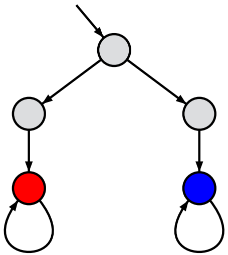
Overview

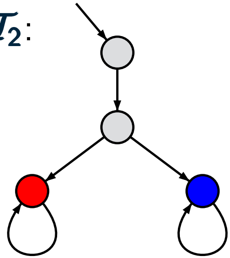
- Introduction
- Modelling parallel systems
- Linear Time Properties
- Regular Properties
- Linear Temporal Logic (LTL)
- Computation-Tree Logic
- **Equivalences and Abstraction**

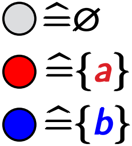
Trace equivalence

$\mathcal{T}_1$:



$\mathcal{T}_2$:





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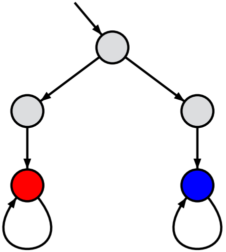

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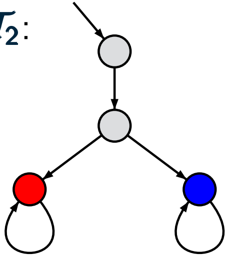

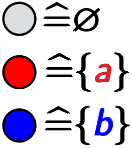

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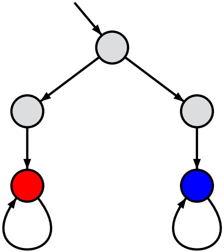

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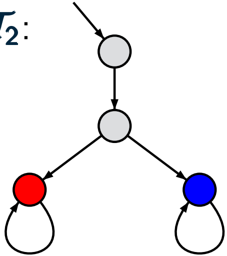

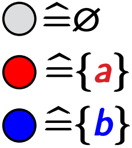

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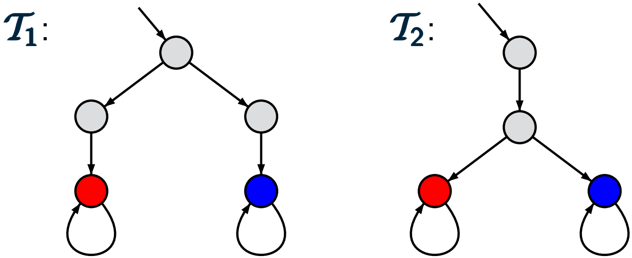

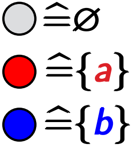

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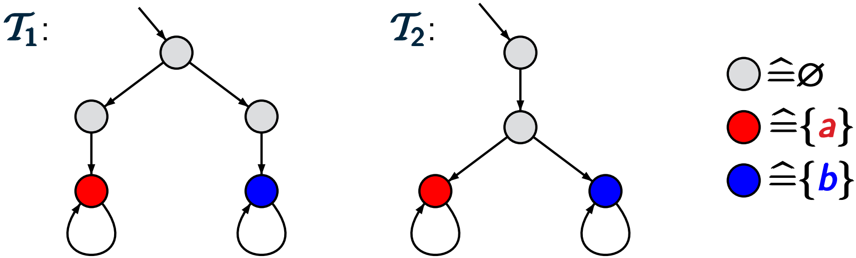
$\mathcal{T}_1 \not\models \Phi$ and $\mathcal{T}_2 \models \Phi$

Trace equivalence is not compatible with CTL



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Implementation relations

- for the design of complex systems
  - $\rightsquigarrow$ comparison of **2** transition systems

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> use equivalence relation $\sim$ for the states of a single transition system $\mathcal{T}$ and analyze the quotient $\mathcal{T}/\sim$

goal: define the equivalence $\sim$ in such a way that

$$\mathcal{T} \models \Phi \quad \text{iff} \quad \mathcal{T}/\sim\, \models \Phi$$

for all "relevant" properties Φ

Linear-time implementation relations

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finite trace inclusion and equivalence:

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* **minimization ???**

Minimization w.r.t. trace equivalence?

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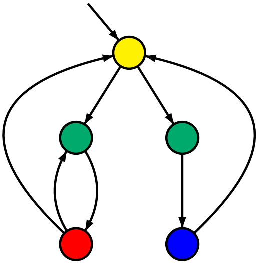

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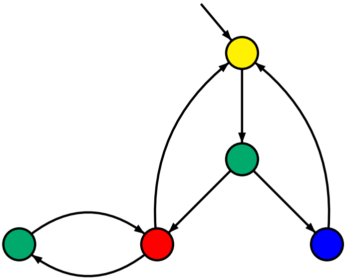

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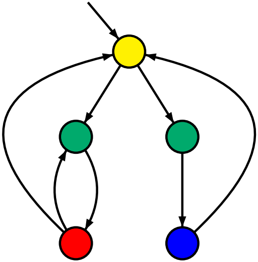

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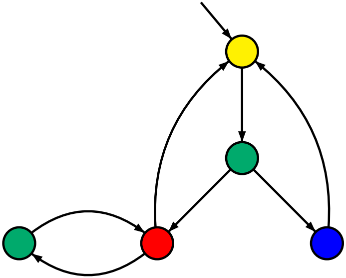

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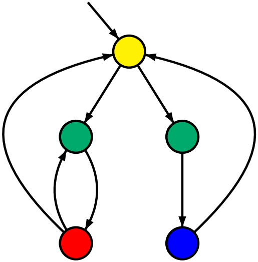

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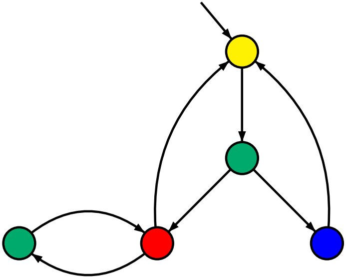

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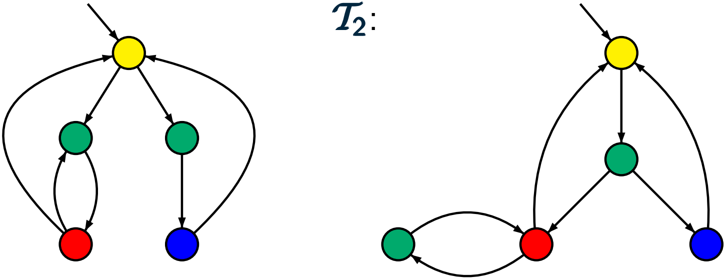

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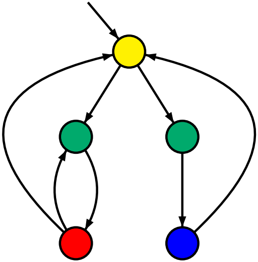

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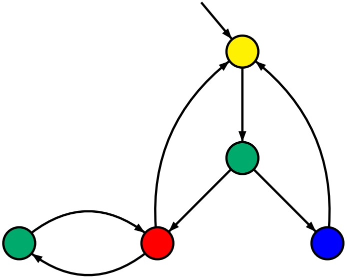

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- there is no smaller TS that is trace-equivalent to $\mathcal{T}_i$

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- strong vs. weak relations
  - strong: reasoning about all transitions
  - weak: abstraction from stutter steps

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

Computation-Tree Logic

Equivalences and Abstraction

- bisimulation ⟵
- CTL, CTL*-equivalence
- computing the bisimulation quotient
- abstraction stutter steps
- simulation relations

Bisimulation for two transition systems

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let $\mathcal{T}_1 = (S_1, Act_1, \rightarrow_1, S_{0,1}, AP, L_1)$,

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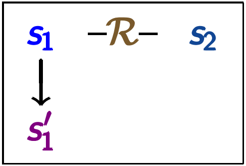

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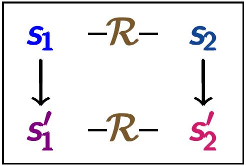

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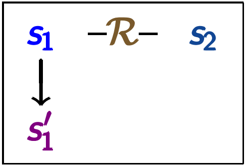

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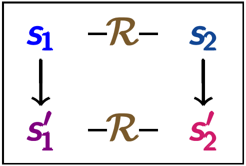

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$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_1' & & \end{array} \quad \text{can be completed to} \quad \begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \downarrow \\ s_1' & -\mathcal{R}- & s_2' \end{array}$$

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and such that the following initial condition holds:

(I) $\forall s_{0,1} \in S_{0,1} \, \exists s_{0,2} \in S_{0,2}$ s.t. $(s_{0,1}, s_{0,2}) \in \mathcal{R}$

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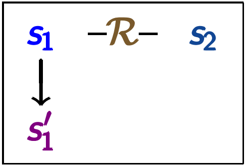

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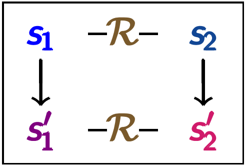

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(2) } mutual stepwise
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for state s_1 of $\mathcal{T}_1$ and state s_2 of $\mathcal{T}_2$:

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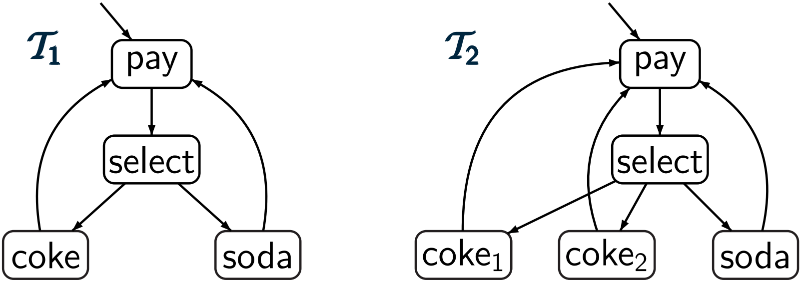
Two beverage machines



$$AP = \{pay, coke, soda\}$$

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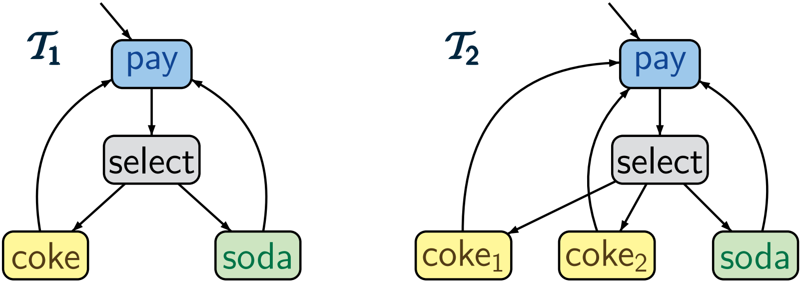

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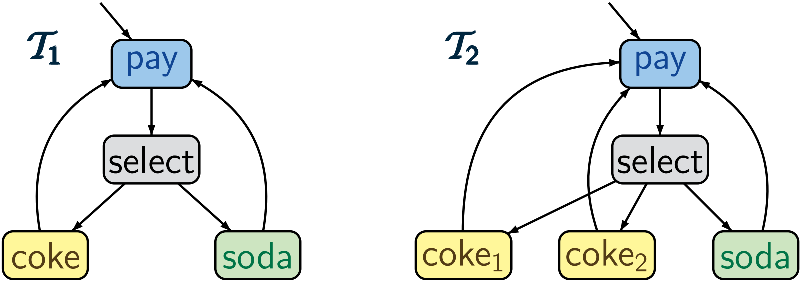

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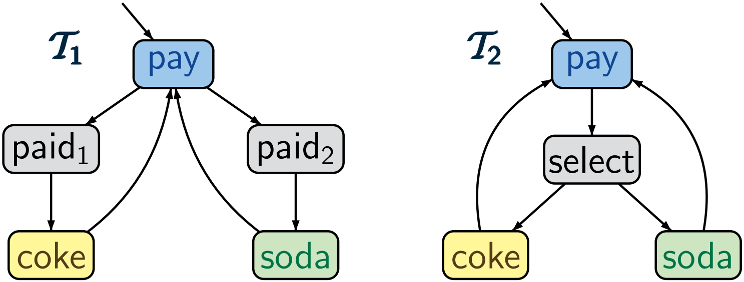
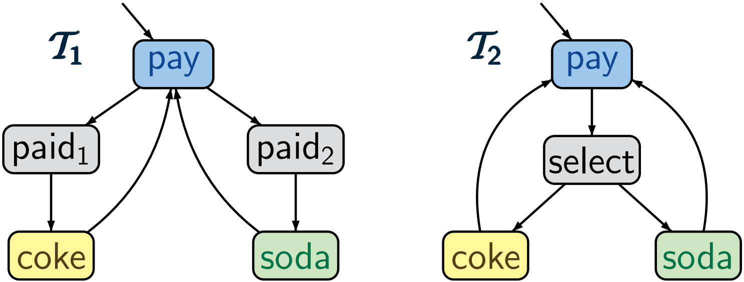
$\mathcal{T}_1 \sim \mathcal{T}_2$ as there is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$:

