Overview

Introduction

Modelling parallel systems

Linear Time Properties

Regular Properties

Linear Temporal Logic (LTL)

- syntax and semantics of LTL ⟵
- automata-based LTL model checking
- complexity of LTL model checking

Computation-Tree Logic

Equivalences and Abstraction

Linear Temporal Logic (LTL)

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$$\varphi \; ::= \; \textit{true} \;\big|\; a \;\big|\; \varphi_1 \wedge \varphi_2 \;\big|\; \neg\varphi \;\big|\; \bigcirc\varphi \;\big|\; \varphi_1 \, \mathsf{U} \, \varphi_2$$

where $a \in AP$ $\qquad \bigcirc \;\widehat{=}\;$ next $\qquad \mathsf{U} \;\widehat{=}\;$ until

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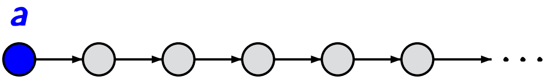

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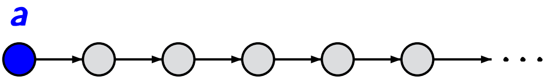

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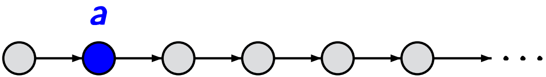

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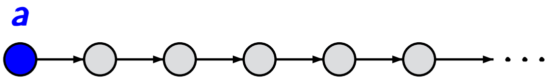

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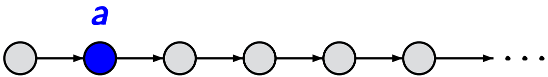
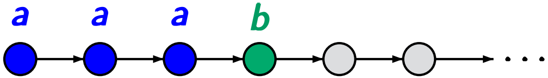


until operator $a \, \mathsf{U} \, b$



Derived operators in LTL

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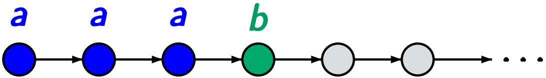

eventually
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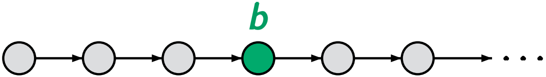

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until operator $a \, \mathsf{U} \, b$

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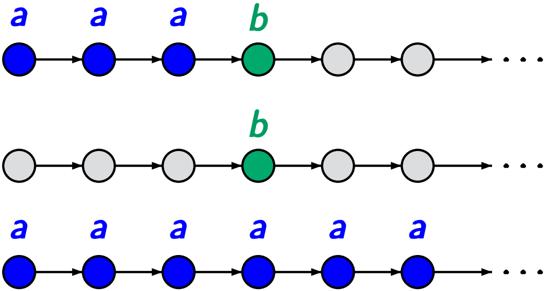
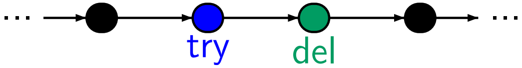


$\Box\,(\text{try\_to\_send} \rightarrow \bigcirc\ \text{delivered})$



Next $\bigcirc$, until U and eventually $\Diamond$

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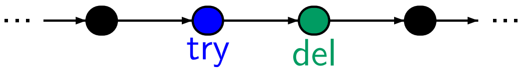

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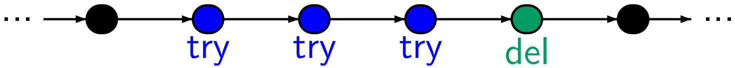

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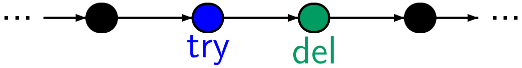

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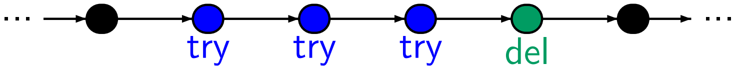

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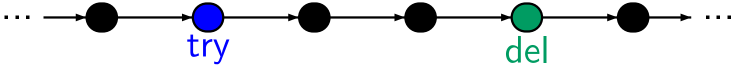

Examples for LTL formulas

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progress property: $\Box(\textit{request} \longrightarrow \Diamond \textit{response})$

traffic light: $\Box(\textit{yellow} \vee \bigcirc\neg \textit{red})$

Infinitely often and eventually forever

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e.g., unconditional fairness $\Box\Diamond \textit{crit}_i$

strong fairness $\Box\Diamond \textit{wait}_i \longrightarrow \Box\Diamond \textit{crit}_i$

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e.g.,	unconditional fairness	$\Box\Diamond \mathit{crit}_i$
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	weak fairness	$\Diamond\Box \mathit{wait}_i \longrightarrow \Box\Diamond \mathit{crit}_i$

LTL-semantics

interpretation of **LTL** formulas over traces, i.e., infinite words over 2^{AP}

formalized by a satisfaction relation $\models$ for

- LTL formulas and
- infinite words $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$

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$\sigma \models \varphi_1 \wedge \varphi_2$ iff $\sigma \models \varphi_1$ and $\sigma \models \varphi_2$

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$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \ldots \models \varphi$

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$\sigma \models \bigcirc\varphi$	iff	$\mathit{suffix}(\sigma, 1) = A_1\, A_2\, A_3 \ldots \models \varphi$
$\sigma \models \varphi_1 \mathsf{U}\, \varphi_2$	iff	there exists $j \geq 0$ such that

$\mathit{suffix}(\sigma, j) = A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi_2$ and

$\mathit{suffix}(\sigma, i) = A_i\, A_{i+1}\, A_{i+2} \ldots \models \varphi_1$ for $0 \leq i < j$

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LT property of formula φ:

$$Words(\varphi) \stackrel{\text{def}}{=} \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

LTL-semantics of derived operators $\Diamond$ and $\Box$

for $\sigma = A_0\, A_1\, A_2 \ldots \in \left(2^{AP}\right)^\omega$:

$$\vdots$$

$\sigma \models \varphi_1 \, \mathsf{U} \, \varphi_2$ iff there exists $j \geq 0$ such that

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\sigma \models \Diamond\varphi & \text{iff} & \text{there exists } j \geq 0 \text{ such that} \\
 & & \quad A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi \\
\sigma \models \Box\varphi & \text{iff} & \text{for all } j \geq 0 \text{ we have:} \\
 & & \quad A_j\, A_{j+1}\, A_{j+2}\, \ldots \models \varphi
\end{array}
$$

LTL semantics over TS

given a TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$

define satisfaction relation $\models$ for

- **LTL** formulas over AP
- the maximal path fragments and states of $\mathcal{T}$

assumption: $\mathcal{T}$ has no terminal states, i.e., all maximal path fragments in $\mathcal{T}$ are infinite

LTL semantics over paths of TS

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given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

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interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

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$$\boxed{\begin{array}{lll} \pi = s_0\, s_1\, s_2 \ldots \models \varphi & \text{iff} & trace(\pi) \models \varphi \\ & \text{iff} & trace(\pi) \in Words(\varphi) \end{array}}$$

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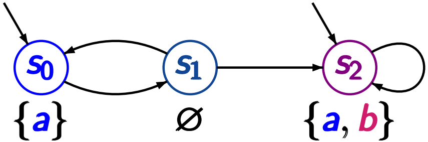
$$\text{iff} \quad trace(\pi) \in Words(\varphi)$$

remind: LT property of an LTL formula:

$$Words(\varphi) = \left\{ \sigma \in \left(2^{AP}\right)^{\omega} : \sigma \models \varphi \right\}$$

Example: LTL-semantics over paths



$AP = \{a, b\}$

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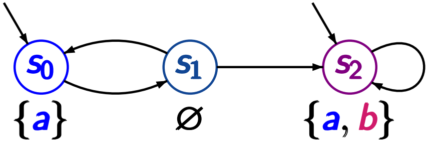

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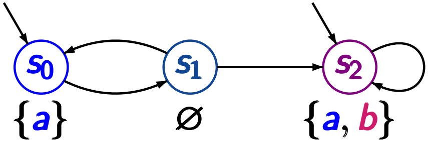

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path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $trace(\pi) = \{a\}\, \varnothing\, \{a, b\}^\omega$

$\pi \models a$

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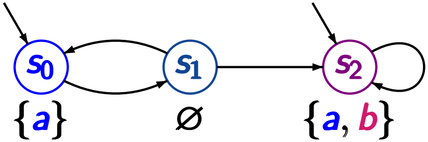

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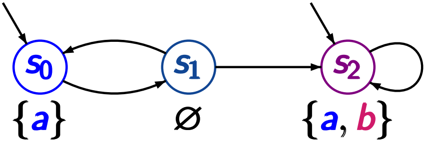

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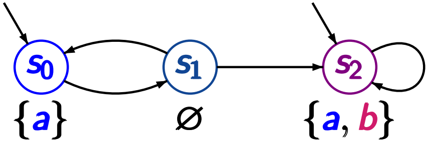
$\pi \models \bigcirc(\neg a \wedge \neg b)$

Example: LTL-semantics over paths



$AP = \{a, b\}$

path $\pi = s_0\, s_1\, s_2\, s_2\, s_2\, s_2 \ldots$ $\quad$ $\mathit{trace}(\pi) = \{a\}\,\varnothing\,\{a, b\}^\omega$

$\pi \models a$, but $\pi \not\models b$ $\quad$ as $L(s_0) = \{a\}$

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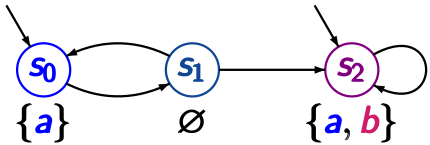

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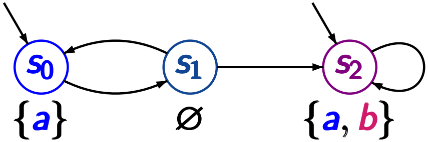

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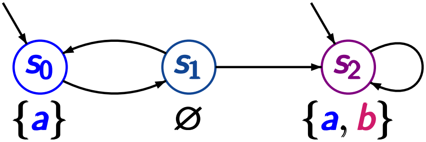

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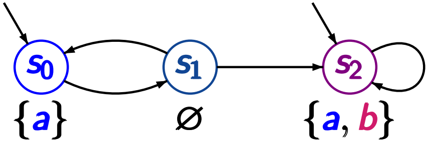

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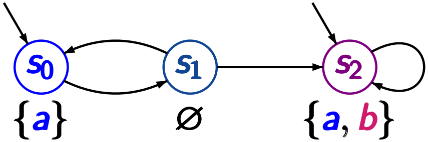

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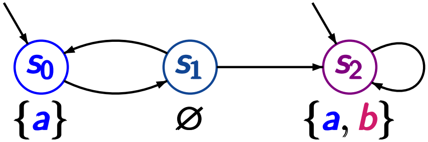
Correct or wrong ?



