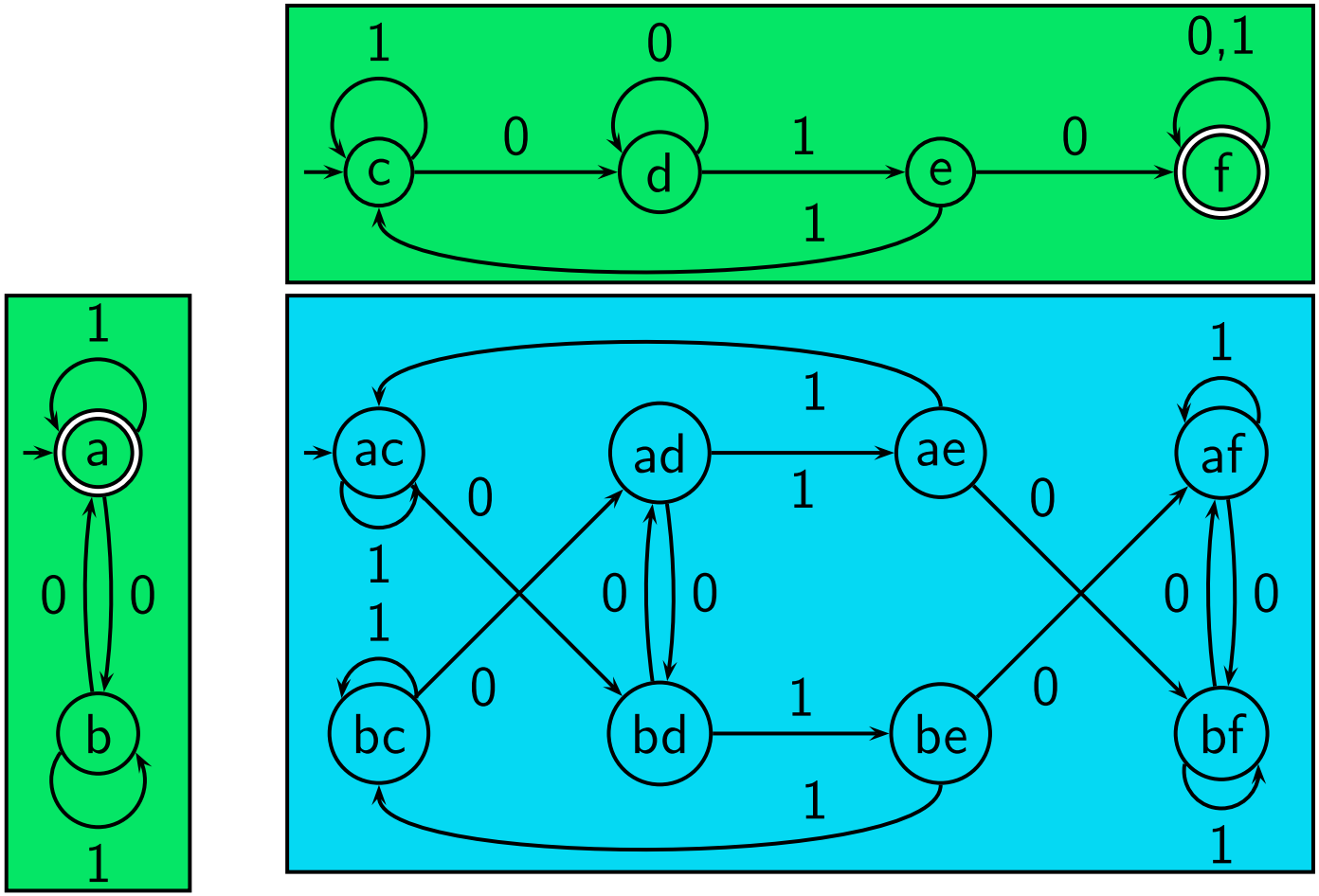

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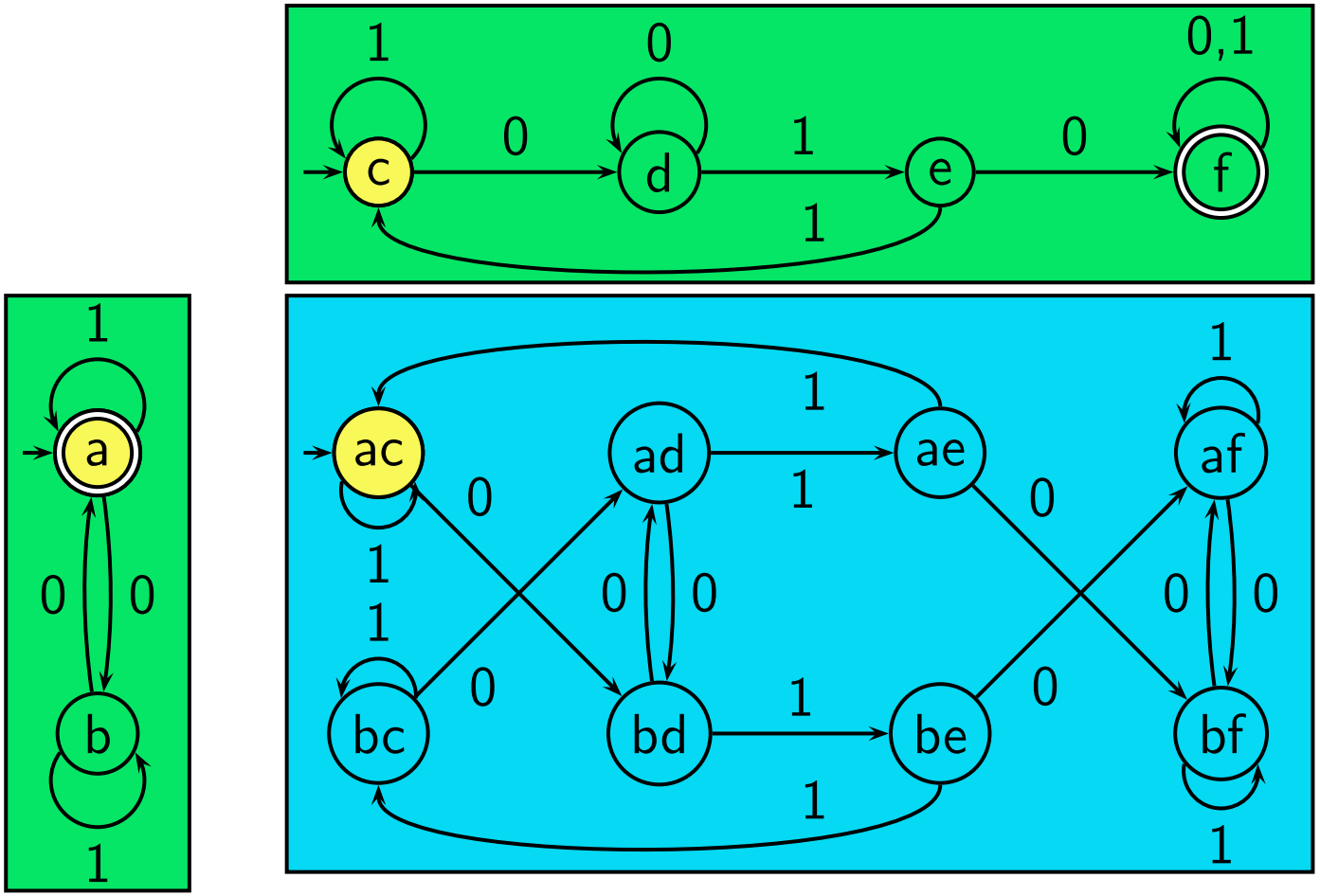

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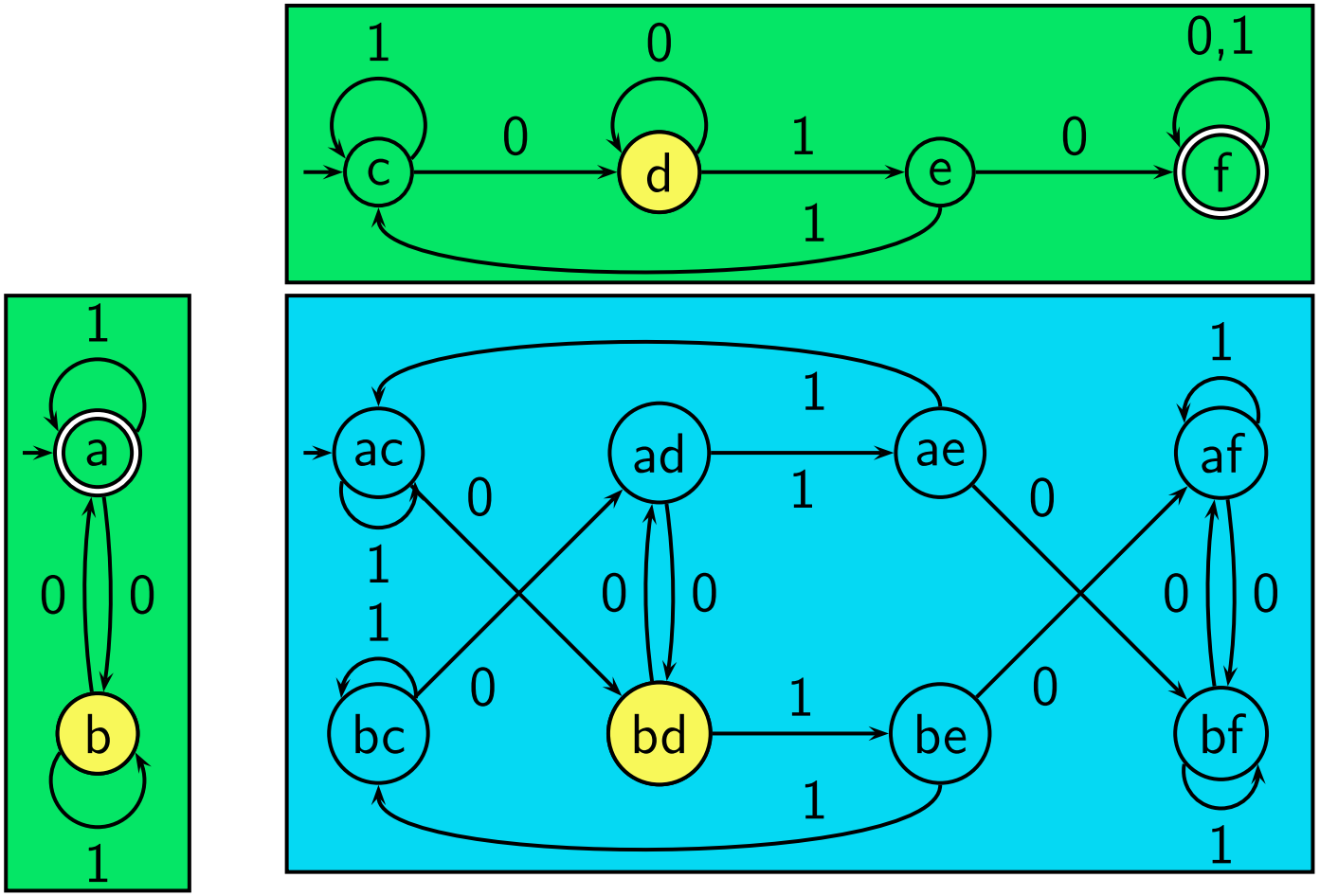

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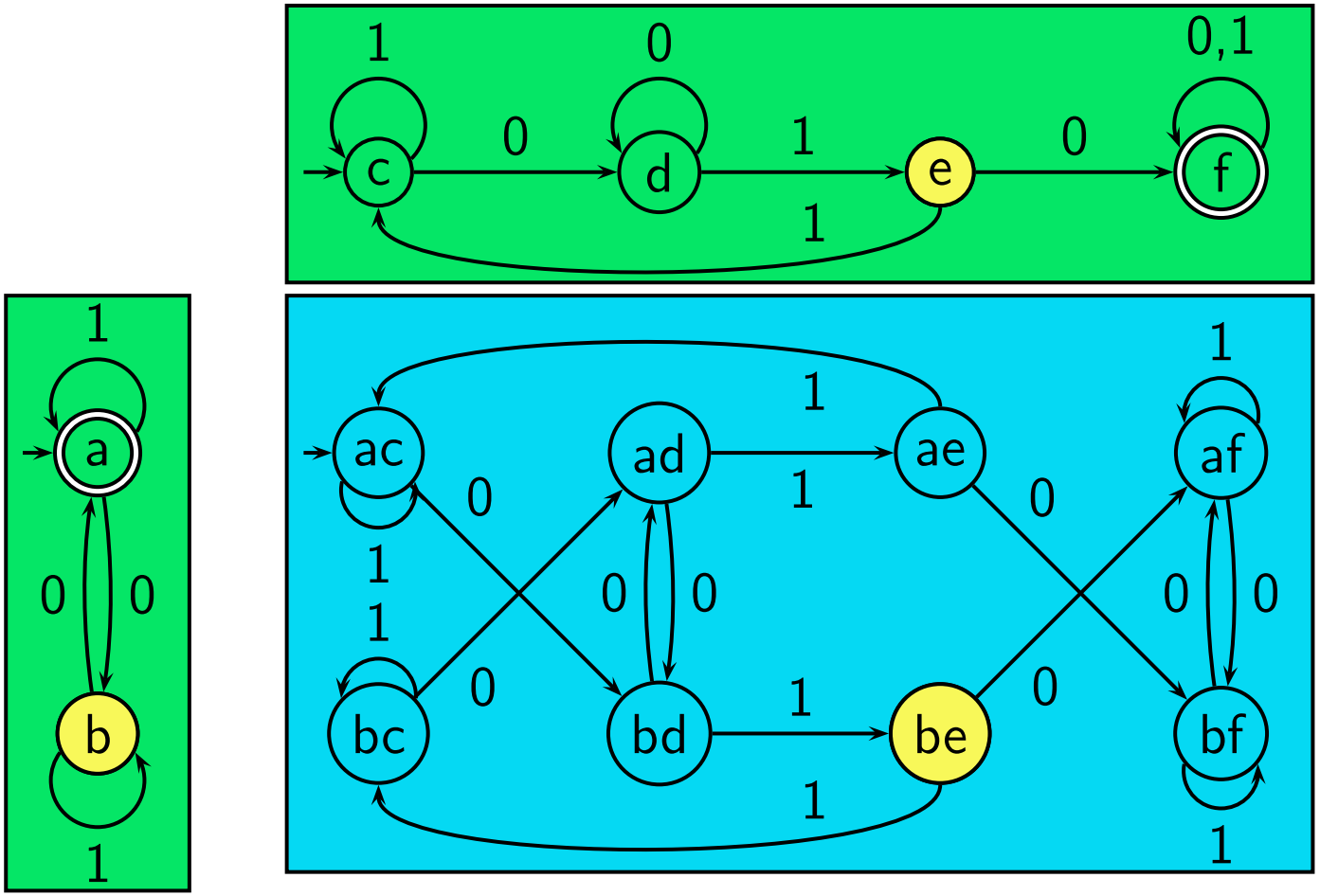

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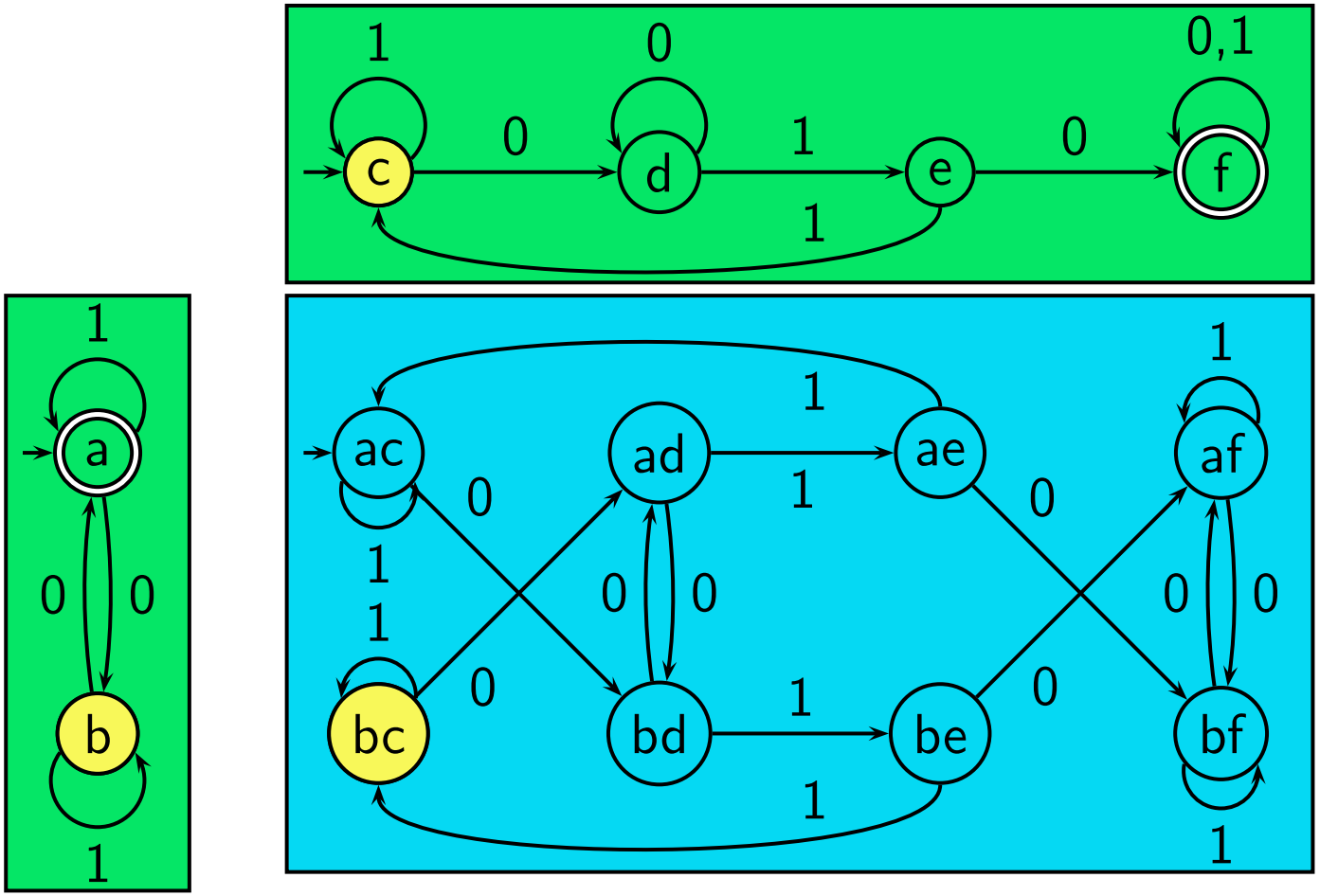
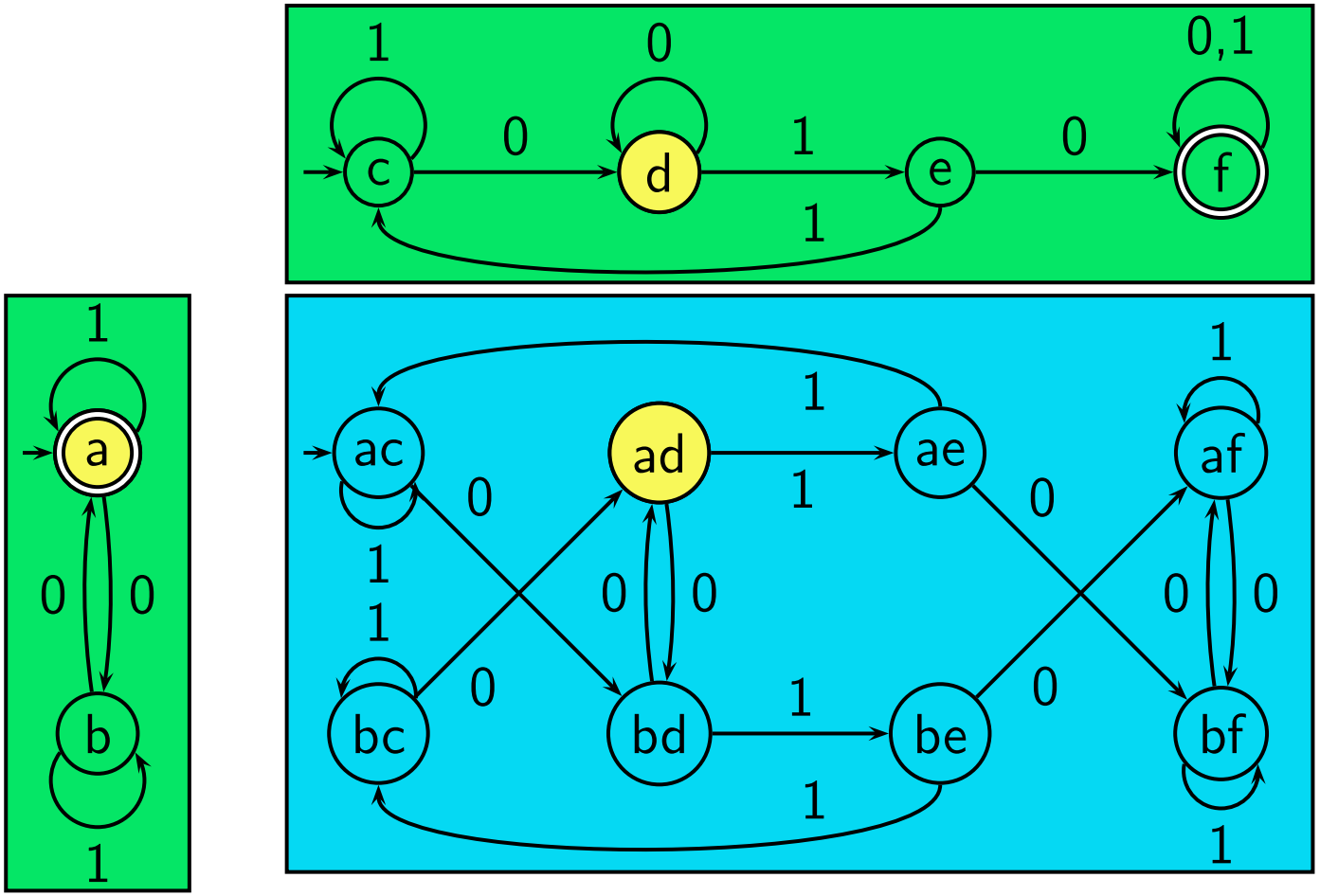


011|0100

Reminder: Product automaton

Product of 2 string automata

- "even number of zeros"
- "contains substring 00"



0110|100

Reminder: Product automaton

Product of 2 string automata

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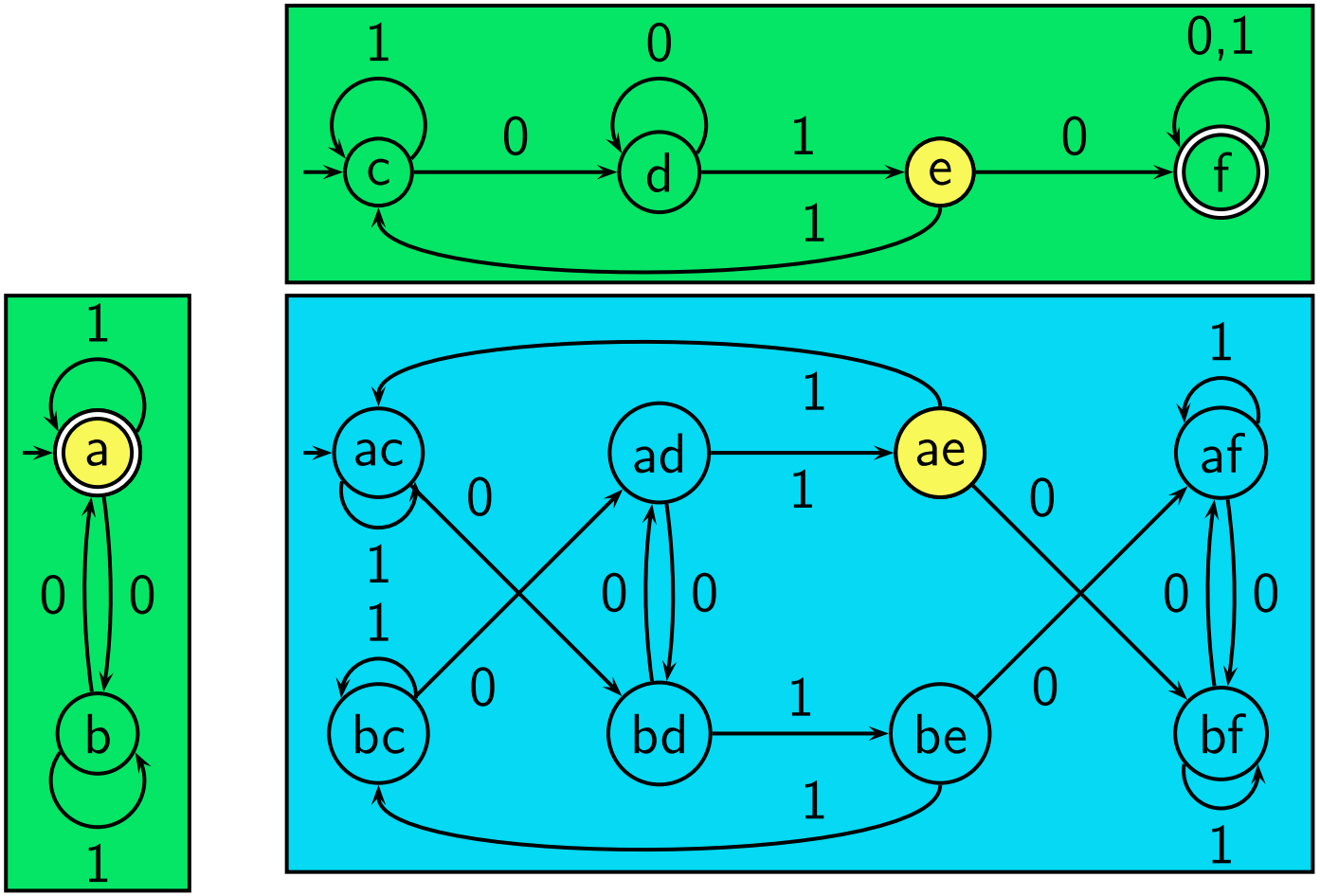

01101|00

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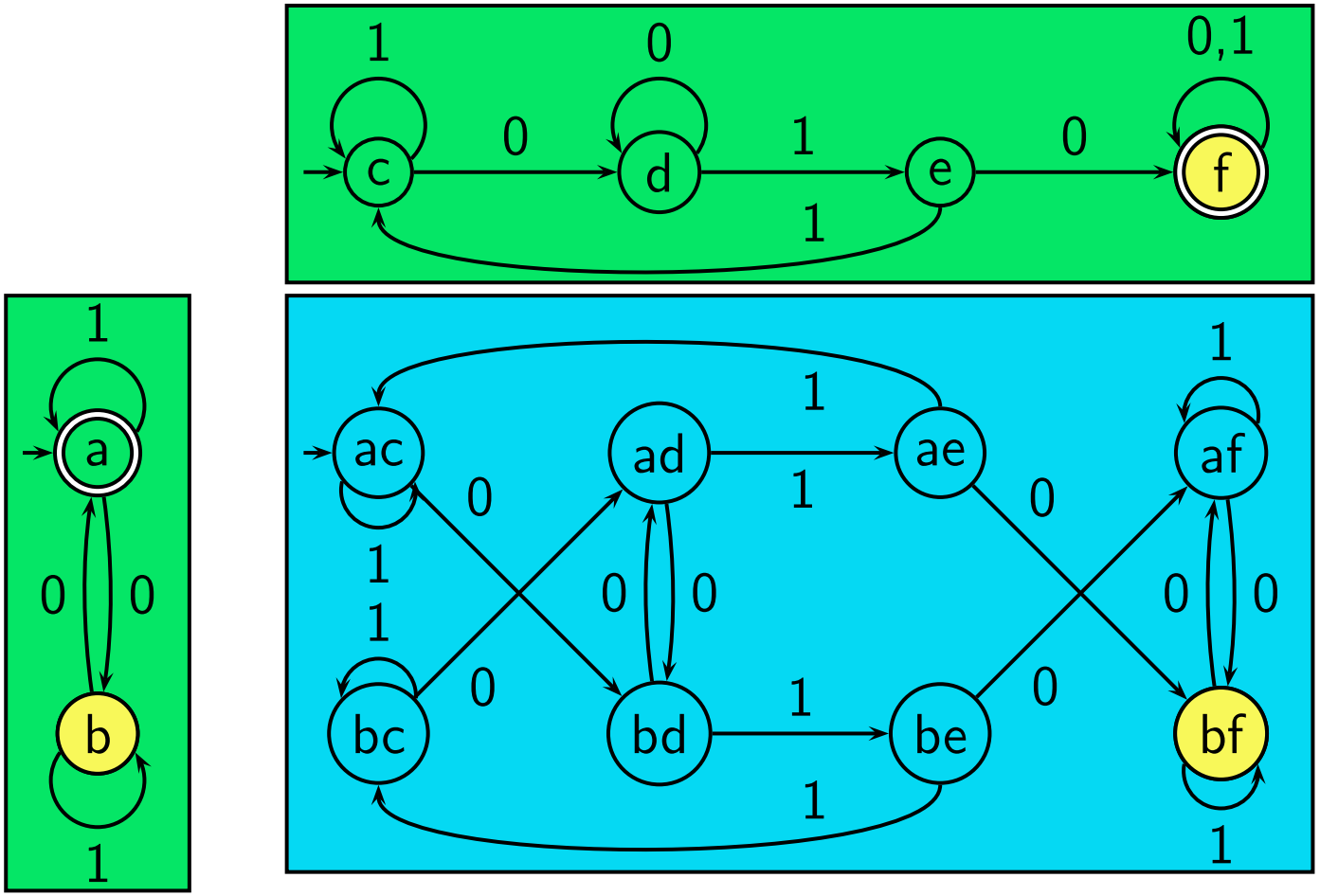

011010|0

Reminder: Product automaton

Product of 2 string automata

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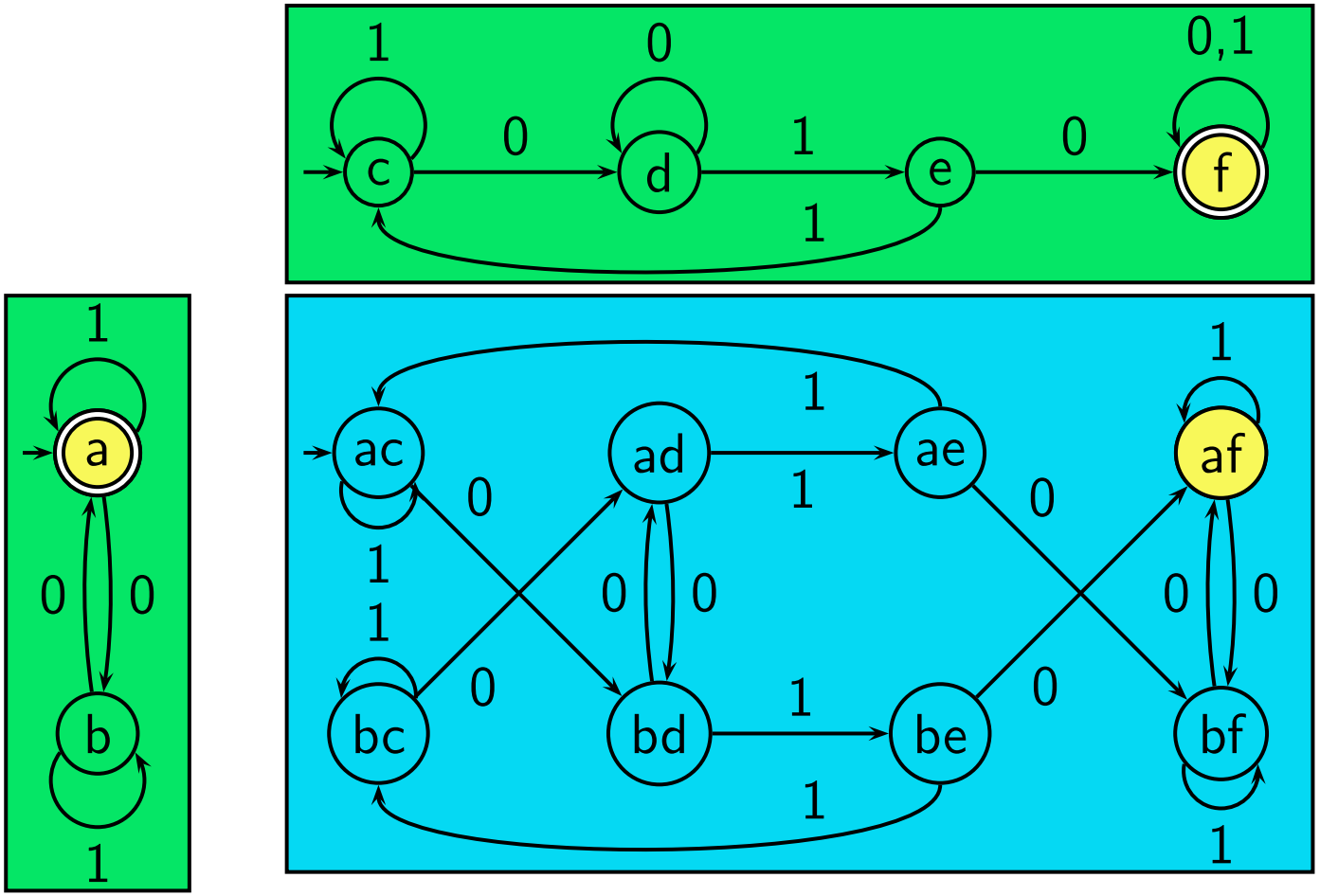


0110100

Containment: Complexity

Deterministic bottom-up tree automata

- Product automaton analogous as for string automata
  - Set of states: $Q_1 \times Q_2$
  - Transitions component-wise
- To check $L(\mathcal{A}_1) \subseteq L(\mathcal{A}_2)$:
  - Compute $\mathcal{B} = \mathcal{A}_1 \times \mathcal{A}_2$
  - Accepting states: $F_1 \times (Q_2 - F_2)$
  - Check whether $L(\mathcal{B}) = \emptyset$
  - If so, $L(\mathcal{A}_1) \subseteq L(\mathcal{A}_2)$ holds

Theorem

Complexity of Containment for deterministic bottom-up tree automata:

$$O(|\mathcal{A}_1| \times |\mathcal{A}_2|)$$

Containment: Complexity (cont.)

Non-deterministic automata

- Naive approach:
  - Make $\mathcal{A}_2$ deterministic (size: $O(2^{|\mathcal{A}_2|})$)
  - Construct product automaton

  $\Rightarrow$ Exponential time

Containment: Complexity (cont.)

Non-deterministic automata

- Naive approach:
  - Make $\mathcal{A}_2$ deterministic (size: $O(2^{|\mathcal{A}_2|})$)
  - Construct product automaton

  $\Rightarrow$ Exponential time

Unfortunately...

There is essentially no better way

Containment: Complexity (cont.)

Non-deterministic automata

- Naive approach:
  - Make $\mathcal{A}_2$ deterministic (size: $O(2^{|\mathcal{A}_2|})$)
  - Construct product automaton

  $\Rightarrow$ Exponential time

Unfortunately...

There is essentially no better way

Theorem [Seidl 1990]

Containment for non-deterministic tree automata is complete for **EXPTIME**

Det. Top-Down Automata: Non-Emptiness

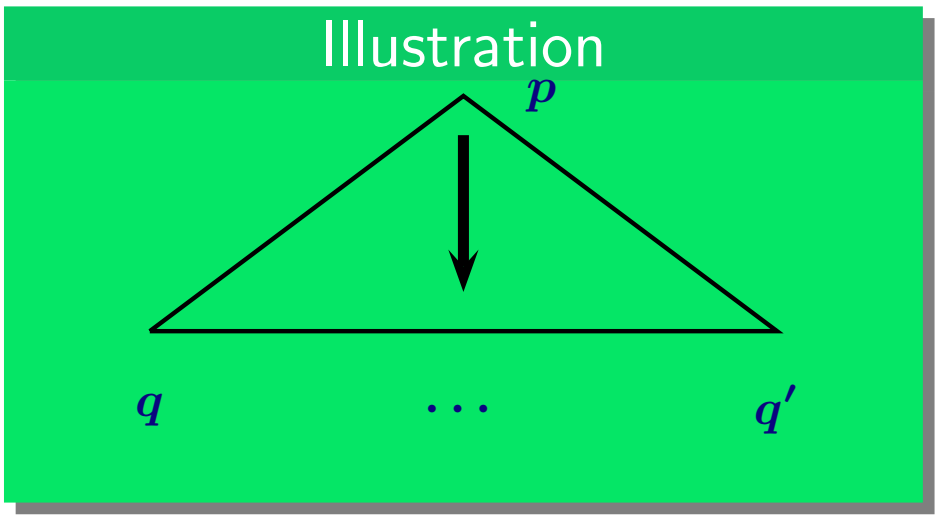
Theorem

Nonemptiness for deterministic top-down automata $\mathcal{A}$ can be checked in polynomial time

Proof idea

Check for each state p of $\mathcal{A}$ and each pair (q, q') of the leaves automaton $\mathcal{B}$:
Is there a tree t such that $\mathcal{A}$ starts from state p and obtains a leave string which brings $\mathcal{B}$ from q to q'?

