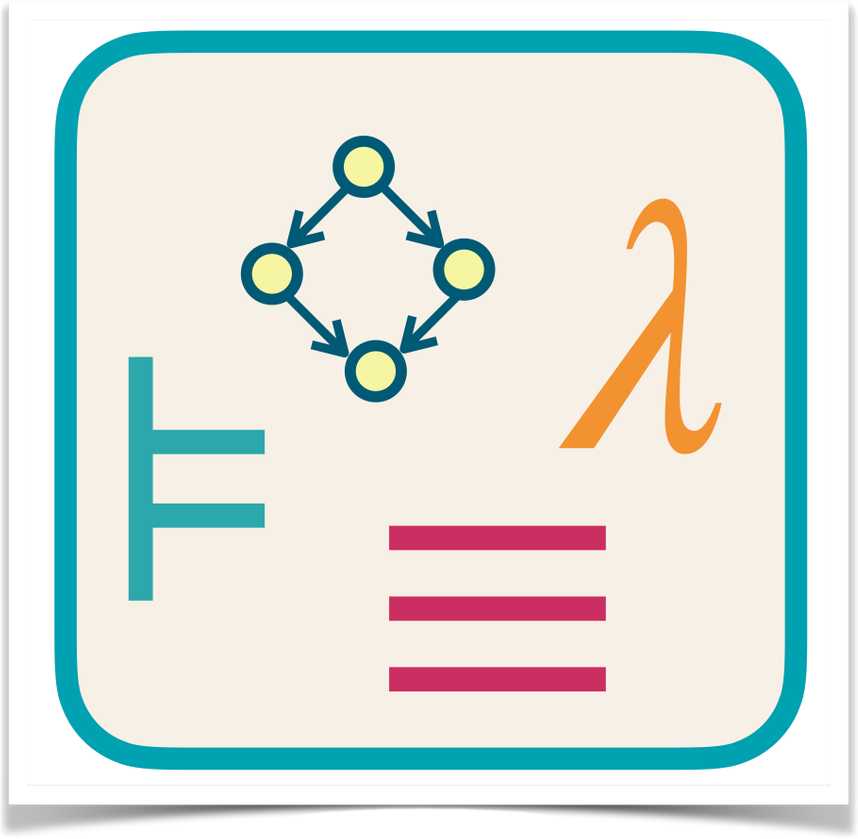

LPP 2026/27 (063AA, 9CFU)

Programming Languages

Fabio Gadducci

and Laboratory

Vincenzo Ciancia

01 - Introduction

Slides by Roberto Bruni

Some quotes

Some quotes

Computer science is no more about computers than astronomy is about telescopes

- Edsger W. Dijkstra [maybe]

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- from the web

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Subjects are divided in two categories:

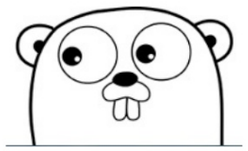
1) too difficult matters, that CANNOT be studied

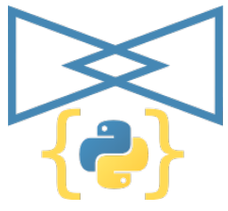
2) easy matters, that DO NOT NEED to be studied

- back of a t-shirt

Objectives

Programming paradigms
(imperative, declarative,
higher order, concurrent,
mobile, stochastic)





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SWI Prolog

Google Go!

ERLANG

Mathematical frameworks (concrete & abstract)
(domains, inference rules, transition systems, λ-calculus, process algebras)

Objectives

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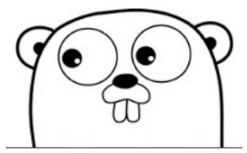

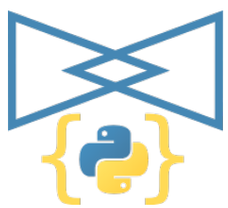

SWI Prolog

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Understand
(recursion, semantics, compositionality)

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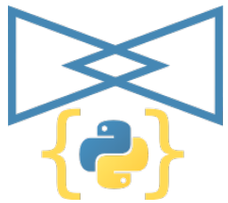

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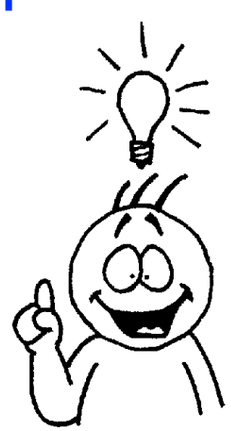

Reason
(induction, modal and temporal logics, behavioural and logical equivalences)

Objectives

Programming paradigms
(imperative, declarative, higher order, concurrent, mobile, stochastic)

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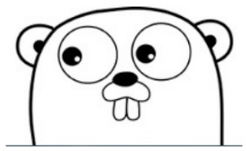

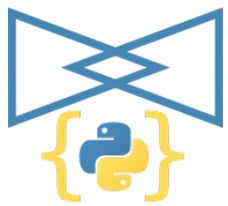

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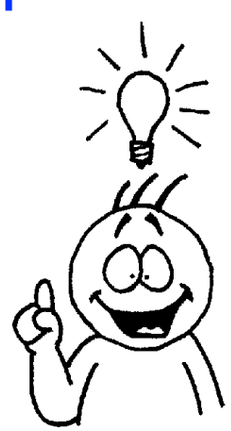
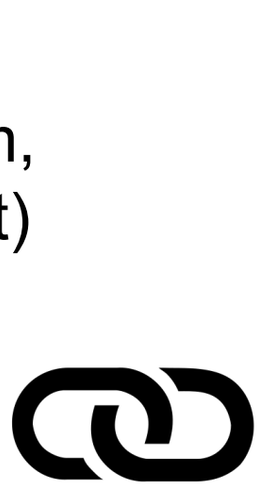


Explain
(correctness, compliance, performance)

