Software Validation and Verification
Fourth Exercise Sheet – Linear Temporal Logic

Exercise 1

Let φ, ψ, π be arbitrary LTL formulae. For each of the following pairs of LTL formulae, determine in which relation they are. More specifically, determine whether they are equivalent, one of them subsumes the other or they are incomparable. Prove your claims.

a) $\Diamond\Box\varphi$ and $\Box\Diamond\varphi$

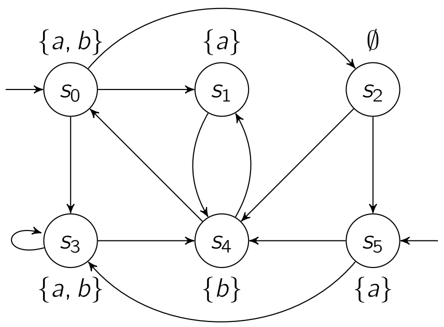
b) $\Diamond\Box\varphi \wedge \Diamond\Box\psi$ and $\Diamond(\Box\varphi \wedge \Box\psi)$

c) $\varphi \wedge \Box(\varphi \rightarrow \bigcirc\Diamond\varphi)$ and $\Box\Diamond\varphi$

d) $(\varphi \mathsf{U} \psi)\mathsf{U}\pi$ and $\varphi\mathsf{U}(\psi\mathsf{U}\pi)$

Exercise 2

Consider the following transition system TS:



Determine whether $\mathsf{TS} \models \varphi_i$ for each of the following properties. Justify your answers.

a) $\varphi_1 = \Box\Diamond a$

b) $\varphi_2 = \Diamond\Box a$

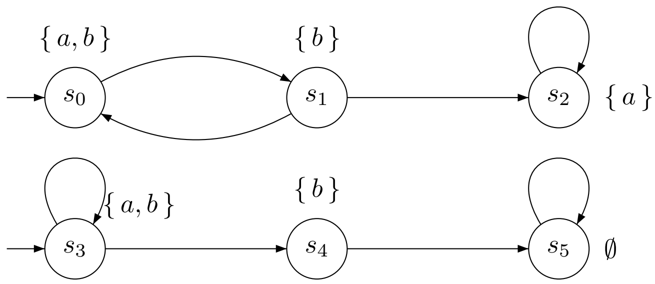
c) $\varphi_3 = a \rightarrow \bigcirc\bigcirc\, a$

d) $\varphi_4 = b \mathsf{R} a$ where $\varphi \mathsf{R} \psi \stackrel{\text{def}}{=} \neg(\neg\varphi \mathsf{U} \neg\psi)$

Exercise 3

Provide a sequence (φ_n) of LTL formulae such that the LTL formulae ψ_n is in PNF (including weak-until), $\varphi_n \equiv \psi_n$, and ψ_n is exponentially longer than φ_n. Use the transformation rules from the lecture.

Exercise 4



Consider the transition system TS above with the set AP $= \{a, b, c\}$ of atomic propositions. Note that this is a single transition system with two initial states. Consider the LTL fairness assumption

$$fair \;=\; \big(\Box\Diamond(a \wedge b) \;\rightarrow\; \Box\Diamond\neg c\big) \;\;\wedge\;\; \big(\Diamond\Box(a \wedge b) \rightarrow \Box\Diamond\neg b\big).$$

a) Determine the fair paths in TS, i.e., the initial, infinite paths satisfying *fair*

b) For each of the following LTL formulae:

$$\begin{aligned} \varphi_1 &= b \mathsf{U} \Box\neg b \\ \varphi_2 &= b \mathsf{W} \Box\neg b \\ \varphi_3 &= (\bigcirc\bigcirc\, b)\mathsf{U}(\Box\neg b) \end{aligned}$$

determine whether TS $\models_{fair} \varphi_i$. In case TS $\not\models_{fair} \varphi_i$, indicate a path $\pi \in Paths(\text{TS})$ for which $\pi \not\models \varphi_i$.

c) Redo the previous task but ignore the fairness assumption.

Exercise 5

Let $\varphi = (a \wedge \bigcirc a) \mathsf{U} (a \wedge \neg \bigcirc a)$ be an LTL–formula over $AP = \{a\}$.

1. Compute all elementary sets with respect to φ.
2. Construct the GNBA $\mathcal{G}_\varphi$ according to the algorithm from the lecture such that $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \mathit{Words}(\varphi)$.
3. Give an ω-regular expression E such that $\mathcal{L}_\omega(\mathcal{G}_\varphi) = \mathcal{L}_\omega(E)$.