Illustration



Det. Top-Down Automata: Containment

Theorem

Containment for deterministic top-down automata $\mathcal{A}$ can be checked in polynomial time

Proof idea

- Tree automata $\mathcal{A}_1$, $\mathcal{A}_2$ with leaves automata $\mathcal{B}_1, \mathcal{B}_2$
- Check
  - for each pair (p_1, p_2) of states of $\mathcal{A}_1$ and $\mathcal{A}_2$ and
  - for each two pairs (q_1, q_1') and (q_2, q_2') of $\mathcal{B}_1$ and $\mathcal{B}_2$, resp.:

  Is there a tree t such that for both $i = 1$, $i = 2$: T_i starts from state p_i and obtains a leave string which brings $\mathcal{B}_i$ from q_i to q_i'?

Summary

Complexities of basic algorithmic problems

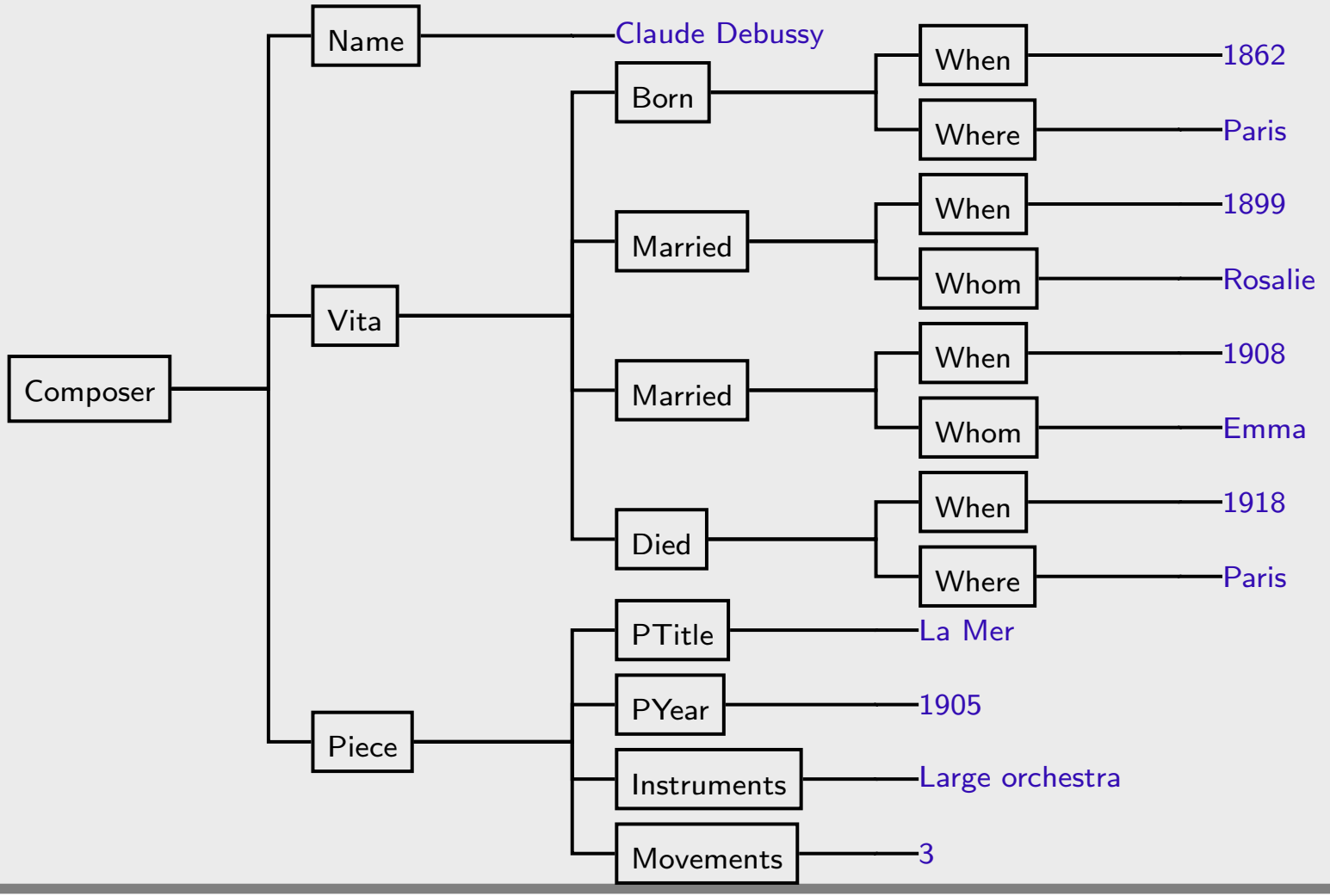
Model	Membership	Non-emptiness	Containment
Det. top-down TA	**LOGSPACE**	**PTIME**	**PTIME**
Det. bottom-up TA	**LOGDCFL**	**PTIME**	**PTIME**
Nondet. bottom-up TA	**LOGCFL**	**PTIME**	**EXPTIME**
Nondet. top-down TA	**LOGCFL**	**PTIME**	**EXPTIME**
Det. TWA	**LOGSPACE**	**PTIME** (∗)	**PTIME** (∗)
Nondet. TWA	**NLOGSPACE**	**PTIME** (∗)	**EXPTIME** (∗)

(∗: upper bounds)

Contents

From Ranked to Unranked Trees

Example Tree



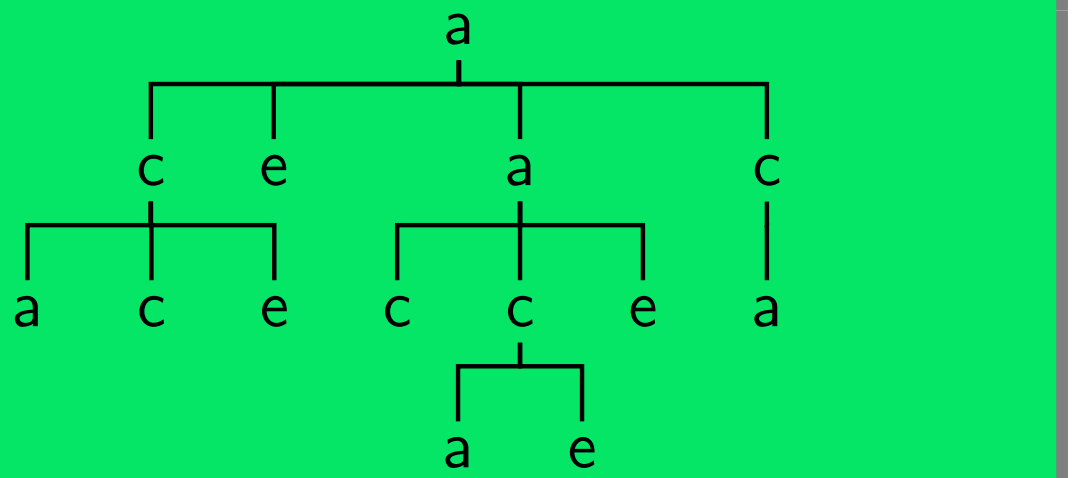
From Ranked to Unranked Trees

Agenda

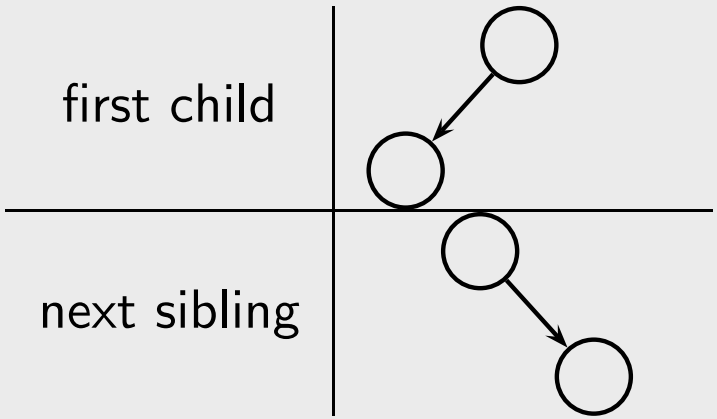
- Now we move from ranked to unranked trees
- There is a basic choice:
  - Either: we encode unranked trees as binary trees and go on with ranked automata
  - Or: we adapt the ranked automata models
- In both cases: not many surprises, most results remain

Encoding Unranked Trees as Binary Trees

Example: Unranked Tree

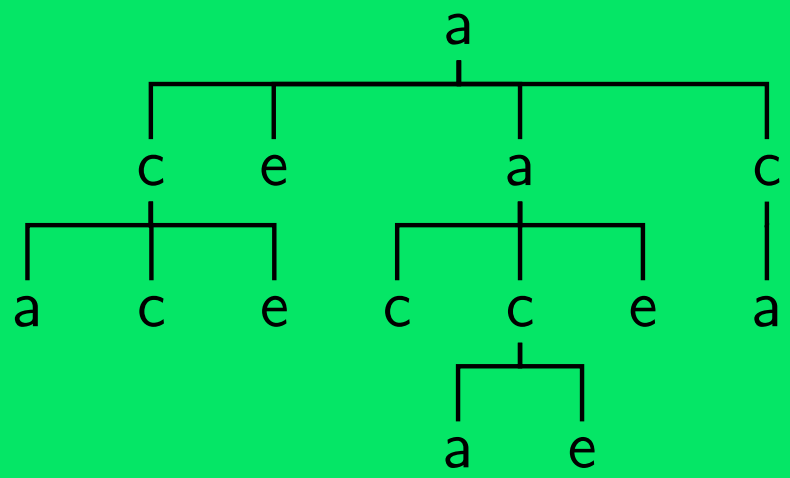


Encoding via ...

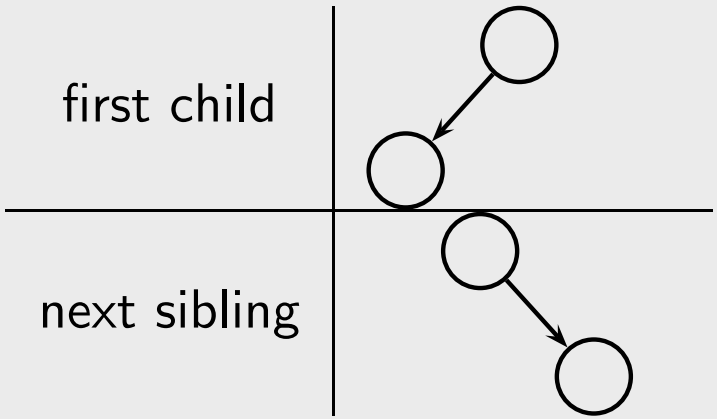


Encoding Unranked Trees as Binary Trees

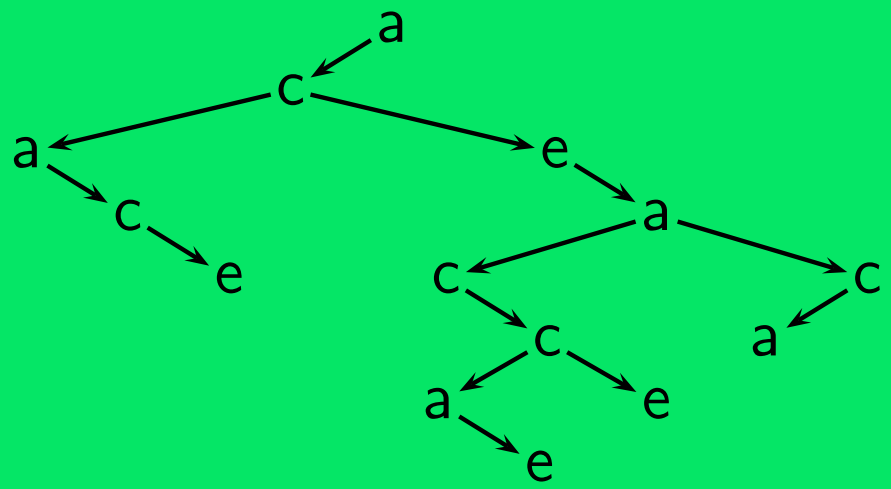
Example: Unranked Tree



Encoding via ...

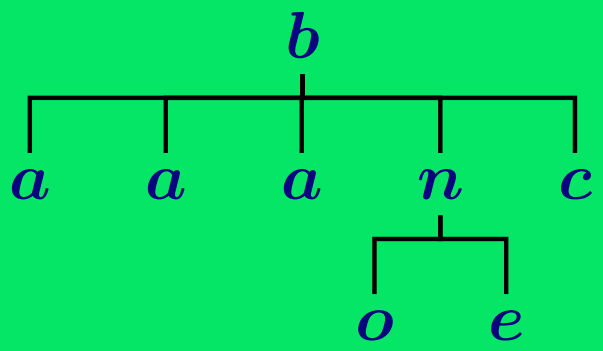


... as Binary Tree

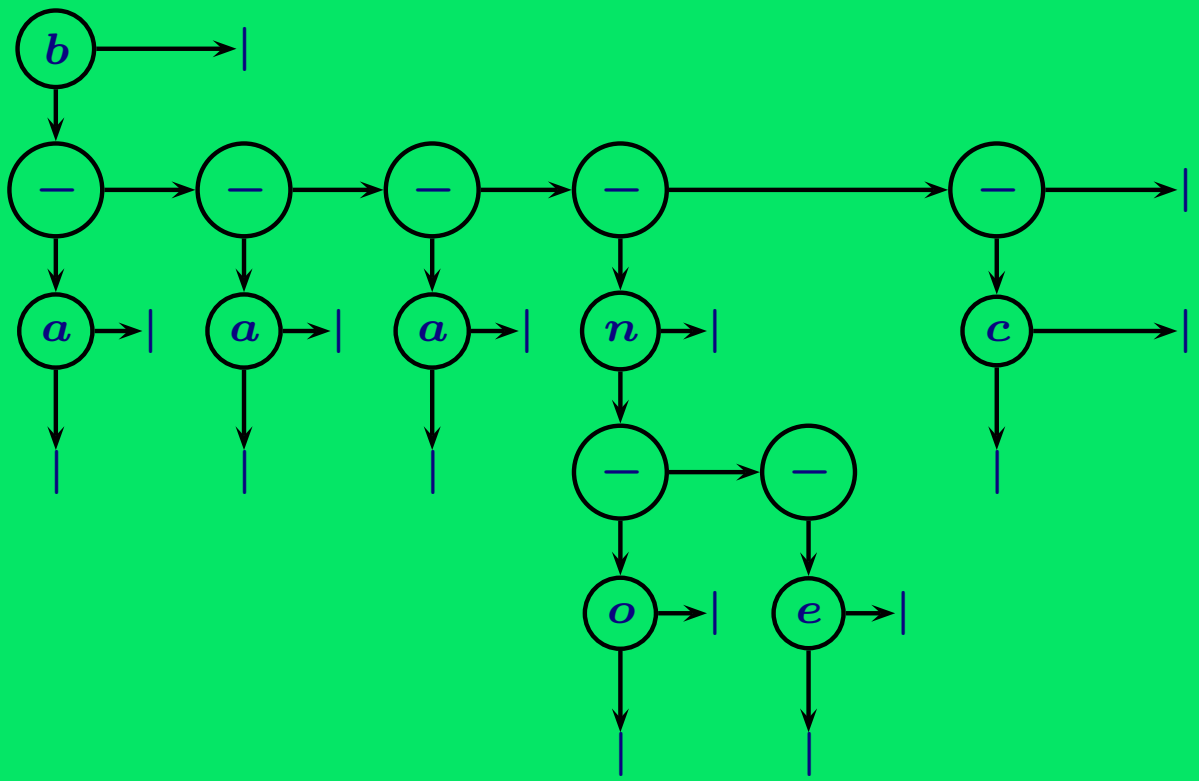


Encoding Unranked Trees as Binary Trees (cont.)

Example: Unranked Tree



... if path expressions matter (Milo,Suciu,Vianu 00)



Binary vs. unranked trees

Remark

- There are still other ways to encode unranked trees as binary trees

$\longrightarrow$ e.g., [Carme, Niehren, Tommasi 04]

- We consider automata for unranked trees next

Unranked Trees: Formal Definition

Definition

A **(finite) tree domain** V over $\mathbf{N}$ is a (finite) subset of $\mathbf{N}^*$, such that if $v \cdot i \in V$, where $v \in \mathbf{N}^*$ and $i \in \mathbf{N}$,

- then $v \in V$
- and $v \cdot (i-1) \in V$, if $i > 1$

Note

ε represents the root

Definition

A **labelled tree t** is a pair (V, λ), where V is a tree domain over $\mathbf{N}$, and λ is a function from V to the set Σ of labels.

Remark

XML tags can be captured by the set Σ of labels. But what about text?

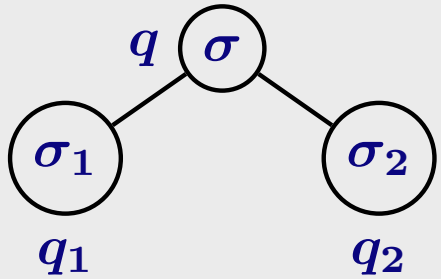
- This depends on the context
- E.g., for type checking, text is irrelevant.
- In many applications, the relevant information about text nodes can be represented by predicates, e.g., whether the `name` $=$ `'Debussy'`.

From Ranked to Unranked Tree Automata

Ranked trees

Transitions are described by finite sets:

$\delta(\sigma, q) = \{(q_1, q_2), (q_3, q_4), \ldots\}$



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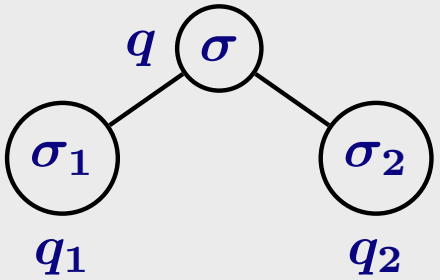
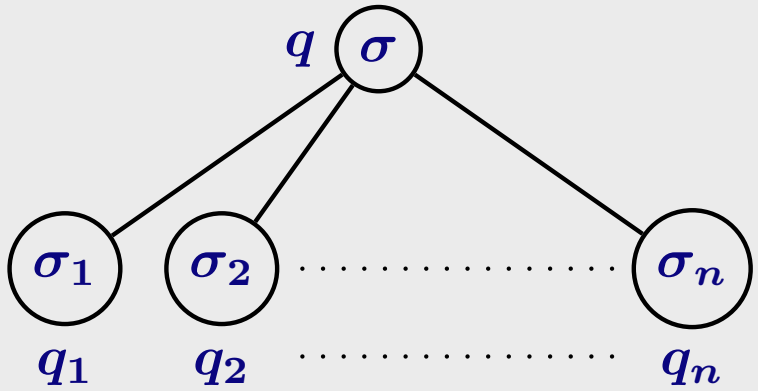


Unranked trees



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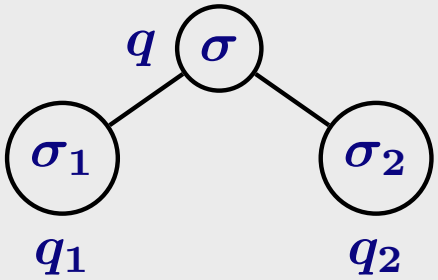
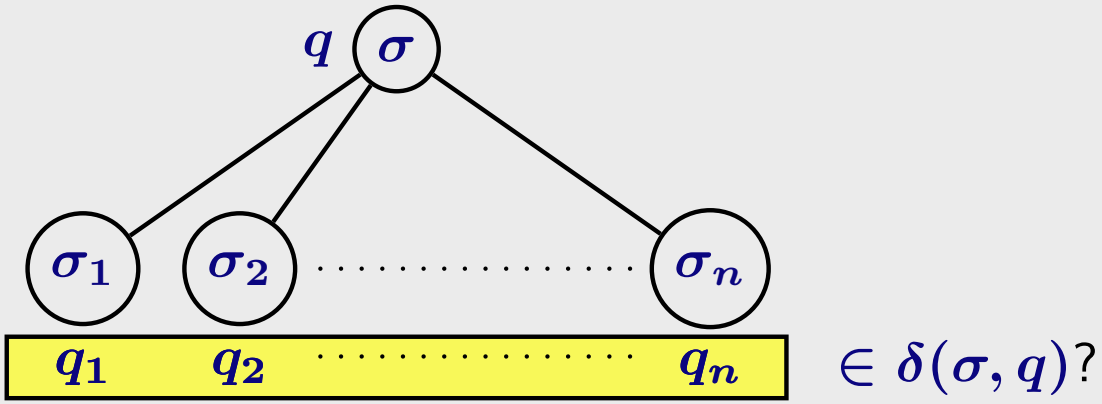


Unranked trees



From Ranked to Unranked Tree Automata

Ranked trees

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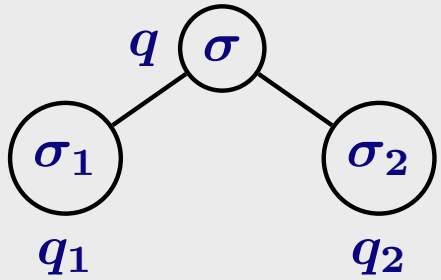
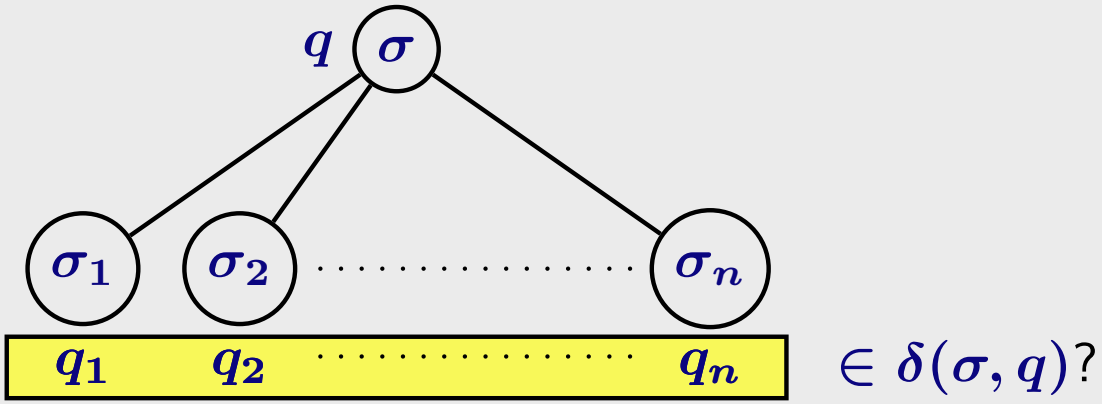


Unranked trees



$\in \delta(\sigma, q)$?

$\delta(\sigma, q)$

- For unranked trees, $\delta(\sigma, q)$ is a regular language
- $\delta(\sigma, q)$ can be specified by regular expression or finite string automaton

  [Brüggemann-Klein, Murata, Wood 2001]

Representation of $\delta(\sigma, q)$

Remark

- Representation of $\delta(\sigma, q)$ has influence on complexity
- Natural choice:
  - For nondeterministic tree automata:
    represent $\delta(\sigma, q)$ by NFAs or regular expressions
  - For deterministic tree automata:
    represent $\delta(\sigma, q)$ by DFAs

$\Rightarrow$ Same complexity results as for ranked trees

Regular sets of unranked trees

Theorem

The following are equivalent for a set L of unranked trees:

(a) L is accepted by a nondeterministic bottom-up automaton

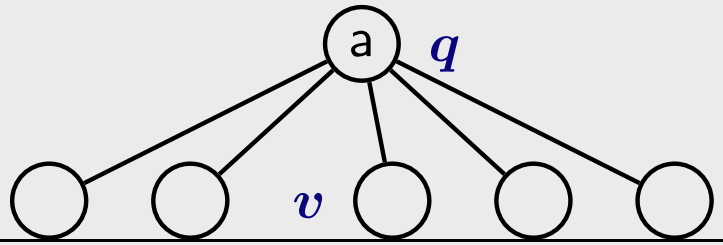
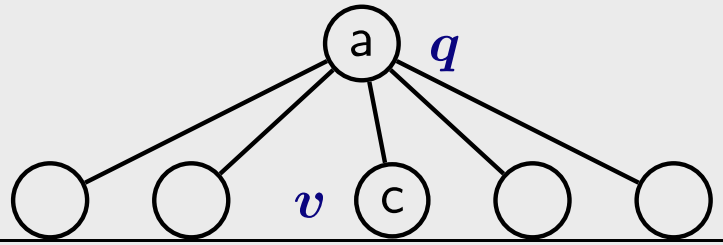
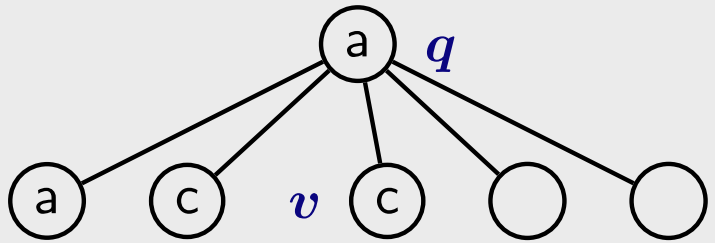
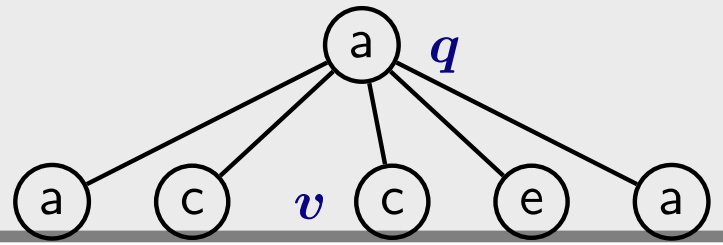
(b) L is accepted by a deterministic bottom-up automaton

(c) L is accepted by a nondeterministic top-down automaton

(d) L is characterized by an MSO-formula

Deterministic Top-Down Automata

State at v might depend on ...

state and symbol of parent	
state and symbol of parent and symbol of v	 simple
state and symbol of parent and symbols at v and its left siblings	 left-siblings aware
state and symbol of parent and symbols at v and its siblings	

Checking Existence of Paths

Fact

A simple deterministic top-down automaton can check the existence of vertical paths with regular properties

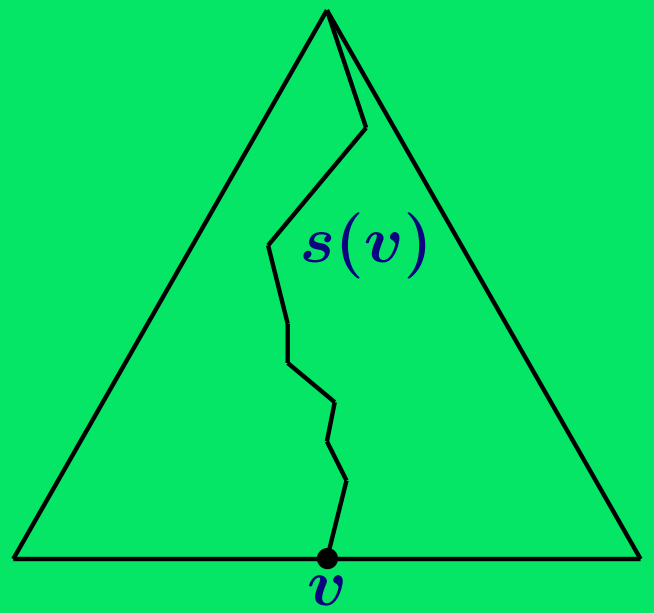
Construction

- For a node v let $s(v)$ denote the sequence of labels from the root to v
- Let $\mathcal{A}$ be a deterministic string automaton
- $\mathcal{A}' :=$ top-down automaton which takes at v state of $\mathcal{A}$ after reading $s(v)$

$\Rightarrow$ $\mathcal{A}'$ is deterministic

- There is a path from the root to a leaf v with $s(v) \in L(\mathcal{A})$ iff $\mathcal{A}'$ assumes at least one state from F at a leave

Illustration



Streaming XML

Similar construction used for `XPath` evaluation on streams [Green et al. 2003]

Contents

Sequential Automata on Unranked Trees

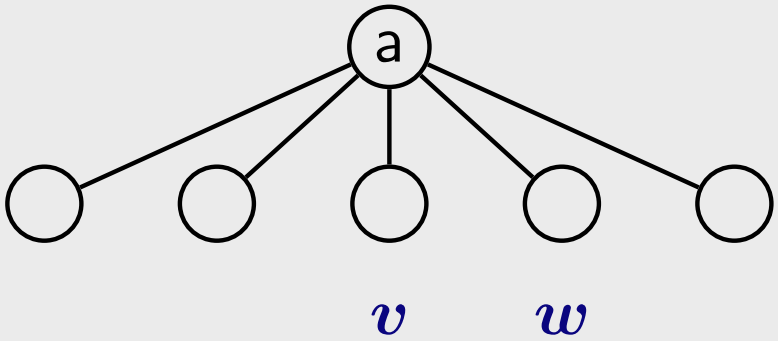
Generalization of Tree-Walk Automata

Allowed transitions:
- `Go up`
- `Go to first child`
- `Go to left sibling`
- `Go to right sibling`

$\longrightarrow$ Caterpillar automata [Brüggemann-Klein, Wood 2000]

Basic design choice

Should a transition to a sibling be aware of the label of the parent?



Contents

Document Automata

A third kind of automata for XML

- **Document automata** are string automata reading XML documents as text
- Tags are represented by symbols from a given alphabet
- Variants:
  - Finite document automata
  - Pushdown document automata
- Useful especially in the context of streaming XML

Theorem [Segoufin, Vianu 02]

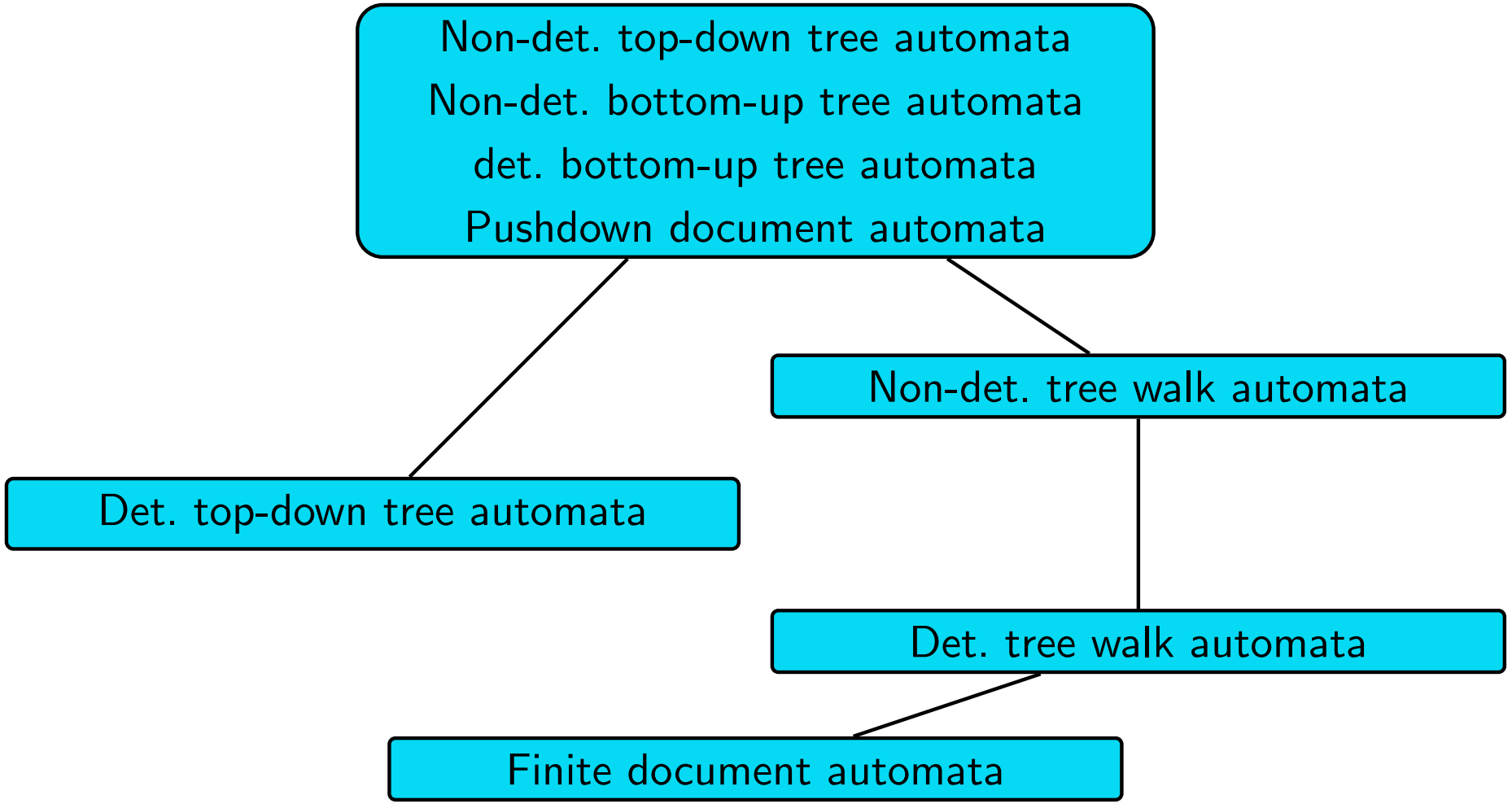
- Regular languages of XML-trees can be recognized by deterministic push-down document automata.
- Depth of push-down is bounded by depth of tree

Summary: Unranked Tree Automata

Summary

- Moving from ranked to unranked automata requires some adaptations
- Transitions can be defined with regular string languages $\delta(\sigma, q)$
- By and large, things work smoothly
- In particular:
  - there is an equally robust notion of regular tree languages
  - The complexities are the same as for ranked automata (if the sets $\delta(\sigma, q)$ are represented in a sensible way)

Refined Overview of Models



Contents

DTDs

Example DTD

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
        Instruments, Movements)>
    ]>
```

Some Facts

- DTDs $\equiv$ generalized context-free grammars

$\longrightarrow$ [Berstel,Boasson 00] provide characterizations

- Additional restriction: one-unambiguous

One-unambiguous Regular Expressions

Definition: One-unambiguous Regular Expression

- Let r be a regular expression
- $r \mapsto r'$: number the symbols of r from left to right
- $w \in L(r) \longleftrightarrow$ there is a numbered string $w' \in L(r')$
- r is **one-unambiguous** if

$$ux_iv \in L(r'),\ uy_jw \in L(r'),\ i \neq j \Rightarrow x \neq y$$

Example

- $(a+b)^*ac+c \mapsto (a_1+b_2)^*a_3c_4+c_5$
- $babbac \in L(r)$ and $b_2a_1b_2b_2a_3c_4 \in L(r')$
- $(a+b)^*ac+c$ is not one-unambiguous because $b_2b_2a_3c_4 \in L(r')$ and $b_2b_2a_1a_3c_4 \in L(r')$
- $(b^*a)^*c$ is one-unambiguous

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- $babbac \in L(r)$ and $b_2a_1b_2b_2a_3c_4 \in L(r')$
- $(a+b)^*ac+c$ is not one-unambiguous because $b_2b_2\,a_3\,c_4 \in L(r')$ and $b_2b_2\,a_1\,a_3c_4 \in L(r')$
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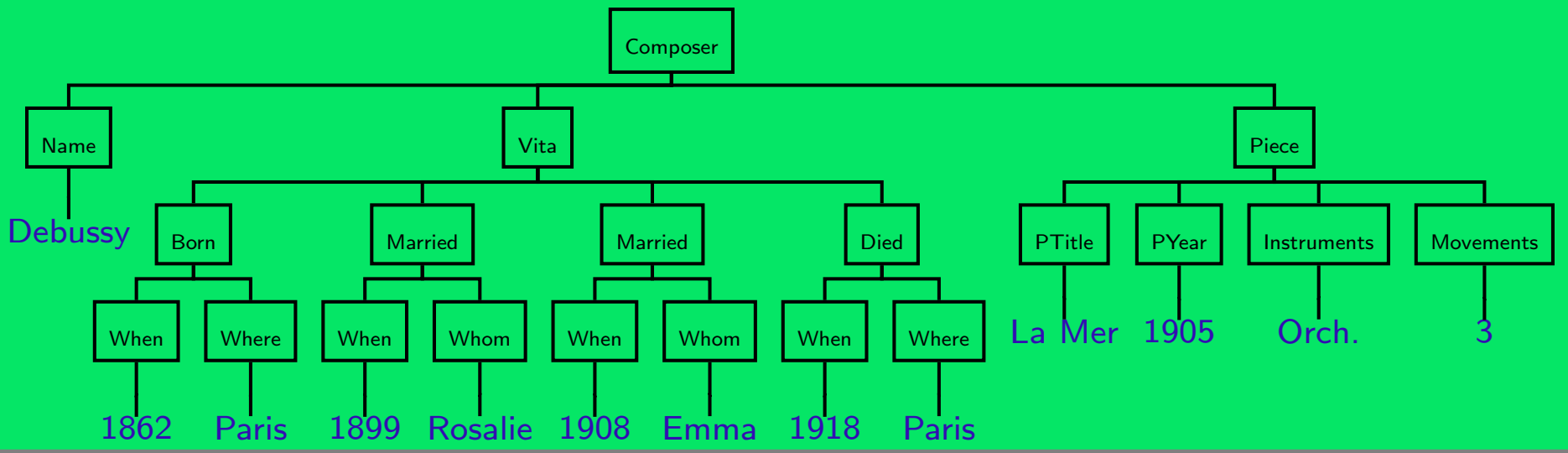
- $(a+b)^*ac+c$ is not one-unambiguous because $b_2b_2\,a_3\,c_4 \in L(r')$ and $b_2b_2\,a_1\,a_3c_4 \in L(r')$
- $(b^*a)^*c$ is one-unambiguous

Restriction

- Expressions in DTDs have to be one-unambiguous
- Inherited from SGML

Validation wrt a DTD

Example Tree



Example DTD

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
         Instruments, Movements)>
    ]>
```

Validation Algorithm

For each node:

Check that the children are ok wrt the parent's rule

Validation wrt a DTD (cont.)