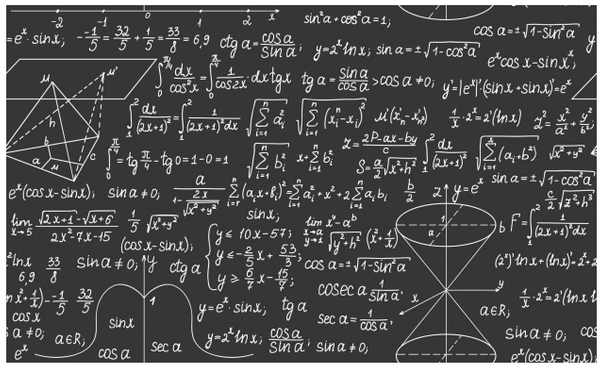
The approach

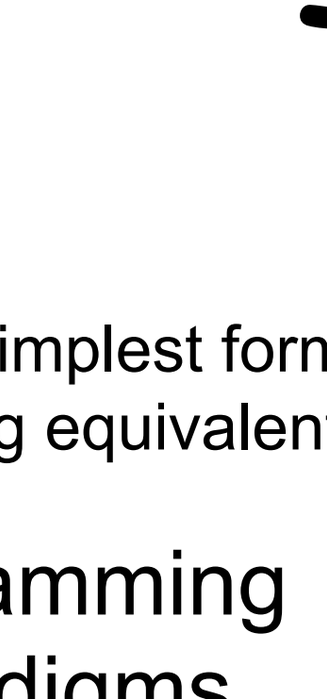
(in their simplest form,
still Turing equivalent)

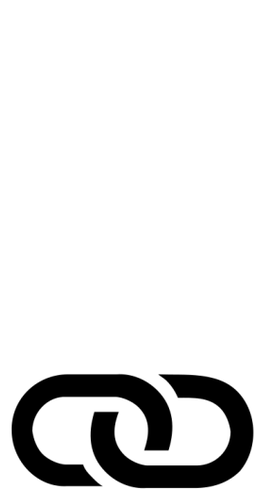
programming paradigms



mathematical frameworks

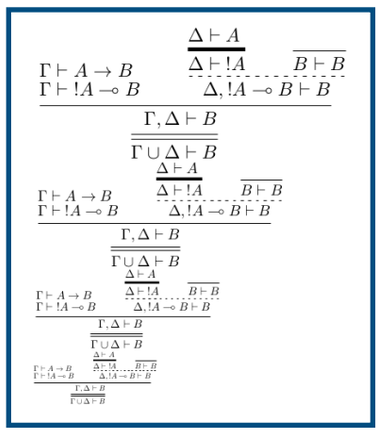






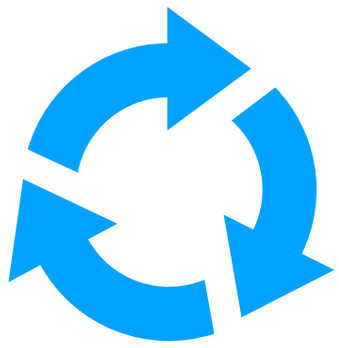
meta-properties
+
proof techniques

(for all programs
or just
some classes of programs)



models

programs



specifications

The approach

(in their simplest form,
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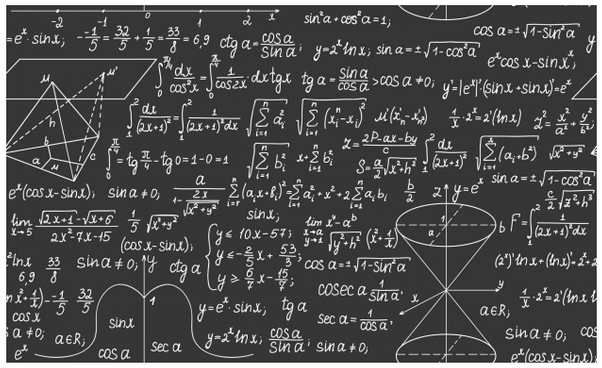
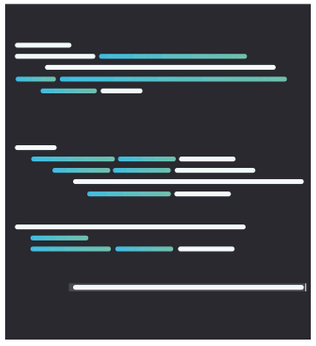
programming paradigms



mathematical frameworks



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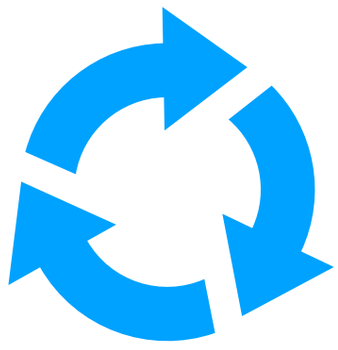

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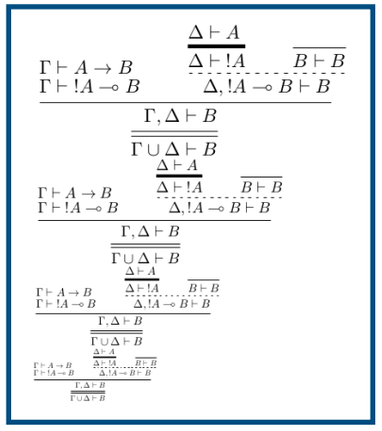


