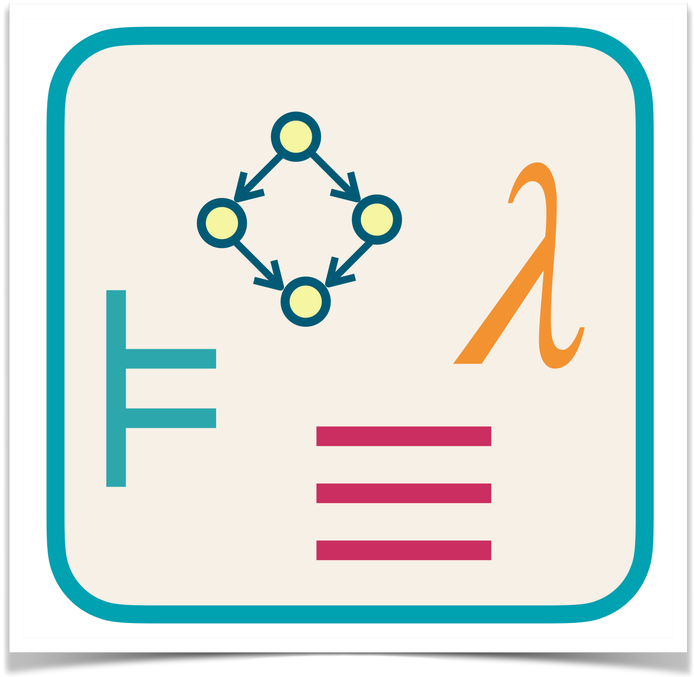


LPL 2026/27 (063AA, 9CFU)

Programming Languages

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and Laboratory

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03e - Exercises on HOFL

[**Ex. 1**] Determinare il tipo del termine HOFL

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ x.\ ((\lambda y.\ \mathbf{if}\ y\ \mathbf{then}\ 0\ \mathbf{else}\ 0)\ x).$$

Poi calcolare la sua forma canonica (lazy).

[Ex. 2] Determinare il tipo del termine

$$map \stackrel{\text{def}}{=} \lambda f.\ \lambda x.\ ((f\ \mathbf{fst}(x)), (f\ \mathbf{snd}(x)))$$

Poi calcolare le forme canoniche (lazy) dei termini seguenti.

$t_1 \stackrel{\text{def}}{=} map\ (\lambda z.\ 2 \times z)\ (1, 2)$ $\qquad$ $t_2 \stackrel{\text{def}}{=} \mathbf{fst}\ (\ map\ (\lambda z.\ 2 \times z)\ (1, 2)\ )$

[Ex. 3] Let $(D, \sqsubseteq_D)$ be a CPO and $f : D \to D$ be a continuous function. Prove that the set of fixpoints of f is itself a CPO (ordered by $\sqsubseteq_D$).

[**Ex. 4**] (Test for convergence) We would like to modify the denotational semantics of HOFL assigning to the construct

$$\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1$$

- the semantics of t_1 if the semantics of t is $\perp_{\mathbb{Z}_\perp}$, and
- the semantics of t_0 otherwise.

Is it possible? If not, why?

[**Ex. 5**] (Strict conditional) Modify the operational semantics of HOFL by taking the following rules for conditionals:

$$\frac{t \to 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \;\to\; c_0} \qquad \frac{t \to n \quad n \neq 0 \quad t_0 \to c_0 \quad t_1 \to c_1}{\textbf{if } t \textbf{ then } t_0 \textbf{ else } t_1 \;\to\; c_1}.$$

Without changing the denotational semantics, prove that:

1. for any term t and canonical form c, we have $t \to c \;\Rightarrow\; \forall \rho.\ [\![t]\!]\,\rho = [\![c]\!]\,\rho$;
2. in general $t{\Downarrow} \not\Rightarrow t{\downarrow}$ (exhibit a counterexample).

[**Ex. 6**] Determine the type of the HOFL term

$$t \stackrel{\text{def}}{=} \mathbf{rec}\ f.\ (\ \lambda x.1\ ,\ \mathbf{fst}(f)\ 0\ )$$

Then, compute the (lazy) denotational semantics of t.