Observation

- Validation wrt DTDs is a very simple task
- Can be done by
  - Bottom-up automata
  - Deterministic top-down automata
    (if siblings contribute to new state)
  - Deterministic tree-walk automata:
    Just a depth-first left-to-right traversal
- In particular: Validation possible in linear time
  during one pass through the document
  (1-pass validation)
- Further: DTDs are always satisfiable

Containment of DTDs

Lemma [Martens, Neven, Sch. 04]

Containment of DTDs with regular expressions from R is in C

$\Longleftrightarrow$

Containment of regular expressions from R is in C

Corollary

Containment of DTDs (with one-unambiguous regular expressions) is in **PTIME**

Proof sketch

- One-unambiguous regular expressions have deterministic automata of linear size

$\Rightarrow$ Containment of regular expressions r_1, r_2 by product automaton of size $O(|r_1||r_2|)$

Containment of DTDs (cont.)

Question

What if the requirement of being one-unambiguous is dropped?

A classical result

Theorem [Stockmeyer, Meyer 71]

Containment and Equivalence for regular expressions on strings are complete for **PSPACE**

Corollary

Containment of DTDs (with unrestricted regular expressions) is **PSPACE**-complete

Theorem [Martens, Neven, Sch. 04]

Containment and Equivalence for regular expressions are

- **coNP**-complete for concatenations of a, b, c and a^*, b^*, c^*
- **coNP**-complete for concatenations of a, b, c and $a?, b?, c?$
- **PSPACE**-complete for concatenations of a, b, c and $(a^* + b^* + \cdots + c^*)$

Contents

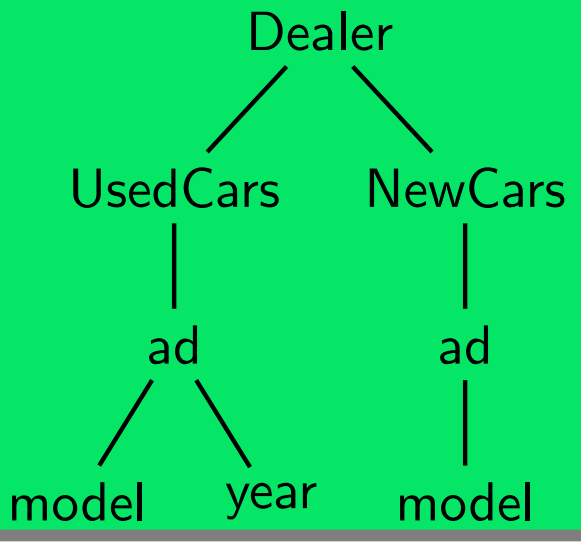
Weakness of DTDs

A classical example

```
<!DOCTYPE Dealer [
    <!ELEMENT Dealer (UsedCars NewCars)>
    <!ELEMENT UsedCars (ad*)>
    <!ELEMENT NewCars (ad*)>
    <!ELEMENT ad ((model, year) | model)> ]>
```

Intention

Intention:



Observation

- Elements with the same name may have different structure in different contexts

$\longrightarrow$ It would be nice to have types for elements

$\longrightarrow$ Specialized DTDs

Specialized DTDs

Definition: [Papakonstantinou, Vianu 2000]

A **specialized DTD** (SDTD) over alphabet Σ is a pair (d, μ), where

- d is a DTD over the alphabet Σ' of **types**
- $\mu : \Sigma' \longrightarrow \Sigma$ maps types to tag names

Note

Concerning the name:

"specialized" refers to types, not to DTDs

Example

Dealer $\longrightarrow$ UsedCars NewCars	μ(Dealer) = Dealer
UsedCars $\longrightarrow$ adUsed*	μ(UsedCars) = UsedCars
NewCars $\longrightarrow$ adNew*	μ(NewCars) = NewCars
adUsed $\longrightarrow$ model year	μ(adUsed) = ad
adNew $\longrightarrow$ model	μ(adNew) = ad

A Further Example

Example: SDTD for Boolean circuit trees

$$
\begin{aligned}
\text{1-AND} &\longrightarrow (\text{1-OR} \mid \text{1-AND} \mid \text{1-leaf})^* \\
\text{1-OR} &\longrightarrow .^* \ (\text{1-OR} \mid \text{1-AND} \mid \text{1-leaf}) \ .^* \\
\text{0-AND} &\longrightarrow .^* \ (\text{0-OR} \mid \text{0-AND} \mid \text{0-leaf}) \ .^* \\
\text{0-OR} &\longrightarrow (\text{0-OR} \mid \text{0-AND} \mid \text{0-leaf})^* \\
\text{1-leaf} &\longrightarrow \epsilon \\
\text{0-leaf} &\longrightarrow \epsilon
\end{aligned}
$$

Tag	μ(Tag)
1-AND	AND
0-AND	AND
1-OR	OR
0-OR	OR
1-leaf	1
0-leaf	0

Specialized DTDs (cont.)

Observation

- A naive validation by exhaustively trying all possible functions μ requires exponential time
- But help comes from automata...
-
-

Specialized DTDs (cont.)

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Specialized DTDs (cont.)

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Specialized DTDs (cont.)

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Theorem

Specialized DTDs capture exactly the regular tree languages

Validation and Typing

Definition: (Validation)

Given: Specialized DTD d, tree t

Qeustion: Is t valid wrt d?

Definition: (Typing)

Given: Specialized DTD d, tree t

Output: Consistent type assignment for the nodes of t

Facts

- Specialized DTDs $\equiv$ regular tree languages

$\longrightarrow$ Validation in linear time by deterministic push-down automata

- Typing in linear time (Bottom-up automaton) the document
- Satisfiability $\equiv$ Non-emptiness of tree automata: **PTIME**

Restrictions of Schemas

Restricted Schemas

(Murata, Lee, Mani 2001) introduced* restrictions on specialized DTDs to ensure efficient validation (*: in a slightly different framework)

- Two types b, b' **compete** if $\mu(b) = \mu(b')$
- A specialized DTD is **single-type** if no competing types occur in the same rule (e.g., $a \longrightarrow bcb'$ is not single-type)
- A specialized DTD is **restrained-competition** if no rule allows strings wbv, $wb'v'$ with competing types b, b' (e.g., $a \longrightarrow c(b + d^*b')$ is not restrained-competition)
- The authors argue that XML-Schema roughly corresponds to single-type SDTDs

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Schema Containment

Schema Containment

Given: Schemas d_1, d_2

Question: Is $L(d_1) \subseteq L(d_2)$?

Observations

- Important, e.g., for data integration
- Recall: Specialized DTDs are essentially non-deterministic tree automata

$\Rightarrow$ Containment of specialized DTDs is in **EXPTIME**

- But the restricted forms have lower complexity
- Complexity of containment depends on the allowed regular expressions

Schema Containment: Complexity

Results (partly from [Martens, Neven, Sch. 04])

Schema type	unrestricted	deterministic expressions
DTDs	**PSPACE**	**PTIME**
single-type SDTDs	**PSPACE**	**PTIME**
restrained-competition SDTDs	**PSPACE**	**PTIME**
unrestricted SDTDs	**EXPTIME**	**EXPTIME**

Observations

- For unrestricted SDTDs the complexity is dominated by tree automata containment
- For the others it is dominated by the sub-task of checking containment for regular expressions

Schema Containment: Complexity

Observations (cont.)

- ... for the others it is dominated by the sub-task of checking containment for regular expressions
- Actually, this observation can be made more precise

Theorem [Martens, Neven, Sch. 04]

For a class $\mathcal{R}$ of regular expressions and a complexity class $\mathcal{C}$, the following are equivalent

(a) The containment problem for $\mathcal{R}$ expressions is in $\mathcal{C}$.

(b) The containment problem for DTDs with regular expressions from $\mathcal{R}$ is in $\mathcal{C}$.

(c) The containment problem for single-type SDTDs with regular expressions from $\mathcal{R}$ is in $\mathcal{C}$.

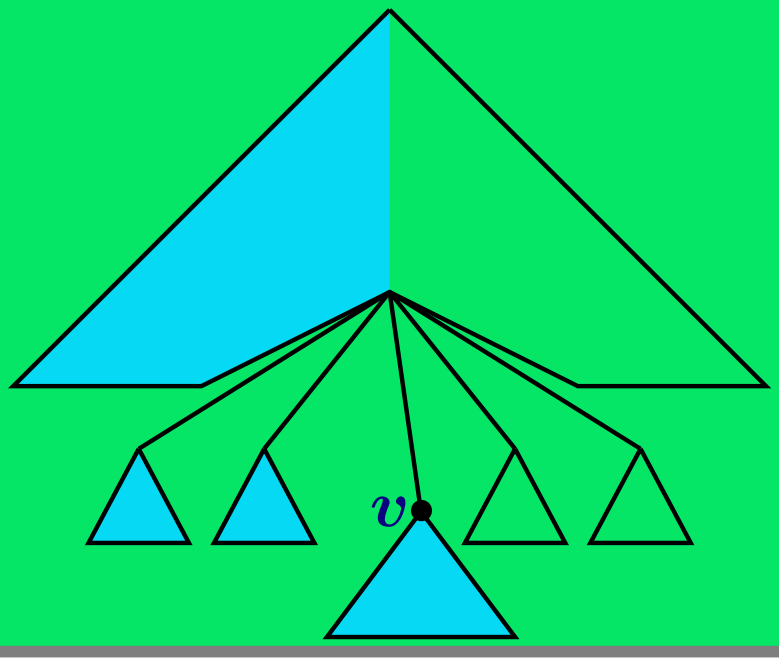
Contents

Typing (cont.)

Observations

- Type of a node $\equiv$ state of deterministic bottom-up automaton
- Deterministic push-down automaton can assign types during 1 pass
- But the type of a node v is determined after visiting its subtree
-

...after visiting subtree

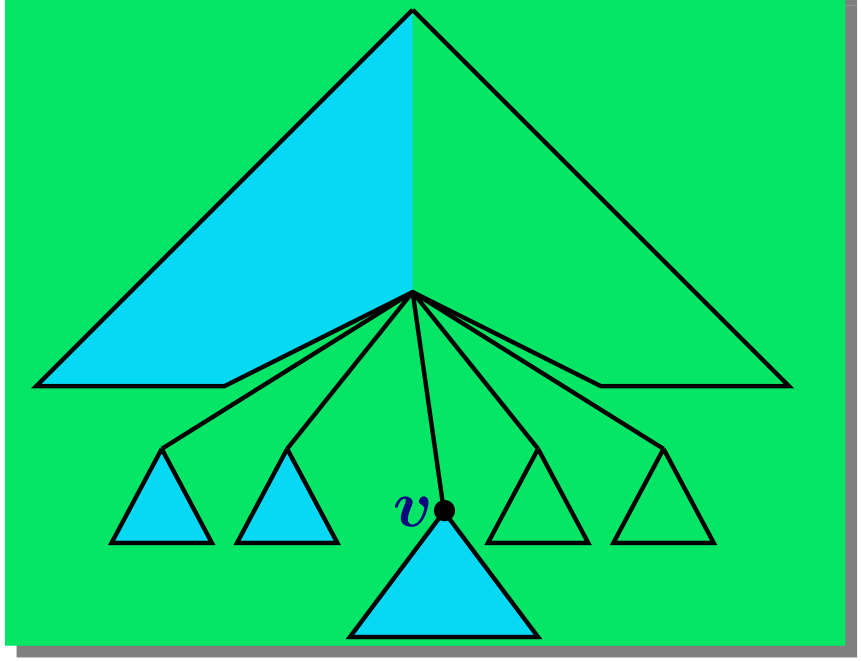


Typing (cont.)

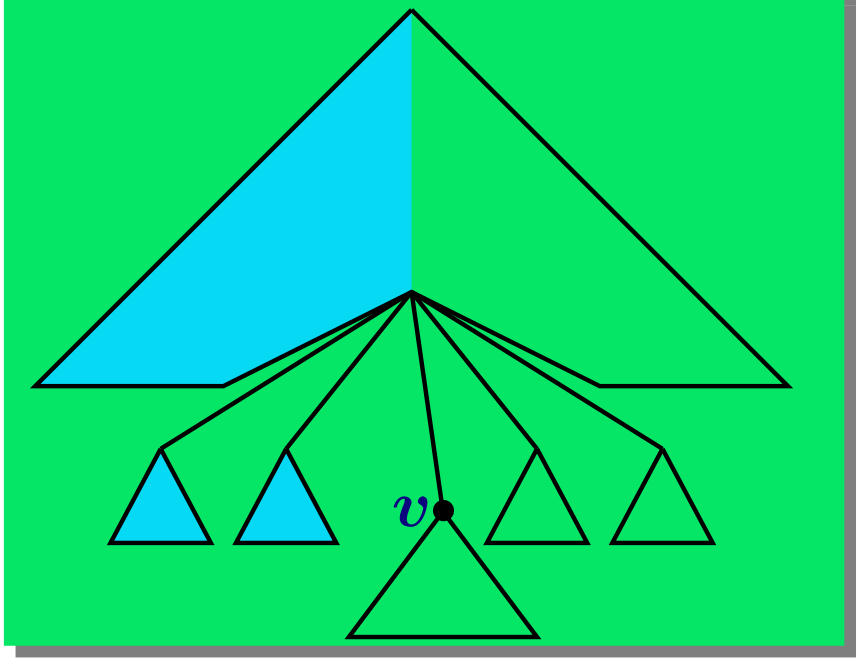
Observations

- Type of a node $\equiv$ state of deterministic bottom-up automaton
- Deterministic push-down automaton can assign types during 1 pass
- But the type of a node v is determined after visiting its subtree
- 1-pass preorder typing:
  determine type of v before visiting the subtree of v

...after visiting subtree



...before visiting subtree

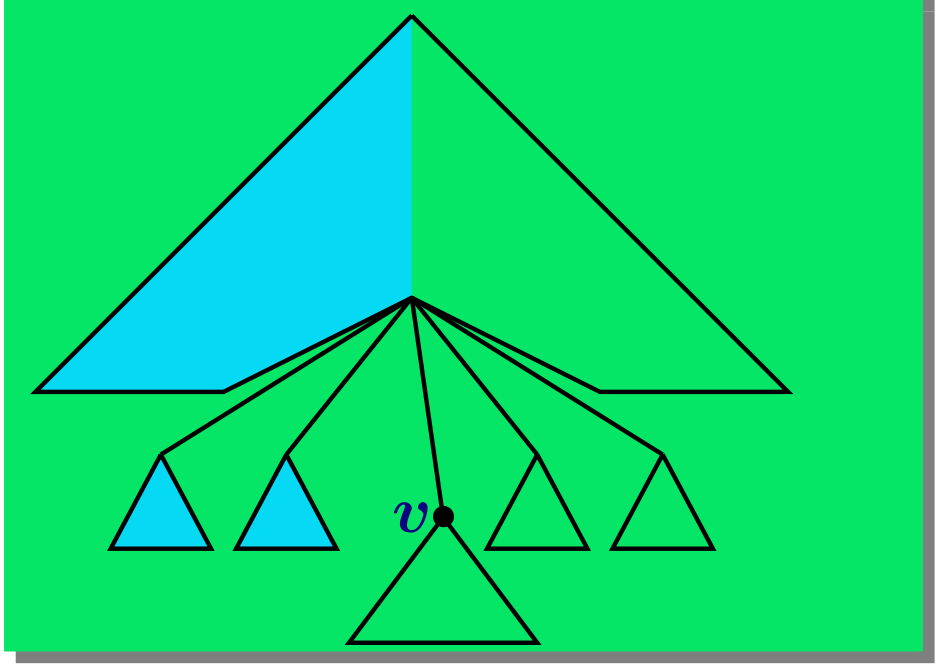


1-Pass Preorder Typing

Question

When would it be important to know the type of v before visiting the subtree of v?

...before visiting subtree



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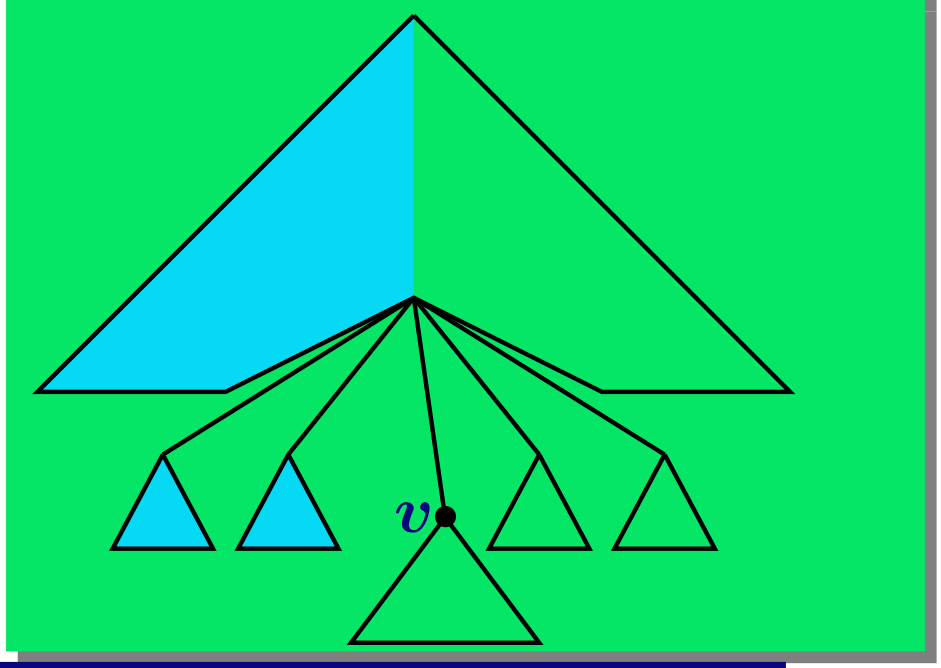

Answer

Whenever the processing proceeds in document order, e.g.:

- Streaming XML: Typing as the first operator in a pipeline
- SAX-based processing

1-Pass Preorder Typing

Question

When would it be important to know the type of v before visiting the subtree of v?

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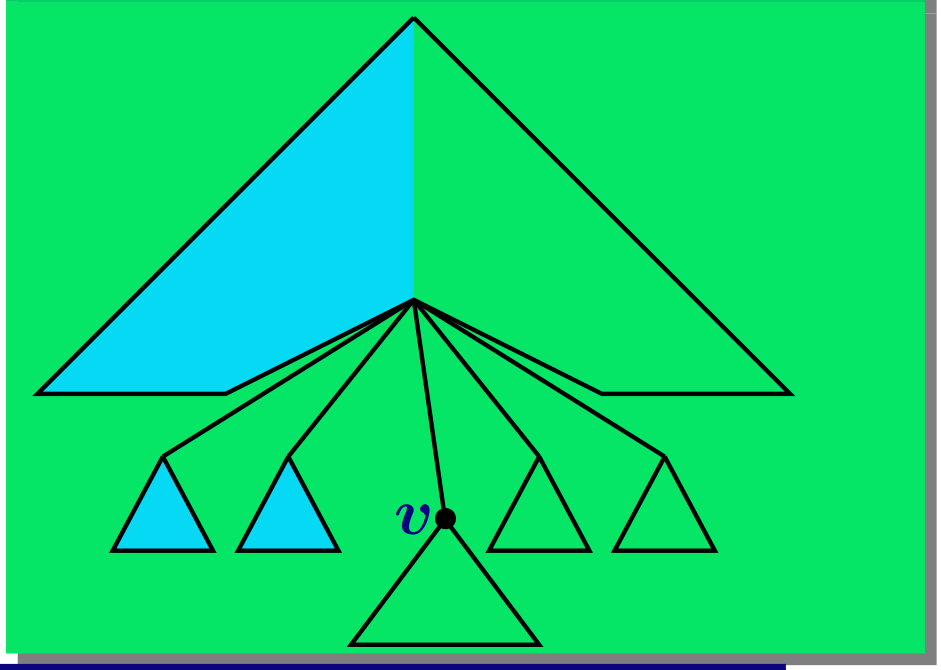

Answer

Whenever the processing proceeds in document order, e.g.:

- Streaming XML: Typing as the first operator in a pipeline
- SAX-based processing

Our next goal

Find out which schemas admit 1-pass preorder typing

1-Pass Preorder Typing (cont.)

Remarks

- The definition of "1-pass preorder typing" does not yet restrict the efficiency of determining the type of a node
- Typing could be 1-pass preorder but very time consuming
- It turns out that essentially this never happens
- Clearly, restrained competition is sufficient for 1-pass preorder typing
- Is it also necessary?

1-Pass Preorder Typing (cont.)

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- Is it also necessary?

Theorem [Martens, Neven, Sch. 2004]

For a regular tree language L the following are equivalent

(a) L can be described by a 1-pass preorder typable SDTD

(b) L can be described by a restrained-competition SDTD

(c) L has linear time 1-pass pre-order typing

(d) L can be preorder-typed by a deterministic pushdown document automaton

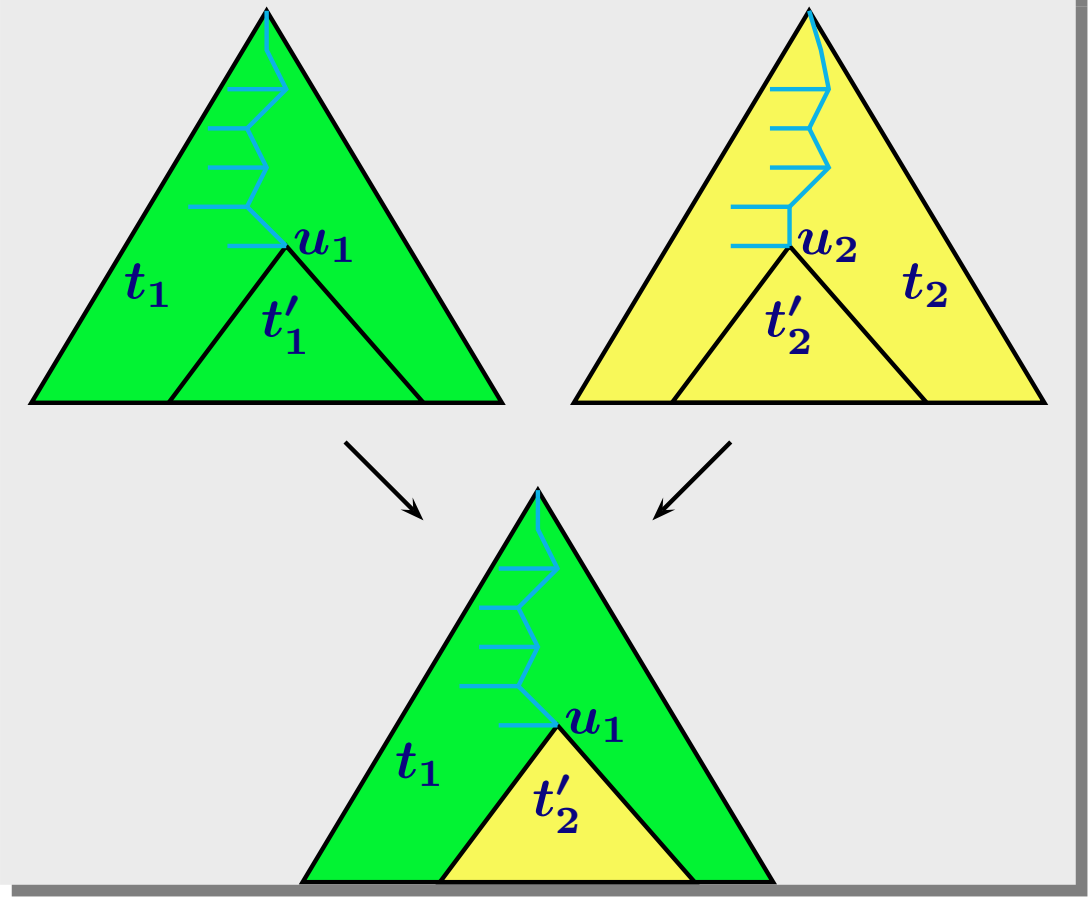
(e) Types for trees in L can be computed by a left-siblings-aware top-down deterministic tree automaton

A Very Robust Class

Further characterizations

- This class has further interesting characterizations
- E.g., by closure under ancestor-sibling-guarded subtree exchange

Illustration



A Related Result

Theorem [Martens, Neven, Sch. 2004]

For a regular tree language L the following are equivalent

(a) L can be described by a single-type SDTD

(b) Types for trees in L can be computed by a simple top-down deterministic tree automaton

(c) L is closed under ancestor-guarded subtree exchange

Summary: Schema Languages

Summary

Expressive power

- Regular tree languages offer a nice framework ($\equiv$ MSO logic!)
- Restrained competition $\equiv$ Deterministic top-down automata

Validation Linear time

Typing

- Linear time
- Efficient 1-pass preorder typing for restrained competition SDTDs

Satisfiability

- DTDs: trivial
- SDTDs: **PTIME**

Containment

- General SDTDs: **EXPTIME**
- Restrained competition SDTDs: **PTIME**

Contents

Node-Selecting Queries

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
⟨Married⟩⟨When⟩ October 1899 ⟨/When⟩⟨Whom⟩ Rosalie⟨/Whom⟩⟨/Married⟩
⟨Married⟩⟨When⟩ January 1908 ⟨/When⟩⟨Whom⟩ Emma ⟨/Whom⟩⟨/Married⟩
⟨Died⟩ ⟨When⟩ March 25, 1918 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩ ⟨/Died⟩
⟨/Vita⟩
⟨Piece⟩
⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
⟨Movements⟩ 3 ⟨/Movements⟩
...
⟨/Piece⟩
...
⟨/Composer⟩
...

Example query

```
//Vita/Died/*
```

Node-Selecting Queries

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⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
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⟨PTitle⟩ La Mer ⟨/PTitle⟩
⟨PYear⟩ 1905 ⟨/PYear⟩
⟨Instruments⟩ Large orchestra ⟨/Instruments⟩
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...
⟨/Piece⟩
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...

Example query

```
//Vita/Died/*
```

Observation

`XPath` expressions define sets of nodes $\longrightarrow$ node-selecting queries

Node-Selecting Queries (cont.)

Question

Is there a class of node-selecting queries, as robust as the regular tree languages?

Node-Selecting Queries (cont.)

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Observation

- There is a simple way to define node selecting queries by monadic second-order formulas:
- Simply use one free variable: $\varphi(x)$
- Is there a corresponding automaton model?
- It is relatively easy to add node selection to nondeterministic bottom-up automata

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Definition: (Nondetermistic bottom-up node-selecting automata)

- Nondeterministic bottom-up automata plus select function:

$$s : Q \times \Sigma \to \{0, 1\}$$

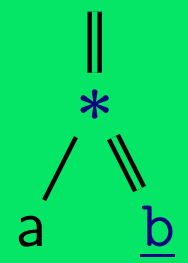
- Node v is in result set for tree t :$\Longleftrightarrow$ there is an accepting computation on t in which v gets a state q such that $s(q, \lambda(v)) = 1$

Example Automaton

Example query

//*[a]//b

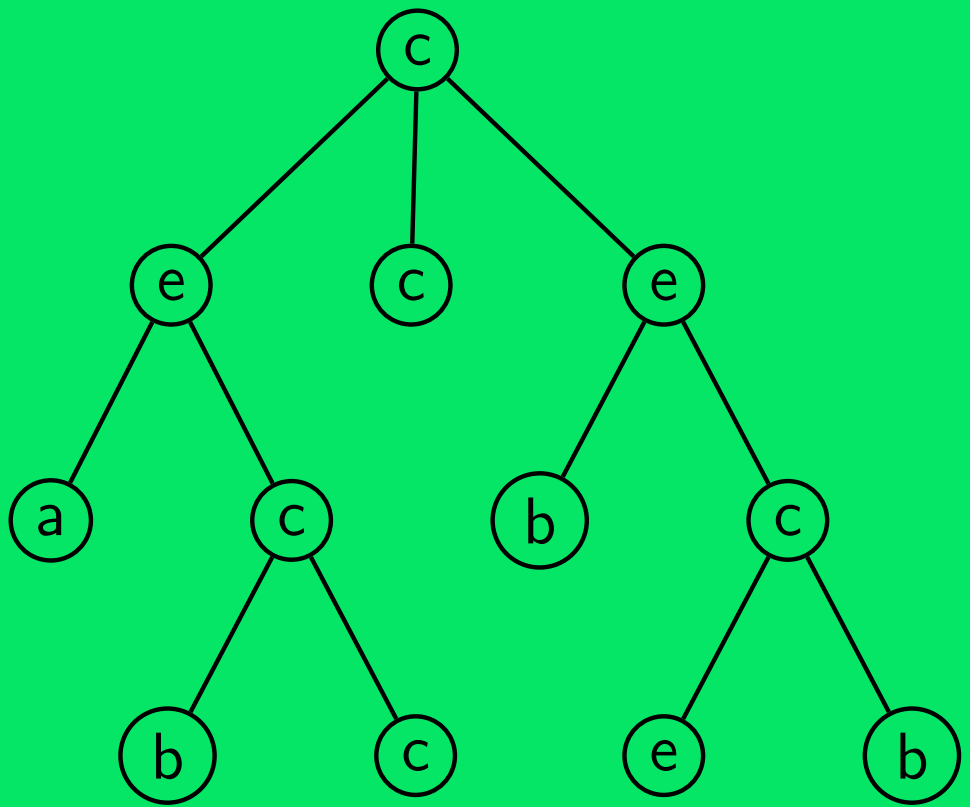
Query tree



Example automaton

- $Q = \{q_0, q_a, q_b\}$
- $L(q_a, a) = Q^*$
- $L(q_b, \sigma) = Q^*$
- $L(q_0, \sigma) =$
  $\epsilon + q_0^* + Q^* q_a Q^*$
- all other sets empty
- $s(q_b, b) = 1$
- all others: 0
- Accepting: q_0

Example tree: Run 1



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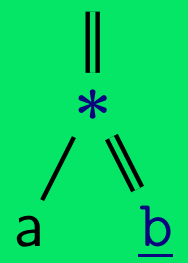

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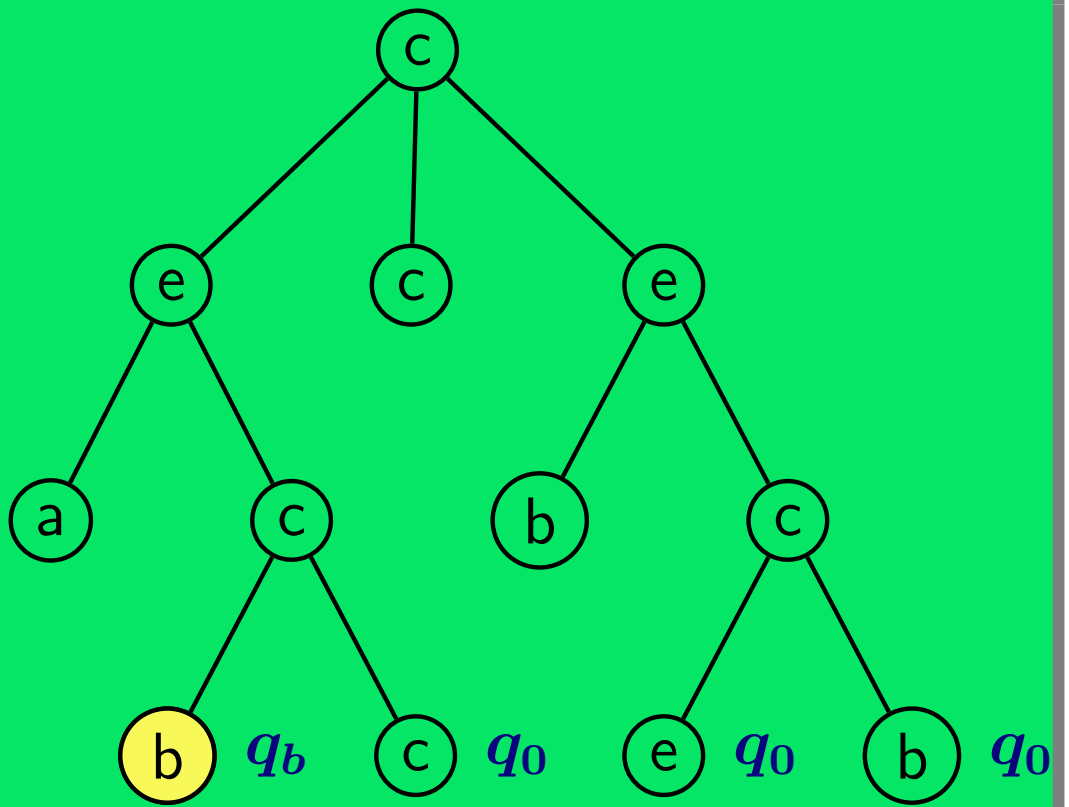

Example Automaton

Example query

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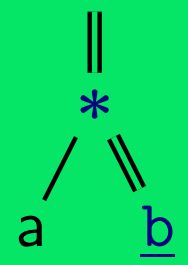
Query tree



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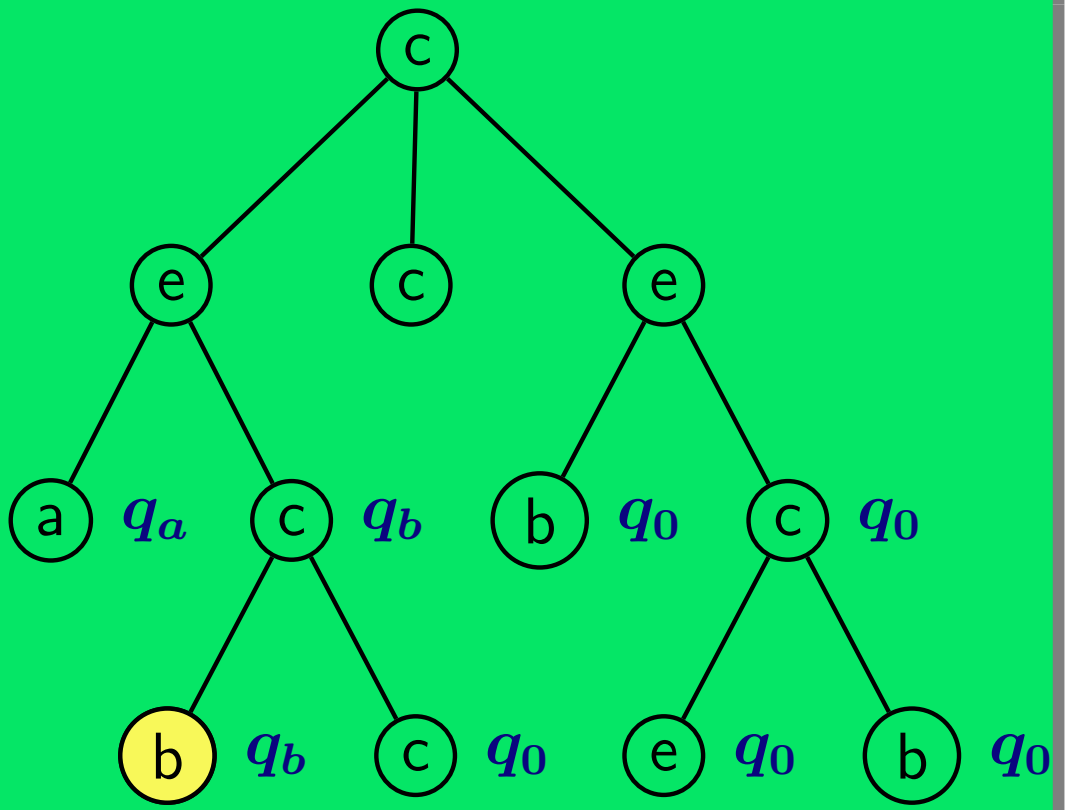