Key question

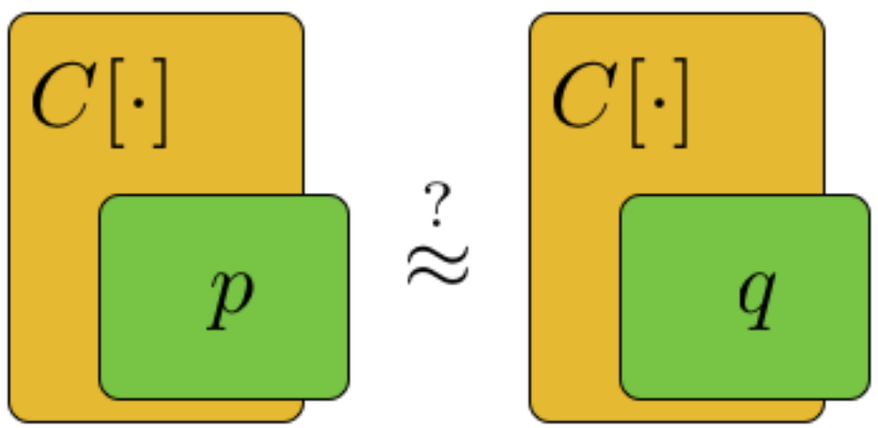
Given two programs *p* and *q*:

Do they behave the same?

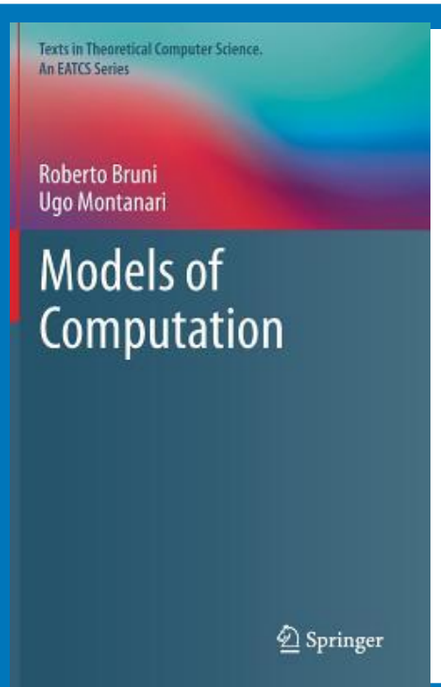
are they "equivalent"?

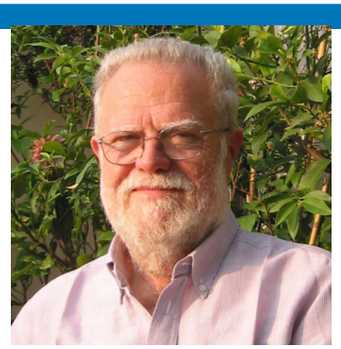
Is it safe to replace one with the other in any context?

are they "congruent"?



Textbooks





Roberto Bruni and Ugo Montanari
Models of Computation
Texts in Theoretical Computer Science (an EATCS series)
https://www.springer.com/book/9783319428987

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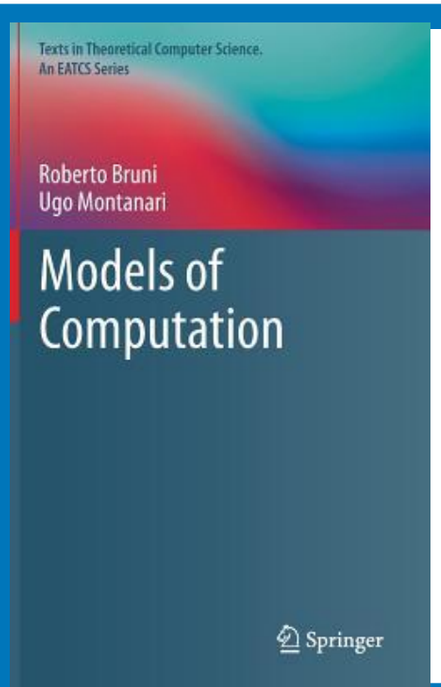

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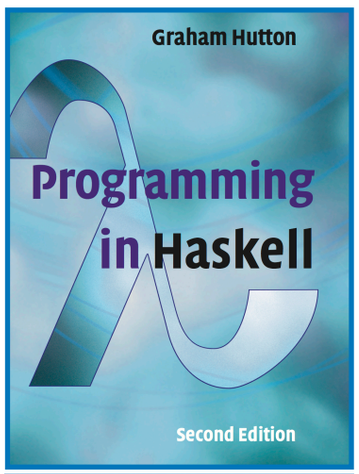

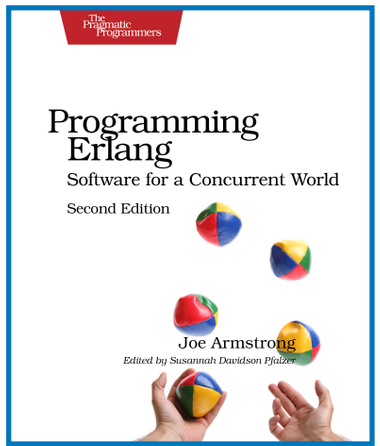

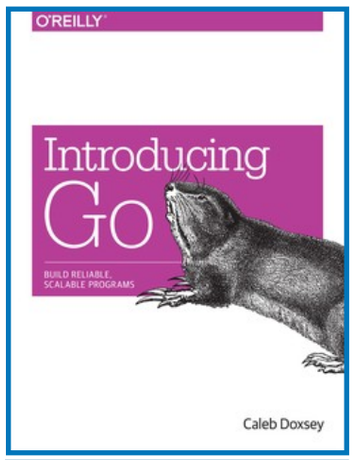

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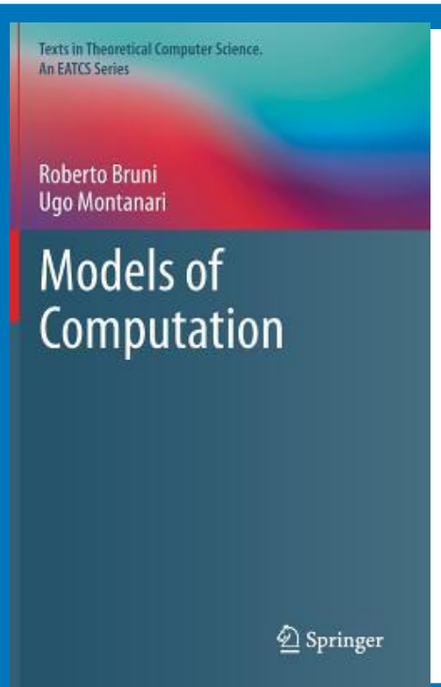