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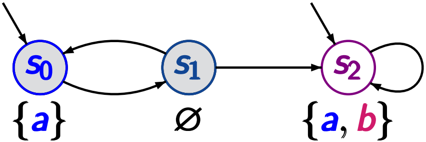

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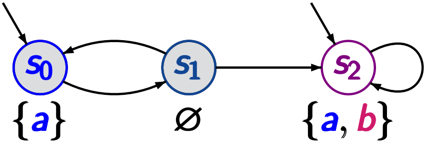

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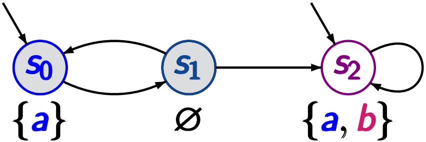

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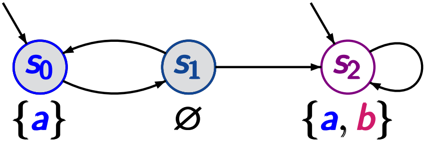

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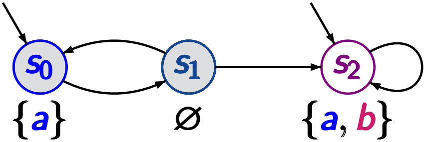
Correct or wrong ?



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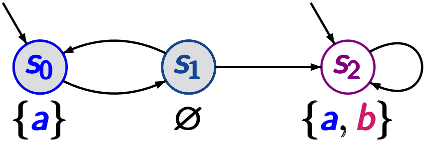

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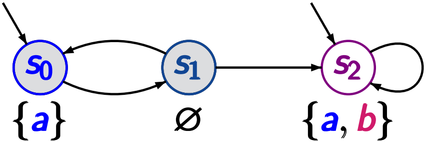

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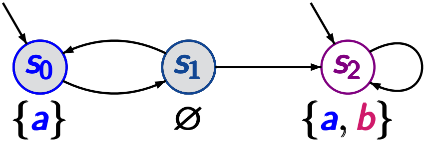

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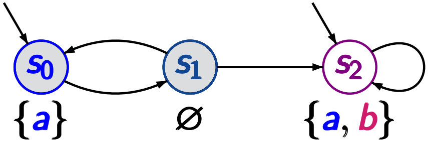
$\pi \models \Box a$?



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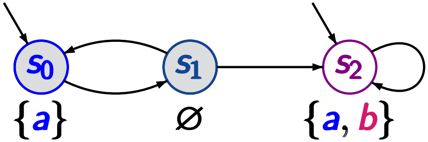
$\pi \not\models \Box a$ $\qquad$ as $s_1 \not\models a$



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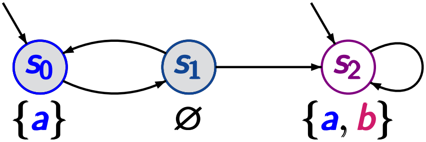
$\pi \models \lozenge b \to (a \,\mathsf{U}\, b)$ $\qquad$ as $\pi \not\models \lozenge b$

$\pi \models \bigcirc \bigcirc \neg b$ $\qquad$ as $s_0 \models \neg b$

$\pi \not\models \square a$ $\qquad$ as $s_1 \not\models a$

$\pi \models \square\lozenge a$?

Correct or wrong ?



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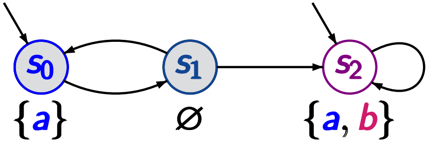

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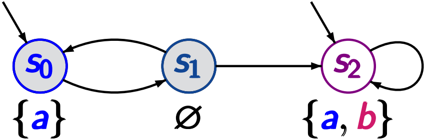

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$\pi \models \Box\Diamond a$ $\qquad$ as $\Box\Diamond \mathrel{\widehat{=}}$ infinitely often

$\pi \not\models \Diamond\Box a$ $\qquad$ as $\Diamond\Box \mathrel{\widehat{=}}$ eventually forever

LTL-semantics of derived operators

for $\sigma = A_0\, A_1\, A_2\, \ldots \in \left(2^{AP}\right)^{\omega}$:

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$\sigma \models \Diamond\varphi$	iff	there exists $j \geq 0$ such that $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
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$\sigma \models \Box\Diamond \varphi$	iff	there are infinitely many $j \geq 0$ s.t. $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$
$\sigma \models \Diamond\Box \varphi$	iff	for almost all $j \geq 0$ we have: $A_j\, A_{j+1}\, A_{j+2} \ldots \models \varphi$

LTL semantics over the states of a TS

LTLSF3.1-SEM-STATES

LTL semantics over the states of a TS

given: TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$
without terminal states

LTL formula φ over AP

interpretation of φ over infinite path fragments

$$\pi = s_0\, s_1\, s_2 \ldots \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi$$

interpretation of φ over states:

$$\boxed{s \models \varphi \quad \text{iff} \quad trace(\pi) \models \varphi \text{ for all } \pi \in Paths(s)}$$

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satisfaction relation for LT properties

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Interpretation of LTL formulas over TS

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LTL formula φ over AP

$\mathcal{T} \models \varphi$ iff $s_0 \models \varphi$ for all $s_0 \in S_0$

iff $trace(\pi) \models \varphi$ for all $\pi \in Paths(\mathcal{T})$

Interpretation of LTL formulas over TS

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LTL formula φ over AP

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Interpretation of LTL formulas over TS

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LTL formula φ over AP

$$
\begin{array}{lll}
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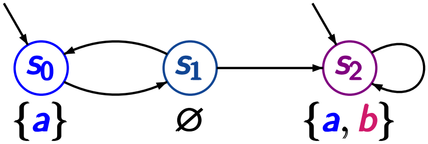
\end{array}
$$

satisfaction relation for LT properties

Which formulas hold for $\mathcal{T}$?



$$AP = \{a, b\}$$

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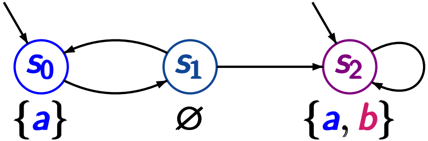

$AP = \{a, b\}$

$\mathcal{T} \models a$

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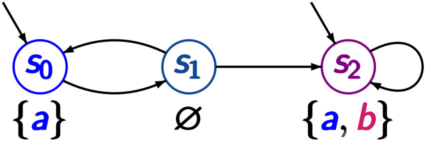
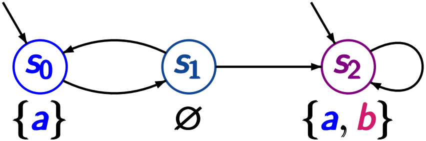


$AP = \{a, b\}$

$\mathcal{T} \models a$ as $s_0 \models a$ and $s_2 \models a$

Which formulas hold for $\mathcal{T}$?



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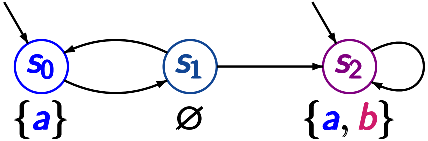

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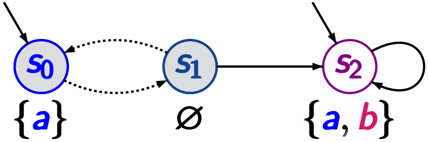

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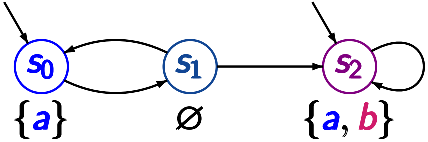

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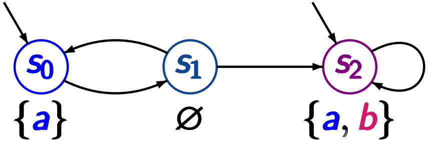

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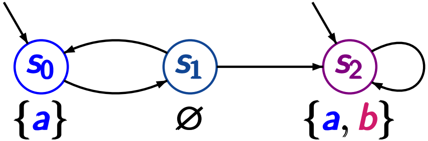

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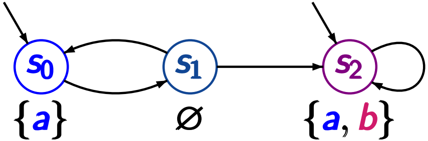

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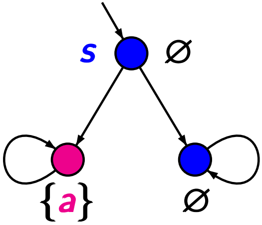

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LTL-formulas for MUTEX protocols

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$$\varphi_{sf} = \Box(wait_1 \rightarrow \Diamond crit_1) \wedge \Box(wait_2 \rightarrow \Diamond crit_2)$$

Provide an LTL formula over $AP = \{a, b\}$ for ...