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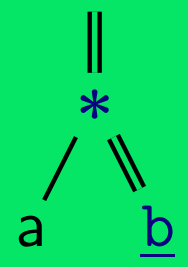
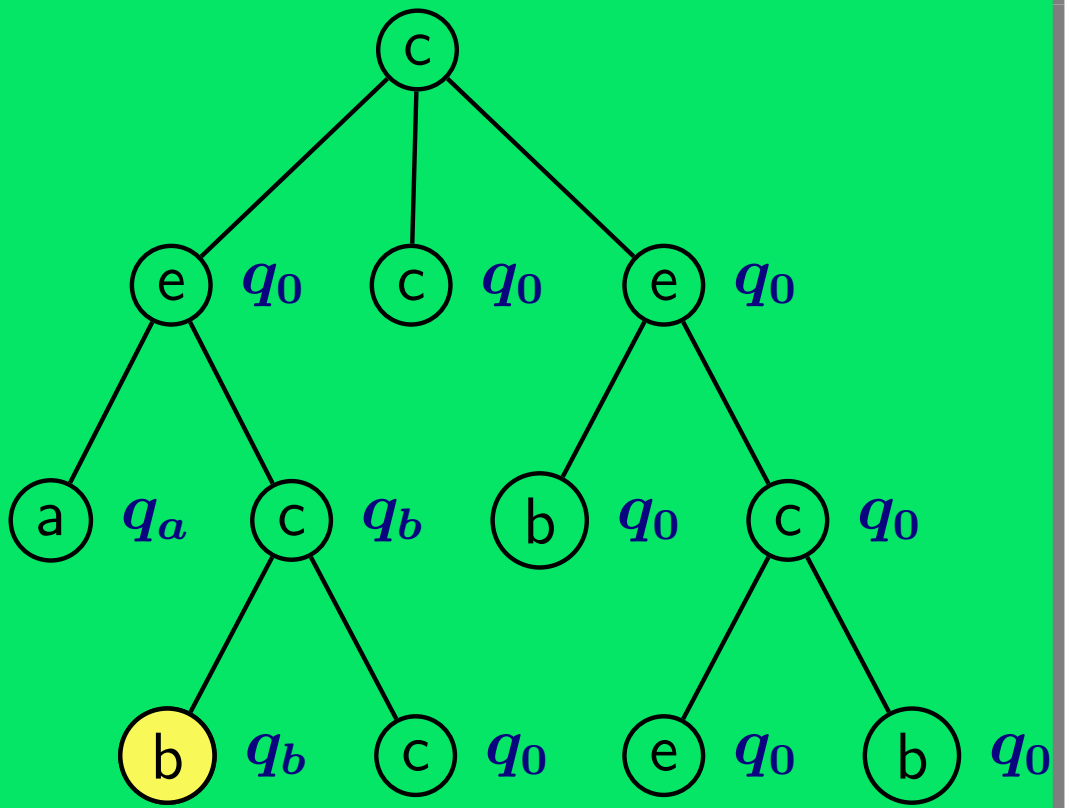
Query tree



Example automaton

- $Q = \{q_0, q_a, q_b\}$
- $L(q_a, a) = Q^*$
- $L(q_b, \sigma) = Q^*$
- $L(q_0, \sigma) =$
  $\epsilon + q_0^* + Q^* q_a Q^*$
- all other sets empty
- $s(q_b, b) = 1$
- all others: 0
- Accepting: q_0

Example tree: Run 1

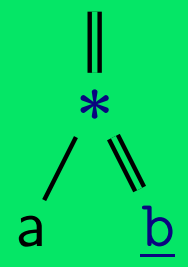


Example Automaton

Example query

//*[a]//b

Query tree



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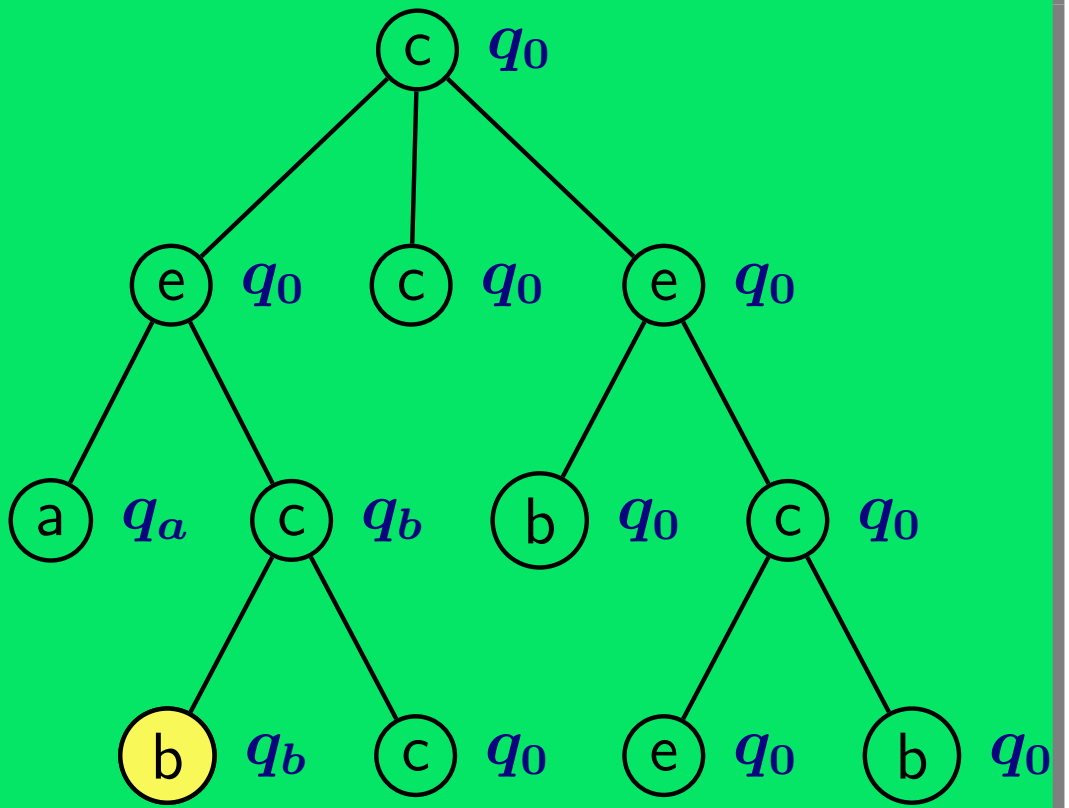
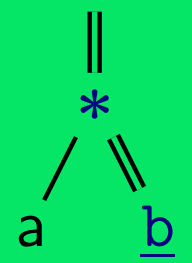


Example Automaton

Example query

//*[a]//b

Query tree



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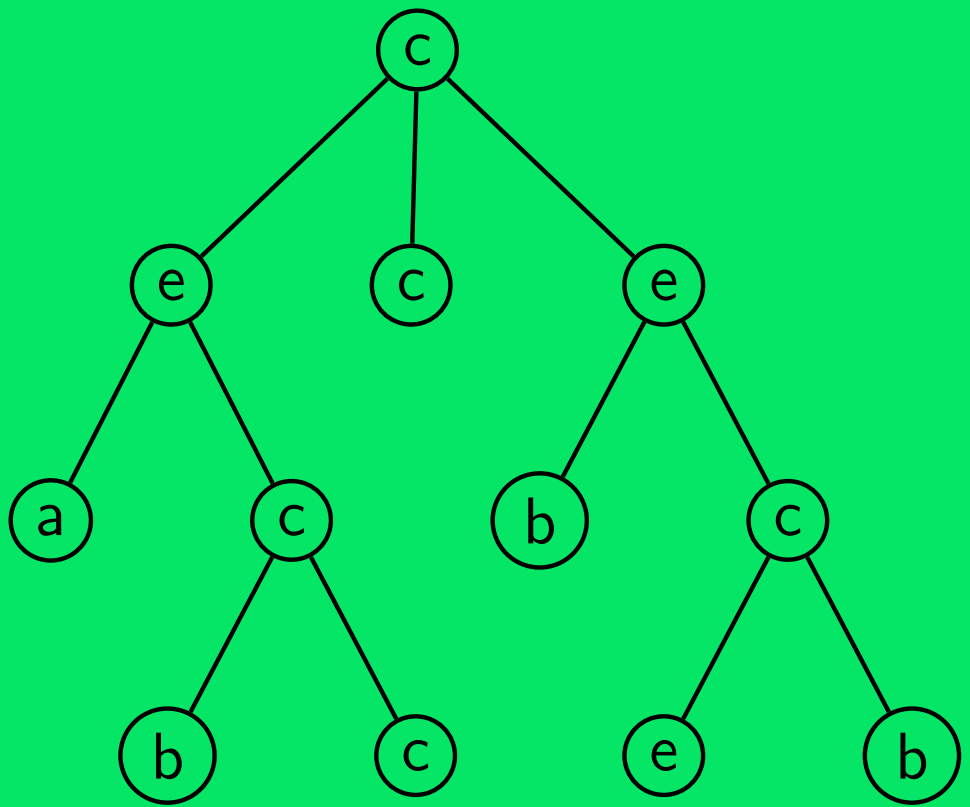
Example tree: Run 2

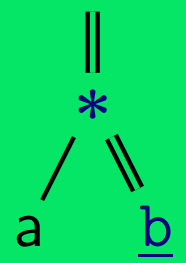


Example Automaton

Example query

//*[a]//b

Query tree



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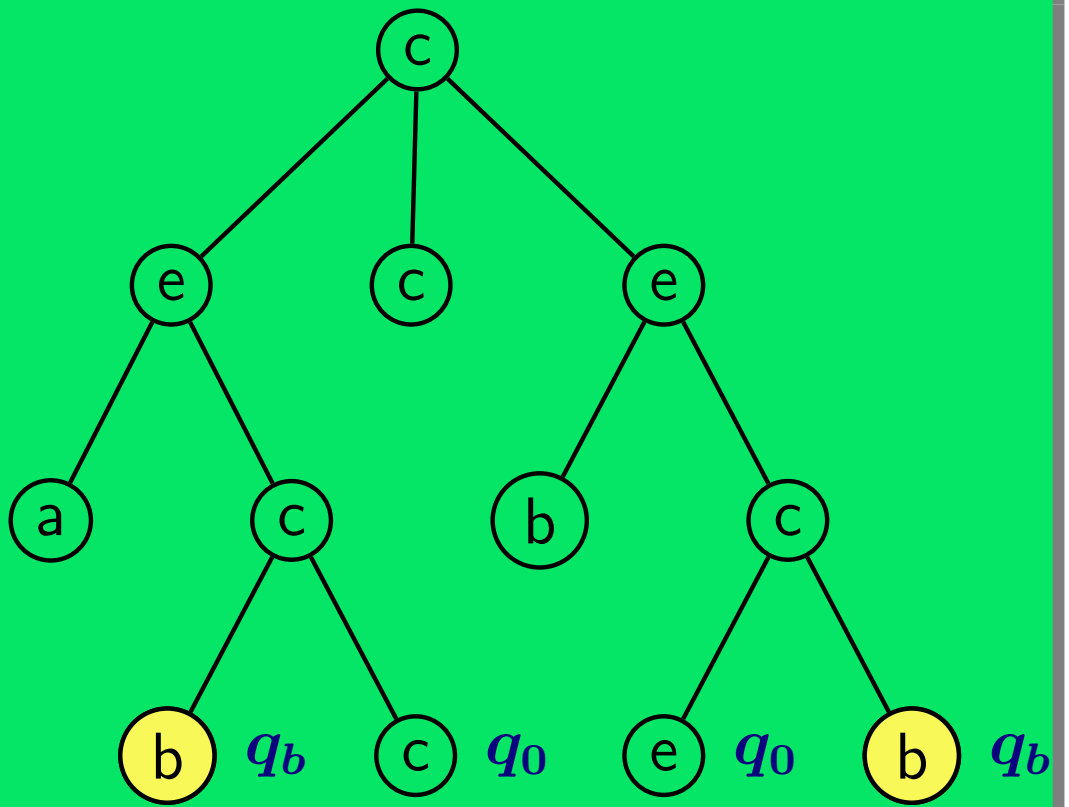
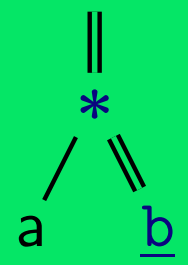


Example Automaton

Example query

//*[a]//b

Query tree



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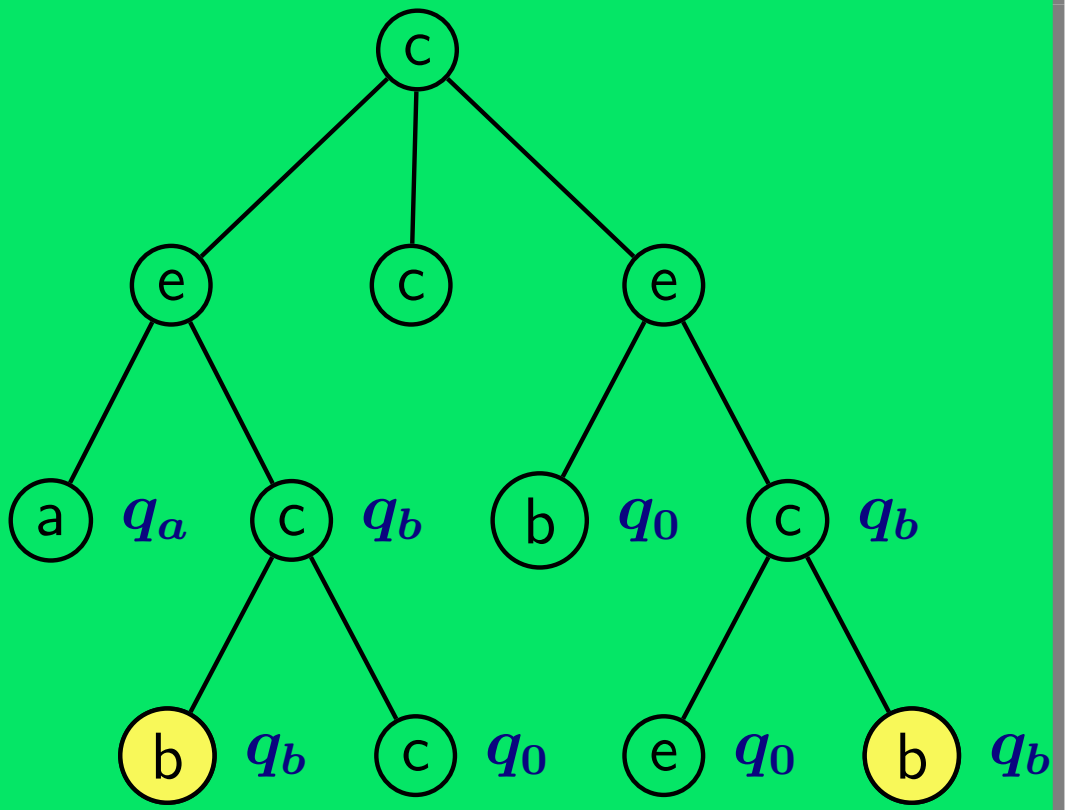

Example Automaton

Example query

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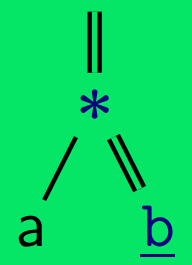
Query tree



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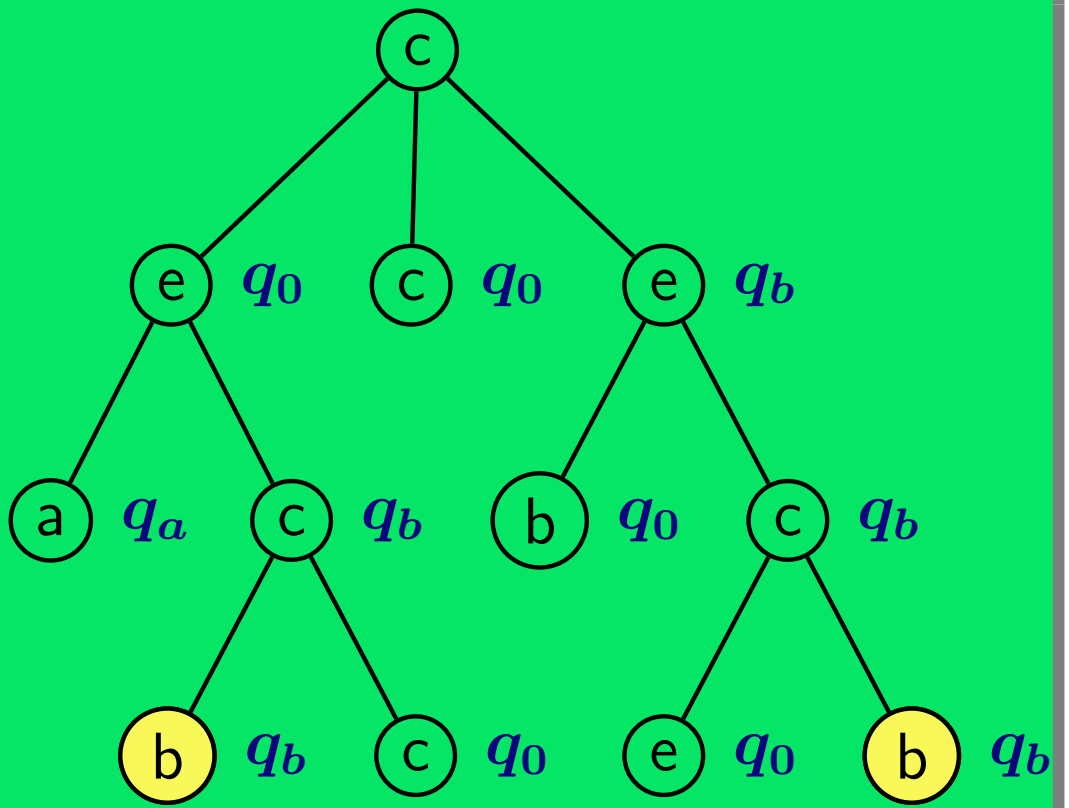

Example Automaton

Example query

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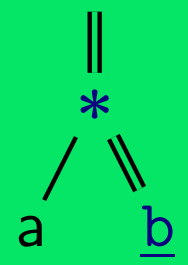

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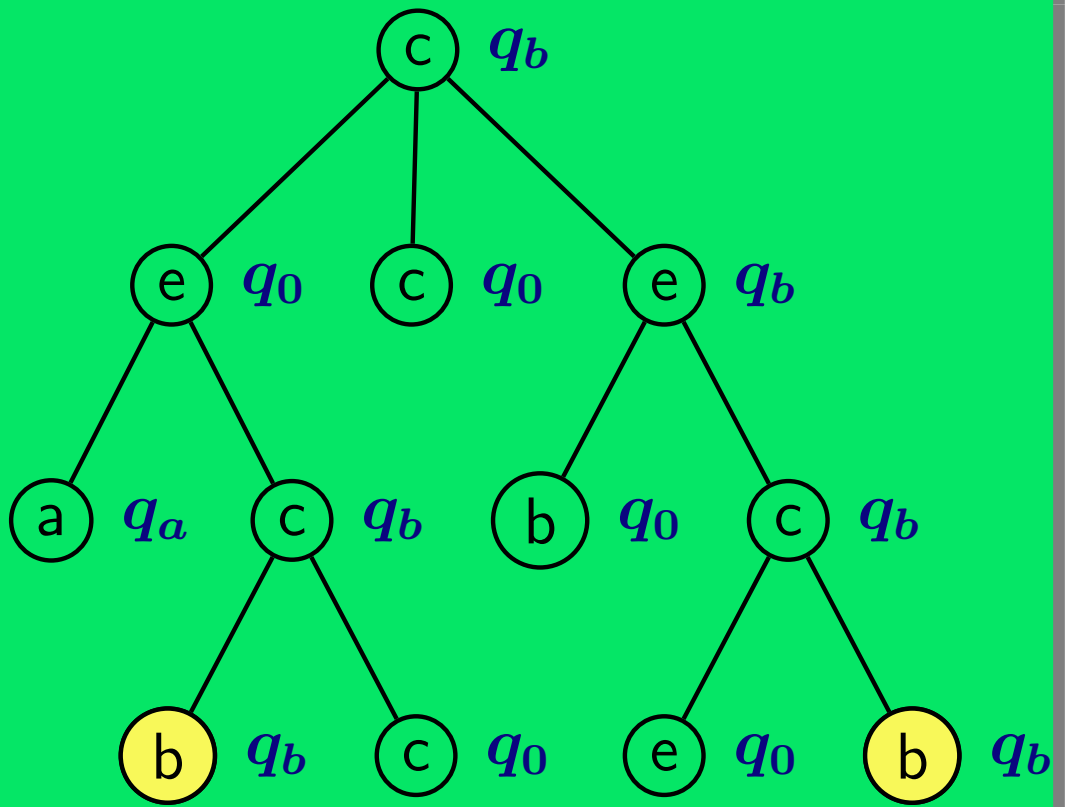


Node-Selecting Automata

Fact

- Existential semantics: a node is in the result if there exists an accepting run which selects it
- Universal semantics: a node is in the result if every accepting run selects it
- Both semantics define the same class of queries

Result

A node selecting query is MSO-definable iff it is expressible by a nondeterministic bottom-up node selecting automaton

Node-Selecting Automata (cont.)

Result

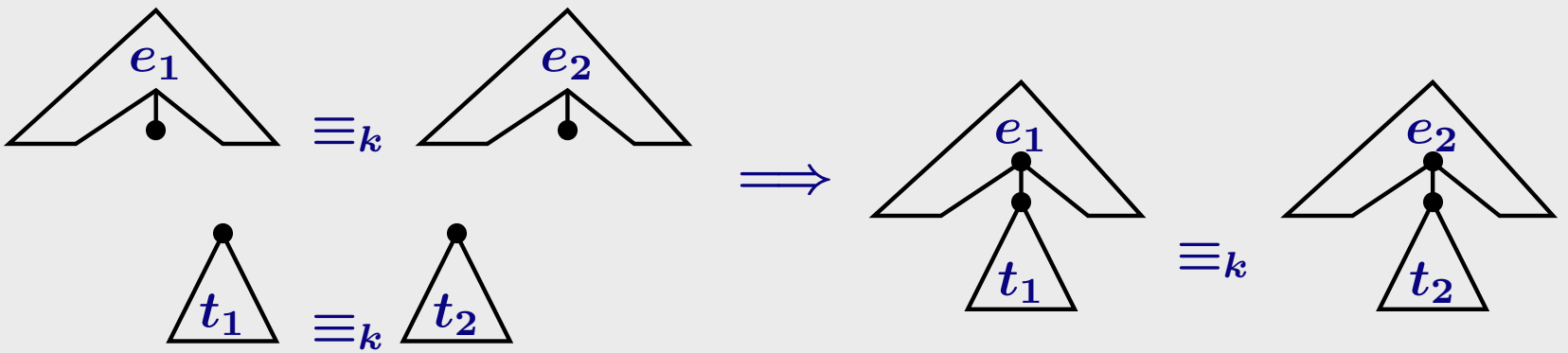
A node selecting query is MSO-definable iff it is expressible by a nondeterministic bottom-up node selecting automaton

Proof idea

- Given formula $\varphi(x)$ of quantifier-depth k and tree t, for each node v the automaton does the following:
  - Compute k-type of subtree at v
  - Guess k-type of "envelope tree" at v
  - Conclude whether v is in the output
  - Check consistency upwards towards the root

$\Rightarrow$ one unique accepting run

Crucial fact



Equivalent Models

More query models

- Unfortunately, the translation from formula to automaton can be prohibitively expensive: number of states $\sim \mathbf{2}^{\mathbf{2}^{\cdot^{\cdot^{\cdot^{\mathbf{2}^{\mathbf{2}^{|\varphi|}}}}}}}\Big\} |\varphi|$
- Actually: If **P** $\neq$ **NP** there is no elementary f, such that MSO-formulas can be evaluated in time $f(|\text{formula}| \times p(|\text{tree}|))$ with polynomial p [Frick, Grohe 2002]

$\longrightarrow$ query languages with better complexity properties needed

- Good candidate: Monadic Datalog [Gottlob, Koch 2002] and its restricted dialects like TMNF
- Further models:
  - Attributed Grammars [Neven, Van den Bussche 1998]
  - μ-formulas [Neumann 1998]
  - Context Grammars [Neumann 1999]
  - Deterministic Node-Selecting Automata [Neven, Sch. 1999]

Node-selecting Queries: Evaluation Complexity

Some facts about query evaluation

- MSO node-selecting queries can be evaluated in two passes through the tree
  - first pass, bottom-up: essentially computes the types of the subtrees
  - second pass, top-down: essentially computes the types of the envelopes and combines it with the subtree information
- This can be implemented by a 2-pass pushdown document automaton which in its first pass attaches information to each node [Neumann, Seidl 1998; Koch 2003]
- In particular: queries can be evaluated in linear time

Node-selecting Queries: Static Analysis

Facts

- Satisfiability: Non-emptiness of node-selecting automata is **PTIME**-complete
- Satisfiability of MSO-queries is non-elementary
- Containment of node-selecting automata is **EXPTIME**-complete

Summary: Node-Selecting Queries

Summary

- There is a natural notion of **regular node-selecting queries** generalizing regular tree languages
- Probably for most practical purposes too strong
- But it offers a useful framework for the study of other classes of queries
- A robust but weaker class of queries is captured by pebble automata

Contents

XPath Fragments

XPath and Important Fragments

- Many fragments of XPath have been defined
- The main fragments we consider are:
  - **Full XPath**: XPath 1.0
    (besides the namespace related functions)
  - **Navigational XPath** [Gottlob, Koch, Pichler 03, Benedikt, Fan, Kuper 03]:
    Location paths along all axes plus Boolean operations
    (no attributes, no relational operators)
  - **Forward XPath**: Navigational XPath restricted to
    `child`, `descendant`, `self`, `descendant-or-self`

XPath

Main Ingredients of Navigational XPath

- **Location Step**:

  $$p = \texttt{Axis :: Node-Test Predicate}^*$$

- **Predicate**: [Expression]
- **Location Path**:

  $$\pi = \texttt{Location Step / Location Path}$$

  More explicitly: $\pi = p_1/\cdots/p_k$
- **Expression**: basically a Boolean combination of location steps

Example

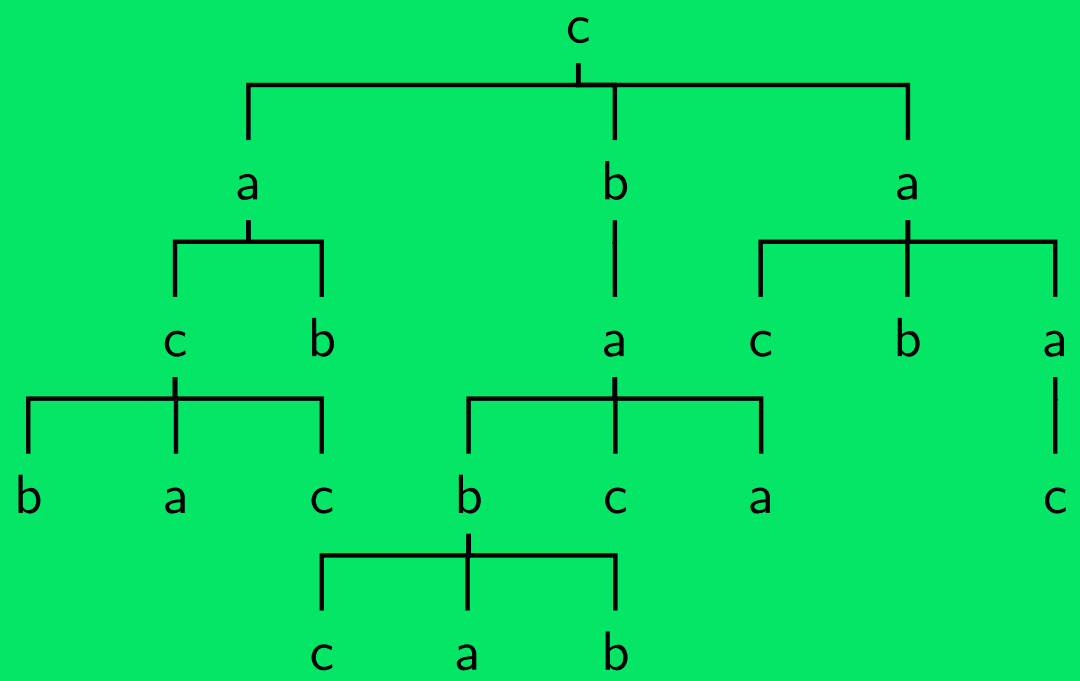
```
/descendant::a/
      child::*[descendant::c and not following-sibling::b]/
            descendant::a
```

XPath Example

Example XPath Expression

```
/desc::a/child::*[desc::c and not foll-sib::b]/desc::a
```

Example Tree



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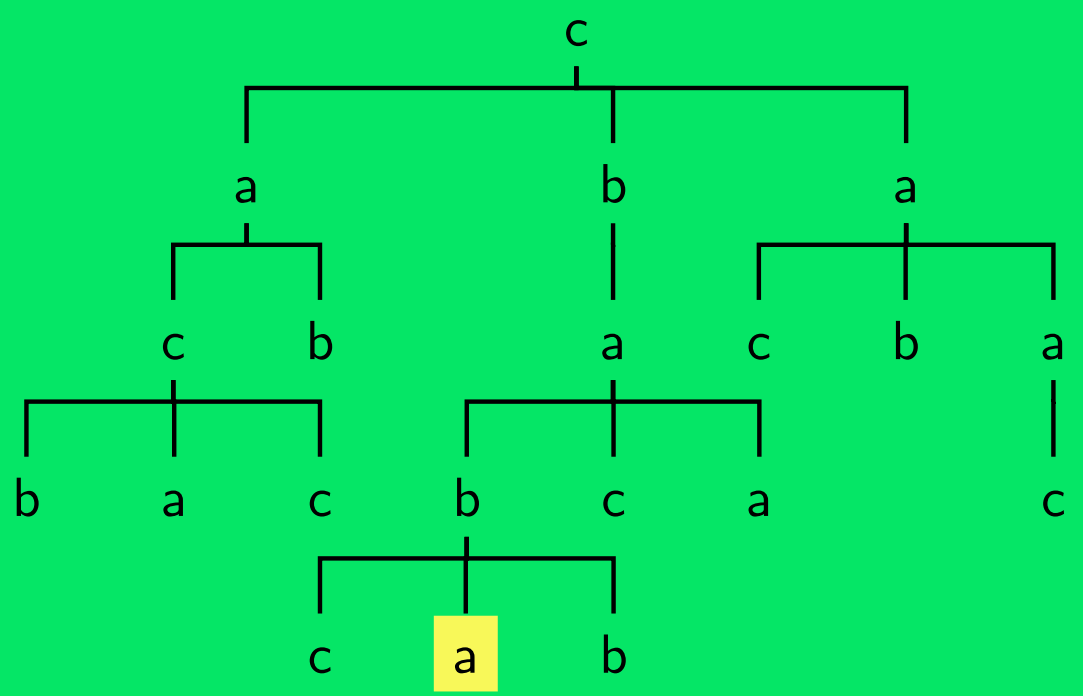


XPath Semantics

XPath Semantics

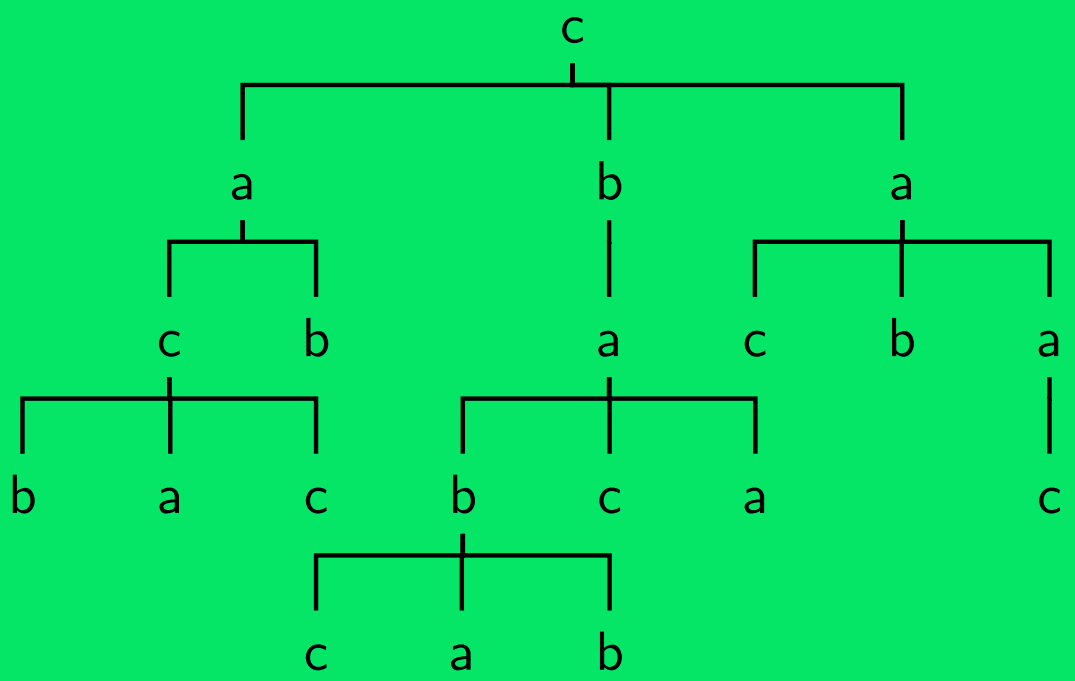
- Result of an expression is a node set or a single value (Boolean, number or string)
- Expressions are evaluated relative to a **context**, in particular relative to a **context node**
- Location step: $p =$ (a :: n q) relative to context node u yields the set $[\![p]\!](u)$ of nodes v such that
  - (u, v) are in a-relation
  - v is labeled according to n (arbitrary, if $n = *$)
  - all predicates of q hold at v
- Extended to sets S of nodes: $[\![p]\!](S) = \cup_{u \in S} [\![p]\!](u)$
- Location path: $[\![p/\pi]\!](S) = [\![\pi]\!]([\![p]\!](S))$

Example Revisited

Example XPath Expression

```
/desc::a/child::*[desc::c and not foll-sib::b]/desc::a
```

Example Tree



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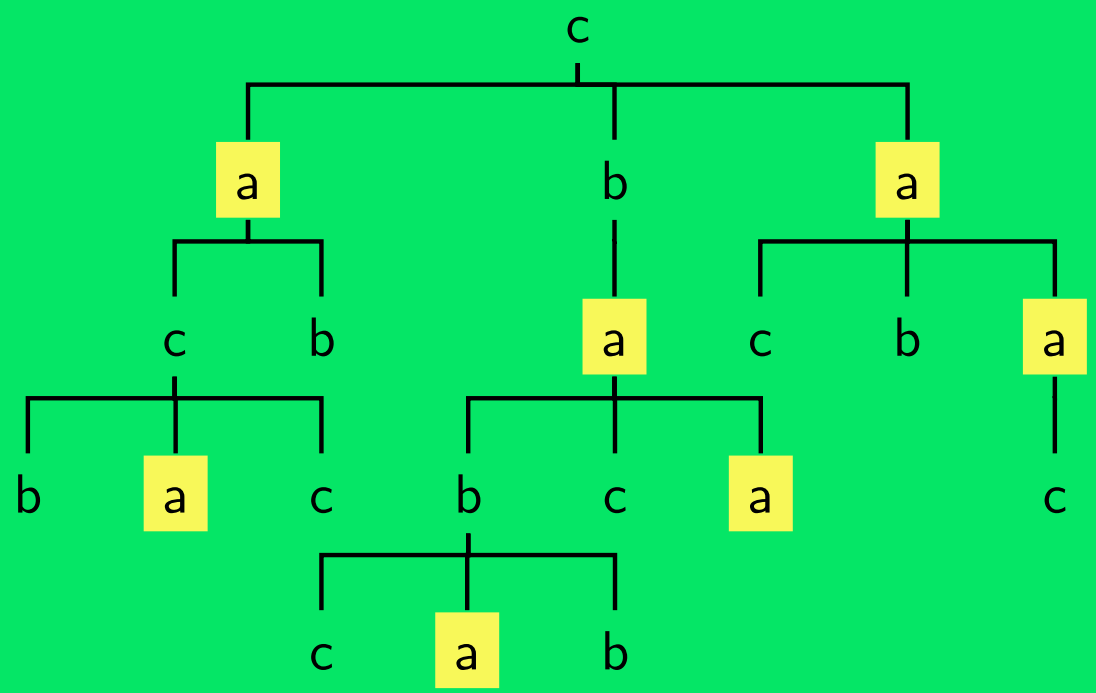