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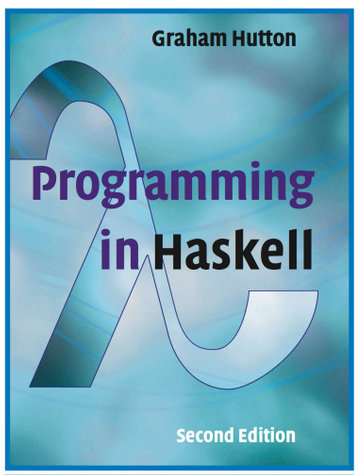

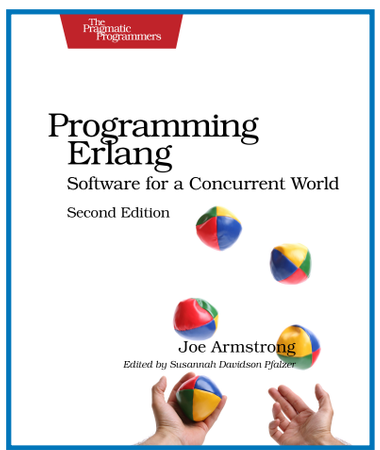

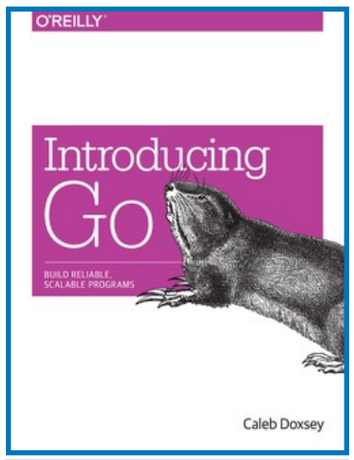

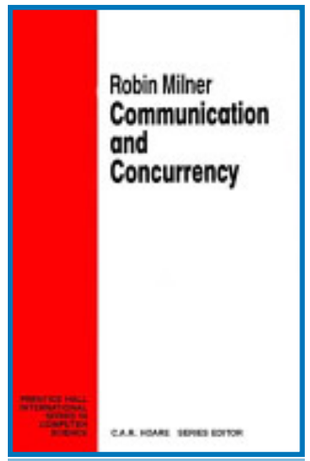

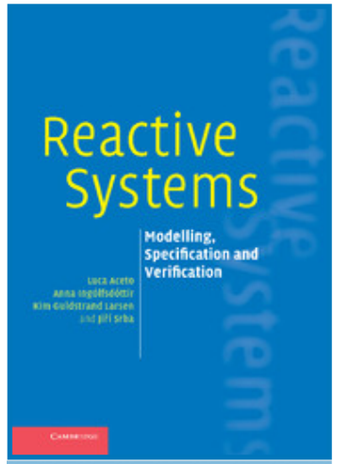

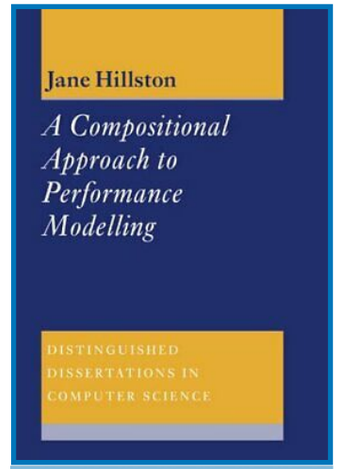

Course activities

attend classrooms:
ask questions!

Course activities

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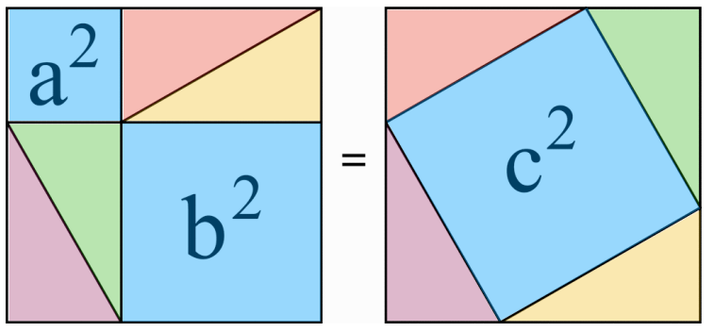
learn theorems



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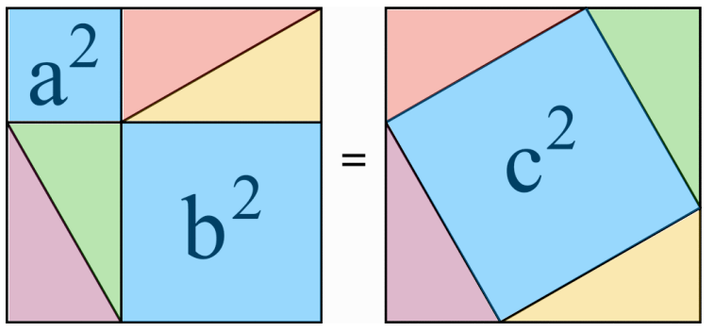

solve ALL your
homeworks
(at least try to)

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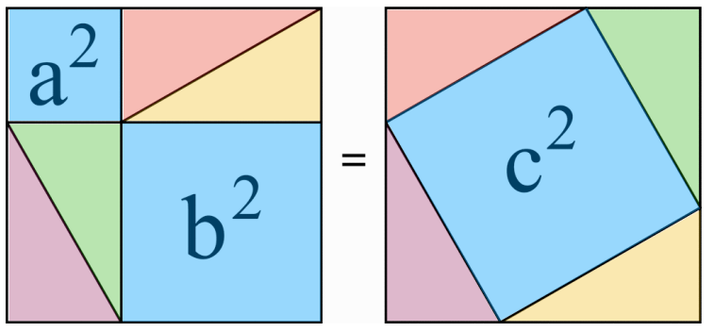

solve ALL your
homeworks
(at least try to)

give the exam

Be proactive!