$\{$ (pay,pay), (select,select), (soda,soda)
(coke,$coke_1$), (coke,$coke_2$) $\}$

Two beverage machines



$$AP = \{pay, coke, soda\}$$



$AP = \{pay, coke, soda\}$

$\mathcal{T}_1 \not\sim \mathcal{T}_2$

because there is no state in $\mathcal{T}_1$ that has both

- a successor labeled with coke and
- a successor labeled with soda

Simulation condition of bisimulations

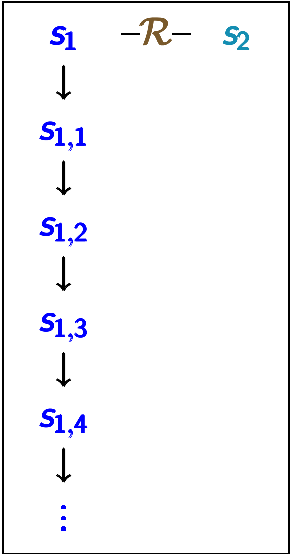
BSEQOR5.1-9-BIS

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_1' & & \end{array}$$

can be completed to

$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \downarrow \\ s_1' & -\mathcal{R}- & s_2' \end{array}$$

Path lifting for bisimulation $\mathcal{R}$

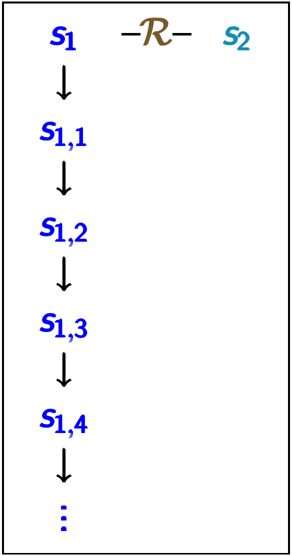


Path lifting for bisimulation $\mathcal{R}$

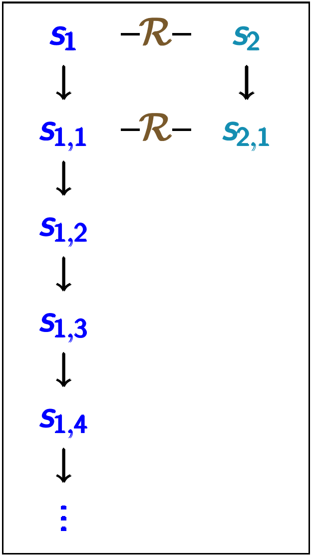
$$\begin{array}{ccc} s_1 & -\mathcal{R}- & s_2 \\ \downarrow & & \\ s_{1,1} & & \\ \downarrow & & \\ s_{1,2} & & \\ \downarrow & & \\ s_{1,3} & & \\ \downarrow & & \\ s_{1,4} & & \\ \downarrow & & \\ \vdots & & \end{array}$$

can be
completed to

Path lifting for bisimulation $\mathcal{R}$



can be completed to



Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

Path lifting for bisimulation $\mathcal{R}$

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \\
s_{1,1} & & \\
\downarrow & & \\
s_{1,2} & & \\
\downarrow & & \\
s_{1,3} & & \\
\downarrow & & \\
s_{1,4} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be completed to

$$
\begin{array}{ccc}
s_1 & -\mathcal{R}- & s_2 \\
\downarrow & & \downarrow \\
s_{1,1} & -\mathcal{R}- & s_{2,1} \\
\downarrow & & \downarrow \\
s_{1,2} & -\mathcal{R}- & s_{2,2} \\
\downarrow & & \downarrow \\
s_{1,3} & -\mathcal{R}- & s_{2,3} \\
\downarrow & & \downarrow \\
s_{1,4} & -\mathcal{R}- & s_{2,4} \\
\downarrow & & \downarrow \\
\vdots & & \vdots
\end{array}
$$

Properties of bisimulation equivalence

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$\sim$ is an equivalence

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$\sim$ is an equivalence, i.e.,

- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$

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↑

If S is the state space of $\mathcal{T}$ then

$$\mathcal{R} = \{(s, s) : s \in S\}$$

is a bisimulation for $(\mathcal{T}, \mathcal{T})$

$\sim$ is an equivalence, i.e.,

- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$
- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$

Properties of bisimulation equivalence

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- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$
- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$

$\uparrow$

If $\mathcal{R}$ is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$ then

$$\mathcal{R}^{-1} = \{(s_2, s_1) : (s_1, s_2) \in \mathcal{R}\}$$

is a bisimulation for $(\mathcal{T}_2, \mathcal{T}_1)$

Properties of bisimulation equivalence

$\sim$ is an equivalence, i.e.,

- reflexivity: $\mathcal{T} \sim \mathcal{T}$ for all transition systems $\mathcal{T}$
- symmetry: $\mathcal{T}_1 \sim \mathcal{T}_2$ implies $\mathcal{T}_2 \sim \mathcal{T}_1$
- transitivity: if $\mathcal{T}_1 \sim \mathcal{T}_2$ and $\mathcal{T}_2 \sim \mathcal{T}_3$ then $\mathcal{T}_1 \sim \mathcal{T}_3$

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↑

Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

Properties of bisimulation equivalence

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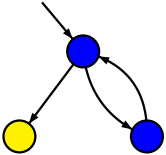
Let $\mathcal{R}_{1,2}$ be a bisimulation for $(\mathcal{T}_1, \mathcal{T}_2)$,
$\mathcal{R}_{2,3}$ be a bisimulation for $(\mathcal{T}_2, \mathcal{T}_3)$.

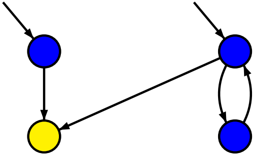
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_3) : \exists s_2 \text{ s.t. } (s_1, s_2) \in \mathcal{R}_{1,2} \text{ and } (s_2, s_3) \in \mathcal{R}_{2,3} \}$$

is a bisimulation for $(\mathcal{T}_1, \mathcal{T}_3)$

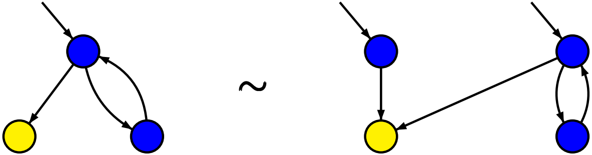
Correct or wrong?



$\sim$

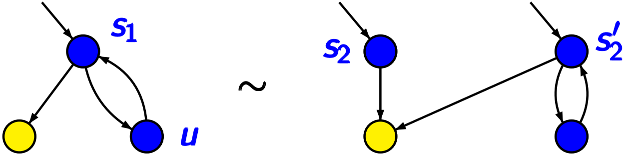


Correct or wrong?



wrong

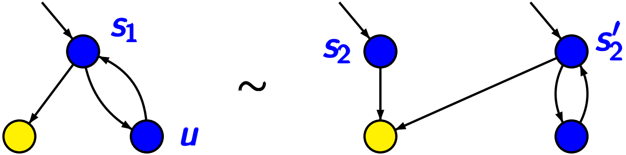
Correct or wrong?



wrong

$s_1 \longrightarrow u$, but $s_2 \not\longrightarrow$ *blue* (thus $s_1 \not\sim s_2$)

Correct or wrong?



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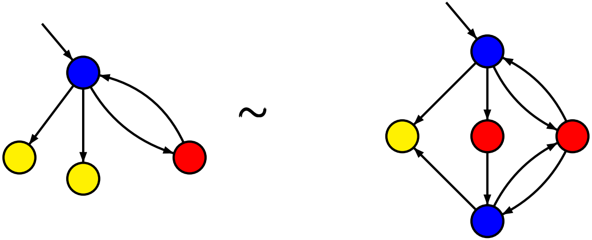

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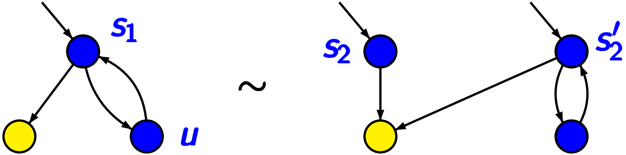
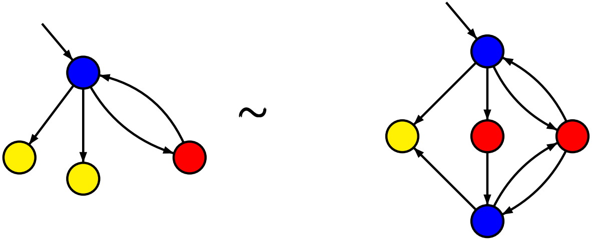


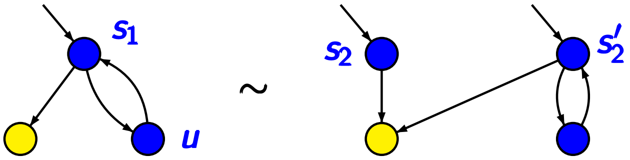
wrong

$s_1 \longrightarrow u$, but $s_2 \not\longrightarrow \textit{blue}$ (thus $s_1 \not\sim s_2$)



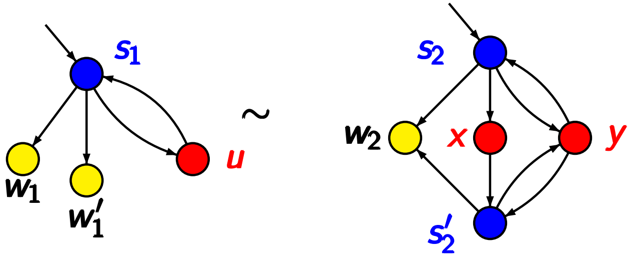
correct

Correct or wrong?



wrong

$s_1 \longrightarrow u$, but $s_2 \not\longrightarrow blue$ (thus $s_1 \not\sim s_2$)

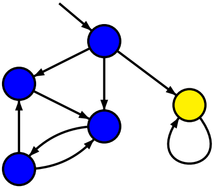


correct

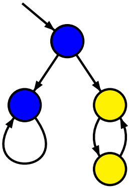
bisimulation:

$$\{(w_1, w_2), (w_1', w_2), (s_1, s_2), (s_1, s_2'), (u, x), (u, y)\}$$

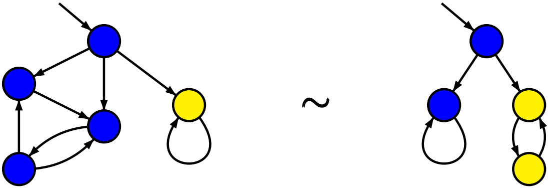
Correct or wrong?



$\sim$

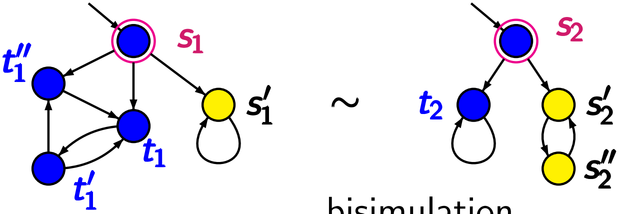


Correct or wrong?



correct

Correct or wrong?