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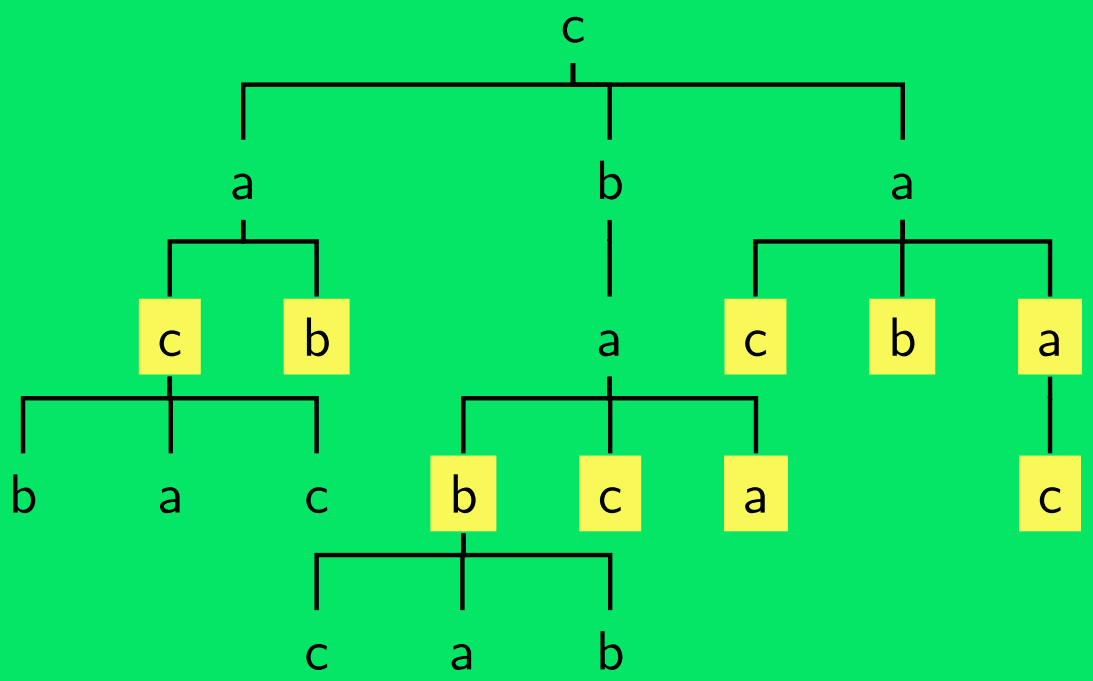

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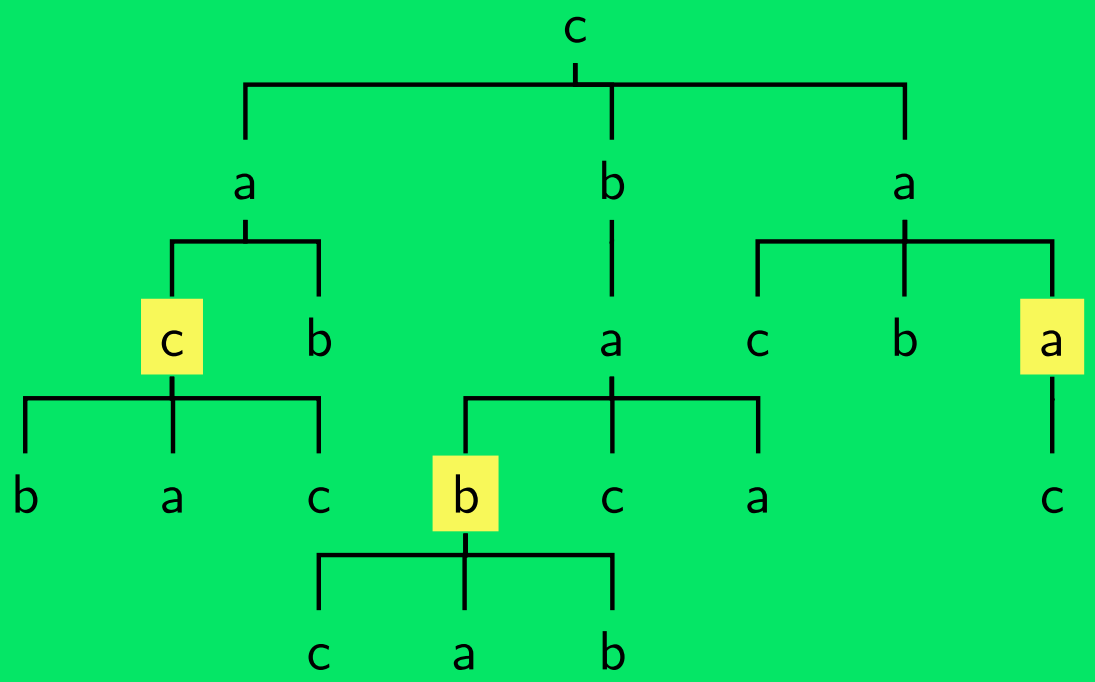

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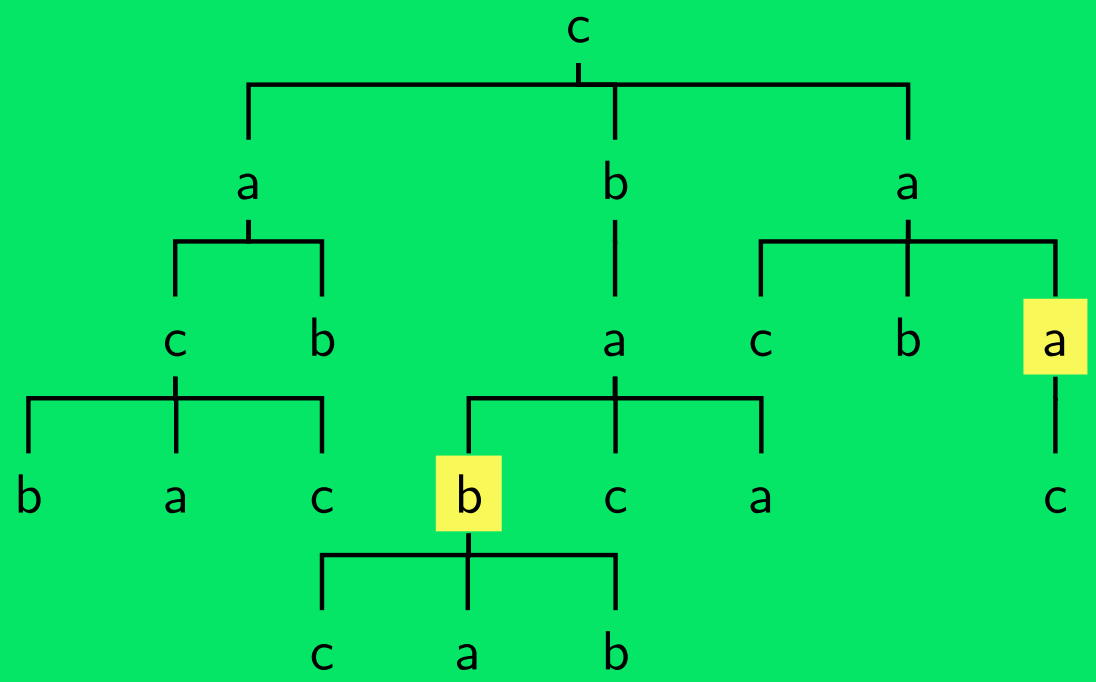
```

Example Tree



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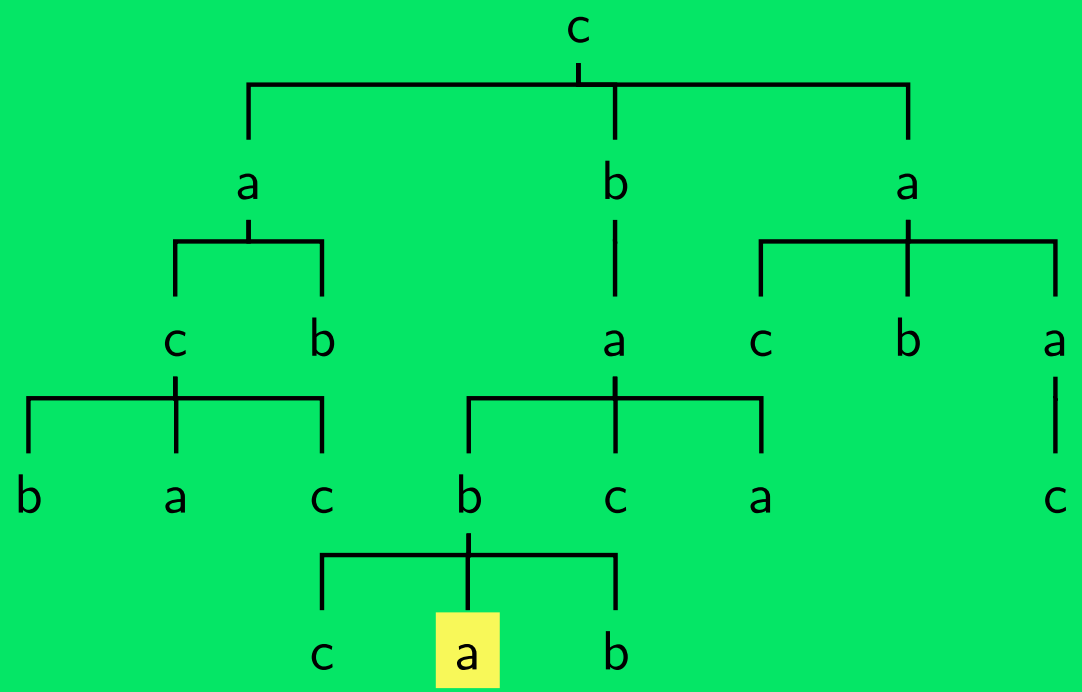
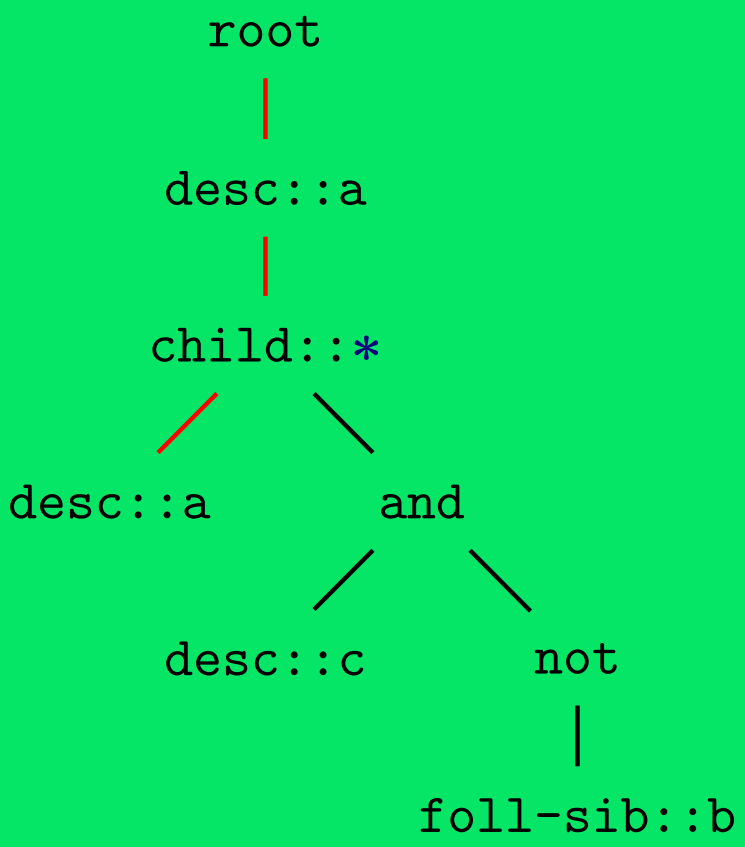


XPath Expression as Query Tree

Example XPath Expression

```
/desc::a/child::*[desc::c and not foll-sib::b]/desc::a
```

Example Query Tree



A simplified Notation [Benedikt, Fan, Kuper 03]

Notation

- $\downarrow$, $\uparrow$, $\rightarrow$, $\leftarrow$, $\circlearrowleft$:
  `child`, `parent`, next-sibling, previous-sibling, `self`
- $\downarrow^+$, $\uparrow^+$, $\rightarrow^+$, $\leftarrow^+$:
  `descendant`, `ancestor`, `following-sibling`, `preceding-sibling`
- $\downarrow^*$, $\uparrow^*$, $\rightarrow^*$, $\leftarrow^*$:
  `descendant-or-self`, `ancestor-or-self`, following-sibling-or-self, preceding-sibling-or-self

Example

- `child::a/descendant::c/following-sibling::*/parent::b` can be expressed as $\downarrow$/a/$\downarrow^+$/c/$\rightarrow$/$\uparrow$/b
- The `following`-axis can be expressed via $\uparrow^*/\rightarrow^+/\downarrow^*$

Contents

Xpath and Existential First-order Logic

Characterizations of XPath [Benedikt, Fan, Kuper 03]

- Navigational XPath (without not and and) corresponds to positive existential first-order logic
- Different XPath axes correspond to different signatures

Proof idea

- Basic idea:
  For each node v of the query tree: guess a node $h(u)$ in the document tree and check that h is a "homomorphism"
- Main difficulty in proof:
  Deal with conjunctions of conditions

Further Results on

- closure properties
- axiomatizations of equivalence

Backward vs. Forward Axes

Elimination of Backward Axes [Olteanu et al. 02]

- In absolute `XPath` expressions, all backward axes can be eliminated
- Two sets of rewrite rules:
  - introducing equality expressions, linear time (and size)
  - without equality expressions, possibly exponential size

Xpath and First-Order Logic

Reminder

Navigational XPath without negation corresponds to positive existential first-order logic

Question: What is needed to capture full first-order logic?

Xpath and First-Order Logic

Reminder

Navigational XPath without negation corresponds to positive existential first-order logic

Question: What is needed to capture full first-order logic?

Conditional axes

Conditional axes:

Expressions of the kind P^+, where P is an expression

Example

$(\texttt{child} :: a[\texttt{desc} :: b \text{ or } \texttt{child} :: c])^+$

holds between u and v if

- v is a descendant of u and
- all intermediate nodes
  - are labelled with a and
  - have a c-child or a b-descendant

Xpath and First-Order Logic (cont.)

Theorem [Marx 04]

Navigational `XPath` with conditional axes corresponds exactly to first-order logic (wrt node-selecting queries)

Proof idea

The proof uses a decomposition technique similar to the proof that LTL corresponds to first-order logic over linear structures [Gabbay et al. 80]

Pebble Automata and XPath

Definition: Pebble Automata

- Extension of tree-walk automata by fixed number of pebbles
- Only pebble with highest number (current pebble) can move, depending on state, number of pebbles symbols under pebbles and incidence of pebbles
- Possible pebble movements:
  - stay, go to left sibling, go to right sibling, go to parent
  - lift current pebble or place new pebble at current position
- Nondeterminism possible

Fact

Deterministic pebble automata capture navigational XPath queries

Proof idea

For each node of the query tree:

cycle through all possible nodes of the document tree

Automata and Logic

Some observations

- On strings, MSO logic and (unary) transitive closure logic (TC-logic) coincide
- On trees
  - MSO $\equiv$ parallel automata
  - TC-logic $\equiv$ pebble automata (i.e., strongest sequential automata)
- Whether on trees MSO $\equiv$ TC-logic is open
- The relationship between logics and automata models between FO and TC-logic is largely unexplored:
  - Tree-walk automata,
  - FO-logic $+$ regular expressions
  - Conditional `XPath` $+$ arbitrary star operator
  - ...

Contents

XPath Query Evaluation

Naive Evaluation

Procedure Eval($p_1/\cdots/p_n$,v)

$S := [\![p_1]\!]v$

IF $n = 1$ RETURN S ELSE $S' := \emptyset$

FOR $u \in S$ DO $S' := S' \cup$ Eval($p_2/\cdots/p_n$,u)

RETURN S'

XPath Query Evaluation

Naive Evaluation

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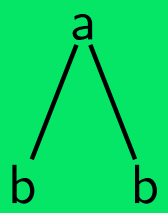
RETURN S'

Complexity

- $T(p_1/\cdots/p_n, t) = O(\text{size of } t) \times T(p_2/\cdots/p_n, t)$
- Could be exponential
- Experiments (reported in [Gottlob, Koch, Pichler 02]) show that available XPath processors had exponential complexity

Example

`/descendant::a(/child::b/parent::a)`^{n} on document

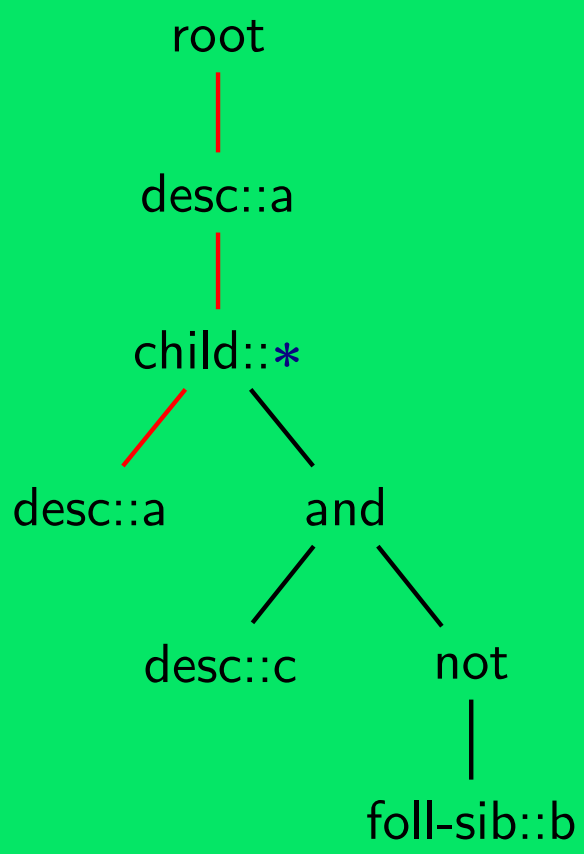


Evaluation of Navigational XPath

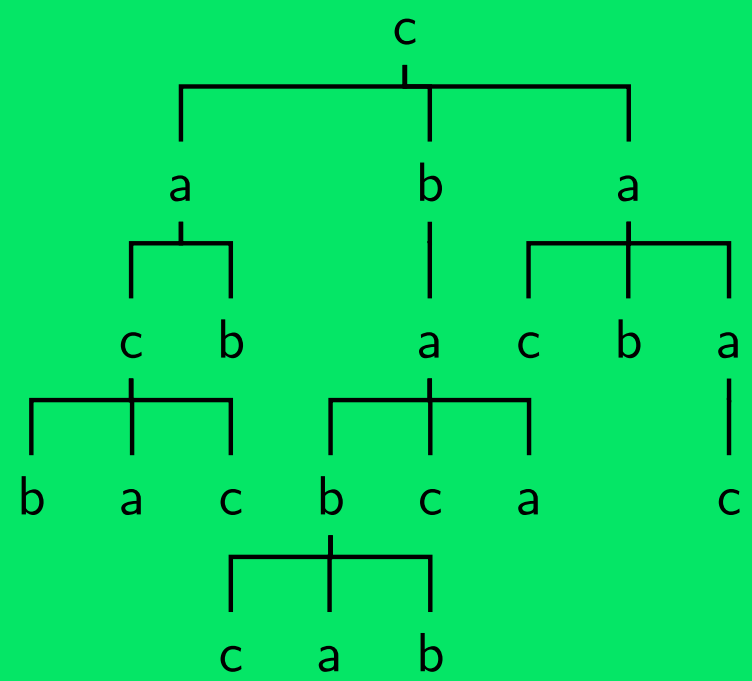
Basic Idea

Combine top-down evaluation of the "main path" with bottom-up evaluation of predicates [Gottlob, Koch, Pichler 02]

Example Query Tree



Example Document

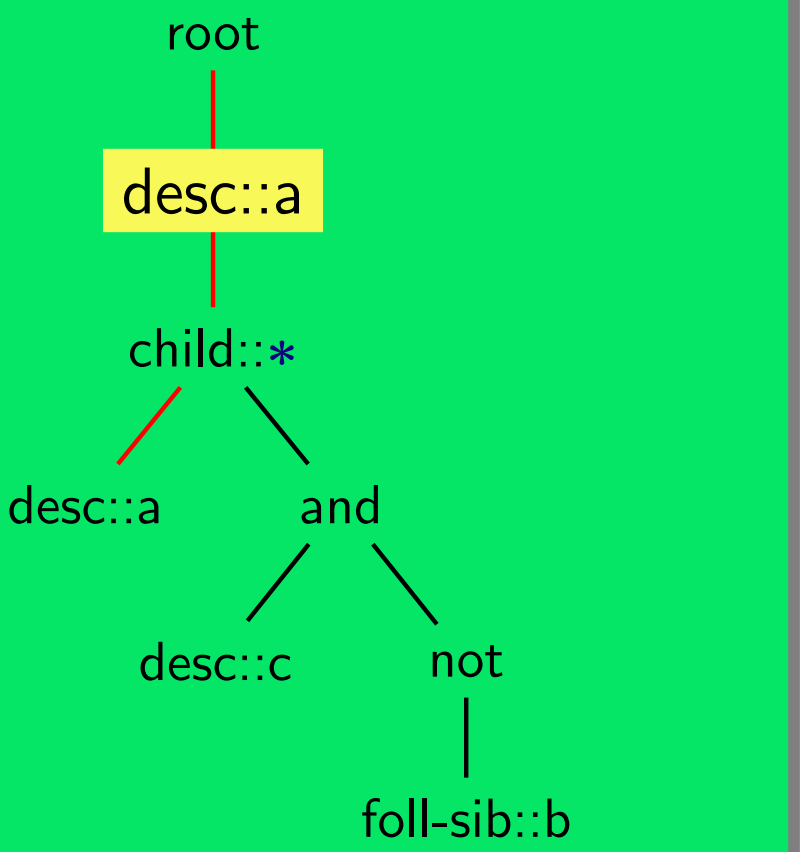


Evaluation of Navigational XPath

Basic Idea

Combine top-down evaluation of the "main path" with bottom-up evaluation of predicates [Gottlob, Koch, Pichler 02]

Example Query Tree



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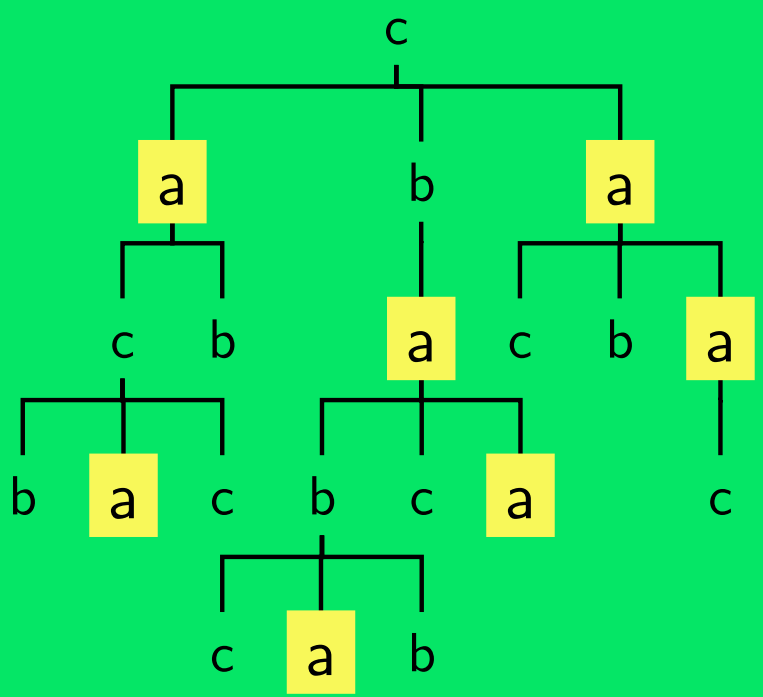

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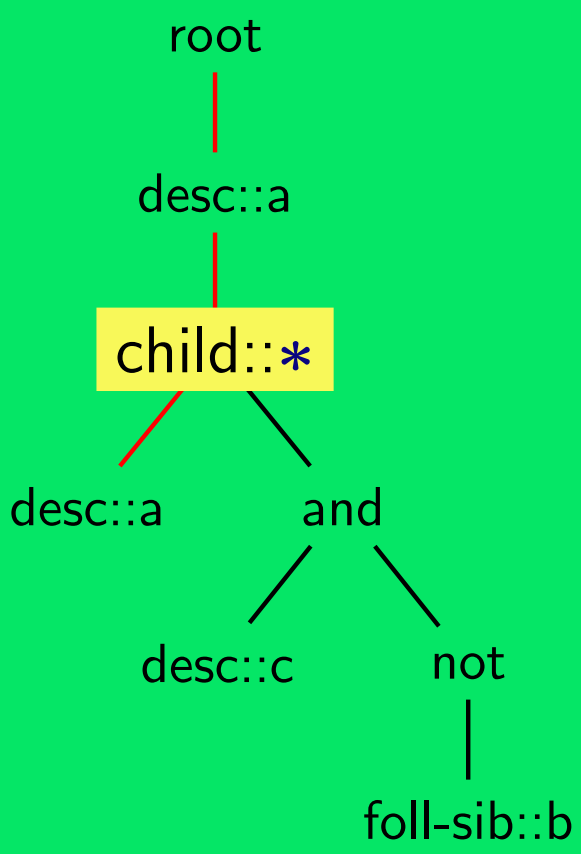

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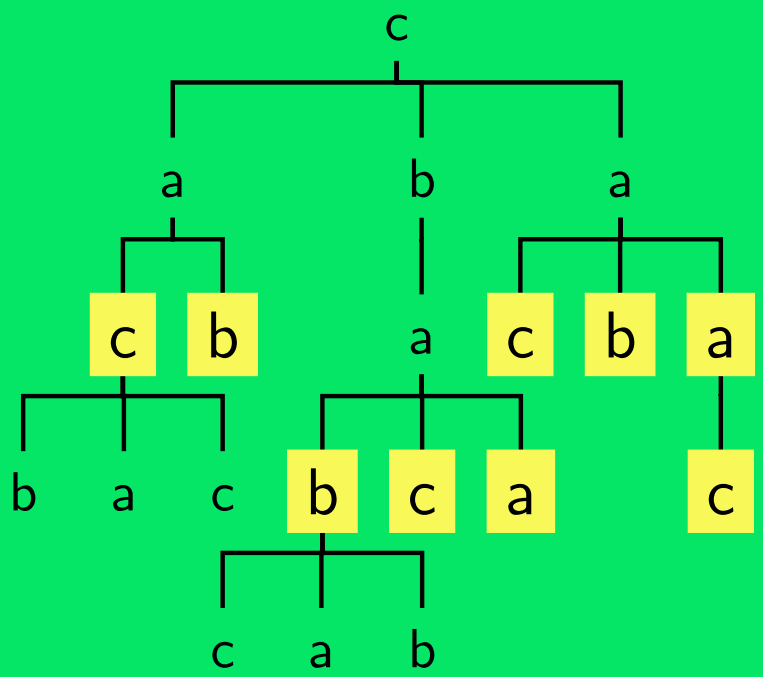

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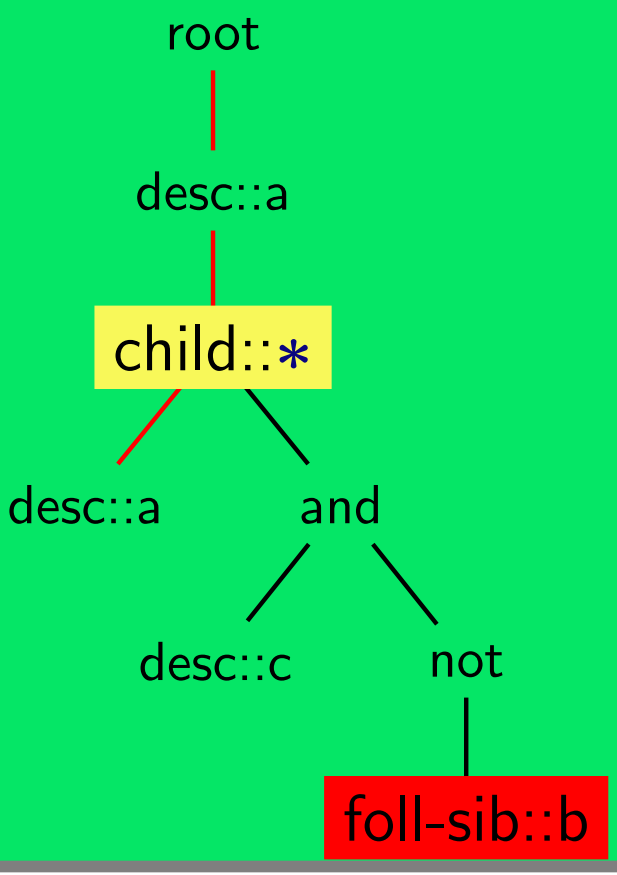

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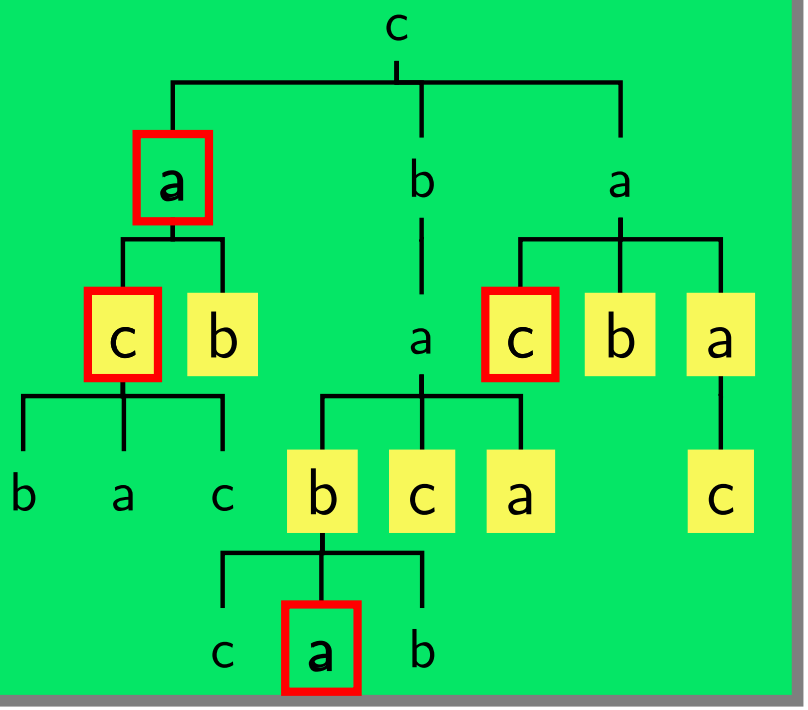

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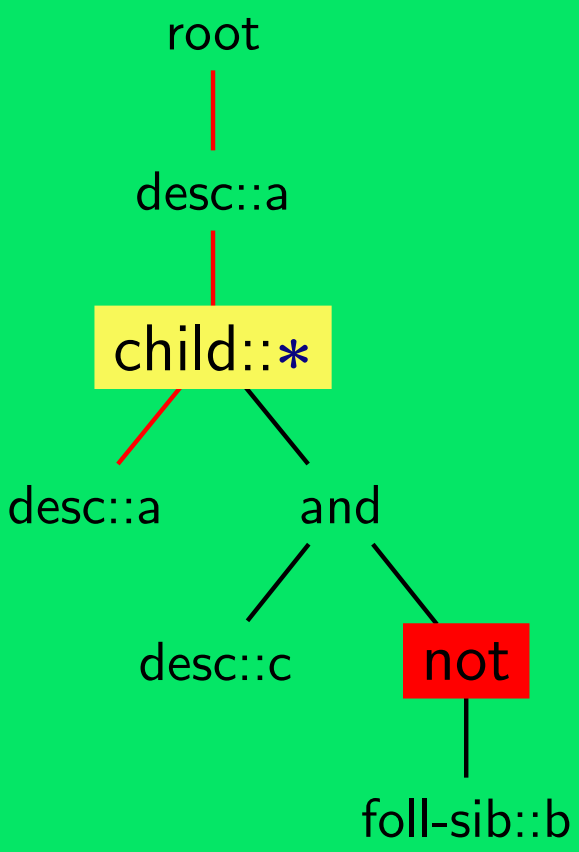

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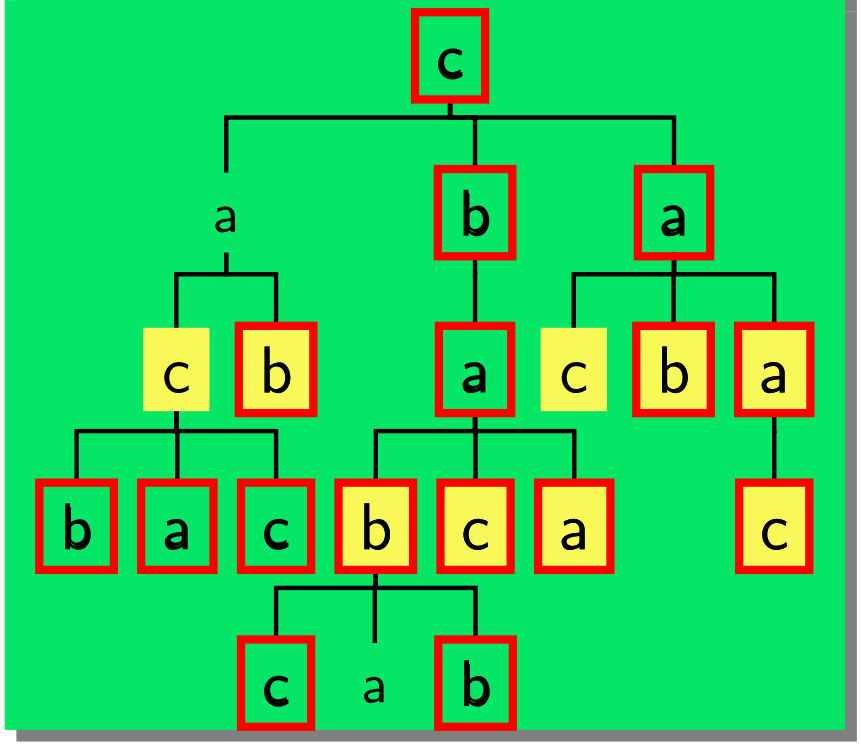

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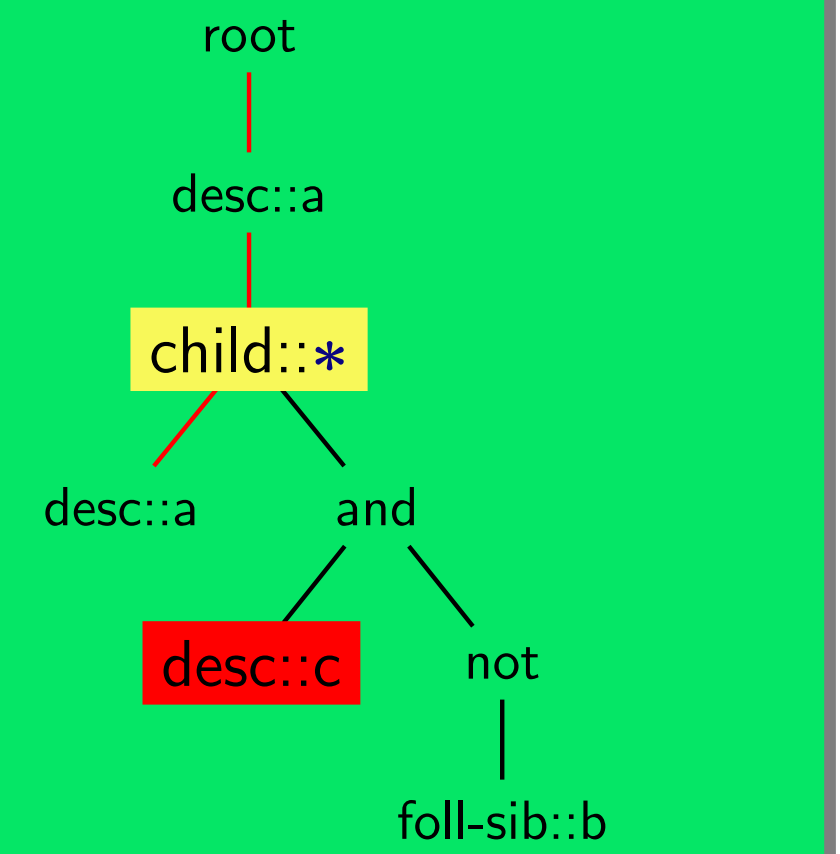

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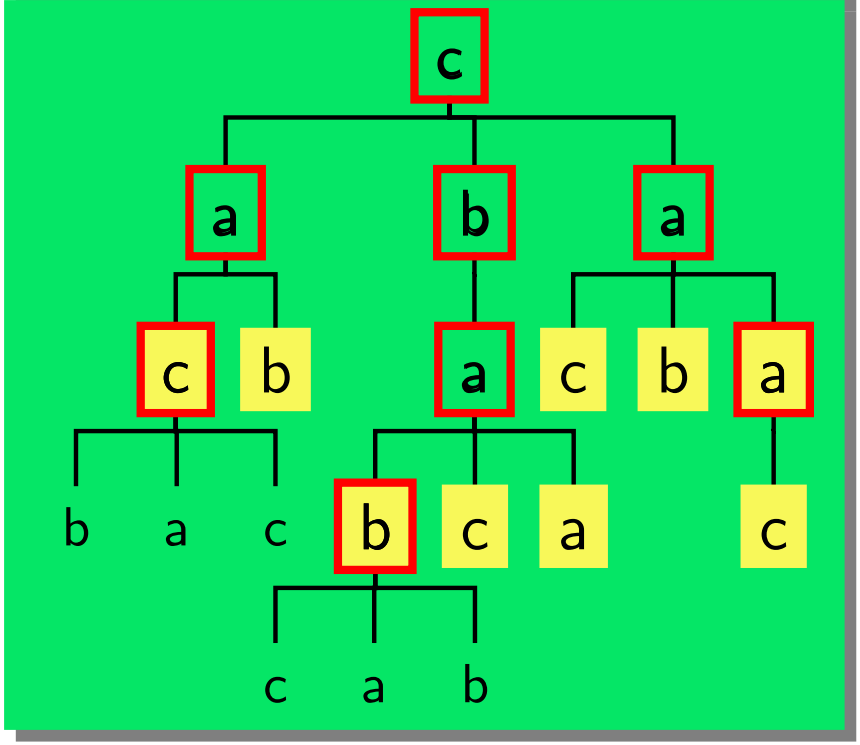

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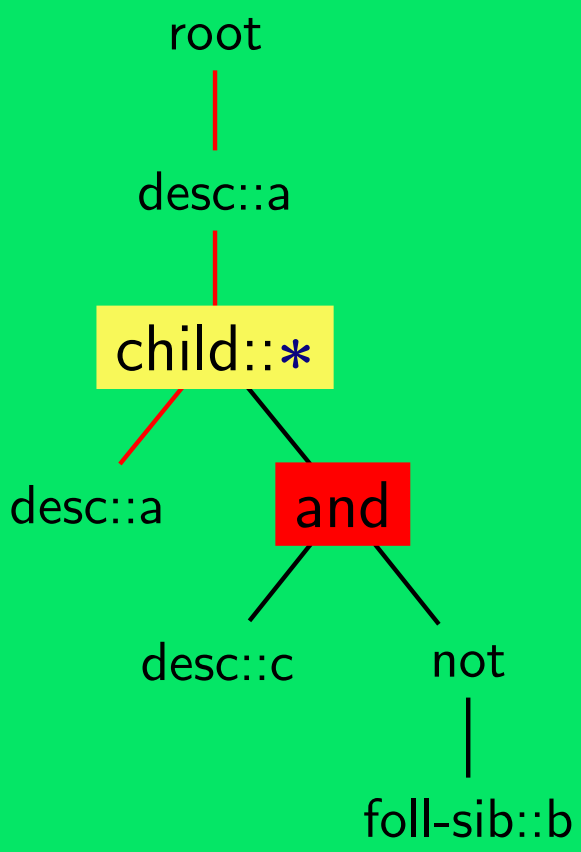

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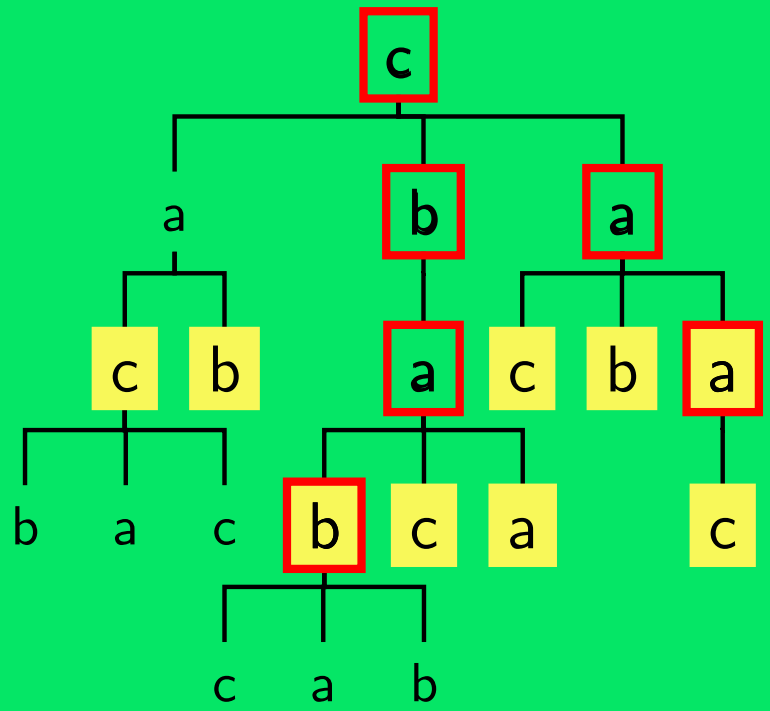

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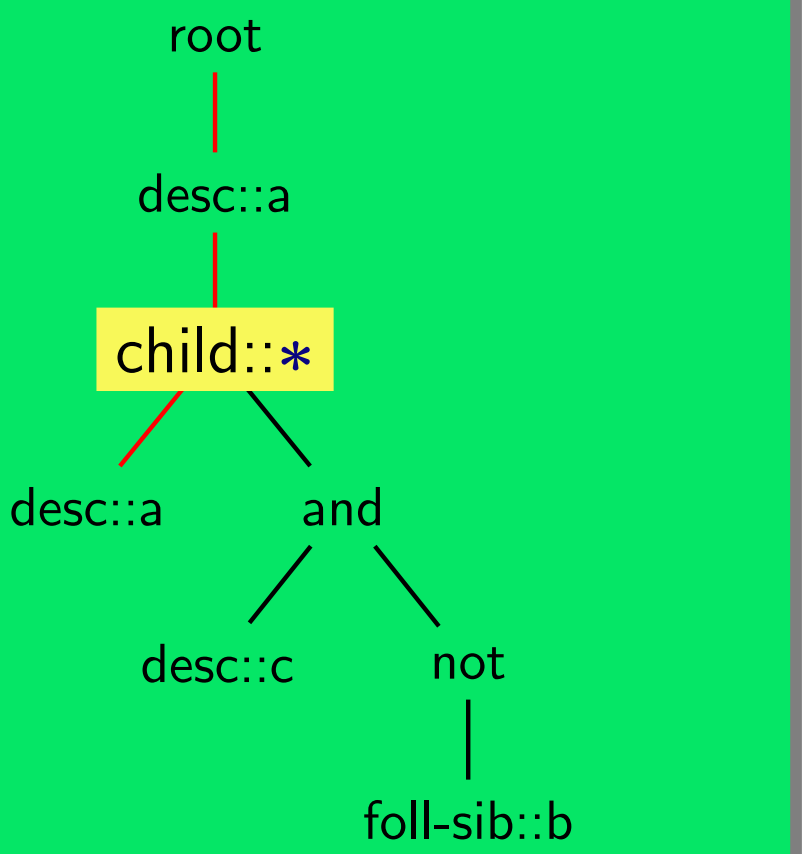

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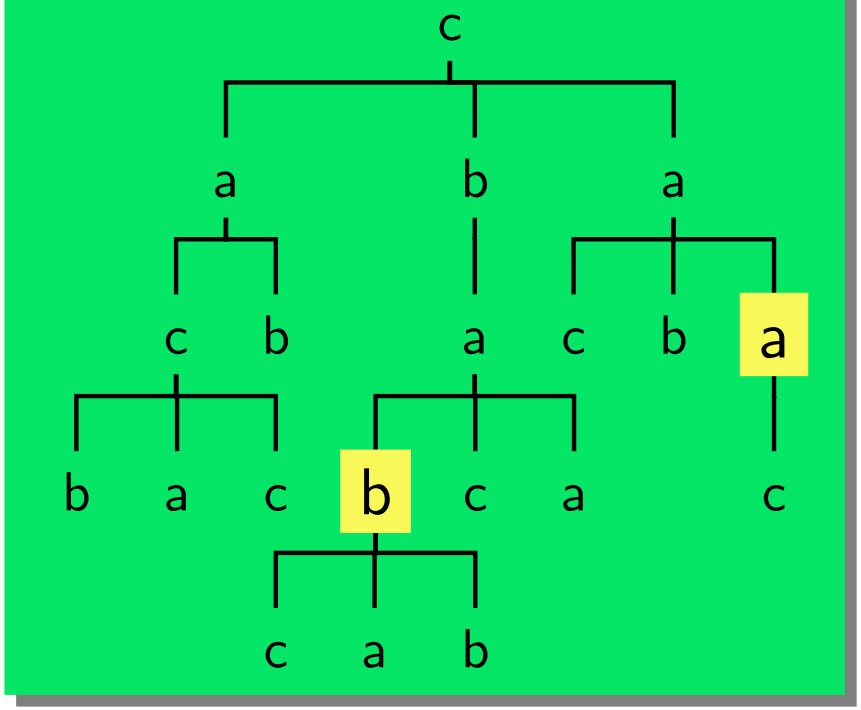

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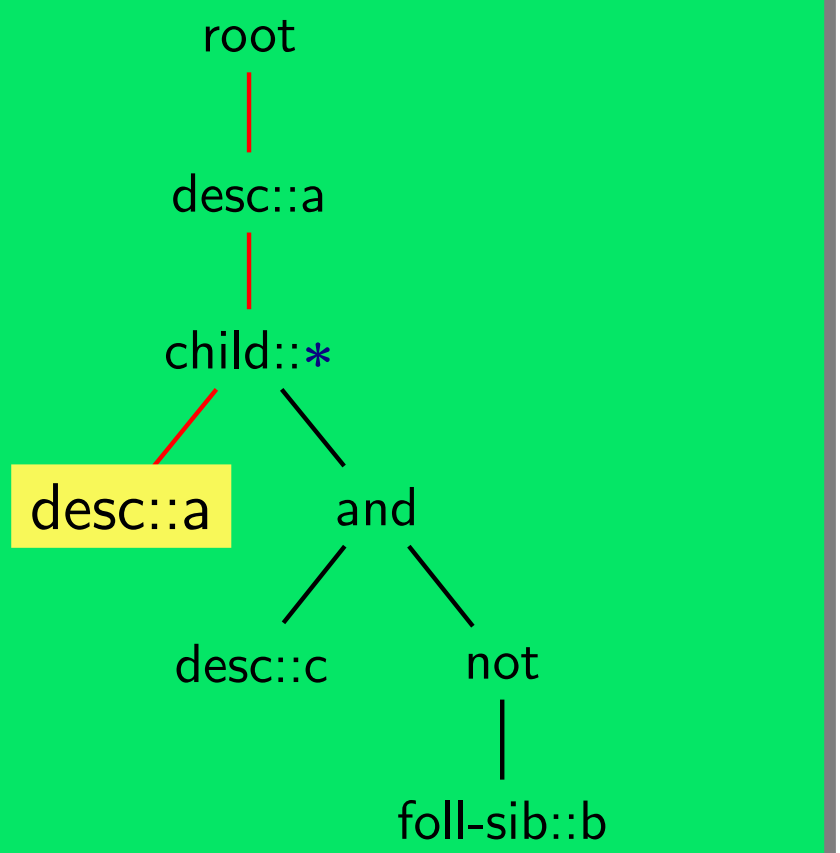

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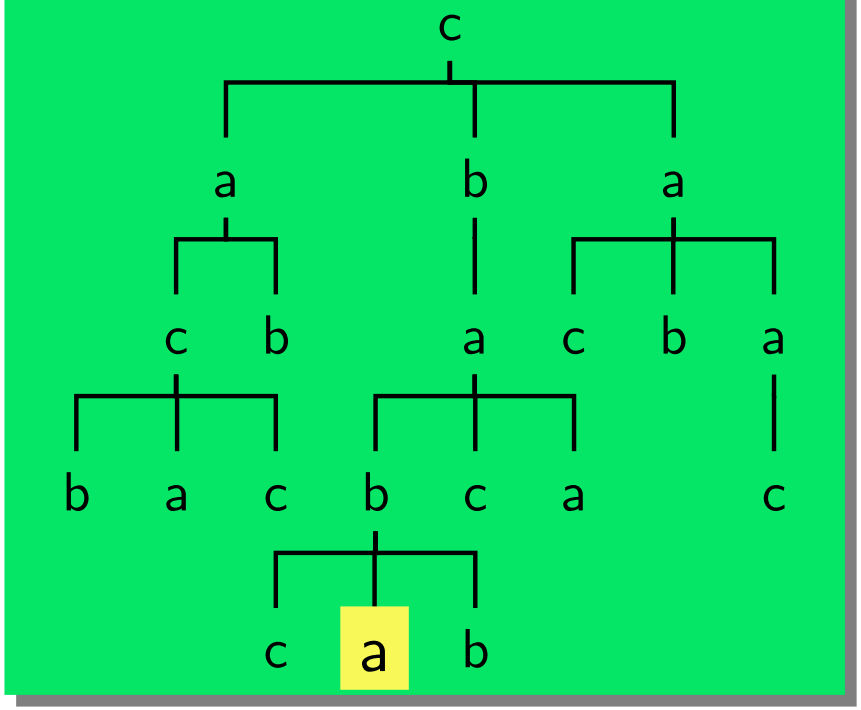

Evaluation of Navigational XPath (cont.)

Evaluation Algorithm for Navigational XPath

Procedure NEval($p_1/\cdots/p_n$,v)

$S' := \{v\}$

FOR $i := 1$ TO n

($* \; p_i = a_i::n_i \; q_i \; *$)

$S' := \{u \mid v \in S', (v,u)$ in a_i-relation, u matches $n_i\}$

Compute $S'' := \{u \mid [\![q_i]\!](u) \neq \emptyset\}$ bottom-up

$S' := S' \cap S''$

RETURN S'

Evaluation of Navigational XPath (cont.)

Evaluation Algorithm for Navigational XPath

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Complexity

- For each node of the query tree: $O(|t|)$ steps
- Overall: $O(\text{query size} \times |t|)$

Beyond Navigational XPath

Example expression

`/desc::a/child::*[desc::c[position() > 1]]/desc::a`

Observations

- In general, a subexpression does not only depend on a context node but also on
  - context position (`position()`)
  - context size (`last()`)

$\longrightarrow$ predicates can no longer be evaluated in a bottom-up fashion

- Basic idea of [Gottlob, Koch, Pichler 02]: Compute the value of each subexpression for each triple (v, i, l) of
  - a node v
  - a position i
  - a size l

Two Algorithms for XPath Evaluation

Results from [Gottlob, Koch, Pichler 02/03]

- The basic idea can be turned into different algorithms:
  - a bottom-up algorithm:
    - Computing the value for each $e, (v, i, l)$ in a dynamic programming fashion
    - Time bound: $O((\text{tree size})^5 \times (\text{query size})^2)$
  - a (mixed) top-down algorithm:
    - Compute as much information as possible in top-down fashion to evaluate subexpressions only for relevant triples (v, i, l)
    - Time bound: $O((\text{tree size})^4 \times (\text{query size})^2)$
- Further time bound for the "extended Wadler fragment": $O((\text{tree size})^2 \times (\text{query size})^2)$

Structural Complexity of XPath Evaluation

Further Results

- In [Gottlob, Koch, Pichler 03] the complexity of XPath evaluation is considered
- Data Complexity:
  - Navigational XPath: **LOGSPACE**-complete (e.g., via pebble automata)
  - Full XPath: also **LOGSPACE** (?)
- Combined Complexity:
  - Navigational XPath: **PTIME**-complete
  - Positive Navigational XPath: **LOGCFL**-complete
  - An even much larger fragment (pXPath) is in **LOGCFL**

Contents

XPath satisfiability

Observation

Not all XPath expressions are satisfiable, e.g.:

`child::a/child::b/following-sibling::c/parent::d`

Question

What is the complexity of checking satisfiability of an XPath expression for different fragments?

XPath satisfiability

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Not all XPath expressions are satisfiable, e.g.:

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What is the complexity of checking satisfiability of an XPath expression for different fragments?

Theorem [Hidders 03]

- Satisfiability for positive navigational XPath expressions is in **NP**
- Even for expressions without Boolean operators it is **NP**-hard
- For relative expressions without Boolean operators it is in **P**

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- For relative expressions without Boolean operators it is in **P**

Remark

As navigational XPath can express star-free regular expressions along a path:

Satisfiability of navigational XPath is non-elementary

XPath Satisfiability: Upper Bound

Theorem [Hidders 03]

Satisfiability for positive navigational XPath expressions is in **NP**

Proof idea

- If an expression e without $\cup$ is satisfiable it has a model of size $\leq |e|$
- For an arbitrary (negation-free) expression guess a disjunct of the disjunctive normal form

XPath Satisfiability: Lower Bound

Theorem [Hidders 03]

Satisfiability for positive navigational XPath expressions without Boolean operators is **NP**-hard

Proof idea

- Reduction from *Bounded Multiple String Matching (BMS)*:
  - Given: Pattern strings $p_1, \ldots, p_n$ over $\{0, 1, *\}$
  - Question: Is there a string over $\{0, 1\}$ of length $|p_1|$ which matches all patterns?
- Example: $*0**1$, $00*1$, $*111$ has solution 00111
- As XPath expression:
  $/\downarrow/\downarrow/0/\downarrow/\downarrow/\downarrow/1\ [\uparrow^*/1/\uparrow/\uparrow/0/\uparrow/0]\ [\uparrow^*/1/\uparrow/1/\uparrow/1]$

Contents

Forward XPath

Example query