Let's spell out definitions together

```
% find the least (non-unitary) divisor d of n>1
% example: for n=21 the result is d=3
d := 0;
x := 2;
while (.....................................................) do {
    if ( n%x == 0 ) then {
        d := x;
    } else {
        x := x + 1;
    }
}
```

Be proactive!

Correct me if I'm wrong

```
% find the index i of the last occurrence of n in a
% example: for n=5, a=[3,5,7,5,9] the result is i=3
i := length(a)-1;
while ( i>0 && n!=a[i] ) do {
    i := i-1;
}
```

Be proactive!

Sometimes tricky questions!

Can you find the the mistake?

1 2 3 4 5 6 7 8 9

Interaction protocol

when I will ask questions such as:

"does the program c satisfy the property p?"

there are the 3 possible answers

- ⭕ yes
- ⭕ no
- ⭕ don't know

you are welcome to take your time... but then
please pick one option whenever I ask questions in these classes

Prerequisites

Basic set theory

$\varnothing$ $A \cap B$ $A \cup B$ $A \setminus B$ $\overline{A}$

$a \in A$ $A \subset B$ $A \subseteq B$ $A \times B$

$a \notin A$ $A \not\subseteq B$ $A \cap B = \emptyset$

Prerequisites

Basic set theory

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$\mathbb{N}$ $\mathbb{Z}$ $\mathbb{Q}$ $\mathbb{R}$ $\mathbb{B}$

$N \subseteq \mathbb{N}$ $N \in \wp(\mathbb{N})$ $S \subseteq \wp(\mathbb{N})$

Prerequisites

Basic set theory: functions, relations

$f : A \to B$

$R \subseteq A \times B$

Prerequisites

Basic set theory: functions, relations

$f : A \to B$

$R \subset A \times B$

sets as functions
(characteristic function)

$f_N : \mathbb{N} \to \mathbb{B}$

$$f_N(n) \triangleq \begin{cases} 1 & n \in N \\ 0 & \text{otherwise} \end{cases}$$

$$N = \{n \mid f_N(n) = 1\}$$

Prerequisites

Basic set theory: functions, relations

$f : A \to B$

$R \subset A \times B$

functions as relations

$R_f \triangleq \{(a, f(a)) \mid a \in A\}$

sets as functions
(characteristic function)

$f_N : \mathbb{N} \to \mathbb{B}$

$$f_N(n) \triangleq \begin{cases} 1 & n \in N \\ 0 & \text{otherwise} \end{cases}$$

$$N = \{n \mid f_N(n) = 1\}$$

Prerequisites

First order logic

ff false 0 F tt true 1 T $P \wedge Q$ $P \vee Q$ $\neg P$

$\exists x.\ P(x)$ $\forall x.\ P(x)$ $P \Rightarrow Q$ $P \Leftrightarrow Q$

Prerequisites

First order logic

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meaning of implication!

$$P \Rightarrow Q$$

$$Q \vee \neg P$$

$$\neg Q \Rightarrow \neg P$$

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meaning of implication!

$$P \Rightarrow Q$$

$$Q \vee \neg P$$

$$\neg Q \Rightarrow \neg P$$

order of quantifiers matters!

$$\forall n \in \mathbb{N}.\ \exists m \in \mathbb{N}.\ n < m$$

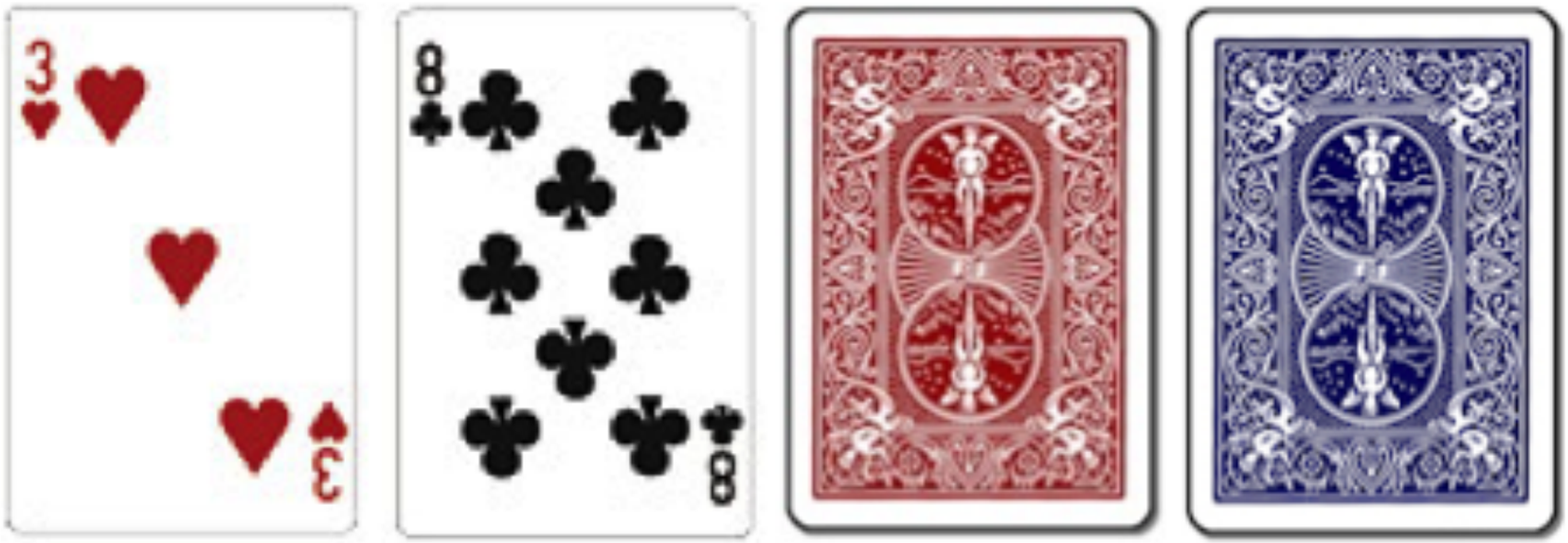
$$\exists m \in \mathbb{N}.\ \forall n \in \mathbb{N}.\ n < m$$