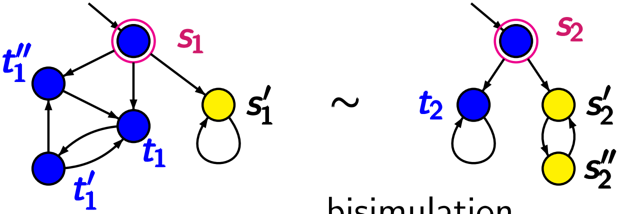


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$

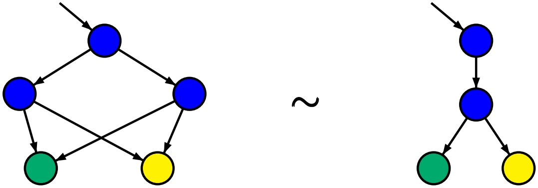
Correct or wrong?



correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$



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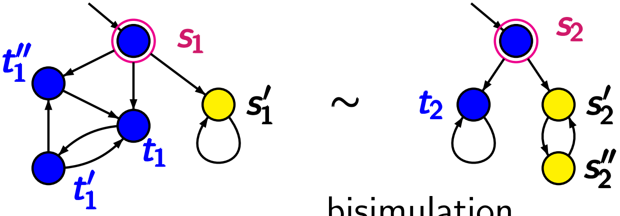
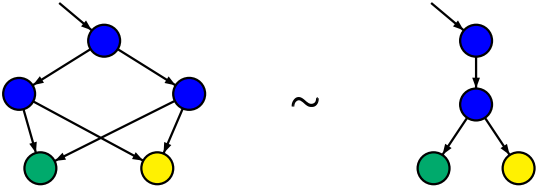


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$



correct

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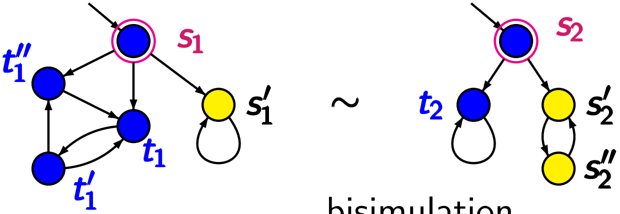
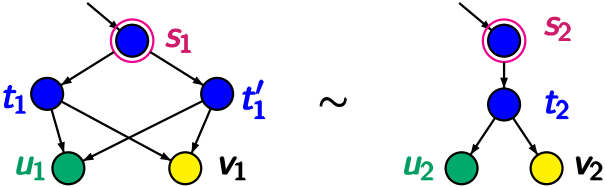


correct

bisimulation

$$\{(s_1, s_2), (s_1', s_2'), (s_1', s_2''), (t_1, t_2), (t_1', t_2), (t_1'', t_2)\}$$



correct

bisimulation: $\{(s_1, s_2), (t_1, t_2), (t_1', t_2), (u_1, u_2), (v_1, v_2)\}$

Bisimulation vs. trace equivalence

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \implies \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \Longrightarrow \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

proof: ... path fragment lifting ...

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proof: ... path fragment lifting ...

$$\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2) \not\Longrightarrow \mathcal{T}_1 \sim \mathcal{T}_2$$

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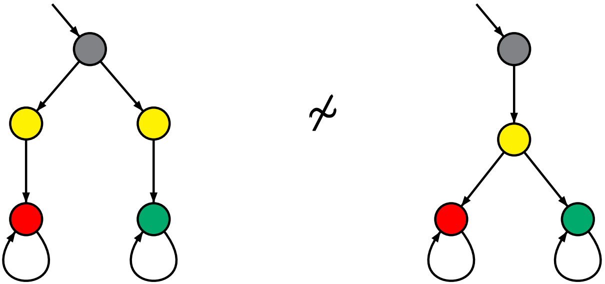

trace equivalent, but not bisimulation equivalent

Bisimulation vs. trace equivalence

$$\mathcal{T}_1 \sim \mathcal{T}_2 \Longrightarrow \mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2)$$

proof: ... path fragment lifting ...

$$\mathit{Traces}(\mathcal{T}_1) = \mathit{Traces}(\mathcal{T}_2) \not\Longrightarrow \mathcal{T}_1 \sim \mathcal{T}_2$$

Trace equivalence is strictly coarser than bisimulation equivalence.

Bisimulation vs. trace equivalence

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Trace equivalence is strictly coarser than bisimulation equivalence.

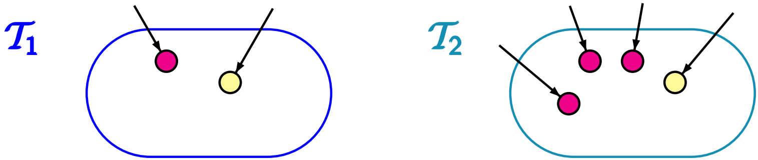
Bisimulation equivalent transition systems satisfy the same **LT properties** (e.g., **LTL** formulas).

Bisimulation equivalence ...

- as a relation that compares 2 transition systems

Bisimulation equivalence ...

- as a relation that compares **2** transition systems



Bisimulation equivalence ...

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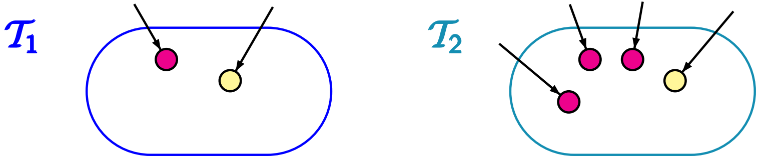
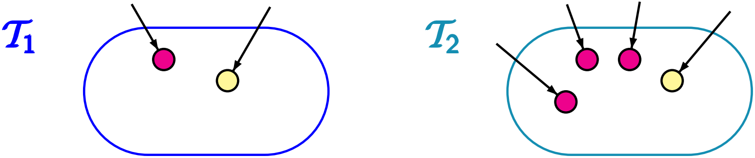


- as a relation on the states of **1** transition system

Bisimulation equivalence ...

- as a relation that compares **2** transition systems



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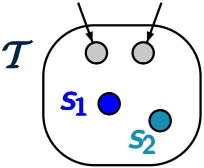

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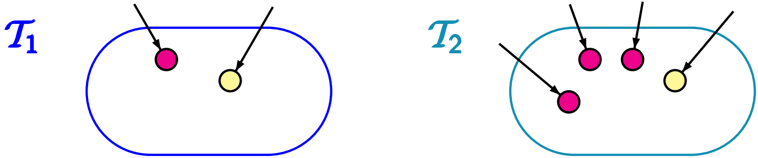

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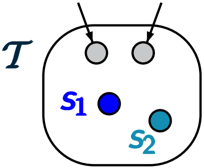
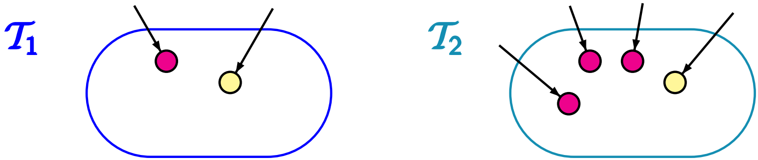


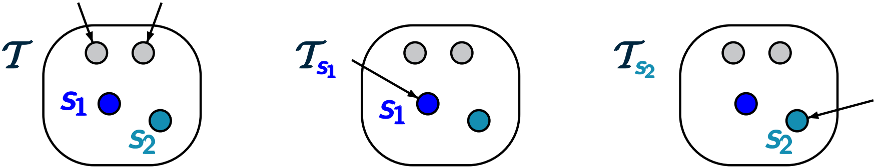
$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$

Bisimulation equivalence ...

- as a relation that compares **2** transition systems



- as a relation on the states of **1** transition system



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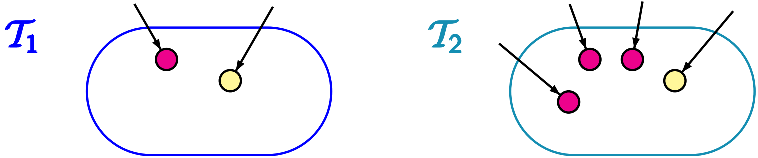
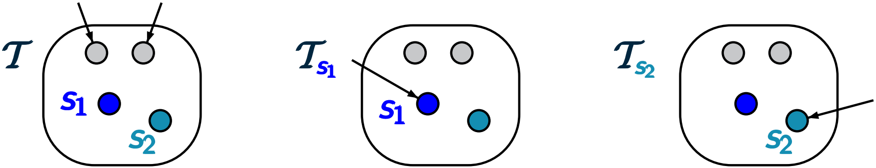
Bisimulation equivalence ...

- as a relation that compares **2** transition systems



- as a relation on the states of **1** transition system



$s_1 \sim s_2$ iff $\mathcal{T}_{s_1} \sim \mathcal{T}_{s_2}$ iff
there exists a bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

Bisimulations on a single TS

Bisimulations on a single TS

Let $\mathcal{T}$ be a TS with proposition set AP.

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A bisimulation for $\mathcal{T}$ is a binary relation $\mathcal{R}$ on the state space of $\mathcal{T}$ s.t. for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) $\forall\, s_1' \in Post(s_1)\ \exists\, s_2' \in Post(s_2)$ s.t. $(s_1', s_2') \in \mathcal{R}$

(3) $\forall\, s_2' \in Post(s_2)\ \exists\, s_1' \in Post(s_1)$ s.t. $(s_1', s_2') \in \mathcal{R}$

Bisimulation equivalence $\sim_{\mathcal{T}}$ on a single TS

Let $\mathcal{T}$ be a TS with proposition set AP.

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(2) $\forall\, s_1' \in Post(s_1)\ \exists\, s_2' \in Post(s_2)$ s.t. $(s_1', s_2') \in \mathcal{R}$

(3) $\forall\, s_2' \in Post(s_2)\ \exists\, s_1' \in Post(s_1)$ s.t. $(s_1', s_2') \in \mathcal{R}$

bisimulation equivalence $\sim_{\mathcal{T}}$:

$s_1 \sim_{\mathcal{T}} s_2$ iff there exists a bisimulation $\mathcal{R}$ for $\mathcal{T}$ s.t. $(s_1, s_2) \in \mathcal{R}$

Bisimulation equivalence

Let $\mathcal{T}$ be a transition system with state space S.

Bisimulation equivalence $\sim_{\mathcal{T}}$ is

- the coarsest bisimulation on $\mathcal{T}$
- and an equivalence on S

Bisimulation equivalence

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Bisimulation equivalence $\sim_{\mathcal{T}}$ is the coarsest equivalence on S s.t. for all states $s_1, s_2 \in S$ with $s_1 \sim_{\mathcal{T}} s_2$:

(1) $L(s_1) = L(s_2)$

(2) each transition of s_1 can be mimicked by a transition of s_2:

$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \\ s_1' & & \end{array} \quad \text{can be completed to} \quad \begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \downarrow \\ s_1' & \sim_{\mathcal{T}} & s_2' \end{array}$$

Two variants of bisimulation equivalence

$\sim$ relation that compares **2** transition systems

$\sim_{\mathcal{T}}$ equivalence on the state space of a single TS $\mathcal{T}$

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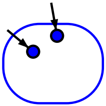
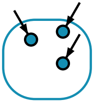
Derivation of $\sim$ from $\sim_{\mathcal{T}}$

given two transition systems $\mathcal{T}_1$ and $\mathcal{T}_2$

$\mathcal{T}_1$ with state space S_1



$\mathcal{T}_2$ with state space S_2



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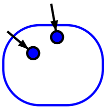

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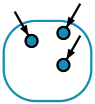

consider $\mathcal{T} = \mathcal{T}_1 \uplus \mathcal{T}_2$
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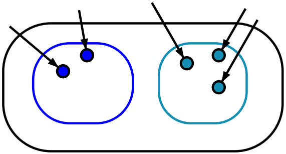

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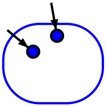

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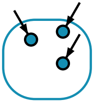

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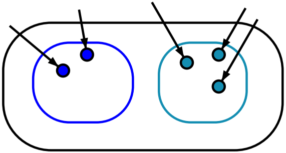

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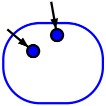

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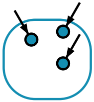

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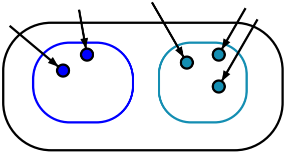

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Bisimulation quotient

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bisimulation quotient $\mathcal{T}/\sim$ arises from $\mathcal{T}$
by collapsing bisimulation equivalent states

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set of bisimulation equivalence classes

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well-defined

by the labeling condition of bisimulations

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$$\mathcal{T} \sim \mathcal{T}/\!\sim$$

Example: interleaving of n printers

parallel system $\mathcal{T} = \underbrace{\mathit{Printer} \mathbin{|||} \mathit{Printer} \mathbin{|||} \ldots \mathbin{|||} \mathit{Printer}}_{n \text{ printer}}$

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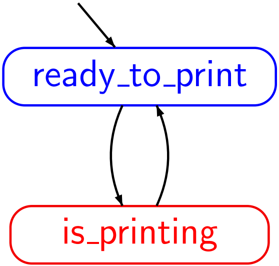
transition system
for each printer