- set of all words $A_0 \, A_1 \, A_2 \, \ldots \in \left(2^{AP}\right)^{\omega}$ such that:

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Self-duality of the next operator

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\end{array}$$

Correct or wrong?

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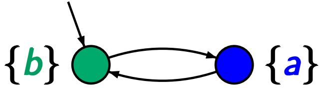

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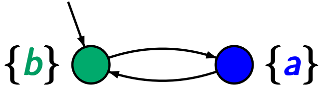

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---

similarly: $\Box(\varphi \wedge \psi) \equiv \Box\varphi \wedge \Box\psi$

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Expansion laws

Expansion law for U

until: $\varphi \, \mathsf{U} \, \psi \;\equiv\; \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$

Expansion laws for U and $\Diamond$

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$$\begin{aligned} \Box \psi &= \neg \Diamond \neg \psi \\ &\equiv \neg(\neg \psi \lor \bigcirc \Diamond \neg \psi) \longleftarrow \boxed{\text{expansion law for } \Diamond} \end{aligned}$$

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\end{aligned}
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Expansion laws are fixed point equations

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...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

consider $\psi = a$

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eventually: $\boxed{\Diamond \psi} \equiv \psi \vee \bigcirc \boxed{\Diamond \psi}$

always: $\boxed{\Box \psi} \equiv \psi \wedge \bigcirc \boxed{\Box \psi}$

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

although

$\Box a \not\equiv \mathit{false}$

Expansion laws are fixed point equations

until: $\varphi \mathbf{U} \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$

least fixed point

eventually: $\Diamond\psi \equiv \psi \vee \bigcirc\Diamond\psi$

least fixed point

always: $\Box\psi \equiv \psi \wedge \bigcirc\Box\psi$

...don't yield a complete characterization, e.g.,

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least fixed point

always: $\Box \psi \equiv \psi \wedge \bigcirc \Box \psi$

greatest fixed point

...don't yield a complete characterization, e.g.,

$$\boxed{\begin{array}{lcl} \mathit{false} & \equiv & a \wedge \bigcirc \mathit{false} \\ \Box a & \equiv & a \wedge \bigcirc \Box a \end{array}}$$

although
$\Box a \not\equiv \mathit{false}$

Expansion law for U

The LTL formula $\chi = \varphi \, \mathsf{U} \, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

Expansion law for U

The LTL formula $\chi = \varphi \,\mathsf{U}\, \psi$ is the least solution of

$$\chi \equiv \psi \vee (\varphi \wedge \bigcirc \chi)$$

i.e., $\mathit{Words}(\varphi \,\mathsf{U}\, \psi)$ least LT-property E s.t.

$$E = \mathit{Words}(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in \mathit{Words}(\varphi) : A_1 A_2 \ldots \in E \right\}$$

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i.e., $Words(\varphi \, \mathsf{U} \, \psi)$ least LT-property E s.t.

$$E = Words(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E \right\}$$

It even holds that $Words(\varphi \, \mathsf{U} \, \psi)$ least LT-property E s.t.

$$(1) \quad Words(\psi) \subseteq E$$

$$(2) \quad \left\{ A_0 A_1 A_2 \ldots \in Words(\varphi) : \; A_1 A_2 \ldots \in E \right\} \subseteq E$$

The weak until operator W

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$$\boxed{\varphi \,\mathsf{W}\, \psi \stackrel{\mathsf{def}}{=} (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

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deriving "always" and "until" from "weak until":

$$\Box\varphi \quad \equiv \quad ?$$

The weak until operator W

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The weak until operator W

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deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \, \mathsf{W} \, \mathit{false}$$

$$\varphi \, \mathsf{U} \, \psi \equiv \ ?$$

The weak until operator W

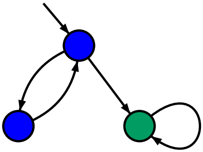
$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

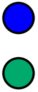
deriving "always" and "until" from "weak until":

$$\Box\varphi \equiv \varphi \,\mathsf{W}\, \mathit{false}$$

$$\varphi \,\mathsf{U}\, \psi \equiv (\varphi \,\mathsf{W}\, \psi) \wedge \Diamond\psi$$

Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?

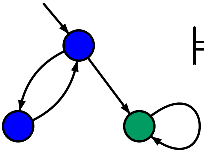




$\widehat{=} \ \{a\}$

$\widehat{=} \ \{b\}$

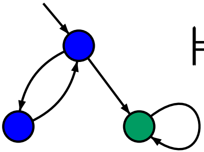
Does $\mathcal{T} \models a \, \mathsf{W} \, b$ hold?



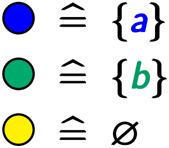
$\models a \, \mathsf{W} \, b$

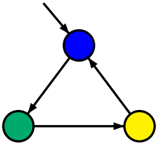
● $\widehat{=}$ $\{a\}$

● $\widehat{=}$ $\{b\}$

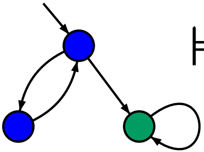


$\models a \,\mathsf{W}\, b$





Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

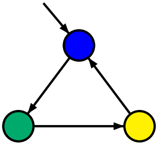


$\models a \,\mathsf{W}\, b$

$\widehat{=}$ $\{a\}$

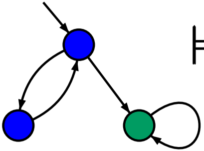
$\widehat{=}$ $\{b\}$

$\widehat{=}$ $\varnothing$



$\models\; a \,\mathsf{W}\, b \quad$ (even $a \,\mathsf{U}\, b$)

Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

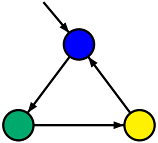


$\models a \,\mathsf{W}\, b$

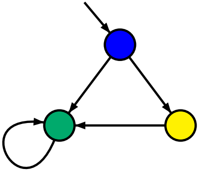
$\widehat{=} \ \{a\}$

$\widehat{=} \ \{b\}$

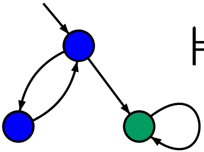
$\widehat{=} \ \varnothing$



$\models \ a \,\mathsf{W}\, b \quad$ (even $a \,\mathsf{U}\, b$)



Does $\mathcal{T} \models a \,\mathsf{W}\, b$ hold?

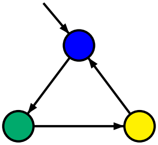


$\models a \,\mathsf{W}\, b$

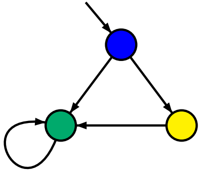
$\bullet_{\text{blue}} \mathrel{\widehat{=}} \{a\}$

$\bullet_{\text{green}} \mathrel{\widehat{=}} \{b\}$

$\bullet_{\text{yellow}} \mathrel{\widehat{=}} \varnothing$



$\models a \,\mathsf{W}\, b$ (even $a \,\mathsf{U}\, b$)



$\not\models a \,\mathsf{W}\, b$

Duality of U and W

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\overset{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

goal: express $\neg(\varphi \, \mathsf{U} \, \psi)$ via **W**, and vice versa

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\stackrel{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \;\vee\; \Box\varphi}$$

$$\begin{aligned} &\neg(\varphi \,\mathsf{U}\, \psi) \\ \equiv\;\; &((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)) \;\vee\; \Box(\varphi \wedge \neg\psi) \end{aligned}$$

$$\boxed{\varphi \,\mathsf{W}\, \psi \;\overset{\text{def}}{=}\; (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv\; & \big((\varphi \wedge \neg\psi) \;\mathsf{U}\; (\neg\varphi \wedge \neg\psi)\big) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\; & (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