Implication is a tricky concept

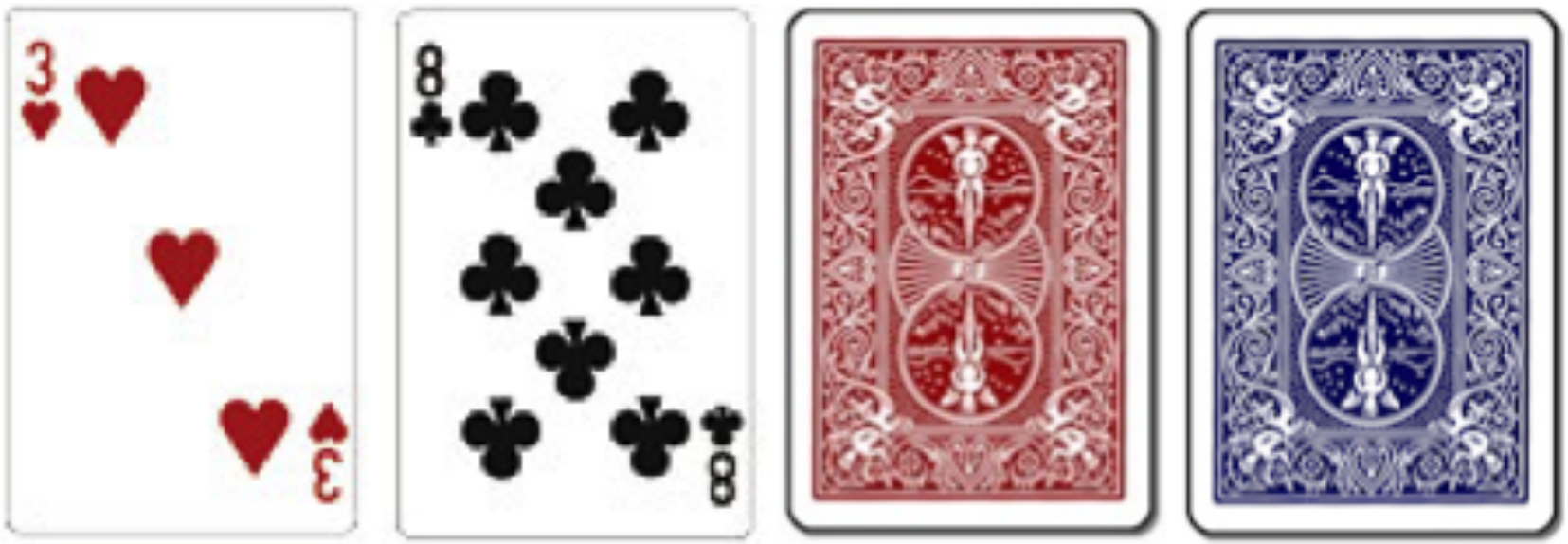
under which circumstances is the following promise broken?

"*If Rob Bery wins the election, I promise to leave the country*"

(Peter Cathart) Wason Selection Task (4 cards game)

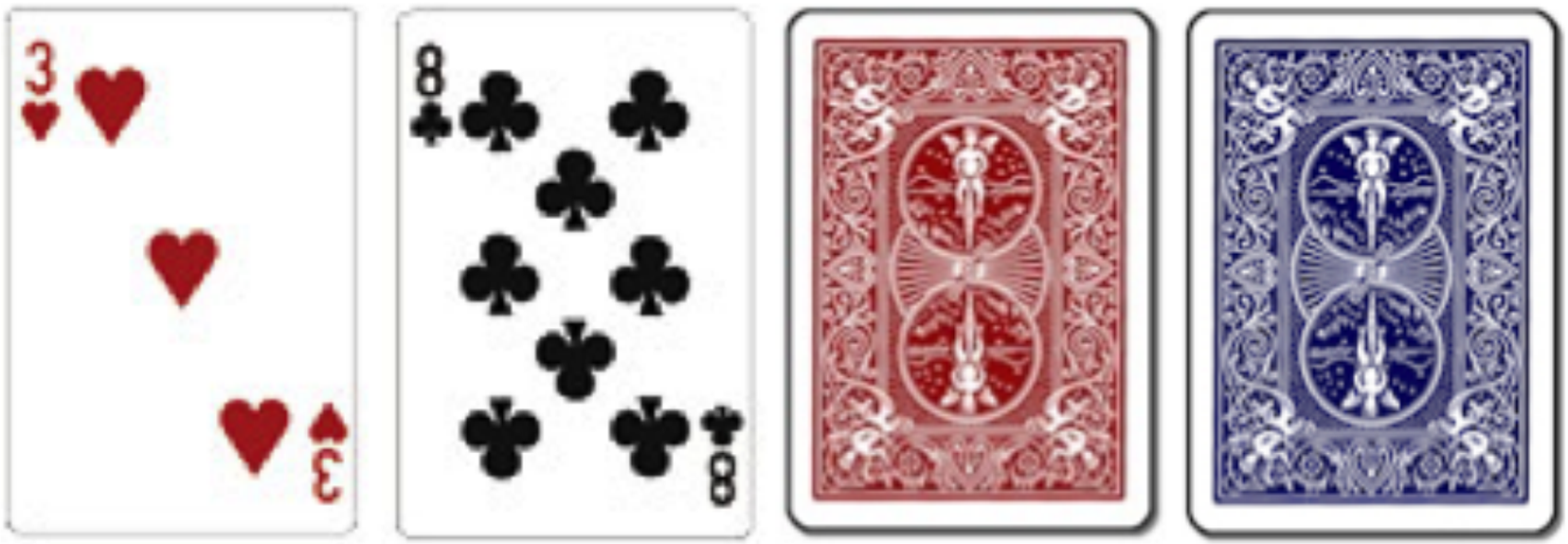


(Peter Cathart) Wason Selection Task (4 cards game)