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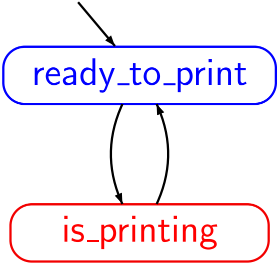
$AP = \{0, 1, \ldots, n\}$ "number of available printers"

transition system
for each printer



Example: n=3 printers

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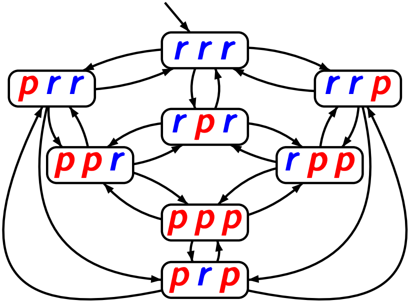

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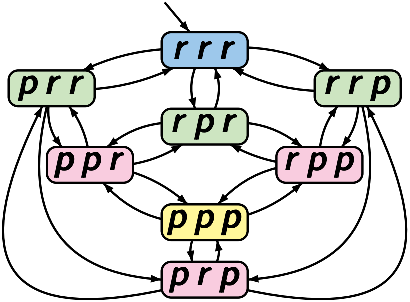

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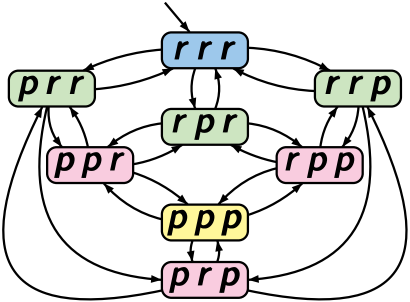

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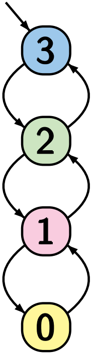

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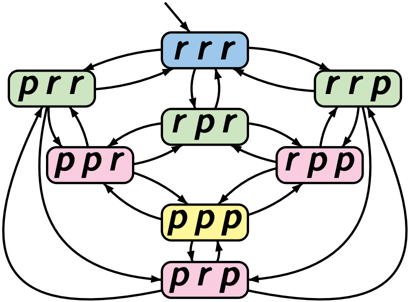

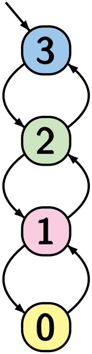

2^n states

$n+1$ states

Mutual exclusion

Mutual exclusion

solutions for mutual exclusion problems:

- semaphore
- Peterson's algorithm

Mutual exclusion: Bakery algorithm

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given two concurrent processes P_1 and P_2

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  if $x_2 < x_1$ then P_2 enters its critical section

  $x_1 = x_2$: cannot happen

Bakery algorithm

protocol for P_1:

```
LOOP FOREVER
    noncritical actions
    x1 := x2+1
    AWAIT (x1 < x2) ∨ (x2=0);
    critical section;
    x1 := 0
END LOOP
```

symmetric protocol for P_2

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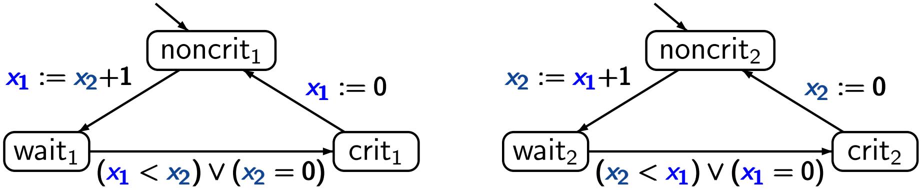
initially:

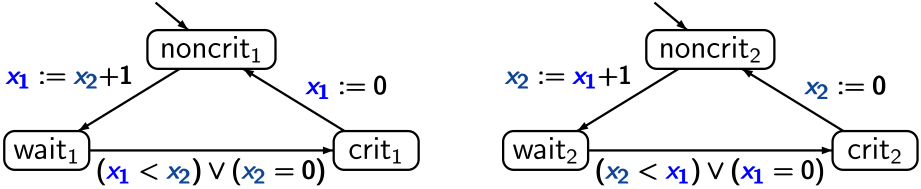
$x_1 = x_2 = 0$

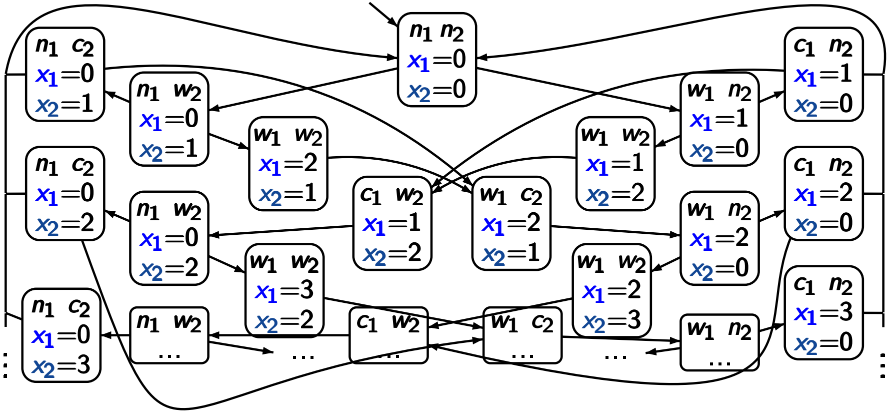
symmetric protocol for P_2

Program graphs for the Bakery algorithm



Transition system for the Bakery algorithm





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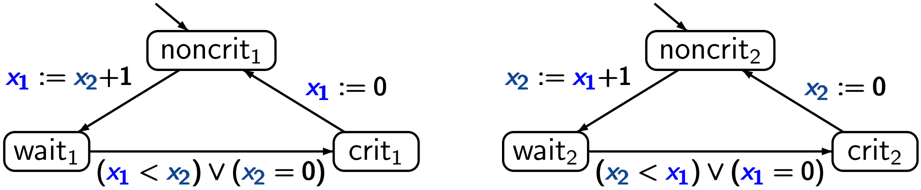

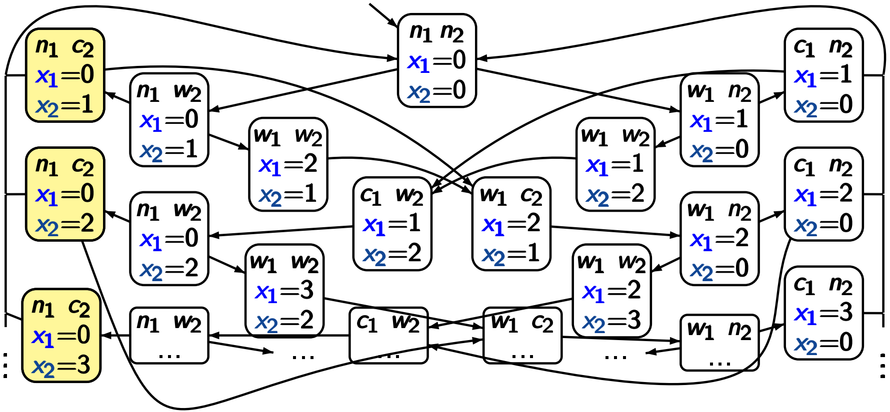

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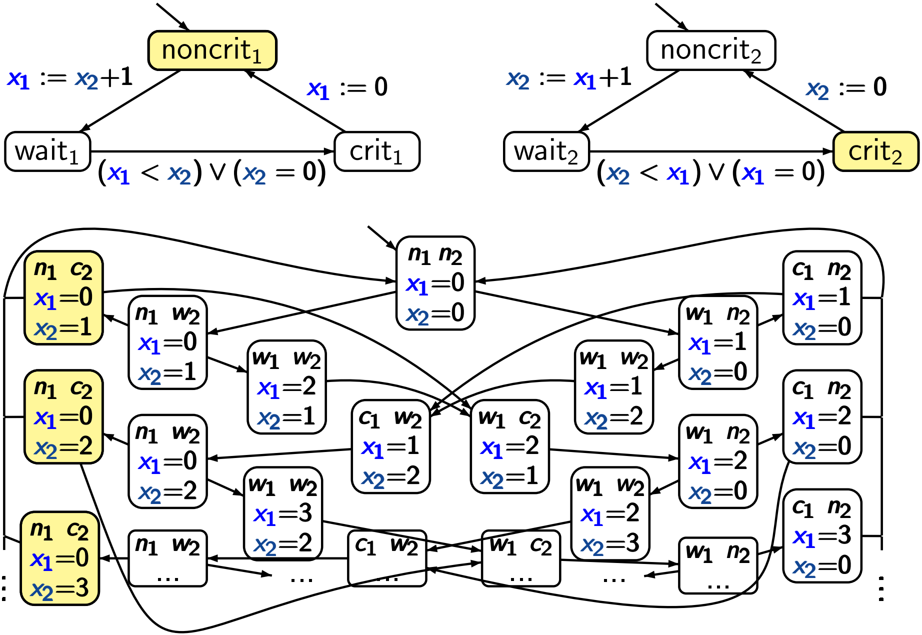

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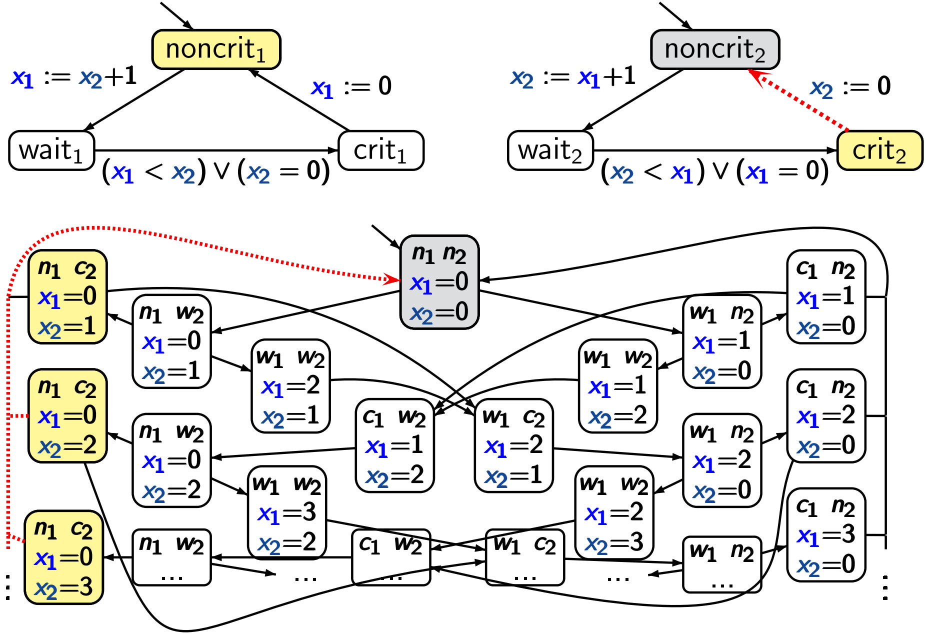

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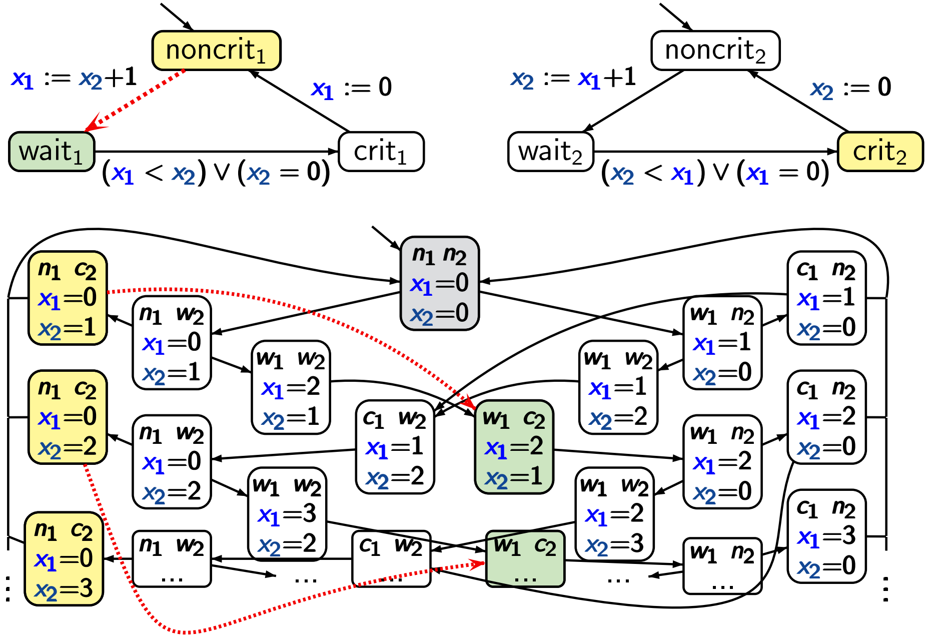