Duality of U and W

$$\boxed{\varphi \,\mathsf{W}\, \psi \overset{\text{def}}{=} (\varphi \,\mathsf{U}\, \psi) \vee \Box\varphi}$$

$$
\begin{aligned}
& \neg(\varphi \,\mathsf{U}\, \psi) \\
\equiv\ & ((\varphi \wedge \neg\psi) \ \mathsf{U}\ (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\ & (\varphi \wedge \neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi) \\
\equiv\ & (\neg\psi) \,\mathsf{W}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}
$$

$$\boxed{\varphi \, \mathsf{W} \, \psi \;\overset{\text{def}}{=}\; (\varphi \, \mathsf{U} \, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
&\neg(\varphi \, \mathsf{U} \, \psi) \\
\equiv\;& ((\varphi \wedge \neg\psi) \; \mathsf{U} \; (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi) \\
\equiv\;& (\varphi \wedge \neg\psi) \; \mathsf{W} \; (\neg\varphi \wedge \neg\psi) \\
\equiv\;& (\neg\psi) \; \mathsf{W} \; (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\neg(\varphi \, \mathsf{U} \, \psi) \equiv (\neg\psi) \, \mathsf{W} \, (\neg\varphi \wedge \neg\psi)$$

$$\neg(\varphi \, \mathsf{W} \, \psi) \equiv \; ?$$

$$\boxed{\varphi\, \mathsf{W}\, \psi \stackrel{\text{def}}{=} (\varphi\, \mathsf{U}\, \psi) \vee \Box\varphi}$$

$$\begin{aligned}
&\neg(\varphi\, \mathsf{U}\, \psi)\\
\equiv\;& ((\varphi \wedge \neg\psi)\ \mathsf{U}\ (\neg\varphi \wedge \neg\psi)) \vee \Box(\varphi \wedge \neg\psi)\\
\equiv\;& (\varphi \wedge \neg\psi)\ \mathsf{W}\ (\neg\varphi \wedge \neg\psi)\\
\equiv\;& (\neg\psi)\ \mathsf{W}\ (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

$$\begin{aligned}
\neg(\varphi\, \mathsf{U}\, \psi) &\equiv (\neg\psi)\, \mathsf{W}\, (\neg\varphi \wedge \neg\psi)\\
\neg(\varphi\, \mathsf{W}\, \psi) &\equiv (\neg\psi)\, \mathsf{U}\, (\neg\varphi \wedge \neg\psi)
\end{aligned}$$

Expansion laws for U and W

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\varphi \,\mathsf{W}\, \psi \equiv \quad ?$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi \equiv \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

Expansion laws for U and W

$\varphi \mathbf{U} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{U} \psi))$	smallest solution
$\varphi \mathbf{W} \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \mathbf{W} \psi))$	largest solution

Words$(\varphi \mathbf{U} \psi)$ smallest LT-property E s.t.

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
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$$\begin{array}{ll} (1) & Words(\psi) \subseteq E \\ (2) & \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\} \subseteq E \end{array}$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
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$$\Updownarrow$$

$$Words(\psi) \cup \left\{ A_0 A_1 A_2 \ldots \in Words(\varphi) : \; A_1 A_2 \ldots \in E \right\} \subseteq E$$

Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
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$$Words(\psi) \cup \{A_0 A_1 A_2 \ldots \in Words(\varphi) : A_1 A_2 \ldots \in E\} \subseteq E$$

$\varphi \, \mathsf{U} \, \psi$	$\equiv$	$\psi \vee (\varphi \wedge \bigcirc(\varphi \, \mathsf{U} \, \psi))$	smallest solution
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$Words(\varphi \, \mathsf{W} \, \psi)$ largest LT-property E s.t.

Expansion laws for U and W

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Expansion laws for U and W

$\varphi \,\mathsf{U}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$	smallest solution
$\varphi \,\mathsf{W}\, \psi \;\equiv\; \psi \vee (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$	largest solution

$Words(\varphi \,\mathsf{U}\, \psi)$ smallest LT-property E s.t.

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$Words(\varphi \,\mathsf{W}\, \psi)$ largest LT-property E s.t.

$$E \subseteq Words(\psi) \cup \{A_0A_1A_2\ldots \in Words(\varphi) : A_1A_2\ldots \in E\}$$

Expansion laws for U and W

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

largest solution

Expansion laws for U, W, $\Diamond$, and $\Box$

$$\varphi \,\mathsf{U}\, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{U}\, \psi))$$

smallest solution

$$\Diamond\psi \quad \equiv \quad \psi \;\vee\; \bigcirc\Diamond\psi$$

smallest solution

$$\varphi \,\mathsf{W}\, \psi \quad \equiv \quad \psi \;\vee\; (\varphi \wedge \bigcirc(\varphi \,\mathsf{W}\, \psi))$$

largest solution

$$\Box\varphi \quad \equiv \quad \varphi \wedge \bigcirc\Box\varphi$$

largest solution

---

remind: $\Diamond\psi = \mathit{true}\,\mathsf{U}\,\psi$, $\quad \Box\varphi \equiv \varphi\,\mathsf{W}\,\mathit{false}$

Positive normal form (PNF)

Positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

PNF for propositional logic

- negation only on the level of literals
- uses for each operator its dual

syntax of propositional formulas in PNF:

$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

PNF for propositional logic

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syntax of propositional formulas in PNF:

$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2$$

$\neg \mathit{true} \ \equiv \ \mathit{false}$	duality of the constant truth values
$\neg(\varphi_1 \wedge \varphi_2) \ \equiv \ \neg\varphi_1 \vee \neg\varphi_2$	duality of $\vee$ and $\wedge$ (de Morgan's law)

LTL in positive normal form (PNF)

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using duality of constants and duality of $\vee$ and $\wedge$

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$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$
$$\bigcirc \varphi + \text{dual operator for } \bigcirc$$

using duality of constants and duality of $\vee$ and $\wedge$

LTL in positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

$$\varphi \ ::= \ \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid$$

$$\bigcirc\varphi \longleftarrow$$ no new operator needed for $\neg\bigcirc$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg\bigcirc\varphi \equiv \bigcirc\neg\varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

$$\varphi \;::=\; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc \varphi \mid \varphi_1 \, \mathsf{U} \, \varphi_2 \; + \text{ dual operator for } \mathsf{U}$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \equiv \bigcirc \neg \varphi$ self-duality of the next operator

LTL in positive normal form (PNF)

- negation only on the level of literals
- uses for each operator its dual

$$\varphi \ ::= \ \mathit{true} \ | \ \mathit{false} \ | \ a \ | \ \neg a \ | \ \varphi_1 \wedge \varphi_2 \ | \ \varphi_1 \vee \varphi_2 \ | \ \bigcirc\varphi \ | \ \varphi_1 \,\mathsf{U}\, \varphi_2 \ | \ \varphi_1 \,\mathsf{W}\, \varphi_2$$

using duality of constants and duality of $\vee$ and $\wedge$

$\neg \bigcirc \varphi \ \equiv \ \bigcirc \neg \varphi$ self-duality of the next operator

$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) \ \equiv \ (\neg\varphi_2) \,\mathsf{W}\, (\neg\varphi_1 \wedge \neg\varphi_2)$

duality of **U** and **W**

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi \; ::= \; \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc \varphi \mid \varphi_1 \mathsf{U} \varphi_2 \mid \varphi_1 \mathsf{W} \varphi_2$$

Derivation of $\Diamond$ and $\Box$ in LTL-PNF

$$\varphi \quad ::= \quad \mathit{true} \mid \mathit{false} \mid a \mid \neg a \mid \varphi_1 \wedge \varphi_2 \mid \varphi_1 \vee \varphi_2 \mid \bigcirc\varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2 \mid \varphi_1 \,\mathsf{W}\, \varphi_2 \mid \Diamond\varphi \mid \Box\varphi$$

$\Diamond$ and $\Box$ can (still) be derived:

$$\Diamond\varphi \stackrel{\text{def}}{=} \mathit{true} \,\mathsf{U}\, \varphi$$

$$\Box\varphi \stackrel{\text{def}}{=} \varphi \,\mathsf{W}\, \mathit{false}$$

Universality of LTL-PNF

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Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

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LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
by successive application of the following rules:

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LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
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$$
\begin{array}{lcl}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \ \vee \ \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2) \, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}
$$

Each LTL formula can be transformed into an equivalent LTL formula in **PNF**

LTL formula $\varphi \rightsquigarrow$ LTL formula in PNF φ'
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$$
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\neg \mathit{true} & \rightsquigarrow & \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \, \mathsf{U} \, \varphi_2) & \rightsquigarrow & (\neg\varphi_2) \, \mathsf{W} (\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}
$$

exponential-blow up is possible

Example: LTL $\rightsquigarrow$ LTL-PNF

$$\begin{array}{lcl}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \\
\neg \bigcirc \varphi & \rightsquigarrow & \bigcirc\neg\varphi \\
\neg(\varphi_1 \,\mathsf{U}\, \varphi_2) & \rightsquigarrow & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)
\end{array}$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	

Example: LTL $\rightsquigarrow$ LTL-PNF

$\neg \mathit{true}$	$\rightsquigarrow$	false	+ analogue rule for $\neg \mathit{false}$
$\neg\neg\varphi$	$\rightsquigarrow$	φ	
$\neg(\varphi_1 \wedge \varphi_2)$	$\rightsquigarrow$	$\neg\varphi_1 \vee \neg\varphi_2$	+ analogue rule for $\neg\vee$
$\neg \bigcirc \varphi$	$\rightsquigarrow$	$\bigcirc\neg\varphi$	
$\neg(\varphi_1 \,\mathsf{U}\, \varphi_2)$	$\rightsquigarrow$	$(\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2)$	
$\neg\Diamond\varphi$	$\rightsquigarrow$	$\Box\neg\varphi$	$\neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi$

$$
\begin{array}{lcll}
\neg \mathit{true} & \leadsto & \mathit{false} & + \text{ analogue rule for } \neg \mathit{false} \\
\neg\neg\varphi & \leadsto & \varphi & \\
\neg(\varphi_1 \wedge \varphi_2) & \leadsto & \neg\varphi_1 \vee \neg\varphi_2 & + \text{ analogue rule for } \neg\vee \\
\neg \bigcirc \varphi & \leadsto & \bigcirc\neg\varphi & \\
\neg(\varphi_1 \mathsf{U} \varphi_2) & \leadsto & (\neg\varphi_2)\,\mathsf{W}(\neg\varphi_1 \wedge \neg\varphi_2) & \\
\neg\Diamond\varphi & \leadsto & \Box\neg\varphi & \neg\Box\varphi \leadsto \Diamond\neg\varphi
\end{array}
$$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$
\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
\neg(\varphi_1 \wedge \varphi_2) & \rightsquigarrow & \neg\varphi_1 \vee \neg\varphi_2 \quad + \text{ analogue rule for } \neg\vee \\
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\end{array}
$$

$$\neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$$\equiv \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c)$$