Which cards must be turned over to make sure the following statement is true?

(Peter Cathart) Wason Selection Task (4 cards game)



1 2 3 4

Which cards must be turned over to make sure the following statement is true?

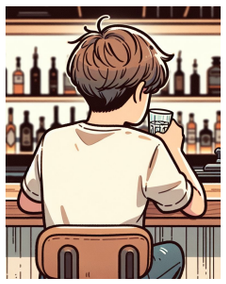
“If the front face of a card bears an even number, then its back face is red.”

(Peter Cathart) Wason Selection Task (with beers)

A policeman enters a local in Florida where a large sign reminds customers that:

“to drink beer you must be over 16 years old”

There are four customers in the local:



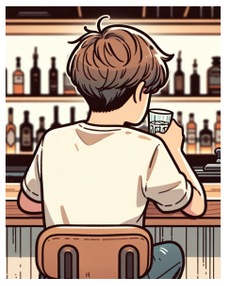
(Peter Cathart) Wason Selection Task (with beers)

A policeman enters a local in Florida where a large sign reminds customers that:

“to drink beer you must be over 16 years old”

There are four customers in the local:

1. a boy who is drinking water,



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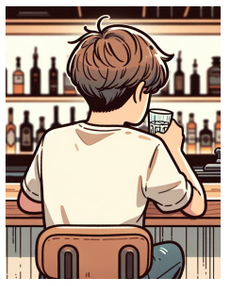
“to drink beer you must be over 16 years old”

There are four customers in the local:

1. a boy who is drinking water,

2. a girl who is drinking beer,



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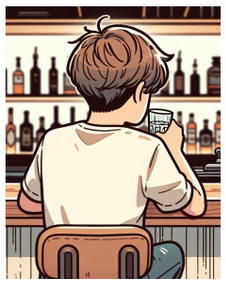

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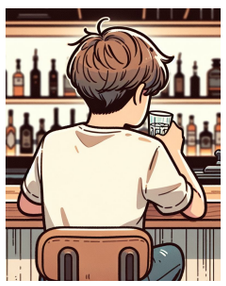

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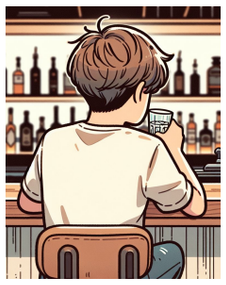
4. a 14-year-old teenager who is drinking at the counter. 

Which customers should the policeman check to verify that the rule is respected?

Prerequisites

Strings and context-free grammars

Alphabet A $\quad A^n \triangleq \underbrace{A \times \cdots \times A}_{n} \quad A^* \triangleq \bigcup_{n \in \mathbb{N}} A^n$

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Strings and context-free grammars

Alphabet A

$$A^n \triangleq \underbrace{A \times \cdots \times A}_{n} \qquad A^* \triangleq \bigcup_{n \in \mathbb{N}} A^n$$

$$\begin{aligned}
\mathbb{B} &= \{0, 1\} \\
\mathbb{B}^0 &= \{\epsilon\} \\
\mathbb{B}^1 &= \{0, 1\} \\
\mathbb{B}^2 &= \{00, 01, 10, 11\} \\
\mathbb{B}^3 &= \{000, 001, 010, 011, 100, 101, 110, 111\} \\
&\ldots \\
\mathbb{B}^* &= \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}
\end{aligned}$$

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$$\mathbb{B}^* = \{\epsilon, 0, 1, 00, 01, 10, 11, 000, \ldots\}$$

$$\mathsf{A} ::= \epsilon \mid 0\ \mathsf{A} \mid 1\ \mathsf{B}$$

$$\mathsf{B} ::= 0\ \mathsf{B} \mid 1\ \mathsf{A}$$

$$\mathsf{A} \to 0\ \mathsf{A} \to 0\ 1\ \mathsf{B} \to 0\ 1\ 1\ \mathsf{A} \to 0\ 1\ 1\ \epsilon = 0\ 1\ 1$$

$\mathcal{L}(\mathsf{A}) = ?$ $\qquad$ $\mathcal{L}(\mathsf{B}) = ?$

Prerequisites

Inductive and recursive definitions

$$
\begin{aligned}
0! &\triangleq 1 \\
(n+1)! &\triangleq n! \cdot (n+1)
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$$f(n) \triangleq \begin{cases} 1 & \text{if } n \leq 1 \\ f(n/2) & \text{if } n > 1 \wedge n\%2 = 0 \\ f(3n+1) & \text{otherwise} \end{cases}$$

$$f(12) = f(6) = f(3) = f(10) = f(5) = f(16) = f(8) = f(4) = f(2) = f(1) = 1$$

Prerequisites

Conjectures vs theorems

a natural number ***p*** is **prime**
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Conjectures vs theorems

a natural number ***p*** is **prime**
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n	Is n prime?	$2^n - 1$	Is $2^n - 1$ prime?
2	yes	3	yes
3	yes	7	yes
4	no: $4 = 2 \cdot 2$	15	no: $15 = 3 \cdot 5$
5	yes	31	yes
6	no: $6 = 2 \cdot 3$	63	no: $63 = 7 \cdot 9$
7	yes	127	yes
8	no: $8 = 2 \cdot 4$	255	no: $255 = 15 \cdot 17$
9	no: $9 = 3 \cdot 3$	511	no: $511 = 7 \cdot 73$
10	no: $10 = 2 \cdot 5$	1023	no: $1023 = 31 \cdot 33$