```
//Vita/Died/*
```

Example document

⟨Composer⟩
⟨Name⟩ Claude Debussy ⟨/Name⟩
⟨Vita⟩
⟨Born⟩ ⟨When⟩ August 22, 1862 ⟨/When⟩⟨Where⟩ Paris ⟨/Where⟩⟨/Born⟩
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Abbreviated Syntax for Forward XPath

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Abbreviated Syntax for Forward XPath

Another example query

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(/*[Name]//When) | (//Where)
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More XPath operators

Operator	Meaning
p/q	child
$p//q$	descendant
$p[q]$	filter
$*$	wildcard
$p \mid q$	disjunction

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Question

Does `//Vita/Died/*` always select a subset of positions of `(/*[Name]//When) | (//Where)` ?

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Does `//Vita/Died/*` always select a subset of positions of `(/*[Name]//When) | (//Where)` ?

Answer

No!

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Answer

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Counter-example

⟨Vita⟩
  ⟨Died⟩
    ⟨How⟩ Heart disease ⟨/How⟩
  ⟨/Died⟩
⟨/Vita⟩

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Does `//Vita/Died/*` always select a subset of positions of `(/*[Name]//When) | (//Where)`?

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  ⟨/Died⟩
⟨/Vita⟩

Further question

But what if the type of documents is constrained?

XPath Containment (cont.)

Fact

For all XML documents of type

```
<!DOCTYPE Composers [
    <!ELEMENT Composers (Composer*)>
    <!ELEMENT Composer (Name, Vita, Piece*)>
    <!ELEMENT Vita (Born, Married*, Died?)>
    <!ELEMENT Born (When, Where)>
    <!ELEMENT Married (When, Whom)>
    <!ELEMENT Died (When, Where)>
    <!ELEMENT Piece (PTitle, PYear,
         Instruments, Movements)>
    ]>
```

the pattern `//Vita/Died/*` always selects a subset of positions of

`(/*[Name]//When) | (//Where)`

XPath Containment: Definition

Definition: Containment for $\texttt{XPath}(S)$

Let S be a set of XPath-operators. The containment problem for $\texttt{XPath}(S)$ is:

Given: $\texttt{XPath}(S)$-expression p, q

Question: Is $p(t) \subseteq q(t)$ for all documents t?

Definition: Containment for $\texttt{XPath}$ (S) with DTD

Let S be a set of XPath-operators. The containment problem for $\texttt{XPath}(S)$ in the presence of DTDs is:

Given: $\texttt{XPath}(S)$-expression p, q, DTD d

Question: Is $p(t) \subseteq q(t)$ for all documents t satisfying $t \models d$?

Observation

These problems are crucial for static analysis and query optimization

Question

For which fragments S are these problems

- decidable?
- efficiently solvable?

Results

General remarks

- The XPath containment problem has been considered for various sets of operators
- Focus on Forward XPath
- Results vary from **PTIME** to "undecidable"
- Various methods have been used:
  - Canonical model technique
  - Homomorphism technique
  - Chase technique
- More about this in [Miklau, Suciu 2002; Deutsch, Tanen 2001; Sch. 2004]
- We will consider automata based techniques

The Automata Technique

Definition: (Relative Containment for XPath (S) wrt DTD)

Let S be a set of XPath-operators. The containment problem for XPath(S) relative to a DTD is:

Given: XPath(S)-expression p, q, DTD d

Question: Is $p(D) \subseteq q(D)$ for all documents D satisfying $D \models d$?

A vague plan

- Construct an automaton $\mathcal{A}_p$ for p
- Construct an automaton $\mathcal{A}_q$ for q
- Construct an automaton $\mathcal{A}_d$ for d
- Combine these automata suitably to get an automaton which accepts all counter-example documents

A Simplification

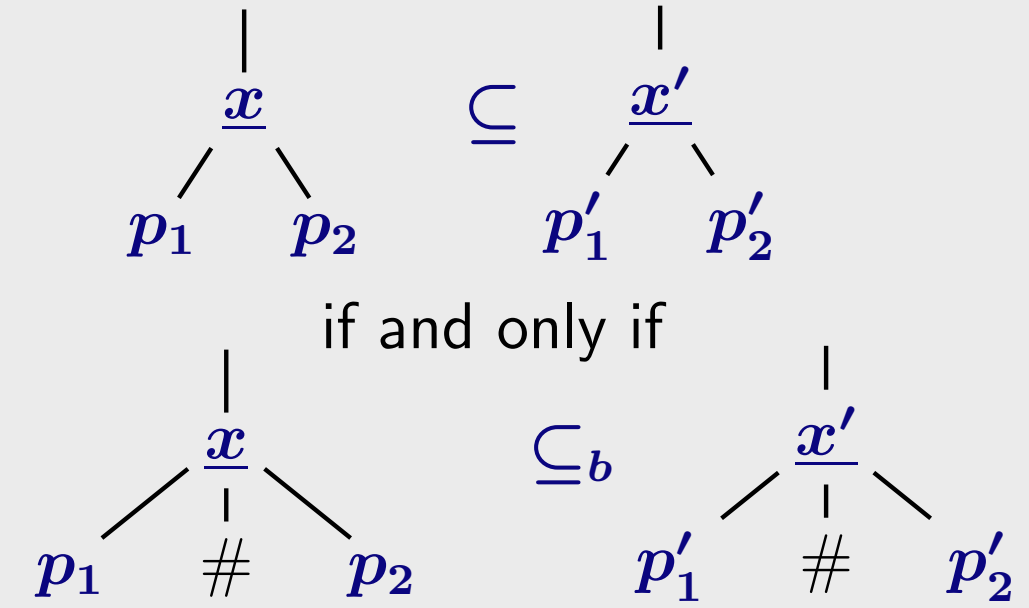
Definition: (Boolean containment)

$p \subseteq_b q$:$\Longleftrightarrow$ whenever p selects *some* node in a tree t then q also selects some node in t.

Useful observation [Miklau, Suciu 2002]

In the presence of $[\,]$, Boolean containment has the same complexity as containment.

Crucial idea



XPath Containment: 2 Examples

Result 1 [Neven, Sch. 2003]

The Boolean containment problem for XPath(/, //) in the presence of DTDs is in **PTIME**

Result 2 [Neven, Sch. 2003]

The Boolean containment problem for XPath(/, //, [], *, |) in the presence of DTDs is in **EXPTIME**

Note

Both results are optimal wrt complexity: the problems are complete for these classes

Containment for XPath(/, //) and DTDs

Result 1 [Neven, Sch. 2003]

The Boolean containment problem for XPath(/, //) in the presence of DTDs is in **PTIME**

Proof idea

- XPath(/, //)-expressions can only describe vertical paths in a tree
- Each expression is basically of the form $p_1//p_2//\cdots//p_k$, where each p_i is of the form $l_{i1}/\cdots/l_{im_i}$
- On strings this is a sequence of string matchings corresponding to a regular language L

$\Rightarrow$ Deterministic string automaton of linear size

- Recall: there is a deterministic top-down automaton which checks whether a p-path exists

$\Rightarrow$ Deterministic top-down automaton $\mathcal{A}_p$

$\Rightarrow$ Deterministic top-down automaton $\mathcal{A}_{\overline{q}}$ checking that no q-path exists

Containment for XPath(/, //) and DTDs

Result 1 [Neven, Sch. 2003]

The containment problem for XPath(/, //) in the presence of DTDs is in **PTIME**

Proof idea (cont.)

- Deterministic top-down automaton $\mathcal{A}_p$
- Deterministic top-down automaton $\mathcal{A}_{\overline{q}}$ checking that no q-path exists
- There is a deterministic top-down automaton $\mathcal{A}_d$ checking whether t conforms to d
- $p \subseteq_b q$ in the presence of $d \iff L(\mathcal{A}_p \times \mathcal{A}_{\overline{q}} \times \mathcal{A}_d) = \emptyset$
- The latter can be checked in polynomial time

Containment for XPath(/, //, [], *, |) and DTDs

Result 2 [Neven, Sch. 2003]

The containment problem for XPath(/, //, [], *, |) in the presence of DTDs is in **EXPTIME**

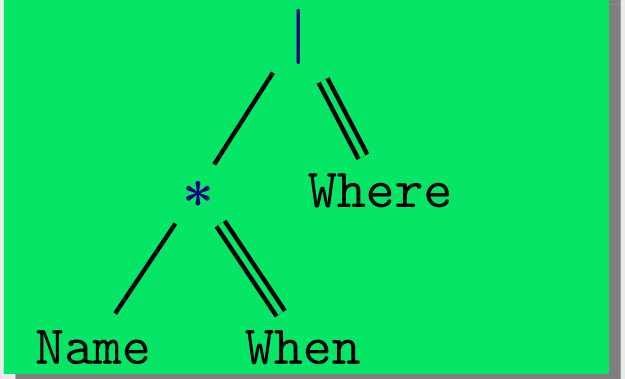
Proof idea

We again represent patterns like