$\longleftarrow$ duality of $\Diamond$ and $\Box$

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$$
\begin{array}{lcl}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
\neg\neg\varphi & \rightsquigarrow & \varphi \\
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\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi
\end{array}
$$

$$
\begin{array}{ll}
 & \neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c) \\
\equiv & \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box} \\
\equiv & \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c) \qquad \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee}
\end{array}
$$

Example: LTL $\rightsquigarrow$ LTL-PNF

$$
\begin{array}{lll}
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\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \rightsquigarrow \Diamond\neg\varphi
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$$

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\begin{array}{lll}
 & \neg\Box((a \,\mathsf{U}\, b) \vee \bigcirc c) & \\
\equiv & \Diamond\neg((a \,\mathsf{U}\, b) \vee \bigcirc c) & \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box} \\
\equiv & \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \neg\bigcirc c) & \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee} \\
\equiv & \Diamond(\neg(a \,\mathsf{U}\, b) \wedge \bigcirc\neg c) & \longleftarrow \boxed{\text{self-duality of } \bigcirc}
\end{array}
$$

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$$
\begin{array}{lll}
\neg \mathit{true} & \rightsquigarrow & \mathit{false} \quad + \text{ analogue rule for } \neg \mathit{false} \\
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\neg\Diamond\varphi & \rightsquigarrow & \Box\neg\varphi \qquad \neg\Box\varphi \;\rightsquigarrow\; \Diamond\neg\varphi
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$$

$$
\begin{array}{lll}
 & \neg\Box((a \,\mathsf{U}\, b) \;\vee\; \bigcirc c) & \\
\equiv & \Diamond\neg((a \,\mathsf{U}\, b) \;\vee\; \bigcirc c) & \longleftarrow \boxed{\text{duality of } \Diamond \text{ and } \Box} \\
\equiv & \Diamond(\neg(a \,\mathsf{U}\, b) \;\wedge\; \neg\bigcirc c) & \longleftarrow \boxed{\text{duality of } \wedge \text{ and } \vee} \\
\equiv & \Diamond((\neg b)\,\mathsf{W}(\neg a \wedge \neg b) \;\wedge\; \bigcirc\neg c) & \longleftarrow \boxed{\text{duality of } \mathsf{U} \text{ and } \mathsf{W}}
\end{array}
$$

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& \neg\Box((a \,\mathsf{U}\, b) \;\vee\; \bigcirc c) \\
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\equiv & \Diamond((\neg b)\,\mathsf{W}(\neg a \wedge \neg b) \;\wedge\; \bigcirc\neg c) \longleftarrow \boxed{\textbf{PNF}}
\end{array}$$

Fairness in LTL

Recall: action-based fairness

Recall: action-based fairness

fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

$$\mathcal{F} = (\mathcal{F}_{ucond}, \mathcal{F}_{strong}, \mathcal{F}_{weak})$$

where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

$\mathcal{F}_{ucond}$	unconditional fairness assumption
$\mathcal{F}_{strong}$	strong fairness assumption
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Recall: action-based fairness

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execution $s_0 \xrightarrow{\alpha_1} s_1 \xrightarrow{\alpha_2} s_2 \xrightarrow{\alpha_3} \ldots$ $\mathcal{F}$-fair if fairness assumption for TS $\mathcal{T} = (S, Act, \rightarrow, S_0, AP, L)$:

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- for all $A \in \mathcal{F}_{ucond}$: $\overset{\infty}{\exists}\, i \geq 1.\, \alpha_i \in A$

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$$\overset{\infty}{\exists} i \geq 1.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists} i \geq 1.\, \alpha_i \in A$$

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- for all $A \in \mathcal{F}_{weak}$:

  $$\overset{\infty}{\forall} i \geq 1.\, A \cap Act(s_i) \neq \varnothing \implies \overset{\infty}{\exists} i \geq 1.\, \alpha_i \in A$$

Recall: action-based fairness

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where $\mathcal{F}_{ucond}$, $\mathcal{F}_{strong}$, $\mathcal{F}_{weak} \subseteq 2^{Act}$

satisfaction relation for LT-properties under fairness:

$\mathcal{T} \models_{\mathcal{F}} E$ iff for all $\mathcal{F}$-fair paths π of $\mathcal{T}$: $trace(\pi) \in E$

Process fairness is LTL-definable

Process fairness is LTL-definable

$$\varphi \ ::= \ \mathit{true} \mid a \mid \varphi_1 \wedge \varphi_2 \mid \neg\varphi \mid \bigcirc \varphi \mid \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually $\Diamond\varphi \overset{\text{def}}{=} \mathit{true}\,\mathsf{U}\,\varphi$

always $\Box\varphi \overset{\text{def}}{=} \neg\Diamond\neg\varphi$

infinitely often $\Box\Diamond\varphi$

eventually forever $\Diamond\Box\varphi$

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$$\varphi \ ::= \ \mathit{true} \ \big| \ a \ \big| \ \varphi_1 \wedge \varphi_2 \ \big| \ \neg\varphi \ \big| \ \bigcirc \varphi \ \big| \ \varphi_1 \,\mathsf{U}\, \varphi_2$$

eventually	$\Diamond\varphi$	$\stackrel{\text{def}}{=} \ \mathit{true}\,\mathsf{U}\,\varphi$
always	$\Box\varphi$	$\stackrel{\text{def}}{=} \ \neg\Diamond\neg\varphi$
infinitely often	$\Box\Diamond\varphi$	
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e.g., unconditional fairness $\Box\Diamond \mathit{crit}_i$

strong fairness $\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

Process fairness is LTL-definable

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e.g.,	unconditional fairness	$\Box\Diamond \mathit{crit}_i$
	strong fairness	$\Box\Diamond \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$
	weak fairness	$\Diamond\Box \mathit{wait}_i \rightarrow \Box\Diamond \mathit{crit}_i$

LTL fairness assumptions

LTL fairness assumptions

... are conjunctions of LTL formulas of the form:

- unconditional fairness $\Box\Diamond\phi$
- strong fairness $\Box\Diamond\phi_1 \rightarrow \Box\Diamond\phi_2$
- weak fairness $\Diamond\Box\phi_1 \rightarrow \Box\Diamond\phi_2$

where ϕ_1, ϕ_2, ϕ are propositional formulas

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If *fair* is a LTL fairness assumption, s a state in a TS, and φ an LTL formula then

$$s \models_{\mathit{fair}} \varphi \quad \text{iff} \quad \text{for all } \pi \in \mathit{Paths}(s): \text{ if } \pi \models \mathit{fair} \text{ then } \pi \models \varphi$$

. . . are conjunctions of **LTL formulas** of the form:

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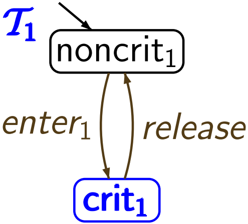
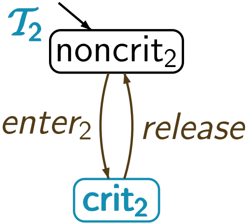
where ϕ_1, ϕ_2, ϕ are propositional formulas

If *fair* is a LTL fairness assumption, s a state in a TS, and φ an LTL formula then

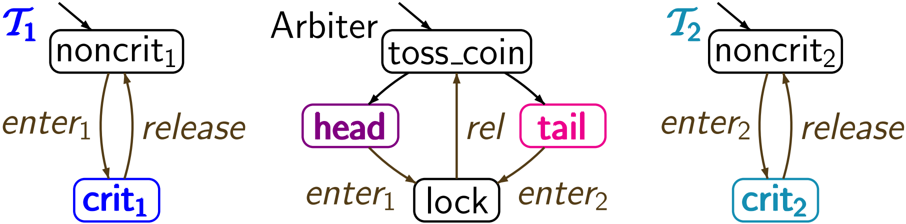
$$
\begin{aligned}
s \models_{fair} \varphi \quad & \text{iff} \quad \text{for all } \pi \in Paths(s): \\
& \qquad\quad \text{if } \pi \models fair \text{ then } \pi \models \varphi \\
& \text{iff} \quad s \models fair \rightarrow \varphi
\end{aligned}
$$