Prerequisites

Conjectures vs theorems

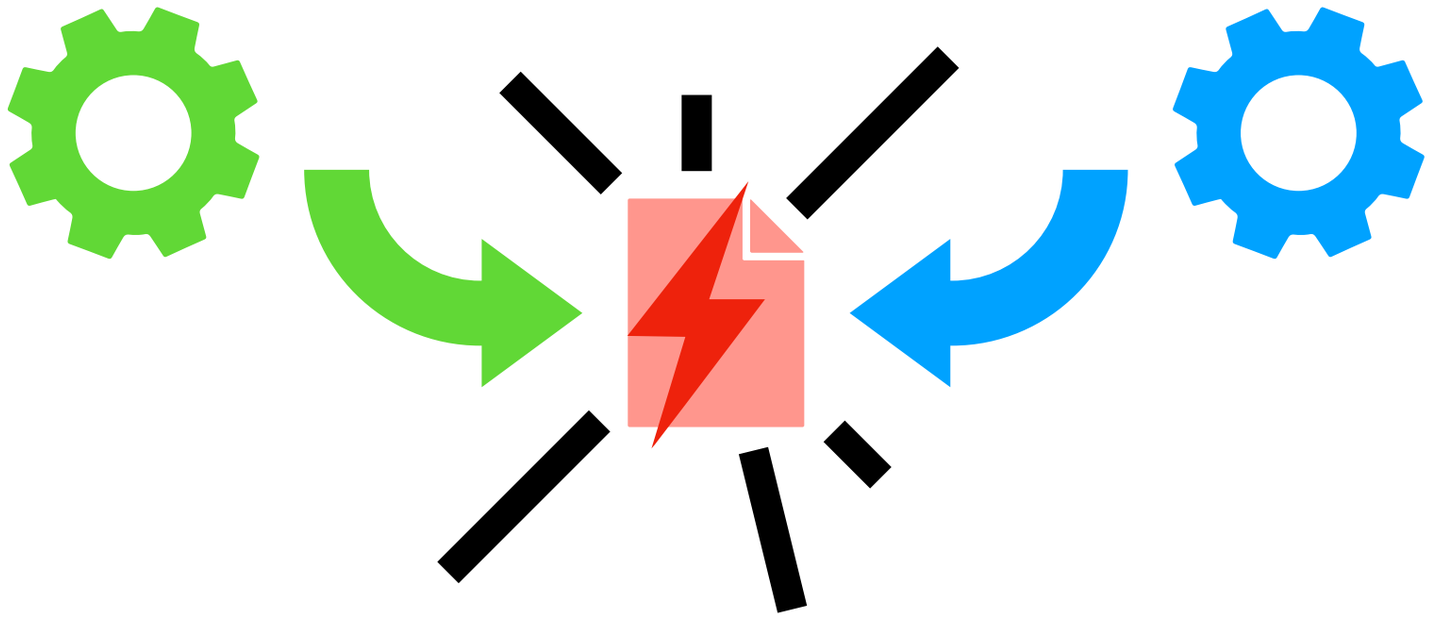
if p is prime
then $2^p - 1$ is prime

if $n > 1$ is not prime
then $2^n - 1$ is not prime

Which way to prove or disprove the above conjectures?

An Appetiser

The problem



Two concurrent processes share a single-use resource

They can communicate using shared memory

We want to guarantee that there are no conflicts when the processes access the resource

No strict alternation of naive turn taking is imposed

Peterson's mutual exclusion algorithm (1981)

```
% Two processes P1, P2
% Two boolean variables b1, b2 (both initially false)
% when Pi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Pk has priority over the other process
%
% Process P1 in pseudocode
```

```
while (true) {
    ...                          % non critical section
    b1 := true ;                 % P1 wants to enter the critical section
    k := 2 ;                     % P1 gives priority to the other process
    while (b2 && k==2) skip ;    % P1 waits its turn
    ...                          % P1 enters the critical section
    b1 := false                  % P1 leaves the critical section
}
```

```
% Process P2 is analogous to P1
```

Which question?

Does Peterson's algorithm work?

What does it mean that "it works"? What do we expect?

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No process will wait forever for the resource

(otherwise the easiest solution is no one gets in)

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If the resource is available, no process is forced to wait

(Bounded Waiting)

No process will wait forever for the resource

(otherwise the easiest solution is no one gets in)

(Mutual Exclusion)

P1 and P2 are never in the critical section at the same time

Hyman's mutual exclusion algorithm (1966)

```
% Two processes H1, H2
% Two boolean variables b1, b2 (both initially false)
% when Hi wants to enter the critical section, then it sets bi to true
% An integer variable k, taking values in {1,2}
% (initial value is arbitrary)
% the process Hk has priority over the other process
%
% Process H1 in pseudocode
```

```
while (true) {
    ...                          % non critical section
    b1 := true ;                 % H1 wants to enter the critical section
    while (k==2) {               % while H2 has priority
        while (b2) skip ;        % H1 waits
        k := 1;                  % H1 sets priority to itself
    }
    ...                          % H1 enters the critical section
    b1 := false                  % H1 leaves the critical section
}
```

```
% Process H2 is analogous to H1
```

The question

Does Peterson's algorithm satisfy mutual exclusion?

Does Hyman's algorithm satisfy mutual exclusion?

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Does Hyman’s algorithm satisfy mutual exclusion?

For the answers be patient and wait November lectures