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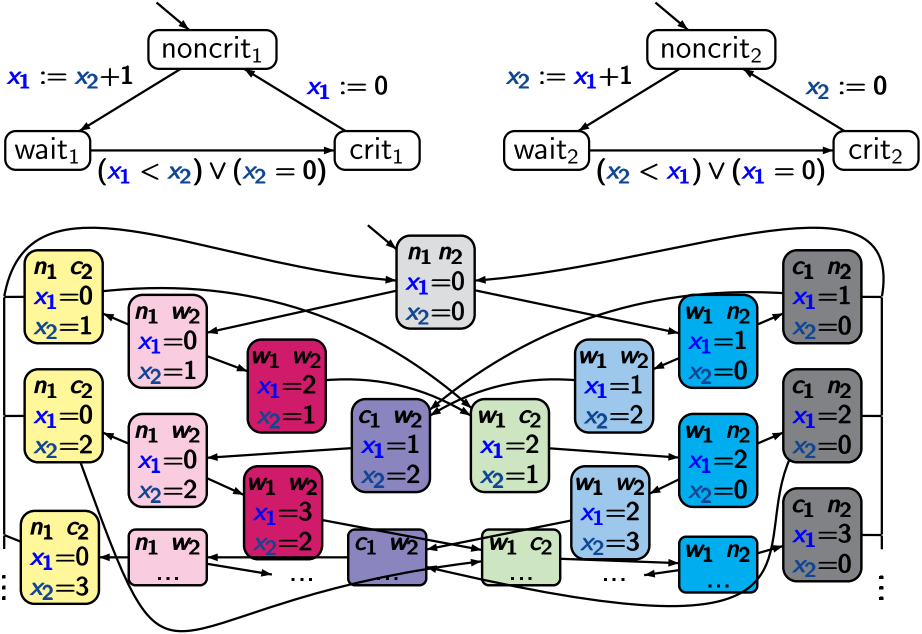
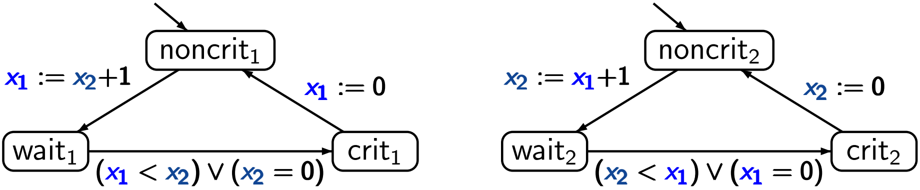
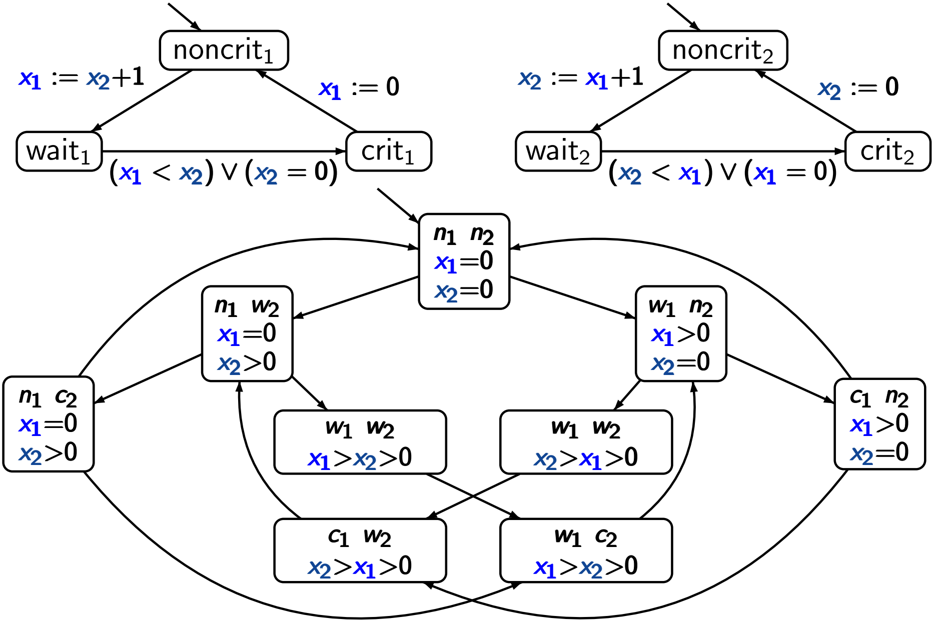


Bakery algorithm: bisimulation quotient



infinite transition system with a
finite bisimulation quotient

Bakery algorithm: bisimulation quotient



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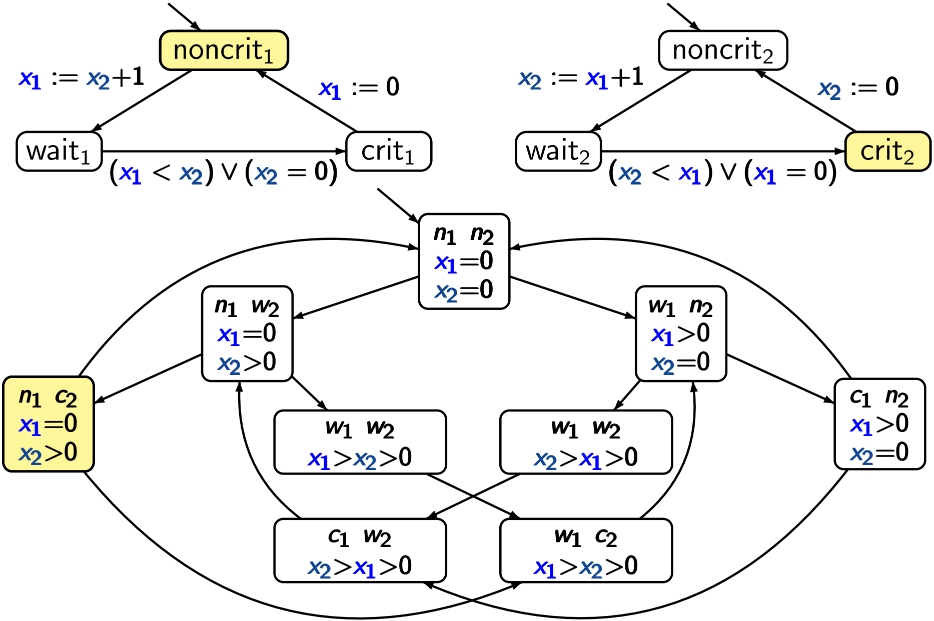

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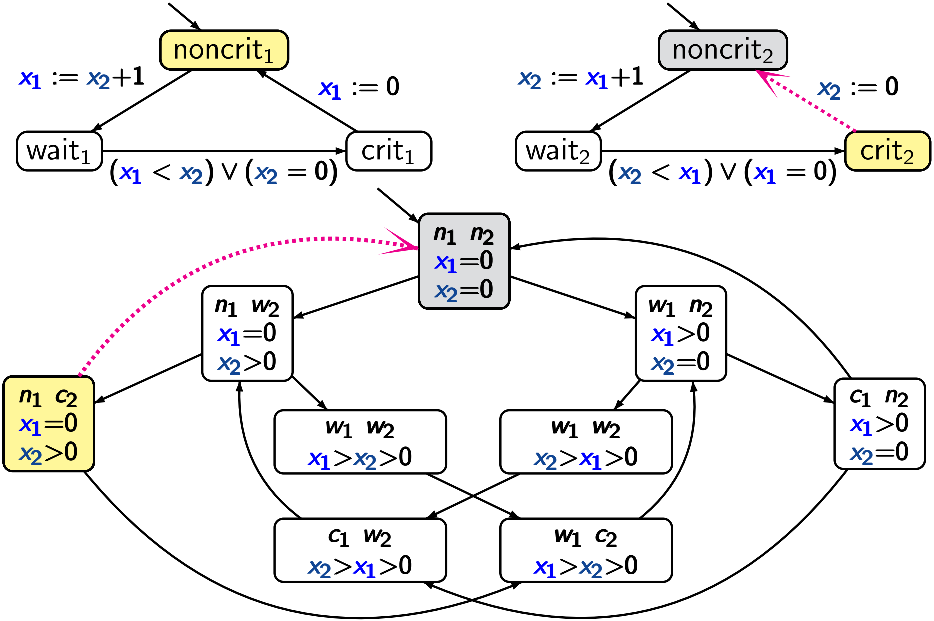

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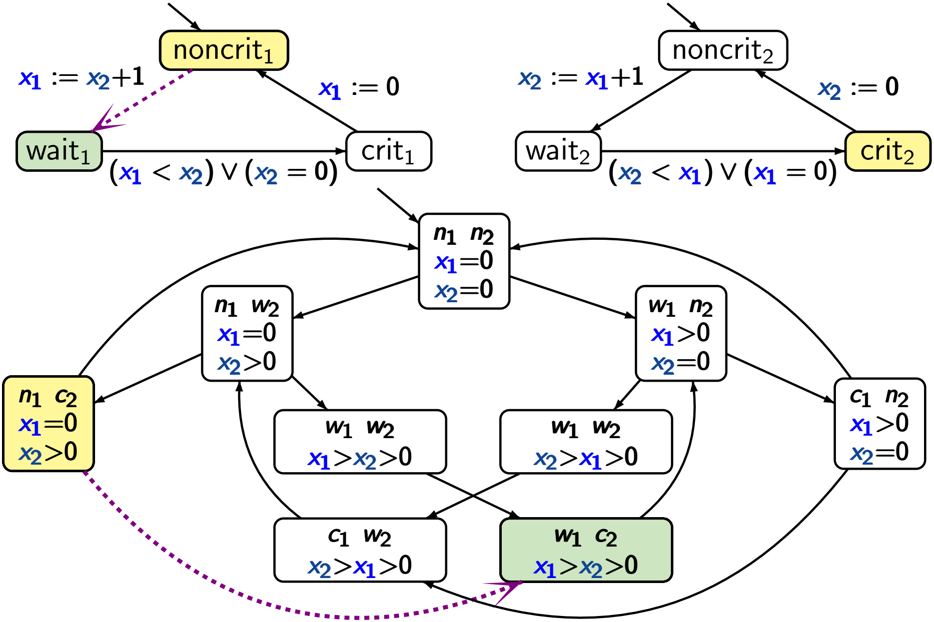

Overview

Recall: CTL*

CTL* state formulas

$$\Phi \ ::= \ \mathit{true} \mid a \mid \Phi_1 \wedge \Phi_2 \mid \neg\Phi \mid \exists\varphi$$

CTL* path formulas

$$\varphi \ ::= \ \Phi \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc\varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2$$

derived operators:

- $\Diamond$, $\Box$, . . . as in **LTL**
- universal quantification: $\forall\varphi \stackrel{\text{def}}{=} \neg\exists\neg\varphi$

Recall: CTL* and CTL

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CTL: sublogic of **CTL***

- with path quantifiers $\exists$ and $\forall$
- restricted syntax of path formulas:
  - *no* boolean combinations of path formulas
  - arguments of temporal operators $\bigcirc$ and U are state formulas

CTL equivalence

CTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

s_1, s_2 are **CTL** equivalent if for all **CTL** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

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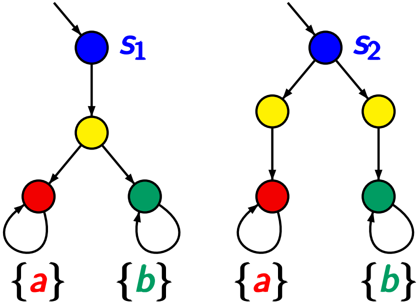

CTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

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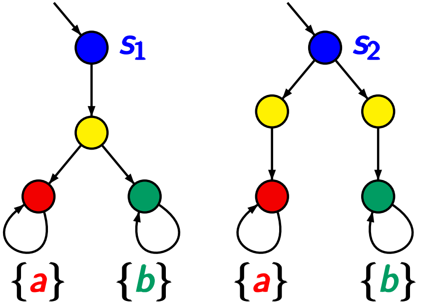

s_1, s_2 are not **CTL** equivalent

$s_1 \models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

$s_2 \not\models \exists\bigcirc(\exists\bigcirc a \wedge \exists\bigcirc b)$

CTL, CTL* and LTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

s_1, s_2 are **CTL** equivalent if for all **CTL** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

analogous definition for **CTL*** and **LTL**

CTL, CTL* and LTL equivalence

Let s_1, s_2 be states of a TS $\mathcal{T}$ without terminal states

s_1, s_2 are **CTL** equivalent if for all **CTL** formulas Φ:

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s_1, s_2 are **CTL*** equivalent if for all **CTL*** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

s_1, s_2 are **LTL** equivalent if for all **LTL** formulas φ:

$$s_1 \models \varphi \quad \text{iff} \quad s_2 \models \varphi$$

CTL/CTL* and bisimulation

CTL/CTL* and bisimulation

bisimulation equivalence

= **CTL** equivalence

= **CTL*** equivalence

CTL/CTL* and bisimulation

	bisimulation equivalence
=	**CTL** equivalence
=	**CTL*** equivalence

$\longleftarrow$ for finite TS

	bisimulation equivalence	
=	**CTL** equivalence	⟵ for finite TS
=	**CTL*** equivalence	

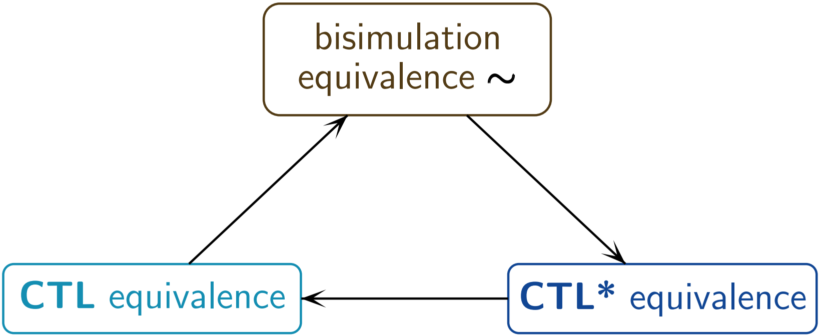
Let $\mathcal{T}$ be a finite TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

$$s_1 \sim_{\mathcal{T}} s_2$$

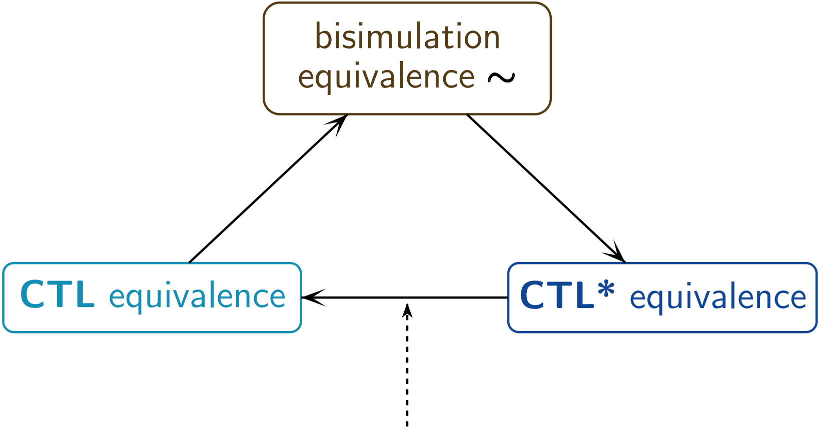
iff s_1 and s_2 are **CTL** equivalent

iff s_1 and s_2 are **CTL*** equivalent

CTL/CTL* and bisimulation

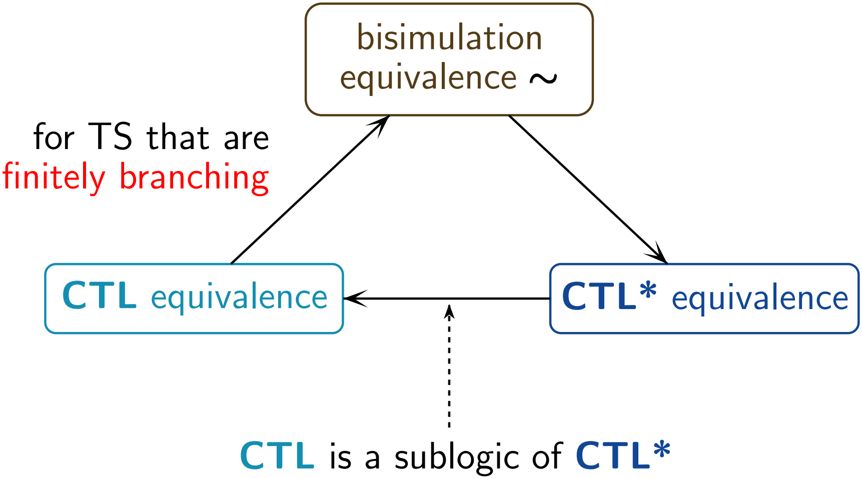


CTL/CTL* and bisimulation

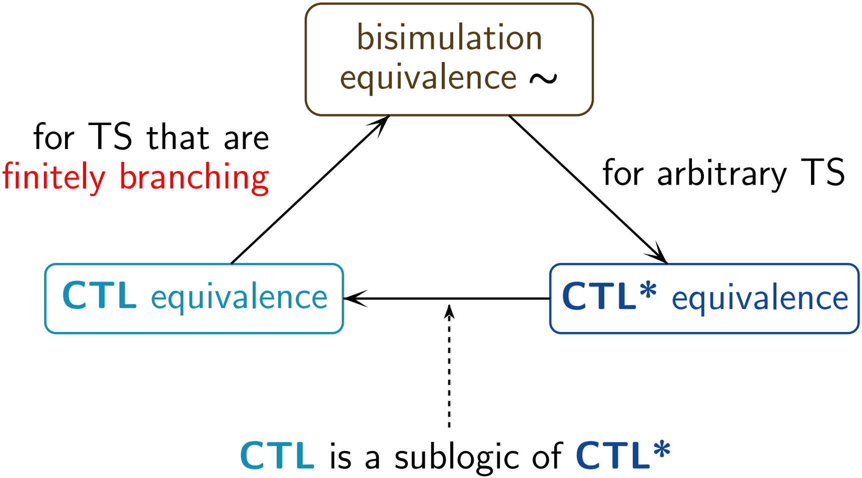


CTL is a sublogic of CTL*

CTL/CTL* and bisimulation



CTL/CTL* and bisimulation



Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For arbitrary (possibly infinite) transition systems without terminal states:

If s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

show by structural induction on **CTL*** formulas:

(a) if s_1, s_2 are states with $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL*** state formulas Φ:

$$s_1 \models \Phi \quad \text{iff} \quad s_2 \models \Phi$$

(b) if π_1, π_2 are paths with $\pi_1 \sim_{\mathcal{T}} \pi_2$ then for all **CTL*** path formulas φ:

$$\pi_1 \models \varphi \quad \text{iff} \quad \pi_2 \models \varphi$$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

show by structural induction on **CTL*** formulas:

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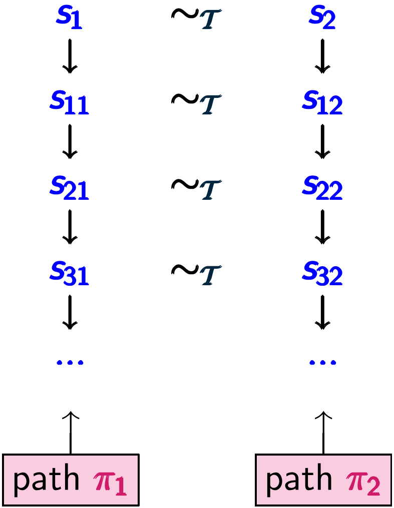
$$\pi_1 \models \varphi \text{ iff } \pi_2 \models \varphi$$

$\pi_1 \sim_{\mathcal{T}} \pi_2 \quad \overset{\text{def}}{\Longleftrightarrow} \quad \pi_1$ and π_2 are statewise bisimulation equivalent

Bisimulation equivalence ⇒ CTL* equivalence

statewise bisimulation equivalent paths:



Bisimulation equivalence ⇒ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Proof by structural induction

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Proof by structural induction

base of induction:

(a) $\Phi = \mathit{true}$ or $\Phi = a \in AP$

(b) $\varphi = \Phi$ for some state formula Φ
s.t. statement (a) holds for Φ

Bisimulation equivalence $\Rightarrow$ CTL* equivalence

For all CTL* state formulas Φ and path formulas φ:

(a) if $s_1 \sim_{\mathcal{T}} s_2$ then: $s_1 \models \Phi$ iff $s_2 \models \Phi$

(b) if $\pi_1 \sim_{\mathcal{T}} \pi_2$ then: $\pi_1 \models \varphi$ iff $\pi_2 \models \varphi$

Proof by structural induction

step of induction:

(a) consider $\Phi = \Phi_1 \wedge \Phi_2$, $\neg\Psi$ or $\exists\varphi$ s.t.

- (a) holds for Φ_1, Φ_2, Ψ
- (b) holds for φ

(b) consider $\varphi = \varphi_1 \wedge \varphi_2$, $\neg\varphi'$, $\bigcirc\varphi'$, $\varphi_1 \mathsf{U} \varphi_2$ s.t.

- (a) holds for $\varphi_1, \varphi_2, \varphi'$

Path lifting for $\sim_{\mathcal{T}}$

$$
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \\
s_{11} & & \\
\downarrow & & \\
s_{21} & & \\
\downarrow & & \\
s_{31} & & \\
\downarrow & & \\
\vdots & &
\end{array}
$$

can be
completed to

$$
\begin{array}{ccc}
s_1 & \sim_{\mathcal{T}} & s_2 \\
\downarrow & & \downarrow \\
s_{11} & \sim_{\mathcal{T}} & s_{12} \\
\downarrow & & \downarrow \\
s_{21} & \sim_{\mathcal{T}} & s_{22} \\
\downarrow & & \downarrow \\
s_{31} & \sim_{\mathcal{T}} & s_{32} \\
\downarrow & & \downarrow \\
\vdots & & \vdots
\end{array}
$$

Path lifting for $\sim_{\mathcal{T}}$

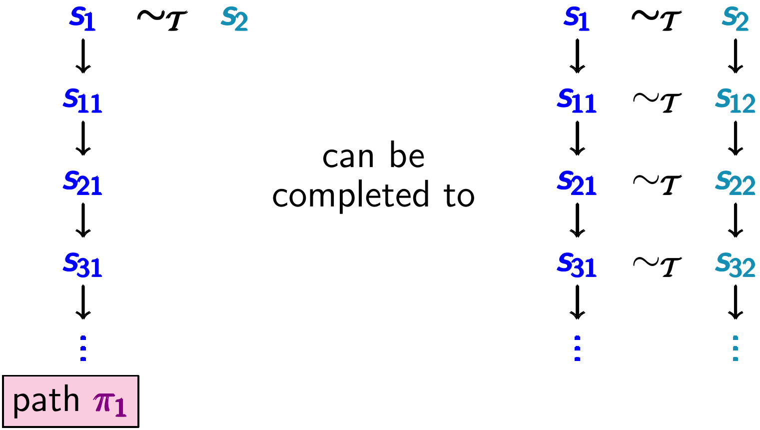
$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \\ s_{11} & & \\ \downarrow & & \\ s_{21} & & \\ \downarrow & & \\ s_{31} & & \\ \downarrow & & \\ \vdots & & \end{array}$$

can be completed to

$$\begin{array}{ccc} s_1 & \sim_{\mathcal{T}} & s_2 \\ \downarrow & & \downarrow \\ s_{11} & \sim_{\mathcal{T}} & s_{12} \\ \downarrow & & \downarrow \\ s_{21} & \sim_{\mathcal{T}} & s_{22} \\ \downarrow & & \downarrow \\ s_{31} & \sim_{\mathcal{T}} & s_{32} \\ \downarrow & & \downarrow \\ \vdots & & \vdots \end{array}$$

If $s_1 \sim_{\mathcal{T}} s_2$ then for all $\pi_1 \in Paths(s_1)$ there exists $\pi_2 \in Paths(s_2)$ with $\pi_1 \sim_{\mathcal{T}} \pi_2$

Path lifting for $\sim_{\mathcal{T}}$



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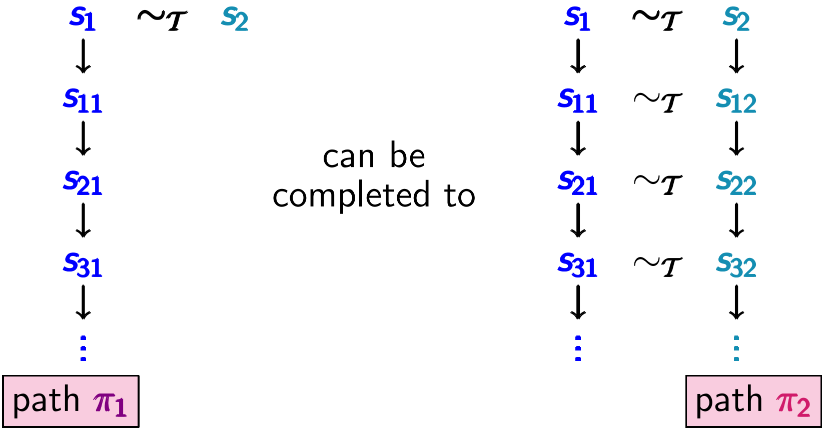
Path lifting for $\sim_{\mathcal{T}}$



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Correct or wrong?