Randomized arbiter for MUTEX

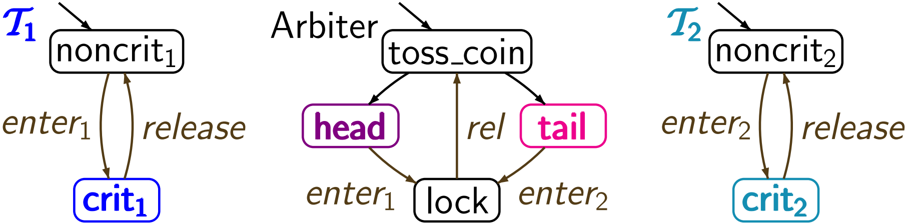




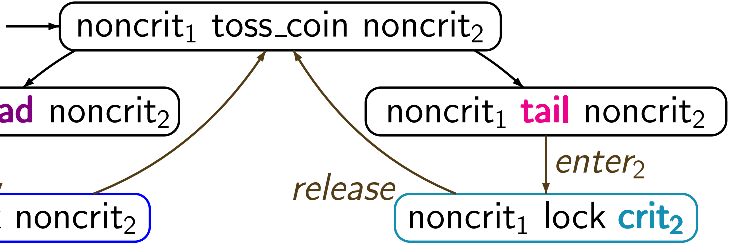
Randomized arbiter for MUTEX



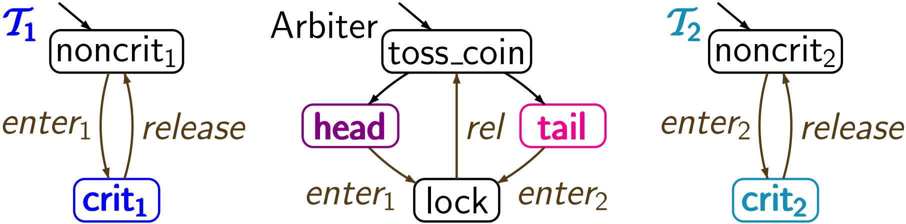
Randomized arbiter for MUTEX



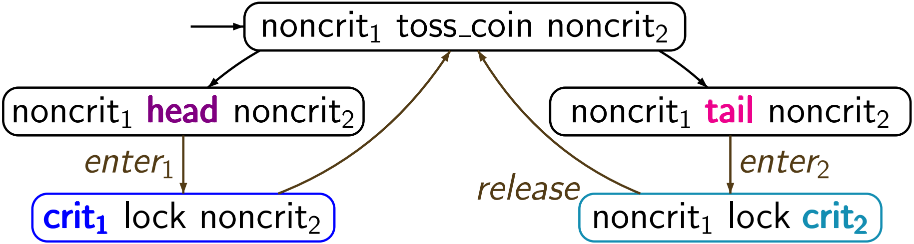
$(\mathcal{T}_1 \ ||| \ \mathcal{T}_2) \parallel Arbiter$



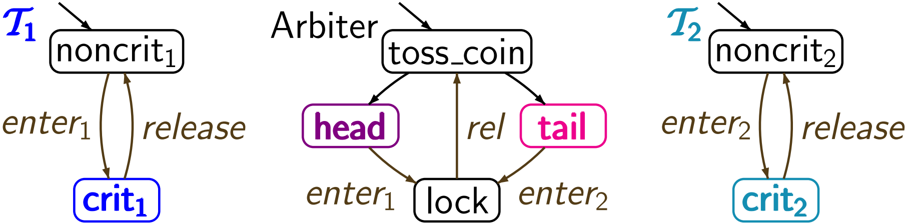
Randomized arbiter for MUTEX



$$(\mathcal{T}_1 \;|||\; \mathcal{T}_2) \,\|\, \textit{Arbiter} \not\models \Box\Diamond \text{crit}_1 \wedge \Box\Diamond \text{crit}_2$$



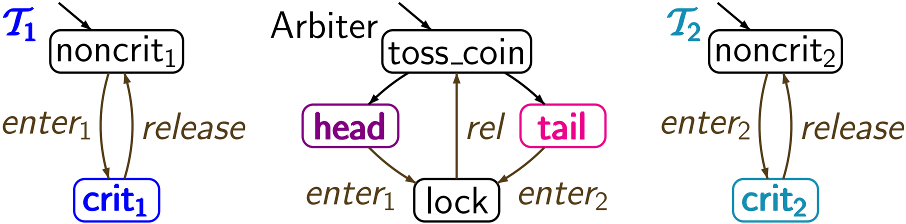
Randomized arbiter for MUTEX



unconditional LTL-fairness:

$$\mathit{fair} = \Box\Diamond\textbf{head} \wedge \Box\Diamond\textbf{tail}$$

Randomized arbiter for MUTEX



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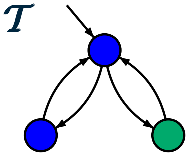
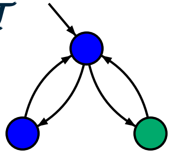


LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

$\mathcal{T}$



LTL fairness assumption

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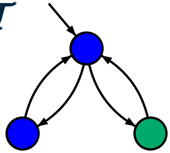
● $\widehat{=} \{a\}$ ● $\widehat{=} \{b\}$

$\mathcal{T} \models_{fair} \bigcirc b$ **?**

Correct or wrong?

$\mathcal{T}$



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$\bullet \mathrel{\widehat{=}} \{a\}$ $\bullet \mathrel{\widehat{=}} \{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as $\bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \bullet \rightarrow \ldots$ is fair

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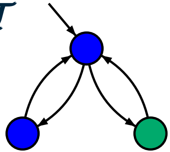

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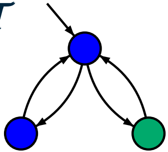
$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ **?**

Correct or wrong?

$\mathcal{T}$



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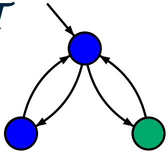
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as ●→●→●→●→●→●→ ... is fair

$\mathcal{T} \models_{fair} a \, \mathsf{U} \, b$ $\surd$

$\mathcal{T}$



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● $\widehat{=}\{a\}$ ● $\widehat{=}\{b\}$

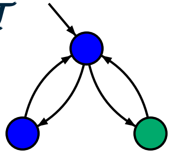
$\mathcal{T} \not\models_{fair} \bigcirc b$ as ●→●→●→●→●→●→ . . . is fair

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ $\checkmark$

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$?

Correct or wrong?

$\mathcal{T}$



LTL fairness assumption
$fair = \Diamond\Box a \rightarrow \Box\Diamond b$

$\textcolor{blue}{\bullet} \,\widehat{=}\, \{a\}$ $\quad \textcolor{green}{\bullet} \,\widehat{=}\, \{b\}$

$\mathcal{T} \not\models_{fair} \bigcirc b$ as $\textcolor{blue}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{green}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{green}{\bullet} \to \ldots$ is fair

$\mathcal{T} \models_{fair} a \,\mathsf{U}\, b$ $\checkmark$

$\mathcal{T} \not\models_{fair} a \,\mathsf{U}\, \Box(b \leftrightarrow \bigcirc a)$

as $\textcolor{blue}{\bullet} \to \textcolor{green}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{blue}{\bullet} \to \textcolor{green}{\bullet} \to \ldots$ is fair

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g., mutual exclusion with arbiter/semaphore

$$\mathcal{T}_{sem} \not\models \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$
$$\mathcal{T}_{sem} \models_{fair} \Box\Diamond crit_1 \wedge \Box\Diamond crit_2$$

for appropriate fairness condition

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for appropriate fairness condition, e.g.,

$$fair = \bigwedge_{i=1,2} \big((\Box\Diamond wait_i \rightarrow \Box\Diamond crit_i) \wedge (\Diamond\Box noncrit_i \rightarrow \Box\Diamond wait_i) \big)$$

LTL-fairness assumptions

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for appropriate fairness condition

- can be verifiable system properties

e.g., Peterson algorithm guarantees strong fairness

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

LTL-fairness assumptions

- can be necessary to prove liveness properties, e.g.,

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  for appropriate fairness condition

- can be verifiable system properties, e.g.,

$$\mathcal{T}_{Pet} \models \Box\Diamond wait_1 \rightarrow \Box\Diamond crit_1$$

- are irrelevant for verifying safety properties

$$\mathcal{T} \models \varphi_{safe} \quad \text{iff} \quad \mathcal{T} \models_{fair} \varphi_{safe}$$

if *fair* is realizable

Correct or wrong?

Each strong **LTL** fairness assumption

$$fair \;=\; \Box\Diamond a \rightarrow \Box\Diamond b$$

is realizable for each TS over $AP = \{a, b, \ldots\}$.

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recall: a fairness condition is called realizable if for each reachable state s there exists a fair path starting in s

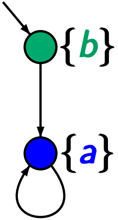
Correct or wrong?

Each strong **LTL** fairness assumption

$$fair \;=\; \Box\Diamond a \rightarrow \Box\Diamond b$$

is realizable for each TS over $AP = \{a, b, \ldots\}$.

wrong



$fair = \Box\Diamond a \rightarrow \Box\Diamond b$

is not realizable

Action-based fairness $\rightsquigarrow$ LTL-fairness

Action-based fairness $\rightsquigarrow$ LTL-fairness

idea: use new atomic propositions ***enabled*(A)** and ***taken*(A)** and extend the labeling function:

$$\mathit{enabled}(A) \in L(s) \quad \text{iff} \quad s \xrightarrow{\alpha} \ldots \text{ for some } \alpha \in A$$

$$\mathit{taken}(A) \in L(s) \quad \text{iff} \quad \text{for all transitions } \ldots \xrightarrow{\alpha} s\text{: } \alpha \in A$$

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- unconditional A-fairness: $\Box\Diamond \mathit{taken}(A)$
- strong A-fairness: $\Box\Diamond \mathit{enabled}(A) \rightarrow \Box\Diamond \mathit{taken}(A)$
- weak A-fairness: $\Diamond\Box \mathit{enabled}(A) \rightarrow \Box\Diamond \mathit{taken}(A)$