```
(/*[Name]//When) | (//Where)
```

as query trees:

Example query tree



Lemma

For each XPath(/, //, [], *, |)-expression p there is a deterministic bottom-up automaton $\mathcal{A}_p$ of exponential size which checks whether in a tree p holds

Containment for XPath(/, //, [], *, |) and DTDs

Lemma

For each XPath(/, //, [], *, |)-expression p there is a deterministic bottom-up automaton $\mathcal{A}_p$ of exponential size which checks whether in a tree p holds

Proof idea for Lemma

- States of $\mathcal{A}_p$ are of the form $(S_/, S_{//})$
- Both $S_/$ and $S_{//}$ are sets of positions of the query tree:
  - $S_/$: positions matching v
  - $S_{//}$: positions matching some node in the subtree of v

Containment for XPath(/, //, [], *, |) and DTDs

Result 2 [Neven, Sch. 2003]

The containment problem for XPath(/, //, [], *, |) in the presence of DTDs is in **EXPTIME**

Proof idea (cont.)

- Construct deterministic bottom-up automaton $\mathcal{A}_p$ of exponential size
- Construct deterministic bottom-up automaton $\mathcal{A}_{\overline{q}}$ of exponential size
- Construct deterministic bottom-up automaton $\mathcal{A}_d$ of exponential size
- $p \subseteq_b q$ in the presence of $d \Longleftrightarrow L(\mathcal{A}_p \times \mathcal{A}_{\overline{q}} \times \mathcal{A}_d) = \emptyset$

$\Rightarrow$ exponential time

Corresponding Lower Bound

Theorem

The containment problem for $\texttt{XPath}(/, //, [], *, |)$ in the presence of DTDs is **EXPTIME**-hard

Proof sketch

Proof by reduction from *Two-player corridor tiling*

Example

Example:

Top row $T =$ | c | a | a | c |

Bottom row $B =$ | a | c | a | c |

Vertical and horizontal constraints:

$V =$ | c | a | c |, | c | c | a |

$H =$ | a | c |, | a | a |, | c | a |

Player I to move

| c | a | a | c |

⋮ ⋮ ⋮ ⋮

| a | c | a | c |

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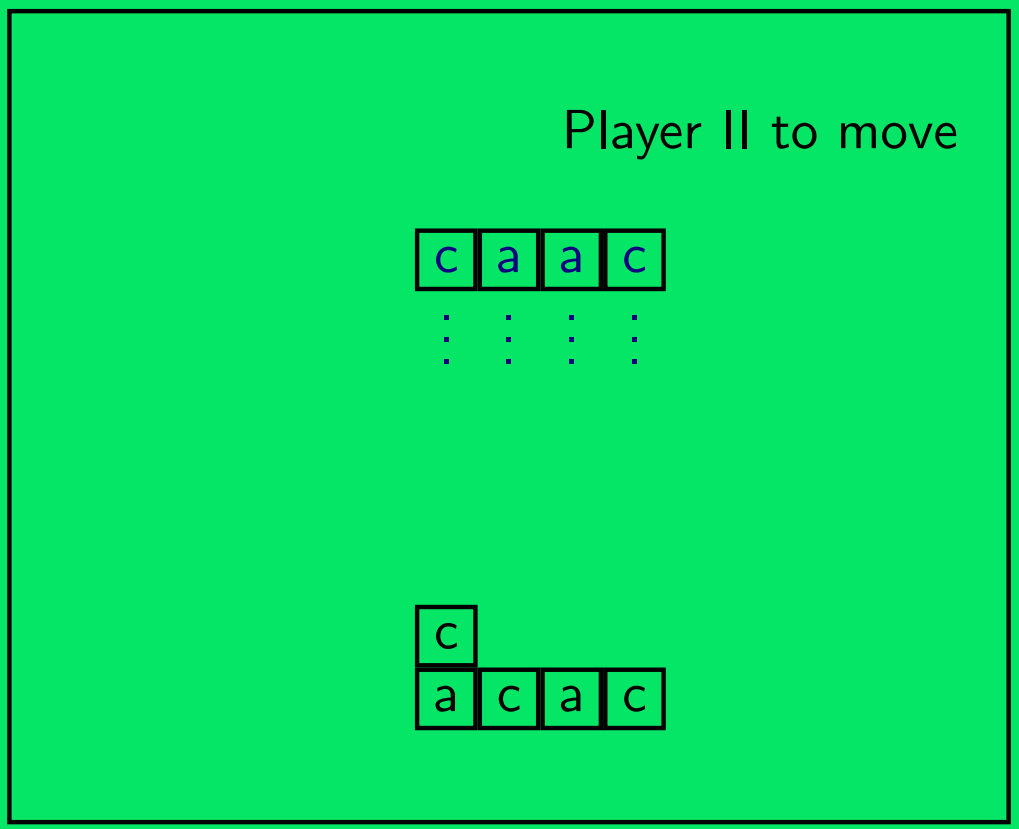

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Player I to move

c	a	a	c
⋮	⋮	⋮	⋮

c	a		
a	c	a	c

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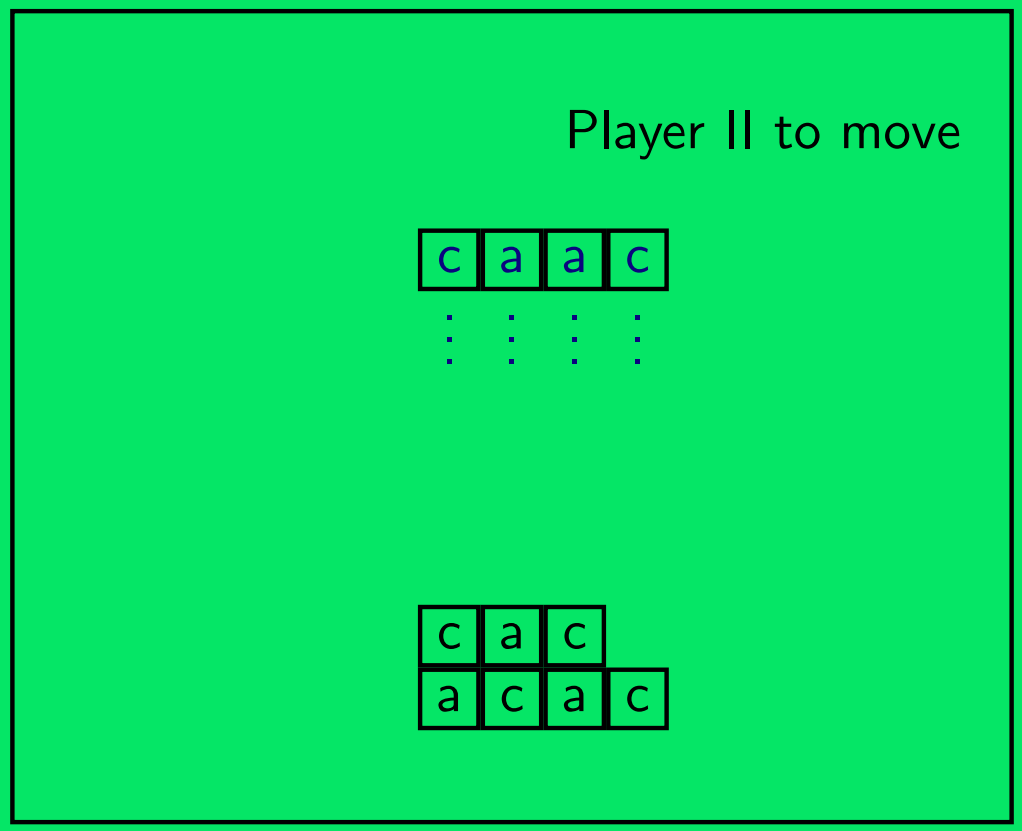

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Player I to move

c	a	a	c
⋮	⋮	⋮	⋮

c	a	c	a
a	c	a	c

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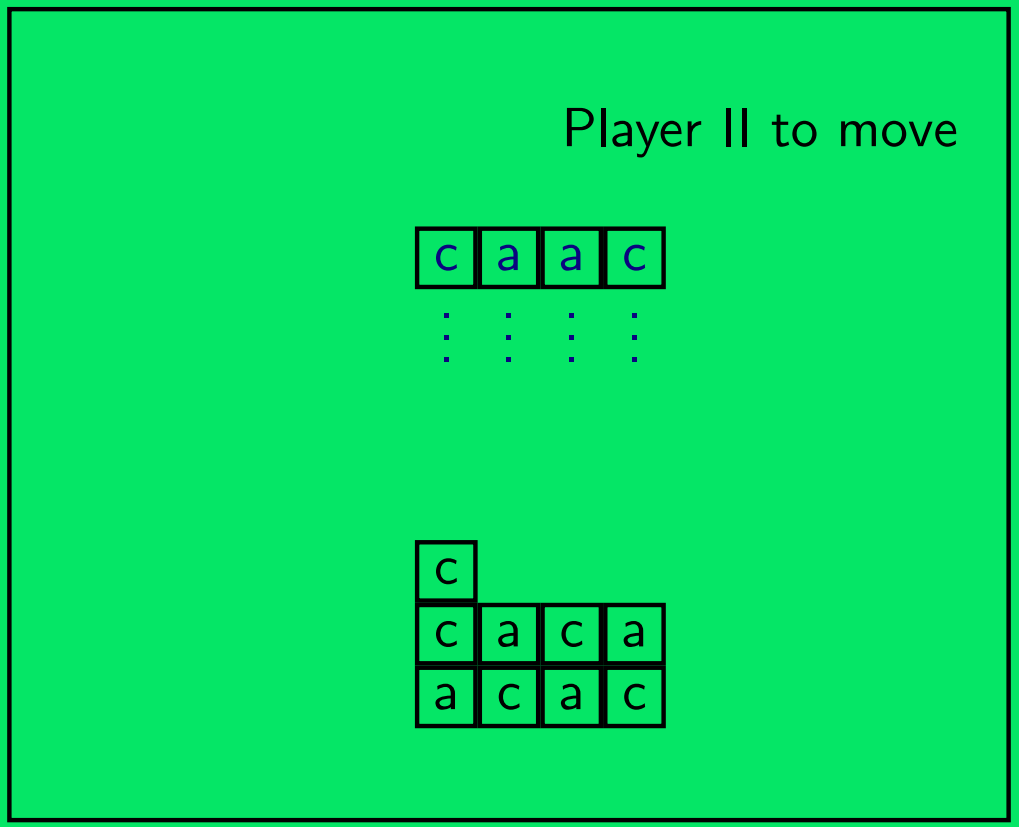

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Vertical and horizontal constraints:

$V =$ | c | a | c |, | c | c | a |

$H =$ | a | c |, | a | a |, | c | a |

Player I to move — Player II lost

c	a	a	c
⋮	⋮	⋮	⋮

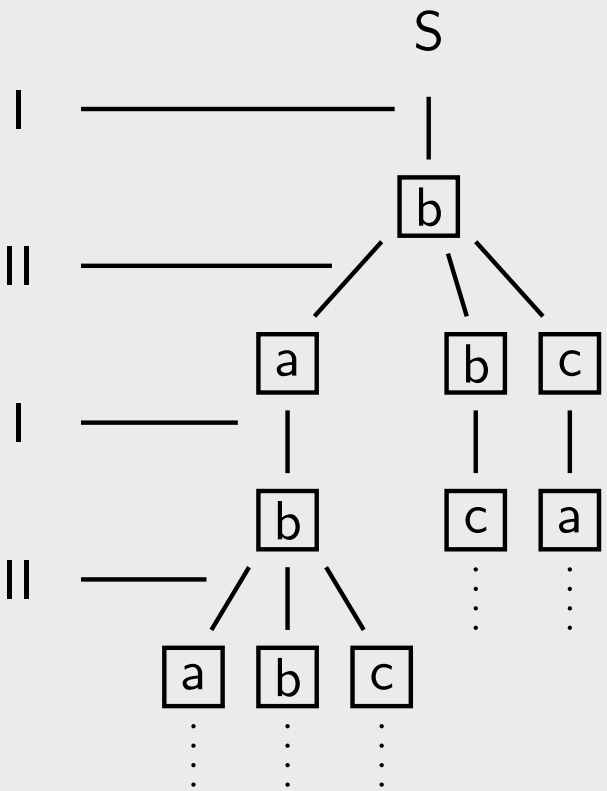
c	c		
c	a	c	a
a	c	a	c

Deciding whether player I has a winning strategy is **EXPTIME**-complete

Strategies as trees

Proof Sketch (cont.)

Tiles over $\{a, b, c\}$



Strategies as trees

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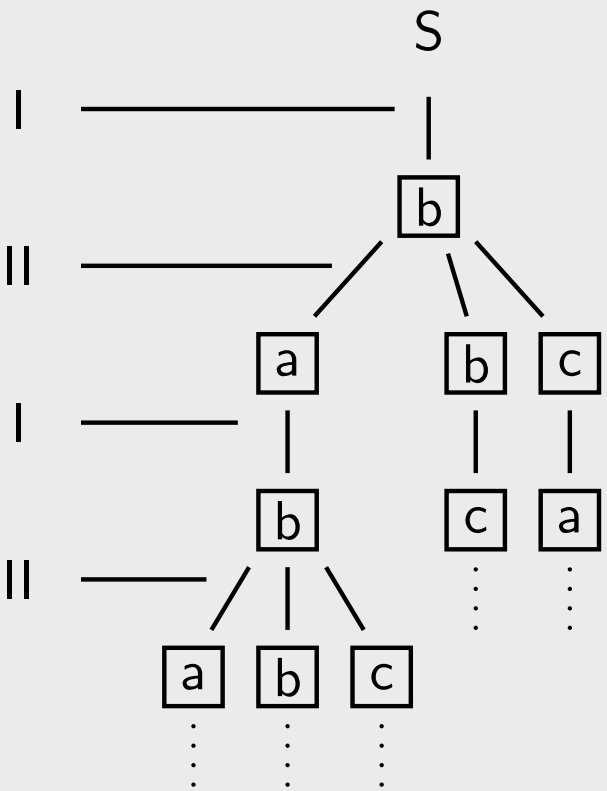


This DTD describes all strategy trees:

$$S \to (a, \mathsf{I}) + (b, \mathsf{I}) + (c, \mathsf{I})$$
$$(\sigma, \mathsf{I}) \to (a, \mathsf{II})(b, \mathsf{II})(c, \mathsf{II}) + \# + \$^{\mathsf{II}}$$
$$(\sigma, \mathsf{II}) \to (a, \mathsf{I}) + (b, \mathsf{I}) + (c, \mathsf{I}) + \# + \$^{\mathsf{I}} + !$$
$$\$^{\mathsf{II}} \to (a, \mathsf{II})(b, \mathsf{II})(c, \mathsf{II})$$
$$\$^{\mathsf{I}} \to (a, \mathsf{I}) + (b, \mathsf{I}) + (c, \mathsf{I})$$

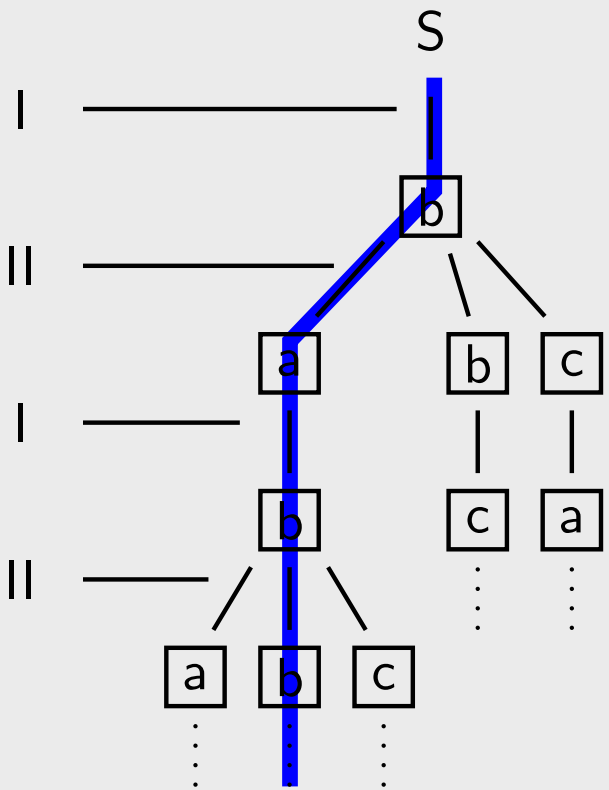
$\$ =$ line separator $\#$ = terminal symbol

! indicates misplaced tile

Strategies as trees

Proof Sketch (cont.)

Tiles over $\{a, b, c\}$



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! indicates misplaced tile

One path corresponds to one game

Winning strategies and paths

Proof Sketch (cont.)

There are various kinds of paths in a game tree:

(a) Legal tilings $\Longrightarrow$ Player I wins

(b) Syntactically wrong: some row of wrong length

(c) II places a wrong tile $\Longrightarrow$ Player I wins

(d) I places a wrong tile $\Longrightarrow$ Player II wins

Player I has a winning strategy

$\Longleftrightarrow$

there is a tree in which all paths are of the form (a) or (c)

We want to construct q such that all paths of the form (b) or (d) are selected

Then: Player I wins iff $/S \not\subseteq q$ wrt DTD

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Problem: if II places a wrong tile, I might be forced to place a wrong tile, too

$\Longrightarrow$ We let player I mark wrong tiles of II by !

$\Longrightarrow$ We have to check that I does this correctly

Path conditions

Proof Sketch (cont.)

Player I has winning strategy $\iff$ $/S \not\subseteq q$

q expresses that one of the following holds

- Player I violates a horizontal constraint:
- Player I violates a vertical constraint:
- Some row does not contain exactly n tiles
- Player I wrongly claims a mistake of II:
- Some more conditions on B and T

Path conditions

Proof Sketch (cont.)

Player I has winning strategy $\iff$ $/S \not\subseteq q$

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  For each $(x, y) \notin H$: $//(x, II)/(y, I)$

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$* =$ OR of all symbols, $\sigma^i = \sigma/\cdots/\sigma$ (i times)

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  For each $(x, y) \notin V$: $//(x, I)/ *^{n+1} /(y, I)$

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$$\mathcal{D}^{n+1} \mid \bigcup_{i=0}^{n-1} (\$^{\mathrm{I}}|\$^{\mathrm{II}}|S)/\mathcal{D}^i/(\$^{\mathrm{I}}|\$^{\mathrm{II}}|\#)$$

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$* =$ OR of all symbols, $\sigma^i = \sigma/\cdots/\sigma$ (i times)

$\mathcal{D} = (d_1, I)|\cdots|(d_m, I)|(d_1, II)|\cdots|(d_m, II)$

Path conditions

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- Some row does not contain exactly n tiles

  $$\mathcal{D}^{n+1} \mid \bigcup_{i=0}^{n-1} (\$^{\mathrm{I}}|\$^{\mathrm{II}}|S)/\mathcal{D}^i/(\$^{\mathrm{I}}|\$^{\mathrm{II}}|\#)$$

- Player I wrongly claims a mistake of II:

  For each $(x, y) \in V, (x', y) \in H$:
  $//(x, II)/ *^n /(x', I)/(y, II)/$!

- Some more conditions on B and T

$*$ = OR of all symbols, $\sigma^i = \sigma/\cdots/\sigma$ (i times)

$\mathcal{D} = (d_1, I)|\cdots|(d_m, I)|(d_1, II)|\cdots|(d_m, II)$

Related work on XPath containment

More Results

- Containment of XPath with / and a subset of $\{//, [], *\}$ was studied in [Miklau and Suciu 2002]:
  - Containment of XPath$(//, [], *)$ is **coNP**-complete even if the number of $*$ or the number of [] is bounded
  - If the number of // is bounded then it is in polynomial time
- XPath containment in the presence of DTDs and simple integrity constraints was investigated in [Deutsch and Tanen 2001]:
  - In general (*unbounded* constraints): undecidable
- More complexity results between **coNP** and undecidable for other fragments and extensions in [Neven and S. 2003]

Some Open Questions

- What's the exact borderline between fragments of XPath with decidable and undecidable containment problem?
- To what extent can the presented result be extended to other axes (siblings, backward)?

Summary: XPath

Summary

Expressive Power

Closely related to first-order logic

Evaluation

- In general: Polynomial time
- Large fragments in linear time
- Structural complexity between **LOGSPACE** and **PTIME**

Satisfiability

- Without negation: **PTIME** or **NP**
- With negation: non-elementary

Containment

- Varying from **PTIME** to undecidable
- Upper bound for positive navigational XPath?

Contents

XSLT Typechecking

Definition: Transformation typechecking

Given: DTDs d_1 and d_2 and a transformation T

Result: Is $T(t)$ valid wrt. d_2, for each document t valid wrt. d_1?

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Question: Is XSLT typechecking decidable?

Question: What is the complexity?

Outline of the Following

- Provide an automata model for XSLT transformations
- Show that the behaviour of these automata can be captured by MSO logic
- Use manipulation of regular tree languages to solve type checking problem

$\longrightarrow$ This part is based on [Milo,Suciu,Vianu 01]

XSLT in more detail

How XSLT Roughly Works

Templates:

⟨`xsl:template name=TName match=pattern mode=MName`⟩

Template application:

⟨`xsl:apply-templates select=Expression mode=MName`⟩

XSLT Processing Whenever `xsl:apply-templates` is called at a node v the following happens:

- Compute set $S(v)$ of nodes, reachable from v via `Expression` (if `select` is not present, $S(v) =$ children of v)
- For each $w \in S(v)$ compute which templates that can be applied to w:
  - w has to match `pattern` of a template
  - the `mode` of the template has to be the same as the mode of `xsl:apply-templates`
- If no template matches, take the default template
- For each $w \in S(v)$ select the best template and apply it.

The process starts at the root of the tree

XSLT: Example

Example Transformation

Remove everything below a c*. Translate* a *below* b *into* d

Example XSLT