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

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correct.

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If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

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wrong.

Correct or wrong?

If s_1, s_2 are not **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

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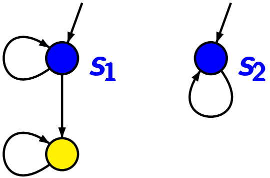

Correct or wrong?

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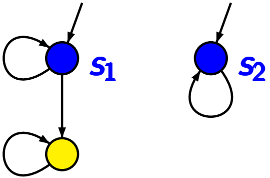
correct.

> If s_1, s_2 are <u>not</u> **LTL** equivalent then there exists a **LTL** formula φ with $s_1 \models \varphi$ and $s_2 \not\models \varphi$

wrong.

$Traces(s_2) \subset Traces(s_1)$



If s_1, s_2 are <u>not</u> **CTL** equivalent then there exists a **CTL** formula Φ with $s_1 \models \Phi$ and $s_2 \not\models \Phi$

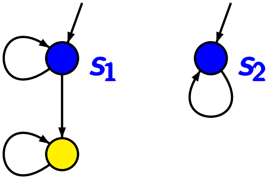
correct.

If s_1, s_2 are <u>not</u> **LTL** equivalent then there exists a **LTL** formula φ with $s_1 \models \varphi$ and $s_2 \not\models \varphi$

wrong.

$Traces(s_2) \subset Traces(s_1)$

hence: $s_1 \models \varphi$ implies $s_2 \models \varphi$



CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

CTL equivalence $\Longrightarrow$ bisimulation equivalence

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>
> if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

Proof: show that

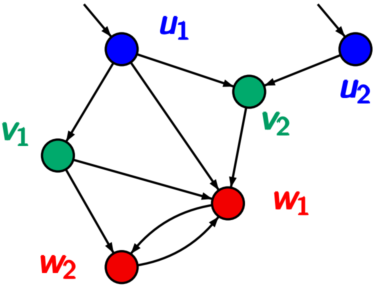
$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation, i.e., for all $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

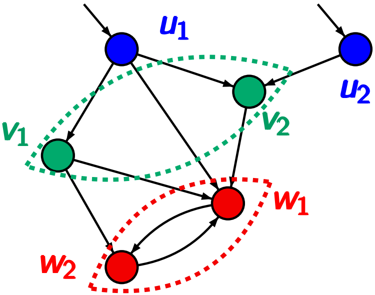
(2) if $s_1 \longrightarrow t_1$ then there exists a transition $s_2 \longrightarrow t_2$ s.t. $(t_1, t_2) \in \mathcal{R}$

Example: CTL master formulas



- (blue) $\widehat{=} \{a\}$
- (red) $\widehat{=} \{b\}$
- (green) $\widehat{=} \varnothing$

Example: CTL master formulas

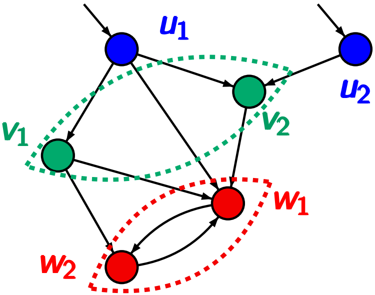


bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

 $\widehat{=} \{a\}$

 $\widehat{=} \{b\}$

 $\widehat{=} \varnothing$



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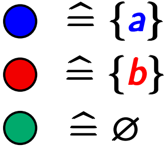
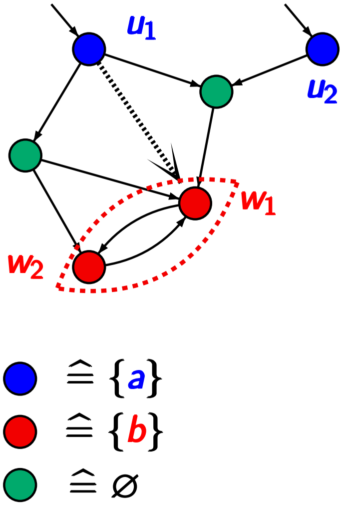
but $u_1 \not\sim_{\mathcal{T}} u_2$



Example: CTL master formulas



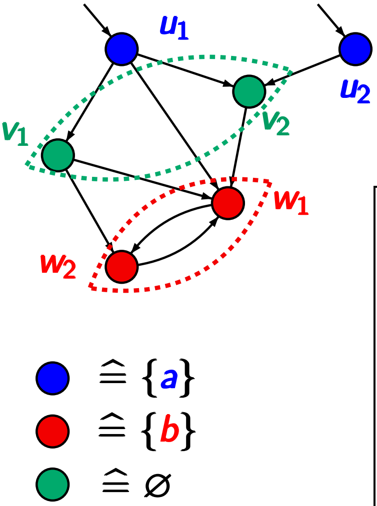
bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

but $u_1 \not\sim_{\mathcal{T}} u_2$

as $u_1 \rightarrow \{w_1, w_2\}$

$u_2 \not\rightarrow \{w_1, w_2\}$

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

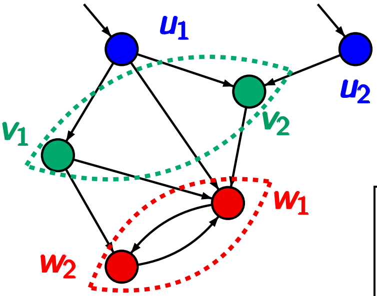
$w_1, w_2 \models$?

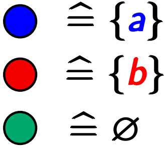
$v_1, v_2 \models$?

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas





bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

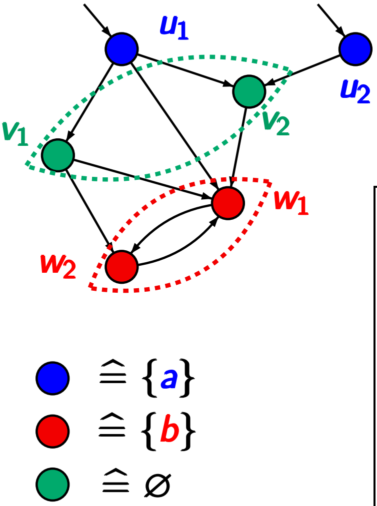
$w_1, w_2 \models b$

$v_1, v_2 \models$?

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \ldots\}$

CTL master formulas:

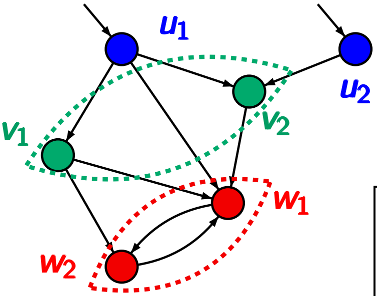
$w_1, w_2 \models b$

$v_1, v_2 \models \neg a \wedge \neg b$

$u_1 \models$?

$u_2 \models$?

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
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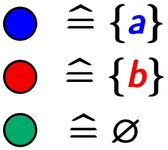


CTL master formulas:

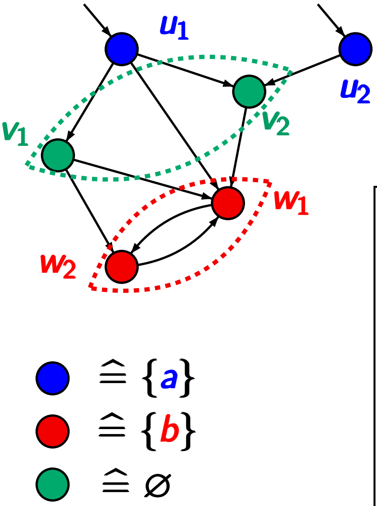
$w_1, w_2 \models b$

$v_1, v_2 \models \neg a \wedge \neg b$

$u_1 \models (\exists \bigcirc b) \wedge a$

$u_2 \models ?$

Example: CTL master formulas



bisimulation equivalence $\sim_{\mathcal{T}}$
$= \{(v_1, v_2), (w_1, w_2), \dots\}$

CTL master formulas:

$w_1, w_2 \models b$

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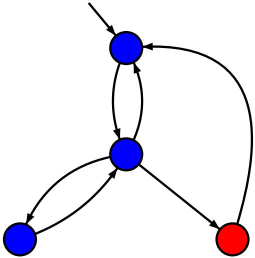
$u_1 \models (\exists \bigcirc b) \wedge a$

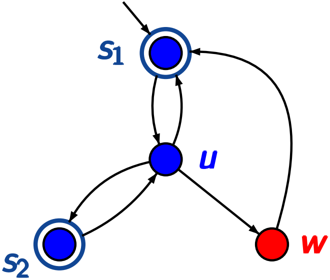
$u_2 \models (\neg \exists \bigcirc b) \wedge a$

...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

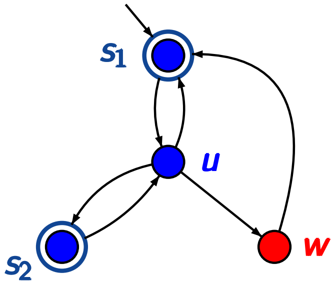
...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

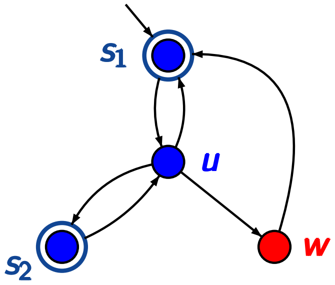
$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

$\Phi_w = ?$

$\Phi_C = ?$ where $C = \{s_1, s_2\}$

$\Phi_u = ?$

...master formulas for $\sim_{\mathcal{T}}$-classes?



$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

$$\Phi_w = red$$

$$\Phi_C = ?$$ where $C = \{s_1, s_2\}$

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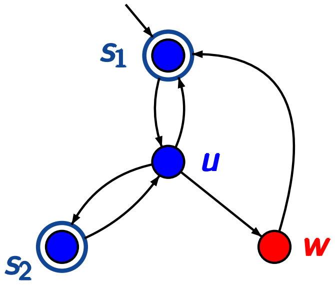
...master formulas for $\sim_{\mathcal{T}}$-classes?



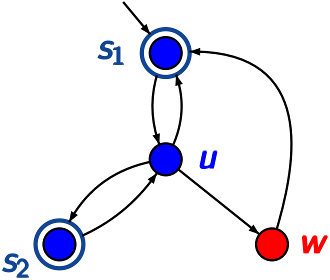
$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

$\Phi_w = red$

$\Phi_C = blue \wedge \forall \bigcirc blue$ where $C = \{s_1, s_2\}$

$\Phi_u = ?$



$AP = \{blue, red\}$

$s_1 \sim_{\mathcal{T}} s_2 \not\sim_{\mathcal{T}} u$

$$\Phi_w = red$$

$$\Phi_C = blue \wedge \forall \bigcirc blue \quad \text{where } C = \{s_1, s_2\}$$

$$\Phi_u = \exists \bigcirc red$$

CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

if s_1, s_2 are **CTL** equivalent then $s_1 \sim_{\mathcal{T}} s_2$

CTL equivalence $\Longrightarrow$ bisimulation equivalence

If $\mathcal{T}$ is a finite TS then, for all states s_1, s_2 in $\mathcal{T}$:

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- wrong for infinite TS

CTL equivalence $\Longrightarrow$ bisimulation equivalence

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↑

possibly infinite-state TS such that

* the number of initial states is finite
* for each state the number of successors is finite

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Proof: as for finite TS. Amounts showing that

$$\mathcal{R} \stackrel{\text{def}}{=} \{ (s_1, s_2) : s_1, s_2 \text{ satisfy the same } \textbf{CTL} \text{ formulas} \}$$

is a bisimulation.