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problem: each state s can have several incoming transitions

$$t \xrightarrow{\alpha} s, \quad u \xrightarrow{\beta} s, \; \ldots$$

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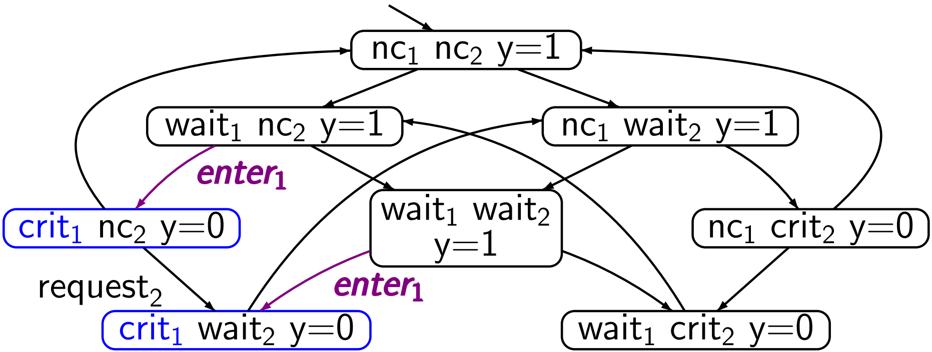
alternative 1: ad-hoc choice of "***taken***-predicate"

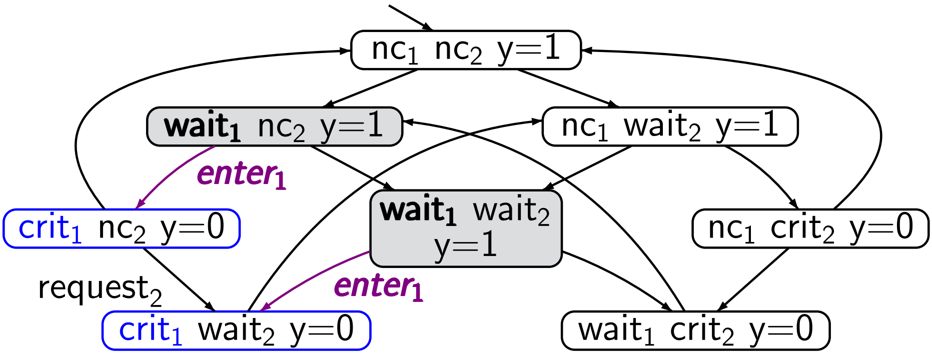
alternative 2: modify the given transition system by adding an action component to the states

Ad-hoc: action fairness ⇝ LTL-fairness



TS for mutual exclusion with semaphore

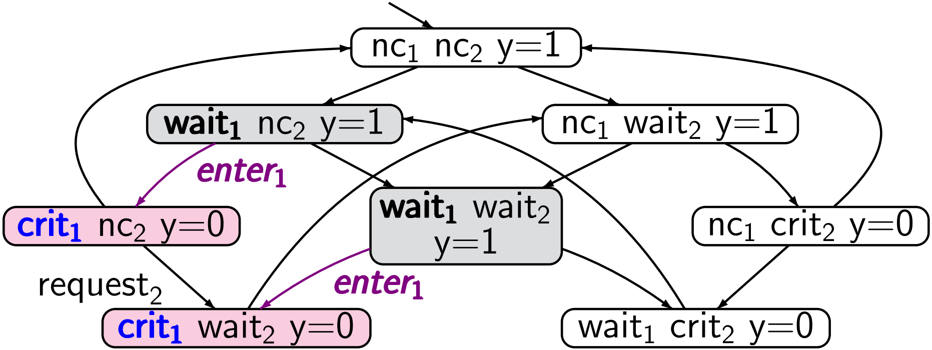
Ad-hoc: action fairness ⇝ LTL-fairness



$\square \;\widehat{=}\; \textit{enabled}(\{\textit{enter}_1\})$

TS for mutual exclusion with semaphore

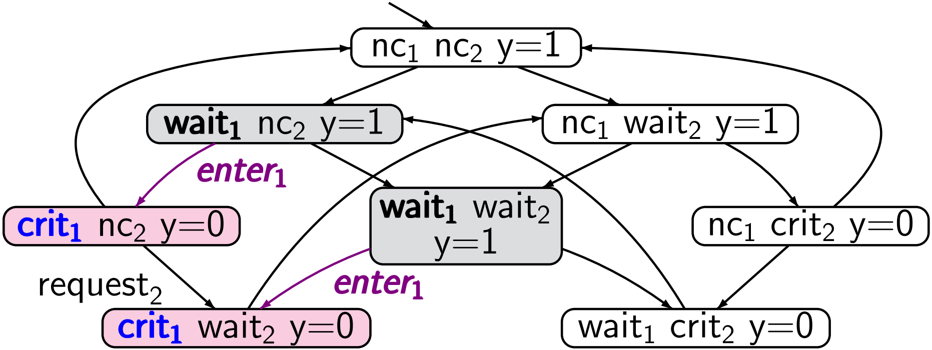
Ad-hoc: action fairness ⇝ LTL-fairness



$\widehat{=}$ *taken*({*enter*$_1$})

$\widehat{=}$ *enabled*({*enter*$_1$})

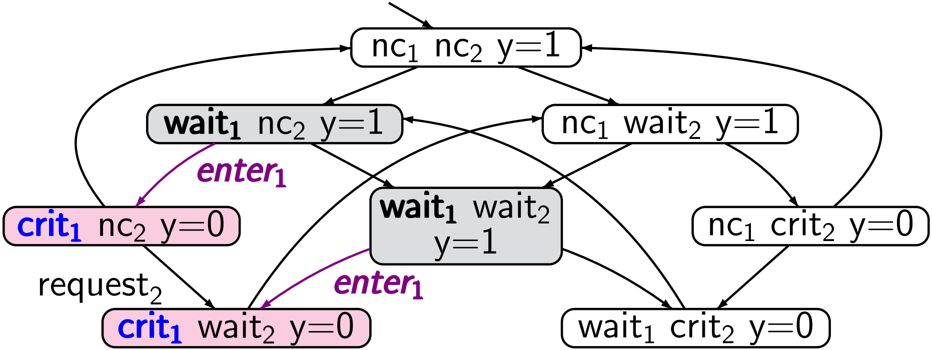
TS for mutual exclusion with semaphore



$\square$ (pink) $\widehat{=}$ $taken(\{enter_1\})$ $\widehat{=}$ $\mathbf{crit_1}$

$\square$ (grey) $\widehat{=}$ $enabled(\{enter_1\})$ $\widehat{=}$ $\mathbf{wait_1} \wedge \neg \mathbf{crit_2}$

Ad-hoc: action fairness $\rightsquigarrow$ LTL-fairness



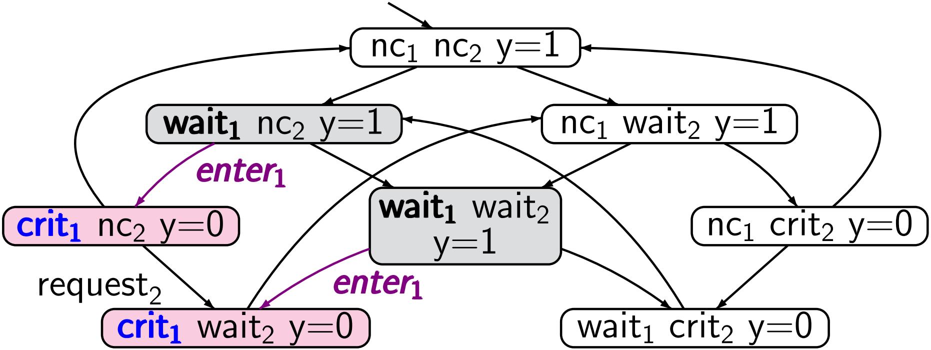
$\widehat{=}$ *taken*({*enter*$_1$}) $\widehat{=}$ **crit$_1$**

$\widehat{=}$ *enabled*({*enter*$_1$}) $\widehat{=}$ **wait$_1$** $\wedge$ $\neg$**crit$_2$**

strong {*enter*$_1$}-fairness: LTL formula

$$\Box\Diamond\, enabled(\{enter_1\}) \rightarrow \Box\Diamond\, taken(\{enter_1\})$$

Ad-hoc: action fairness $\rightsquigarrow$ LTL-fairness



$\widehat{=}$ *taken*($\{$*enter*$_1\}$) $\widehat{=}$ $crit_1$

$\widehat{=}$ *enabled*($\{$*enter*$_1\}$) $\widehat{=}$ $wait_1 \wedge \neg crit_2$

$$\Box\Diamond\, \mathit{enabled}(\{\mathit{enter}_1\}) \;\rightarrow\; \Box\Diamond\, \mathit{taken}(\{\mathit{enter}_1\})$$

$$\widehat{=}\;\; \Box\Diamond\, (wait_1 \wedge \neg crit_2) \;\rightarrow\; \Box\Diamond\, crit_1$$

idea: use new atomic propositions ***enabled***(A) and ***taken***(A) and extend the labeling function:

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Action-based fairness $\rightsquigarrow$ LTL-fairness