```
⟨xsl:template match="a"⟩
   ⟨a⟩ ⟨xsl:apply-templates⟩ ⟨/a⟩
⟨/xsl:template⟩
⟨xsl:template match="a" mode="below"⟩
   ⟨d⟩ ⟨xsl:apply-templates⟩ ⟨/d⟩
⟨/xsl:template⟩
⟨xsl:template match="b"⟩
   ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩
⟨/xsl:template⟩
⟨xsl:template match="b" mode="below"⟩
   ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩
⟨/xsl:template⟩
⟨xsl:template match="c"⟩
   ⟨c⟩ ⟨/c⟩
⟨/xsl:template⟩
⟨xsl:template match="c" mode="below"⟩
   ⟨c⟩ ⟨/c⟩
⟨/xsl:template⟩
```

XSLT: Example

Example Transformation

Remove everything below a c*. Translate* a *below* b *into* d

Example XSLT (Abbreviated)

⟨... match="a"⟩ ⟨a⟩ ⟨xsl:apply-templates⟩ ⟨/a⟩ ⟨/...⟩

⟨... match="a" mode="below"⟩ ⟨d⟩ ⟨xsl:apply-templates⟩ ⟨/d⟩ ⟨/...⟩

⟨... match="b"⟩ ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩ ⟨/...⟩

⟨... match="b" mode="below"⟩ ⟨b⟩ ⟨xsl:apply-templates mode="below"⟩ ⟨/b⟩ ⟨/...⟩

⟨... match="c"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

⟨... match="c" mode="below"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

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⟨... match="c"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

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Example Trees

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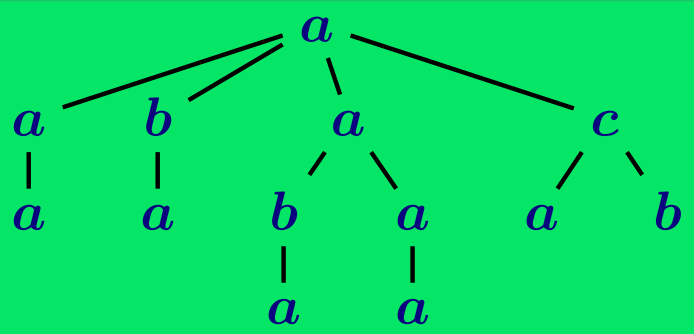
⟨... match="c" mode="below"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

Example Trees



$\Rightarrow$

XSLT: Example

Example Transformation

Remove everything below a c. Translate a below b into d

Example XSLT (Abbreviated)

⟨... match="a"⟩ ⟨a⟩ ⟨xsl:apply-templates⟩ ⟨/a⟩ ⟨/...⟩

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⟨... match="c"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

⟨... match="c" mode="below"⟩ ⟨c⟩ ⟨/c⟩ ⟨/...⟩

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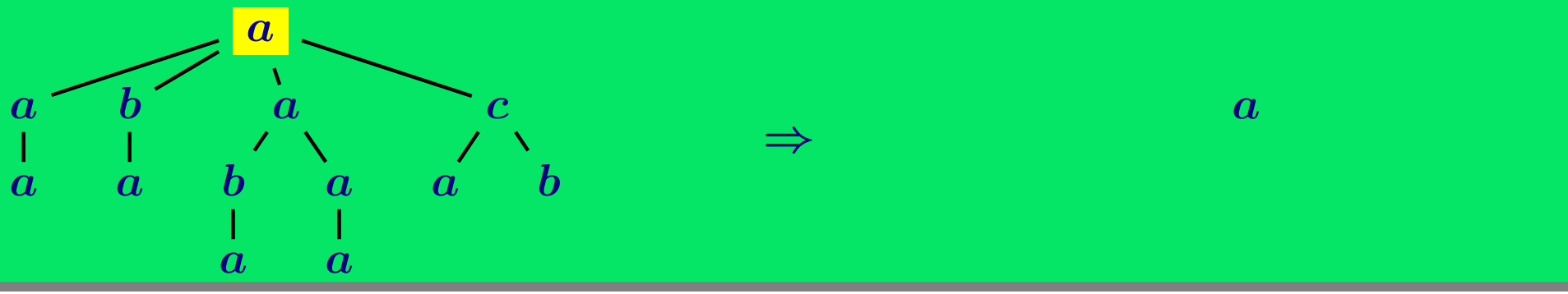

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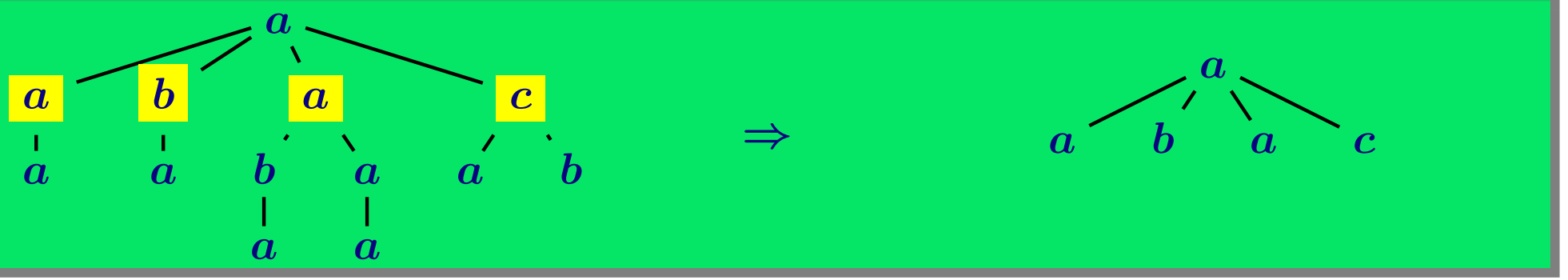

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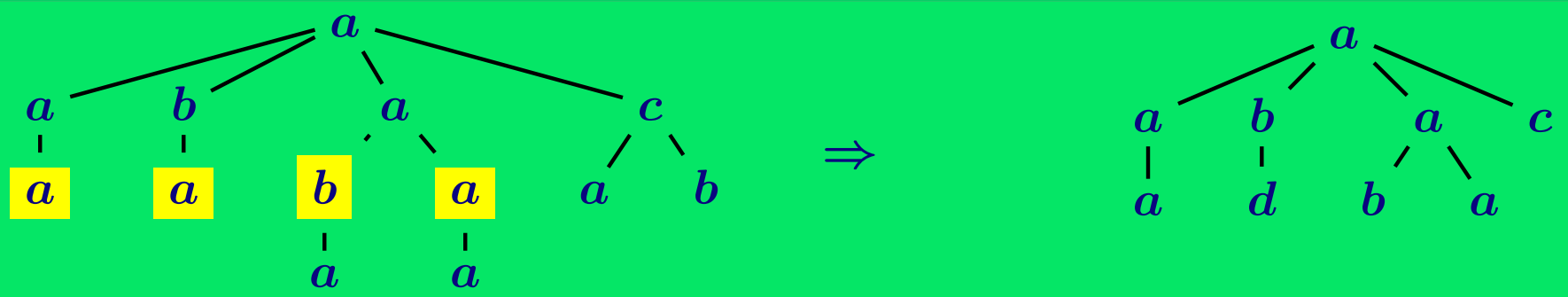

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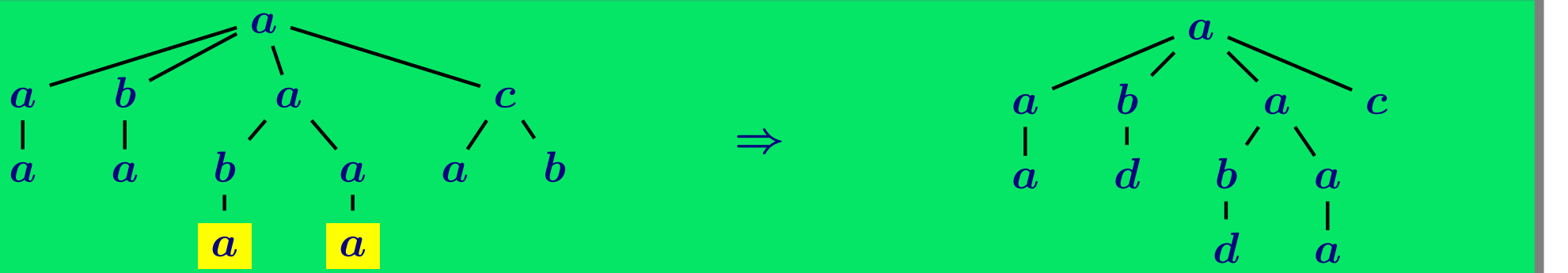

XSLT: More involved example

Remark

The previous example corresponds to top-down tree transducers

Example XSLT

⟨xsl:template match=”/b”⟩
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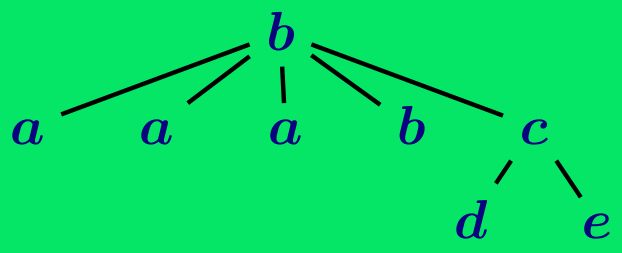

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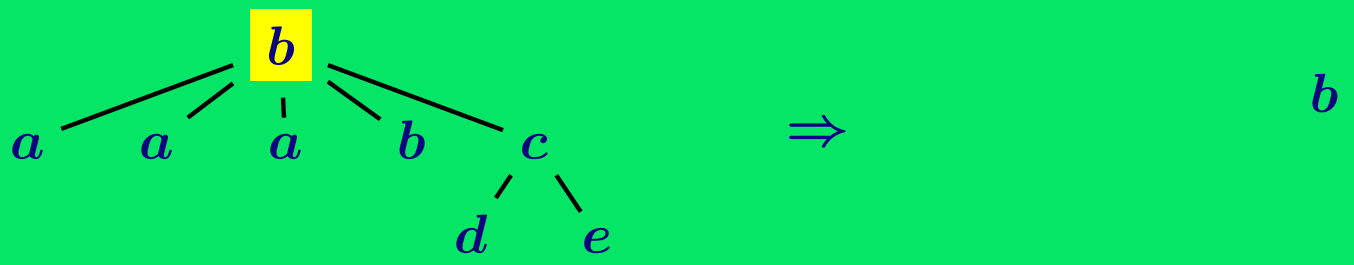

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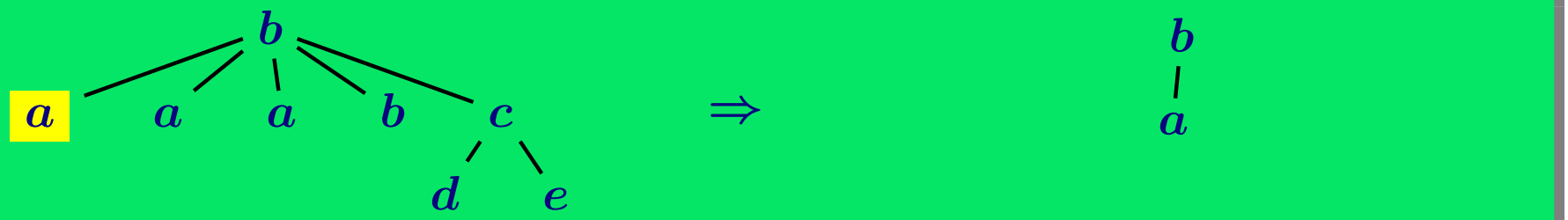

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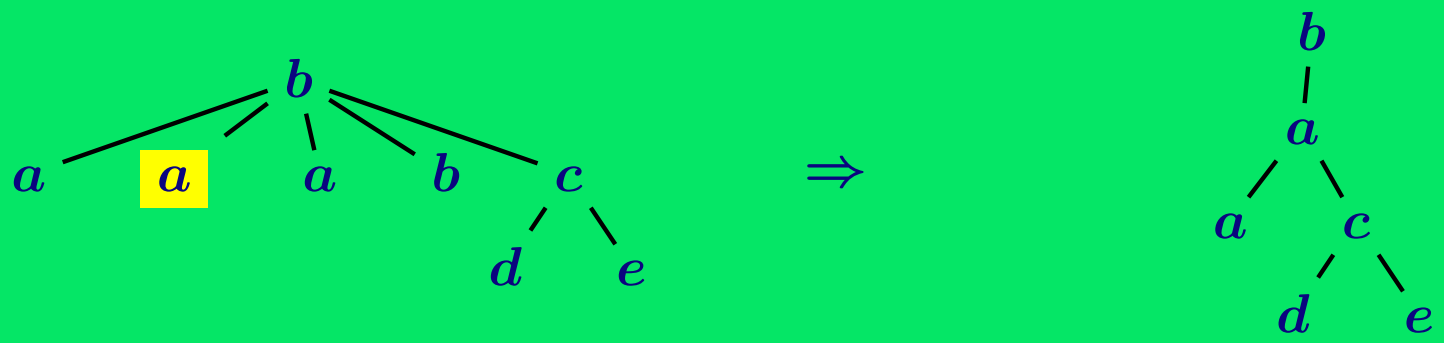

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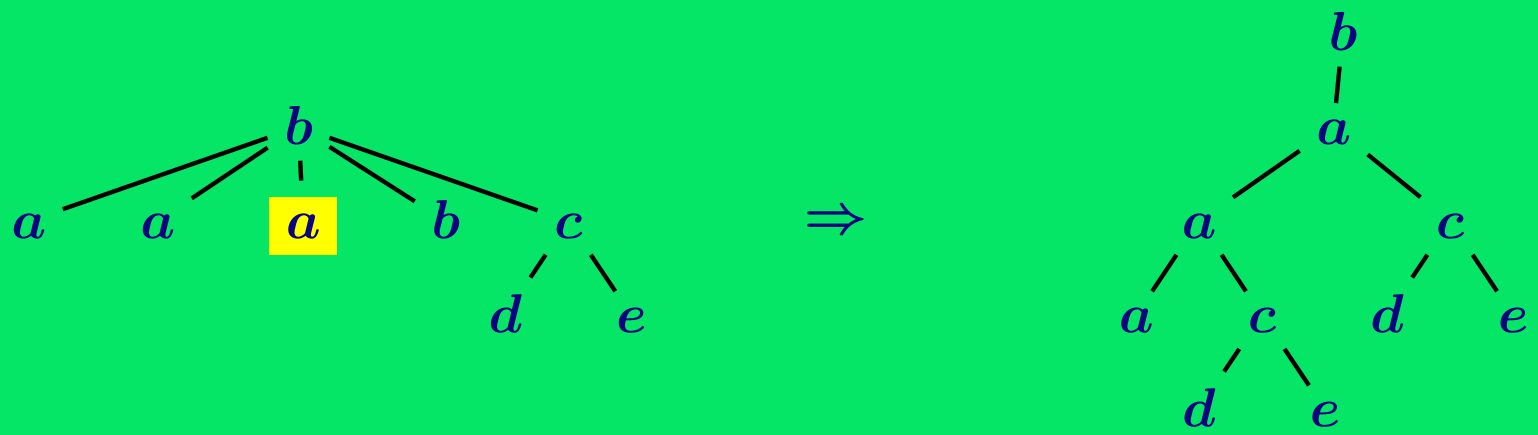

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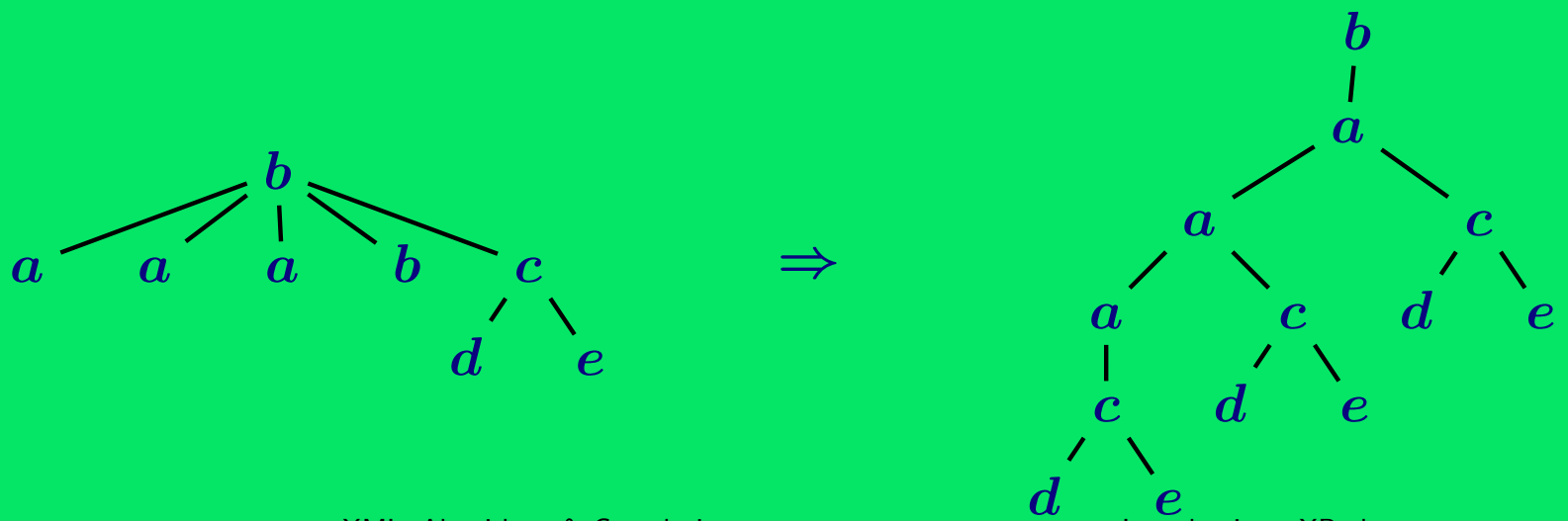

An automaton model for XSLT

Definition: k-pebble Transducer

- Work on binary tree encodings of unranked trees
- Up to k pebbles can be placed on the tree
- Only pebble with highest number (*current pebble*) can move, depending on state, number of pebbles symbols under pebbles and incidence of pebbles
- possible pebble movements:
  - stay
  - go to left child, right child or parent
  - lift current pebble
  - place new pebble on the root
- Nondeterminism allowed
- If current pebble stays it is possible to produce output:
  - a node with two (forthcoming) subtrees; in this case two independent subcomputations (*branches*) are started, which construct the left subtree and right subtree, respectively
  - a leaf; in this case the computation branch stops

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- Nondetermin
- If current pe
  - a node wi
    subcompu
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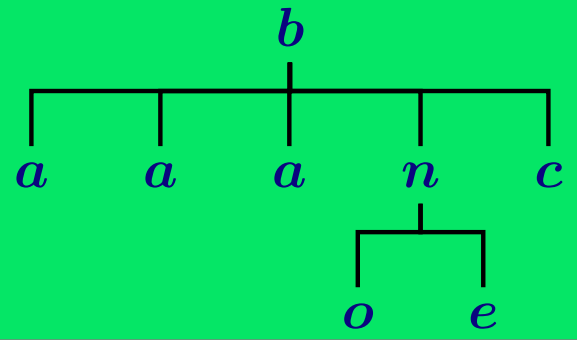
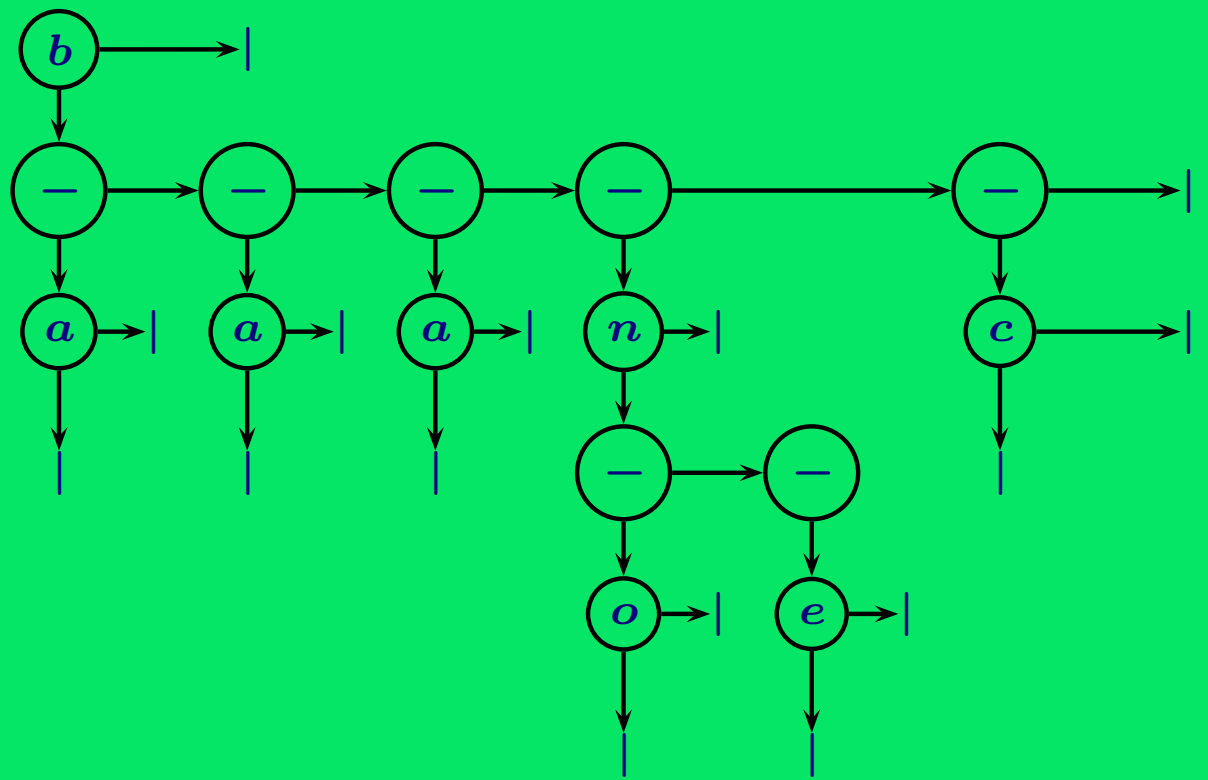
Example: Unranked Tree



... as binary tree



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Computing XSLT transformations by k-pebble transducers

Fact

k-pebble transducers can evaluate most XPath expressions
(and produce as output an encoded version of the result list)
- even with other axes than the forward axis

Proof idea

- Whenever `xsl:apply-templates` is called at a node v the following happens:
  - Cycle through the set $S(v)$ of nodes, reachable from v via `Expression` (if `select` is not present, $S(v) =$ children of v)
  - For each $w \in S(v)$ check which templates can be applied to w:
    - w has to match `pattern` of a template
    - the `mode` of `xsl:apply-templates` is stored in the state of the automaton
  - For each $w \in S(v)$ select the best template and branch into
    - a subcomputation which handles the next node in $S(v)$ (via the right child)
    - a subcomputation which applies the template to the current node
- The computation starts at the root of the tree

Back to the Typechecking Question

Question: Is XSLT typechecking decidable?

Proof idea

- How can we check that $T(t) \in L(d_2)$, for each $t \in L(d_1)$?
- Obvious approach:
  - Compute $T(L(d_1))$
  - Check that $T(L(d_1)) \subseteq L(d_2)$

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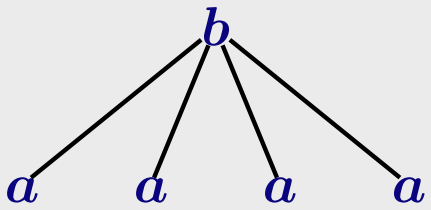
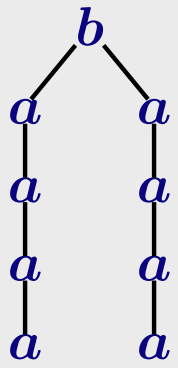
Transform 

into 

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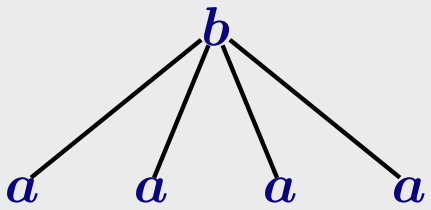
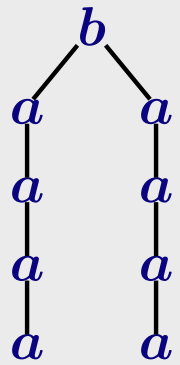
Transform 

into 

- Alternative approach:
  - Compute $T^{-1}(\overline{L(d_2)})$
  - Check $L(d_1) \cap T^{-1}(\overline{L(d_2)}) = \emptyset$

Pebble acceptors

Definition: k-pebble acceptors

- Basically the same as k-pebble transducers
- Instead of output producing steps:
  - *accept*
  - branch into two independent subcomputations
- A tree is accepted if all subcomputations accept

Main Steps of the Proof

(i) $T^{-1}(\overline{L(d_2)})$ is accepted by a k-pebble acceptor

(ii) k-pebble acceptors only accept regular tree languages

Step (i)

Lemma

$T^{-1}(\overline{L(d_2)})$ is accepted by a k-pebble acceptor

Proof

- Let B be a nondeterministic top-down tree automaton which accepts $\overline{L(d_2)}$
- Let T be a k-pebble tree transducer
- We construct k-pebble acceptor A for $T^{-1}(\overline{L(d_2)})$, i.e., an automaton which on input t decides whether there is a tree in $T(t)$ which is accepted by B:
  - Simulate T on t and B
  - Simulate at the same time the behaviour of B on the (virtual) output tree - this is possible as the output tree is produced top-down and can be instantly consumed by B
  - The simulation involves branching, whenever T branches, and produces two new subtrees

Step (ii)

Lemma

k-pebble acceptors only accept regular tree languages

Proof idea

Show that the language of a k-pebble acceptor can be expressed by an MSO-formula:

1. Reduce k-pebble automaton acceptance to AGAP (Alternating Graph Accessibility)
2. Show that AGAP can be expressed in MSO
3. Some adjustments necessary

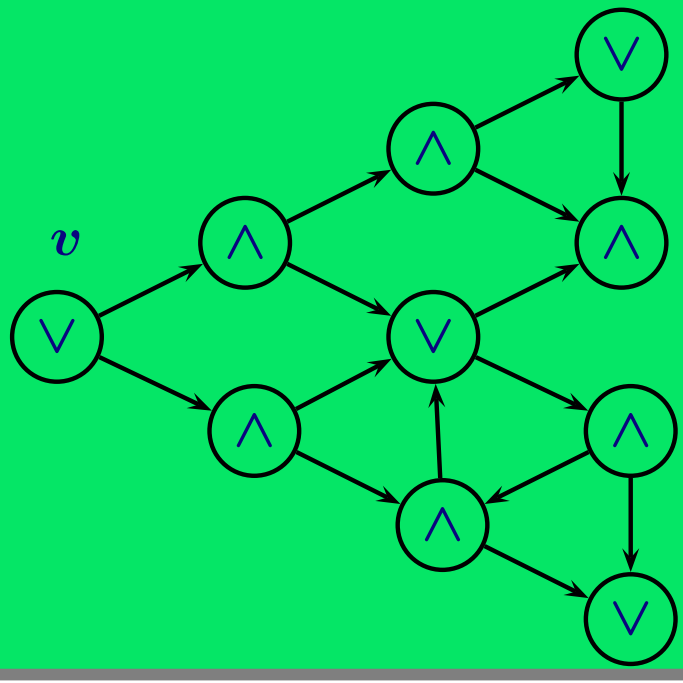
Alternating Graph Accessibility

Definition: Accessible Nodes

Let $G = (V, E)$, $V = V_\wedge \cup V_\vee$. A node w is **accesible** if

- $w \in V_\wedge$ and all successors of w are accessible, or
- $w \in V_\vee$ and at least one successor of w is accessible

Example



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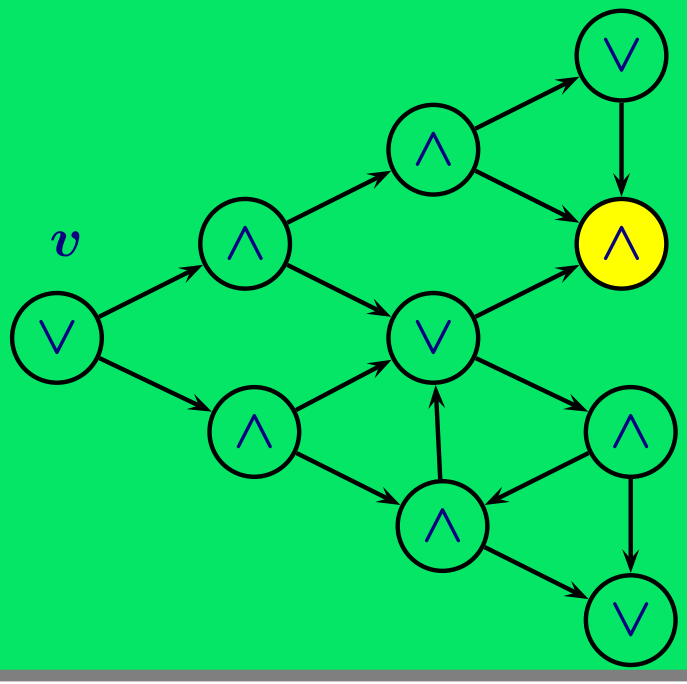

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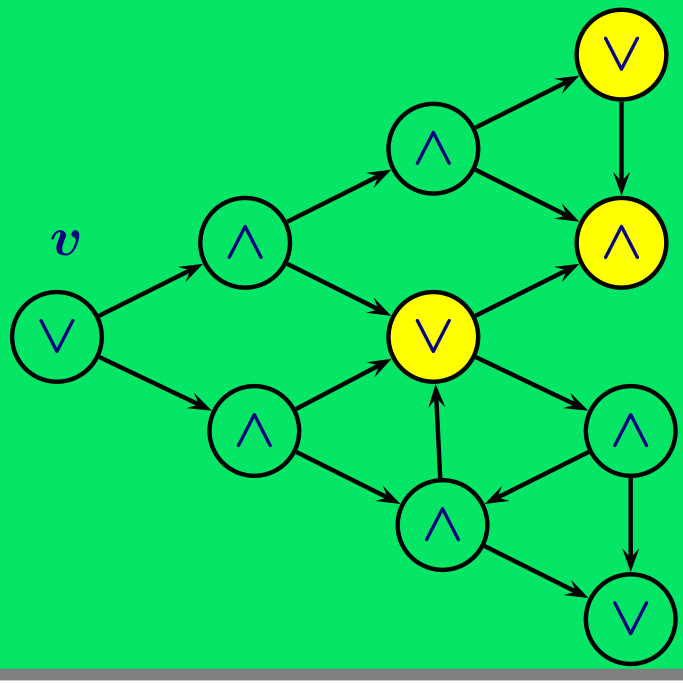

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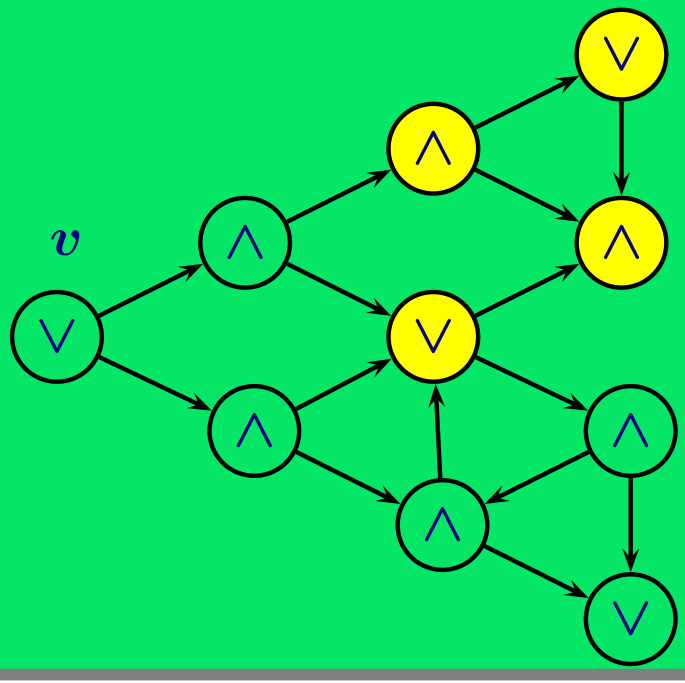

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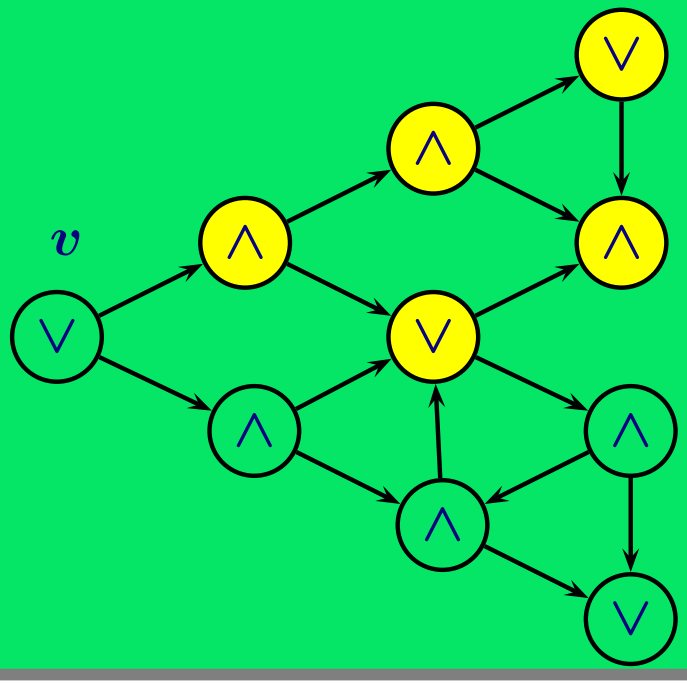


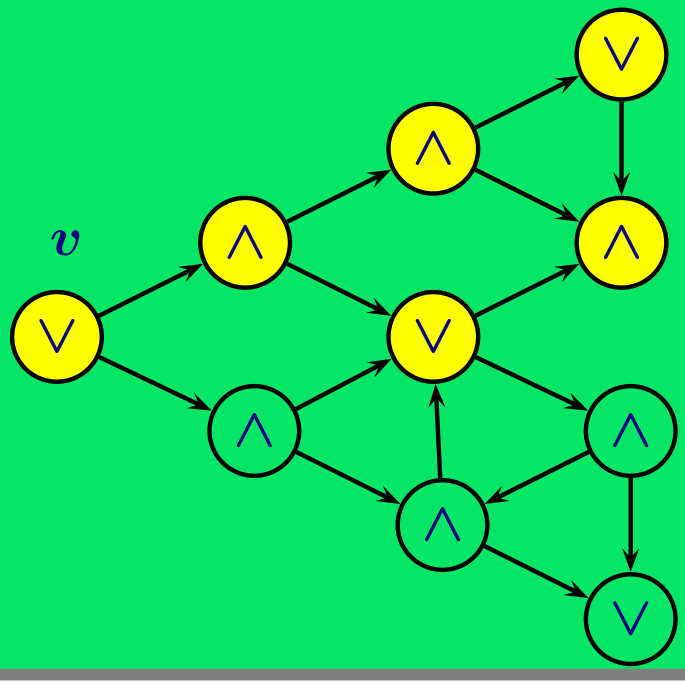
Alternating Graph Accessibility

Definition: Accessible Nodes

Let $G = (V, E)$, $V = V_\wedge \cup V_\vee$. A node w is **accesible** if

- $w \in V_\wedge$ and all successors of w are accessible, or
- $w \in V_\vee$ and at least one successor of w is accessible

Example



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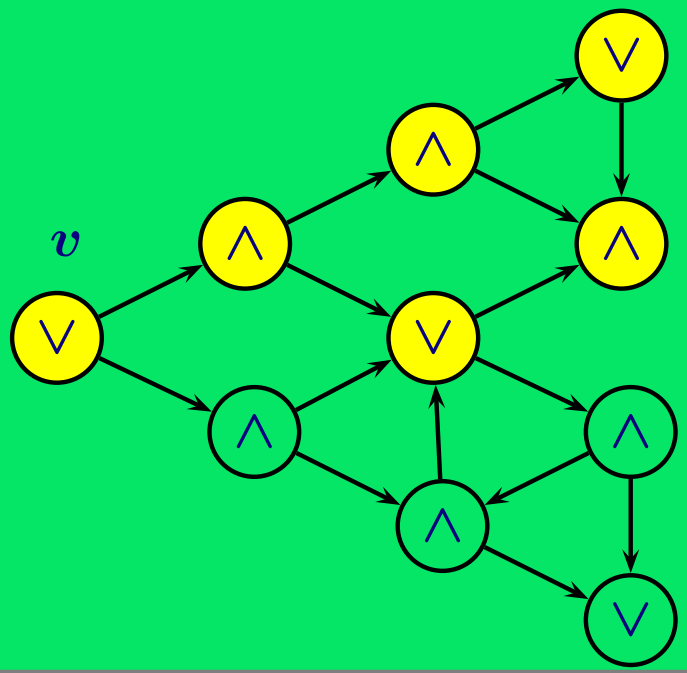


Definition: Alternating Graph Accessibility Problem (AGAP)

Given: Graph $G = (V, E)$, $V = V_\wedge \cup V_\vee$, and $v \in V$

Question: Is v accessible?

Alternating Graph Accessibility (cont.)

Construction of $G_{A,t}$ from Automaton A and Tree t

- Nodes in $V_\vee$ are the configurations of A on t: tuples $[i, q, \theta]$, where $\theta : \{1, \ldots, i\} \to t$
- Nodes in $V_\wedge$ are ϵ and pairs (γ_1, γ_2) of configurations with "the same θ"
- Edges:
  - $(\gamma_1, \gamma_2) \to \gamma_1$, $(\gamma_1, \gamma_2) \to \gamma_2$
  - $\gamma \to \gamma'$, if this is a step of A
  - $\gamma \to \epsilon$, if A can get into the accept state from γ
  - $\gamma \to (\gamma_1, \gamma_2)$ if this is a branching step of A

Fact

A k-pebble acceptor A accepts a tree $t \iff \gamma$ is accessible in $(G_{A,t})$

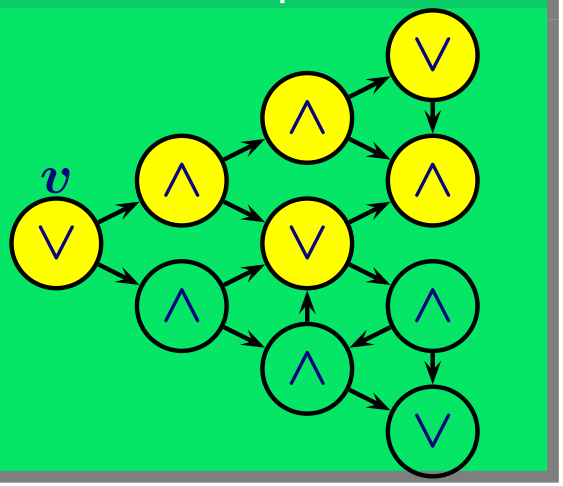
AGAP is MSO-expressible

Definition: Reverse-closed Sets of Nodes

A set S of nodes is **reverse-closed** if the following holds

- if v is in $V_\wedge$ and $w \in S$, for all nodes w with $(v, w) \in E$, then $v \in S$
- if v is in $V_\vee$ and $w \in S$, for some node w with $(v, w) \in E$, then $v \in S$

Example



Fact

Node v is accessible iff it is in every reverse-closed set of nodes.

...as MSO-Formula

v accessible $\equiv \forall S\ (\text{reverse-closed}(S) \rightarrow S(v))$, where

$$\text{reverse-closed}(S) \equiv \forall x\,([V_\wedge(x) \wedge \forall y\,(E(x,y) \rightarrow S(y))] \rightarrow S(x)) \wedge ([V_\vee(x) \wedge \exists y\,(E(x,y) \wedge S(y))] \rightarrow S(x))$$

k-pebble acceptors and MSO

Proof idea

Unfortunately, $G_{A,t}$ has too many nodes to use this directly:

- MSO can only quantify over sets of linear size in the given structure (i.e., t)
- $G_{A,t}$ has $\Omega(|t|^k)$ configurations
- But $G_{A,t}$ has a special structure:
  Nodes are only connected if their number of pebbles is the same ± 1 and they agree in all but at most the last pebble

k-pebble acceptors and MSO (cont.)

Proof (cont.)

- Wlog assume that each state of A is only used for a fixed number of pebbles: $Q = Q_1 \cup \cdots \cup Q_k$, where the states in Q_i are only used, when i pebbles are present
- Further assume that all sets Q_i are of equal size m: $Q_i = \{q_{i1}, \ldots, q_{im}\}$
- $k = 1$:
  - Use one relation S_i^1 for each state q_{1i}
  - Intended meaning of $v \in S_i^1$:
    there is an accepting subcomputation of A starting at v in state q_{1i}
  - $\varphi = \forall S_1^1 \cdots \forall S_m^1$ (reverse-closed $\longrightarrow S_1^1$(root))
  - reverse-closed is a conjunction of subformulas, induced by the transitions of A, e.g.:
    - if $(q_{1i}, a) \longrightarrow$ accept then $\forall x\, Q_a(x) \longrightarrow S_i^1(x)$
    - if $(q_{1i}, a) \longrightarrow (q_{1j}, \text{down-right})$ then
      $\forall x \forall y\, (Q_a(x) \wedge E_r(x, y) \wedge S_j^1(y)) \longrightarrow S_i^1(x)$

k-pebble acceptors and MSO (cont.)

Proof (cont.)

$k = 2$:

- reverse-closed^1 and reverse-closed^2 describe reverse closure for configurations with one and two pebbles, respectively
- reverse-closed^2 expresses the same as reverse-closed before, but with the (immobile) pebble 1 represented by variable x_1
- reverse-closed^1 also refers to subcomputations with a second pebble
- Conjuncts corresponding to simple movements are essentially the same
- Conditions which check whether pebbles are at the same node have to be added
- The following conjuncts are added for pebble placement and lifting:
  - $(q_{2i}, a) \longrightarrow (q_{1j}, \text{lift})$ adds $\forall x_2 \, (Q_a(x_2) \wedge S^1_j(x_1)) \longrightarrow S^2_i(x_2)$ to reverse-closed^2
  - $(q_{1i}, a) \longrightarrow (q_{2j}, \text{place})$ adds $\forall x_1 \, (Q_a(x_1) \wedge \varphi^2) \longrightarrow S^1_i(x_1)$ to reverse-closed^1, where φ^2 is $\forall S^2_1 \cdots \forall S^2_m (\text{reverse-closed}^2 \longrightarrow S^2_j(\text{root}))$

Summary of Proof

Proof (cont.)

- To solve the type checking problem, given d_1, d_2 and T, we can proceed as follows.
  (1) Construct the k-pebble acceptor A for $T^{-1}(\overline{L(d_2)})$
  (2) Transform A into an equivalent MSO formula Φ
  (3) Φ holds for all trees t for which $T(t) \not\subseteq L(d_2)$
  (4) Construct a nondeterministic bottom-up automaton A' equivalent to $\neg\Phi$
  (5) Check that $L(d_1) \subseteq L(A')$
- Hence, the type-checking problem is decidable
- Steps (1) and (4) can be done in poly-time
- Step (2) is exponential in k, FO-quantifier depth of Φ is k, MSO-quantifier depth of Φ is $|Q|$
- Step (3) is non-elementary (exponentiation tower of height k)
- Hence,
  the algorithm for the type-checking problem has a very bad complexity

Summary: Typechecking

Related Work

- If transformations are allowed to compare data values in the input document type checking becomes undecidable very quickly, even for restricted types and transformations [Alon et al. 2001]
- Typechecking for deterministic top-down tree transducers is more tractable. Complexity depends on exact representation of DTDs and restrictions on the transducers: between **PTIME** and **EXPTIME** [Martens and Neven 2003]
- If **P** $\neq$ **NP** there is no elementary f, such that MSO-formulas can be evaluated in time $f(|\text{formula}|) \times p(|\text{tree}|)$ with polynomial p [Frick and Grohe 2002]

Open

- Find (more) transformations with a tractable typechecking problem
- In particular, with data values

Contents

Theory of XQuery

Remarks

- In general, the theoretical foundations of XQuery have to be developed
- Clearly: XQuery is Turing-complete and therefore static analysis is generally impossible
- What about important fragments with better properties?
- E.g., Tree pattern queries
- Here, we concentrate on:
  - Conjunctive queries for trees
  - Some questions related to automata for XQuery

Contents

Conjunctive Queries

Introduction

- Navigational XPath expressions (without or and not) can be written as **conjunctive queries**
- /child::a/desc::*[child::c]/parent::d corresponds to

$$Q(x) = \text{root}(x_1) \wedge \text{child}(x_1, x_2) \wedge L_a(x_2) \wedge \text{desc}(x_2, x_3) \wedge \text{child}(x_3, x_4) \wedge L_c(x_4) \wedge \text{child}(x, x_3) \wedge L_d(x)$$

- Conjunctive Queries can express queries of higher arity:

$$Q(x, y) = \text{child}(x, x_1) \wedge \text{child}(x_1, y)$$

- What is the complexity of evaluating conjunctive queries on trees?
- Data complexity is in **PTIME** (even in **LOGSPACE**):

  Cycle through all valuations of the variables
- What about combined complexity?

A Generic Algorithm

Definition

- **Pre-valuation**: mapping from variables to non-empty sets of nodes
- For a conjunctive query Q a pre-valuation θ is **consistent** if:
  - for each atom $L_\sigma(x)$: $v \in \theta(x) \Rightarrow L_\sigma(v)$
  - for each atom $R(x, y)$:
    - $v \in \theta(x) \Rightarrow \exists u \in \theta(y) R(v, u)$
    - $v \in \theta(y) \Rightarrow \exists u \in \theta(x) R(u, v)$

Example Query

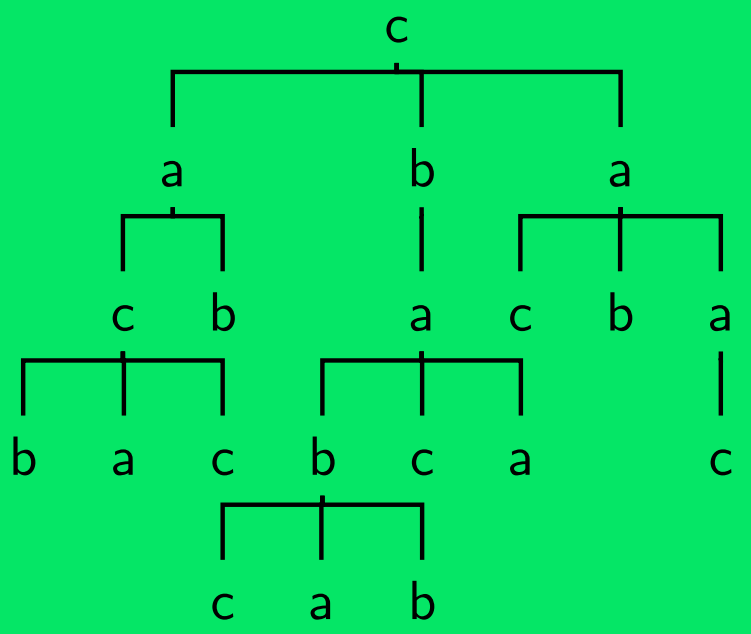
$$Q(x, y) = \texttt{child}(x, y) \wedge L_a(x)$$

Example Pre-Valuation

$\theta(x) = \ldots$

$\theta(y) = \ldots$

Example Document



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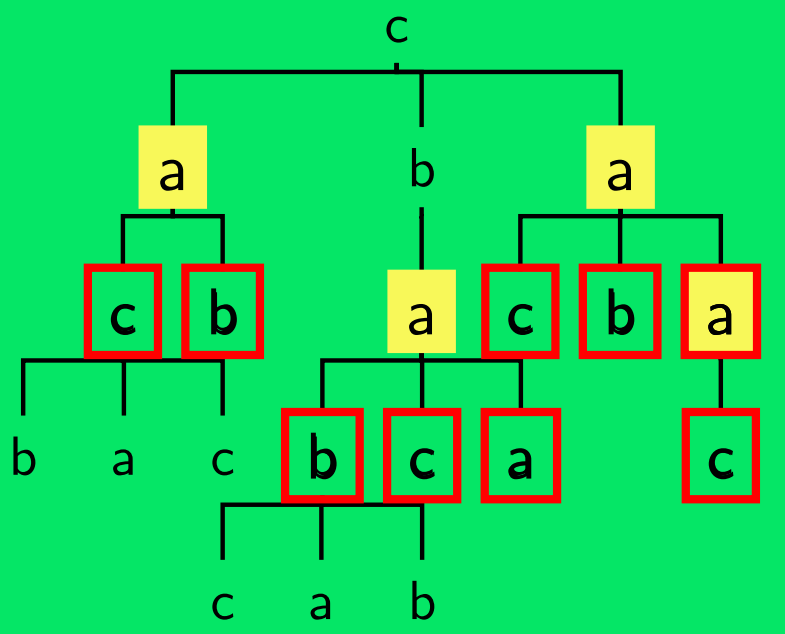

A Generic Algorithm (cont.)

Fact

A maximal consistent pre-valuation θ can be computed in time

$$O(\text{query size} \times \text{tree size})$$

Algorithmic Idea

- Let $<$ be a total order on the nodes
- For query Q and tree t:
  Compute maximal consistent pre-valuation θ
  Define $<$-minimal valuation h via:
  For each variable x:
  $h(x) :=$ minimal node in $\theta(x)$ wrt $<$

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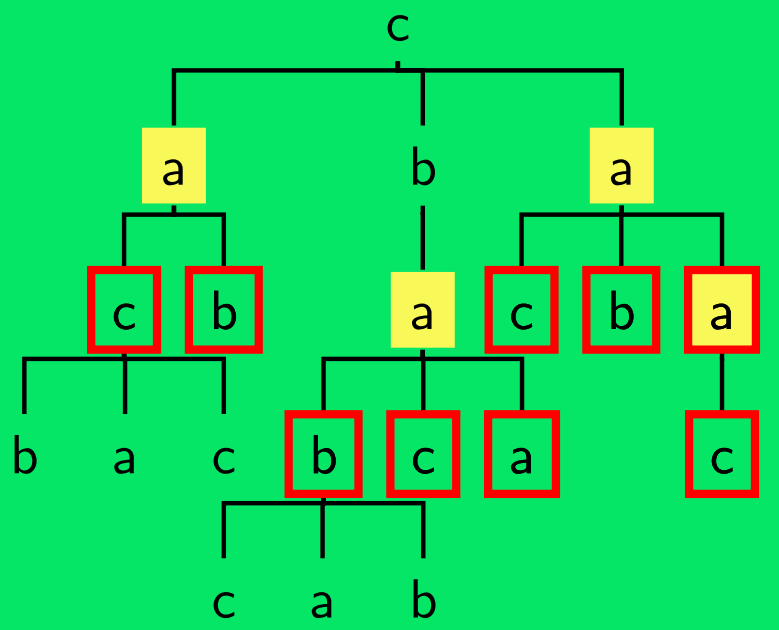

Let $<$ be the breadth-first left-to-right order

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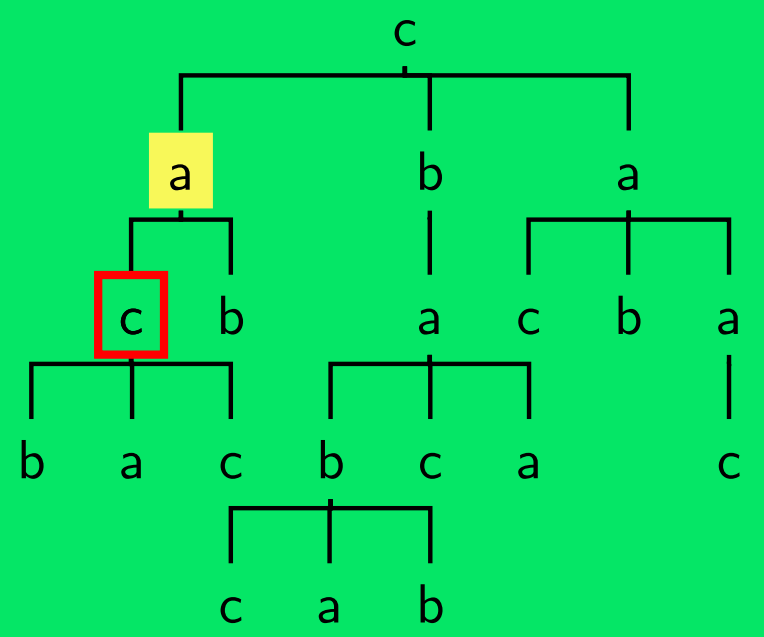

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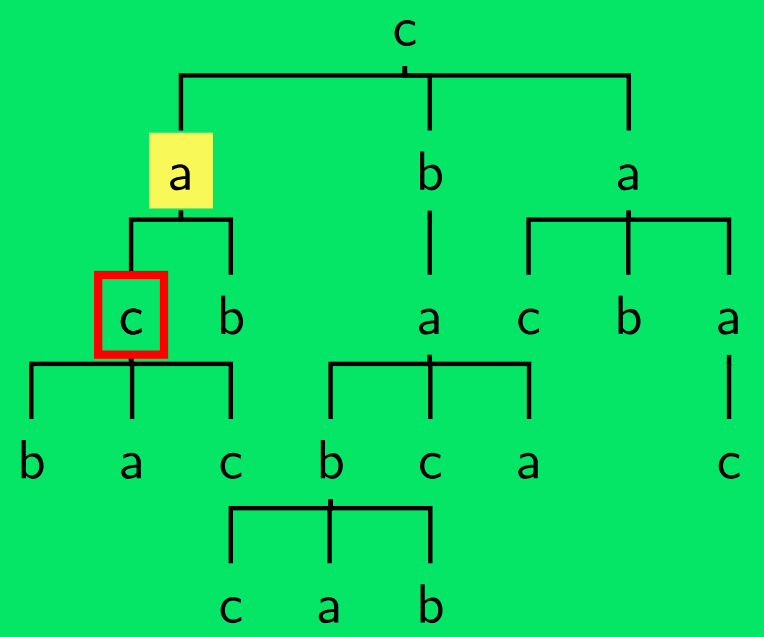

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Question: Is h always a solution?

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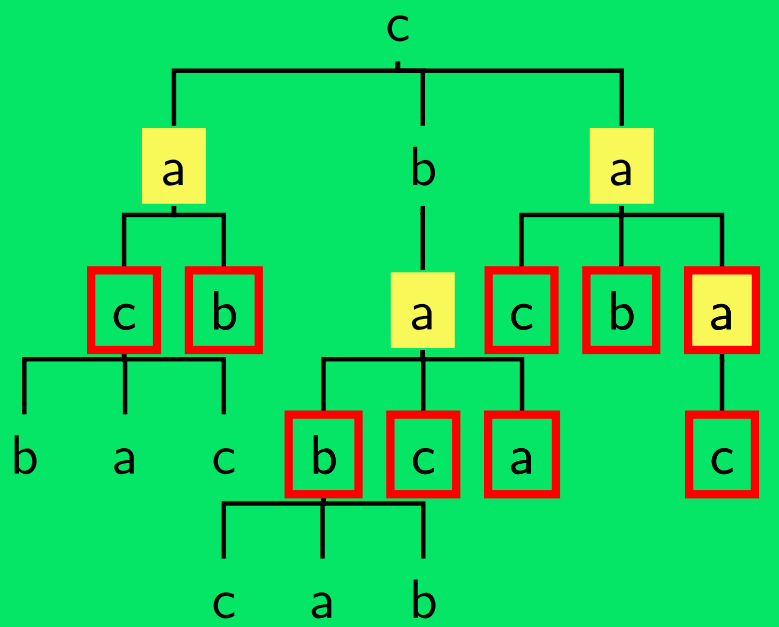

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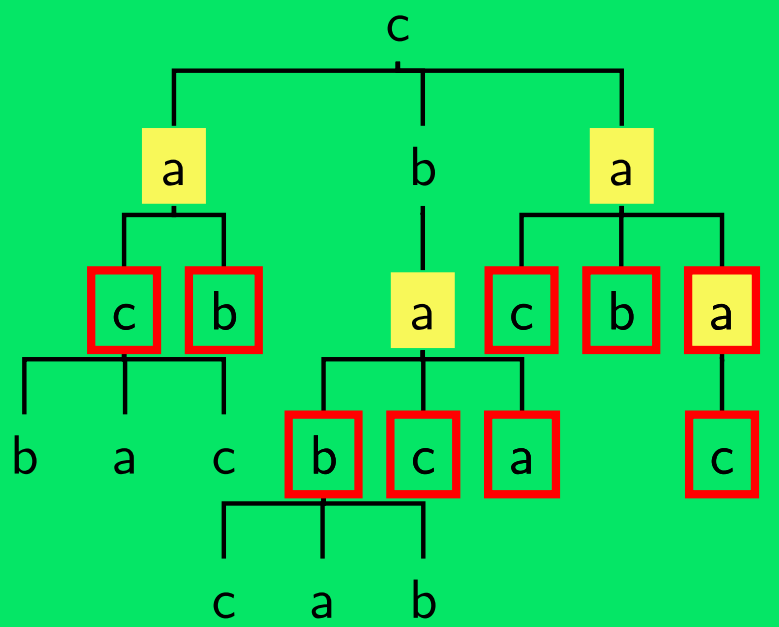

Question: Is h always a solution?

Observations

- Let $u = h(x), v = h(y)$
- As $u \in \theta(x)$ there is v' such that $\texttt{child}(u, v')$
- As $v \in \theta(y)$ there is u' such that $\texttt{child}(u', v)$
- As $u \leq u'$ and $v \leq v'$ we get $\texttt{child}(u, v)$

A Generic Algorithm (cont.)

Definition

A binary relation R is $<$-hemichordal if for all u, u', v, v' with $u < u'$ and $u \leq v \leq v'$

- $R(u, v') \wedge R(u', v) \longrightarrow R(u, v)$ and
- $R(v', u) \wedge R(v, u') \longrightarrow R(v, u)$

Theorem [Gottlob, Koch, Schulz 04]

If the relations of a query Q are $<$-hemichordal and θ is a consistent pre-valuation for Q
then the $<$-minimal valuation for θ is a solution for Q

A Generic Algorithm (cont.)

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If the relations of a query Q are $<$-hemichordal and θ is a consistent pre-valuation for Q
then the $<$-minimal valuation for θ is a solution for Q

Corollary

If the axes used in a conjunctive query Q are $<$-hemichordal then Q can be evaluated in time $O(\text{query size} \times \text{tree size})$

Combined Complexity of Conjunctive Queries

Observation

It is sufficient to consider the axes `child`, `child`$^+$, `child`*, NextSibling, NextSibling$^+$, NextSibling*, `Following`

Theorem [Gottlob, Koch, Schulz 04]

- `child`$^+$ and `child`* are preorder-hemichordal
- `following` is postorder-hemichordal
- `child`, NextSibling, NextSibling$^+$, NextSibling* are breadth-first-left-to-right-hemichordal

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For each of these sets of axes conjunctive queries can be evaluated in time $O(\text{query size} \times \text{tree size})$

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Corollary

For each of these sets of axes conjunctive queries can be evaluated in time $O(\text{query size} \times \text{tree size})$

Amazing Result

For sets of axes not contained in those, the combined complexity of conjunctive query evaluation is **NP**-complete

Contents

Automata and XQuery

So far...

- We have seen that automata are useful for
  - Validation, Typing
  - Navigation
  - Transformation
- What about more general queries?
  - results of higher arity?
  - joins, i.e., comparisons of data values
  - counting
- Are automata useful for XQuery?
- ... for tree pattern queries?

General Queries (cont.)

Higher arity

- Nonemptiness and containment questions can be handled by automata: tuples can be encoded by additional labels
- What about query evaluation for higher arity?

Data values

- When data values in XML documents are taken into account, things become more complicated, e.g.:
  - Even First-order logic becomes undecidable
  - Pebble automata become undecidable
  - Automata with data registers become undecidable when they are allowed to move up and down
- What is the right notion for regular (string) languages over infinite alphabets?
- What are sensible decidable restrictions of logics and automata in the context of data values?

General Queries (cont.)

Counting

- Automata can be equipped with counting facilities, e.g.:
  Presburger tree automata: $\delta(\sigma, q)$ is Boolean combination of
  - regular expressions and
  - quantifier-free Presburger formulas like
    "number of children in state q_1 = number of children in state q_2"
- Nondet. Presburger automata:
  - $\equiv$ MSO logic
  - Whether automaton accepts all trees is undecidable
- Det. Presburger automata:
  - $\equiv$ Presburger μ-formulas
  - Membership test: $O(|\mathcal{A}||t|)$
  - Non-emptiness: **PSPACE**
  - Containment: **PSPACE**

[Seidl, Sch., Muscholl, Habermehl 2004]

Contents

Conclusion

Summary

- Schema languages and `XPath` are well understood
- There are some nice results on transformations
- Theory for `XQuery` still has to be developed

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- Schema languages and `XPath` are well understood
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- Theory for `XQuery` still has to be developed

Finally...

Thanks for your patience