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is a bisimulation, i.e., for $(s_1, s_2) \in \mathcal{R}$:

(1) $L(s_1) = L(s_2)$

(2) if $s_1 \rightarrow t_1$ then there exists a transition $s_2 \rightarrow t_2$ s.t. $(t_1, t_2) \in \mathcal{R}$

Summary: CTL/CTL* and bisimulation

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Let $\mathcal{T}$ be a finitely branching TS without terminal states, and s_1, s_2 states in $\mathcal{T}$. Then:

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iff s_1 and s_2 are **CTL** equivalent

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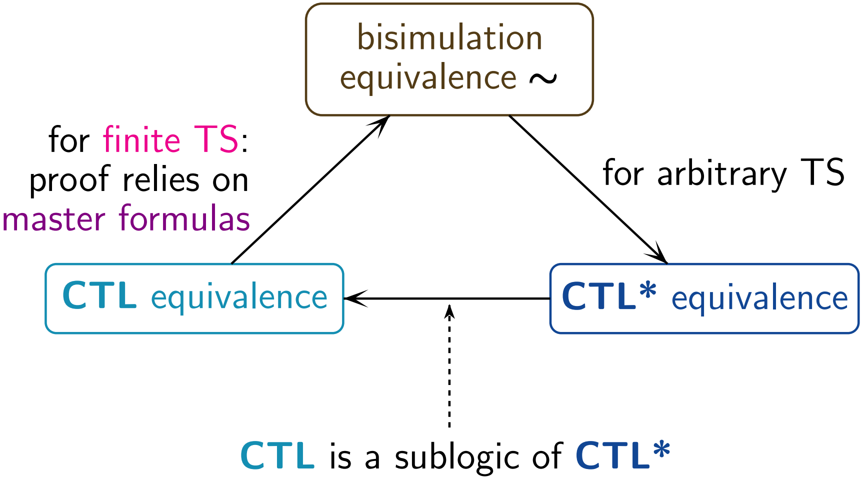
Summary: CTL/CTL* and bisimulation



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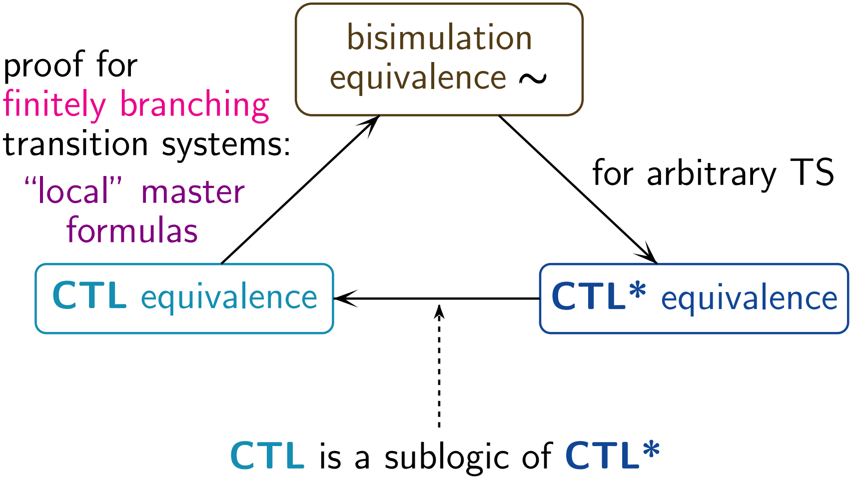


CTL/CTL* and bisimulation for TS

CTL/CTL* and bisimulation for TS

so far: we considered

- **CTL**/**CTL*** equivalence
- bisimulation equivalence $\sim_{\mathcal{T}}$

for the states of a single transition system $\mathcal{T}$

CTL/CTL* and bisimulation for TS

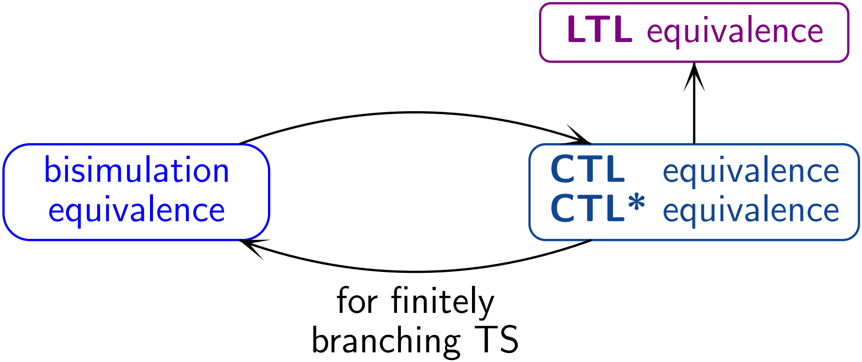
If $\mathcal{T}_1$, $\mathcal{T}_2$ are finitely branching TS over AP without terminal states then:

$\mathcal{T}_1 \sim \mathcal{T}_2$

iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL** formulas

iff $\mathcal{T}_1$ and $\mathcal{T}_2$ satisfy the same **CTL*** formulas

Summary: equivalences



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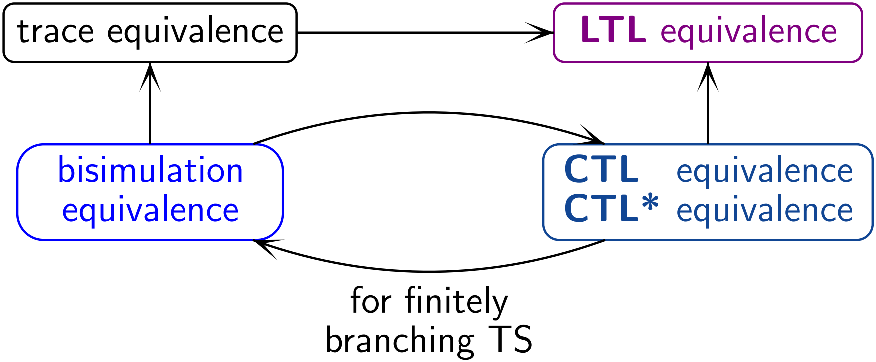

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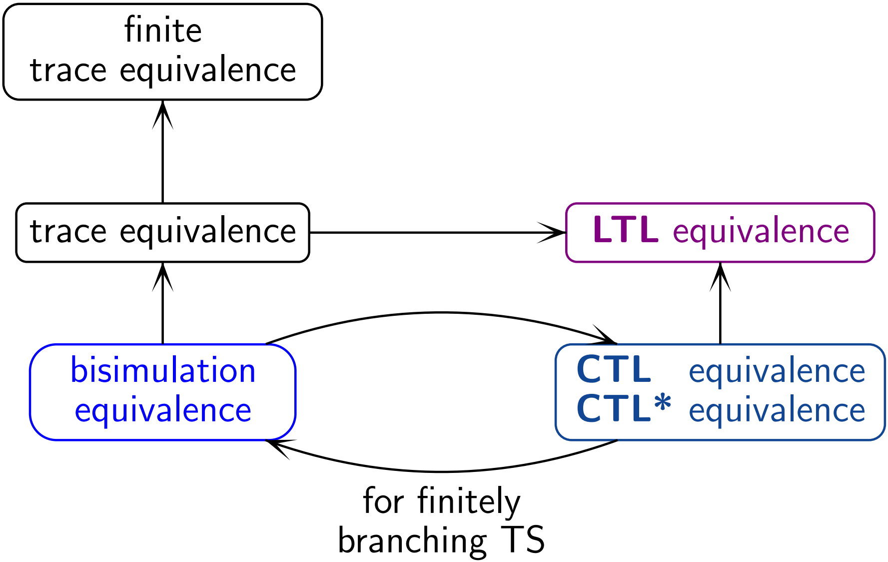

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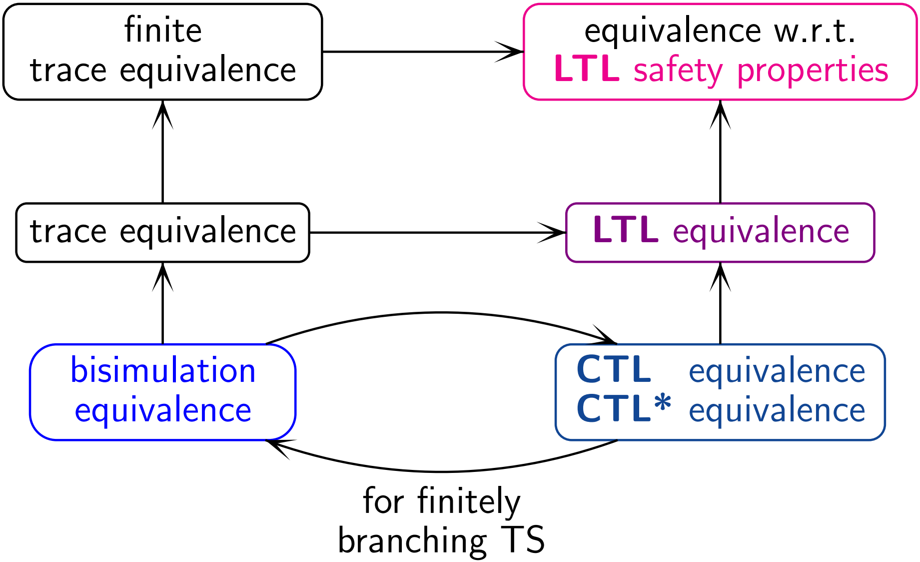

Let $\mathcal{T}$ be a finite TS without terminal states and s_1, s_2 states of $\mathcal{T}$.

If s_1, s_2 satisfy the same $\mathrm{CTL}_{\setminus \mathrm{U}}$ formulas then $s_1 \sim_{\mathcal{T}} s_2$.

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Correct or wrong?

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correct.

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correct. see the proof

"**CTL** equivalence $\Longrightarrow$ bisimulation equivalence"

$\text{CTL}_{\setminus U}$-equivalence $\Rightarrow$ bisimulation equivalence

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$$\text{and} \quad Sat(\Phi_{C,D}) \subseteq C \setminus D$$

Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states.

> $\mathcal{T}$ and its bisimulation quotient $\mathcal{T}/\sim$ satisfy the same **CTL*** formulas.

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correct. Recall that $\mathcal{T} \sim \mathcal{T}/\!\sim$

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correct. Recall that $\mathcal{T} \sim \mathcal{T}/\!\sim$ as

$$\mathcal{R} = \{(s, [s]) : s \in S\}$$

is a bisimulation for $(\mathcal{T}, \mathcal{T}/\!\sim)$

here: $[s] = \sim_{\mathcal{T}}$-equivalence class of state s

Correct or wrong?

Let $\mathcal{T}$ be a finite TS without terminal states and let *fair* be a **CTL** fairness assumption.

If $s_1 \sim_{\mathcal{T}} s_2$ then for all **CTL** formulas Φ:

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$$\Psi = \exists\big(fair \wedge \Box(a \wedge \forall(fair \rightarrow \Diamond b))\big)$$

Correct or wrong?

Let $\mathcal{T}$ be a finite TS over AP without terminal states.

If $s_1 \sim_{\mathcal{T}} s_2$ then for all **LT** properties $E \subseteq (2^{AP})^\omega$:

$$s_1 \models E \quad \text{iff} \quad s_2 \models E$$

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correct.

Note that:

(1) $s_1 \sim_{\mathcal{T}} s_2 \implies Traces(s_1) = Traces(s_2)$

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Note that:

$$(1) \quad s_1 \sim_{\mathcal{T}} s_2 \quad \Longrightarrow \quad Traces(s_1) = Traces(s_2)$$

$$(2) \quad s \models E \quad \Longleftrightarrow \quad Traces(s) \subseteq E$$

Correct or wrong?

Let $\mathcal{F}$ be an action-based strong fairness assumption

e.g., strong fairness for a single action α

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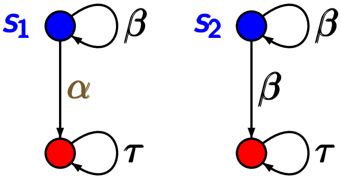

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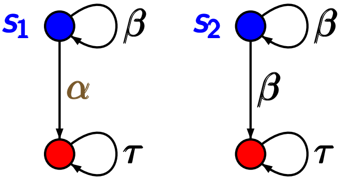
$E \mathrel{\widehat{=}} \Diamond red$



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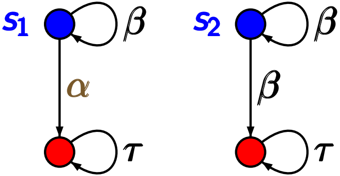
$s_1 \models_{\mathcal{F}} E$

$s_2 \not\models_{\mathcal{F}} E$



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- strong action-based fairness assumptions are realizable