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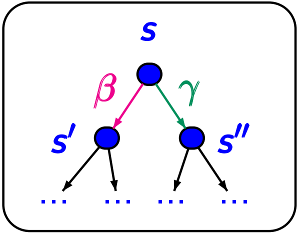
Action-based fairness $\rightsquigarrow$ LTL-fairness

transition system

$\mathcal{T} = (S, Act, \to, \ldots)$



Action-based fairness $\rightsquigarrow$ LTL-fairness

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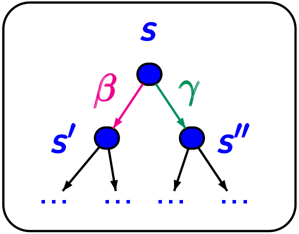
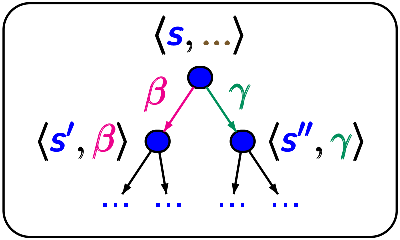


transition system $\mathcal{T}' = (S \times Act, \ldots, AP', L')$



Action-based fairness $\rightsquigarrow$ LTL-fairness

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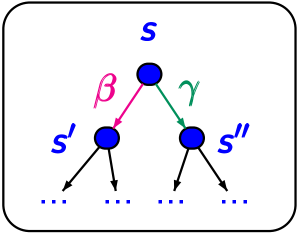

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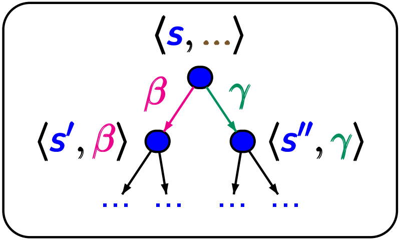

strong A-fairness
for $A \subseteq Act$

strong **LTL**-fairness
$\Box\Diamond enabled(A) \rightarrow \Box\Diamond taken(A)$

Action-based fairness $\rightsquigarrow$ LTL-fairness

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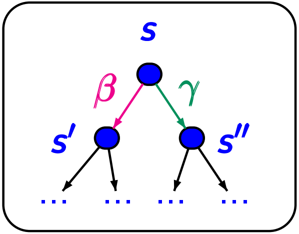

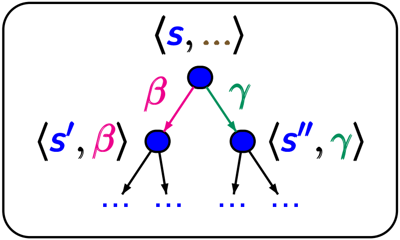

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strong **LTL**-fairness
$\Box\Diamond enabled(A) \to \Box\Diamond taken(A)$

$$enabled(A) \in L'(\langle s, \alpha \rangle) \quad \text{iff} \quad s \xrightarrow{\beta} \ldots \text{ for some } \beta \in A$$

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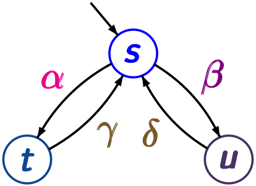
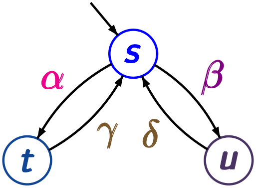
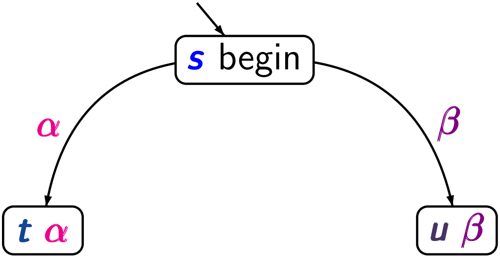
action-based fairness ⇝ LTL-fairness



Example: action fairness $\rightsquigarrow$ LTL-fairness

action-based fairness $\rightsquigarrow$ LTL-fairness





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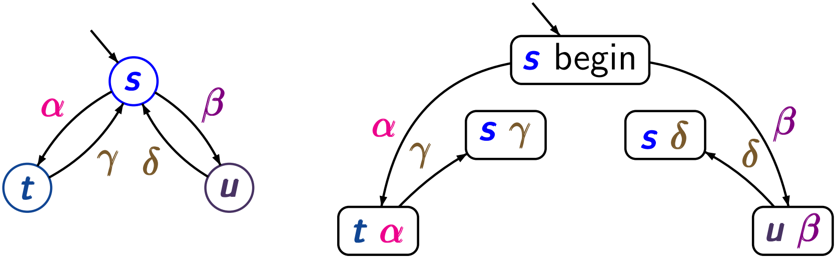

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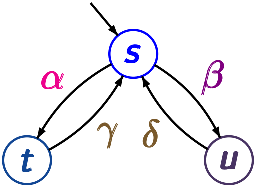

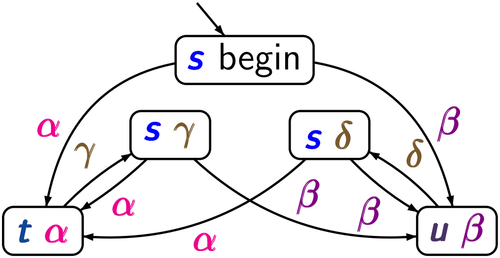

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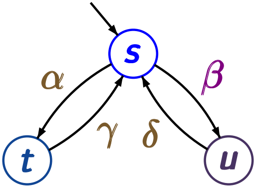
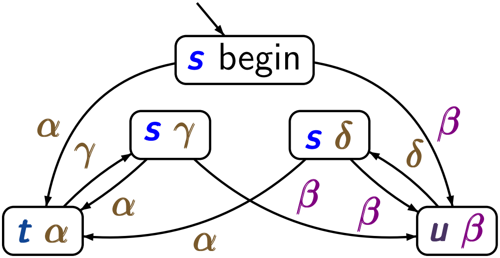
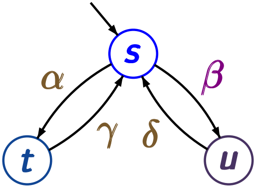


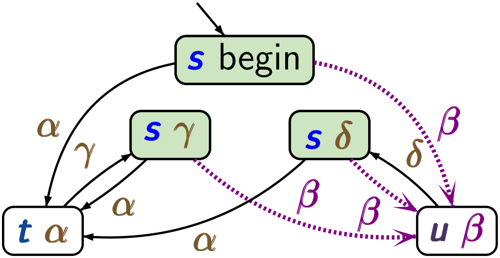


strong fairness for $\{\beta\}$:

$$\Box\Diamond\, \mathit{enabled}(\beta) \rightarrow \Box\Diamond\, \mathit{taken}(\beta)$$

action-based fairness $\rightsquigarrow$ LTL-fairness





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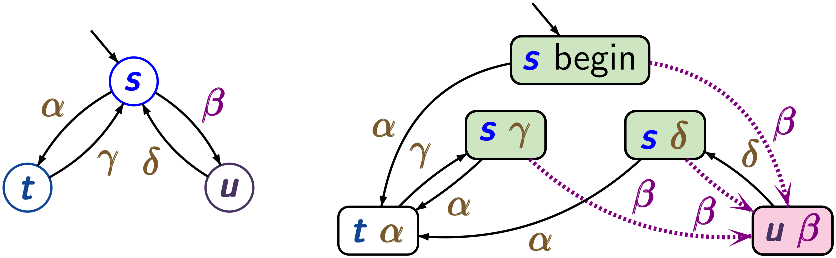
action-based fairness $\rightsquigarrow$ LTL-fairness



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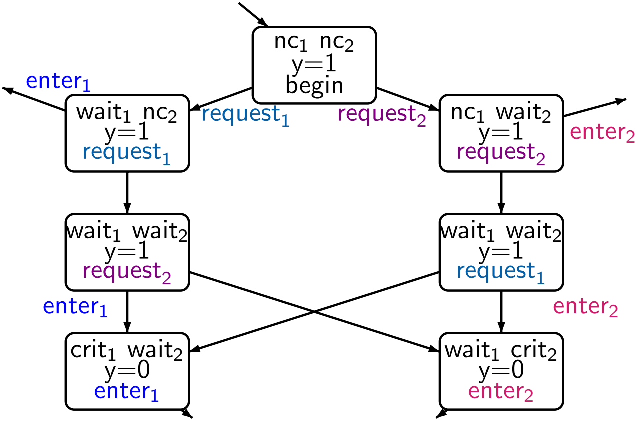
Example: mutual exclusion with semaphore

add additional variable last_action with domain Act ∪ {begin}



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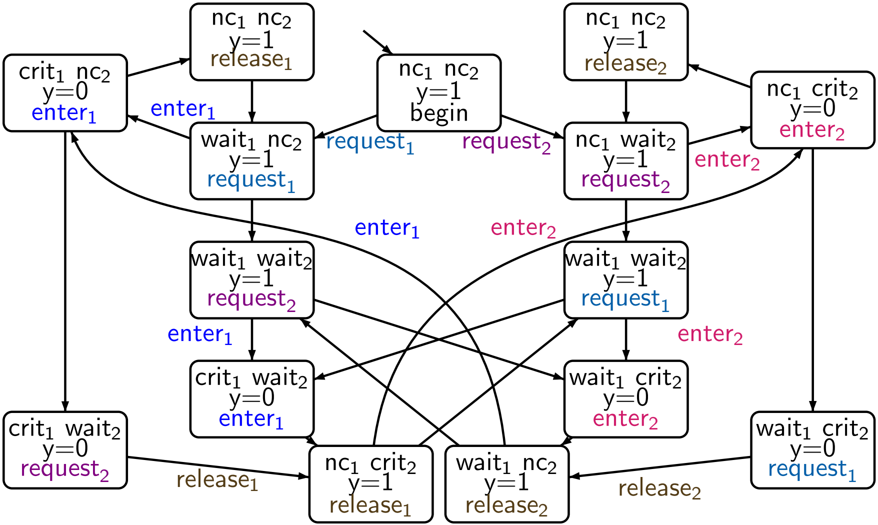

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