Teoria e metodi dell'ottimizzazione

Laurea Magistrale in Matematica - a.a. 2016/17

Basic data

Title: Teoria e metodi dell'ottimizzazione
Semester II - 6 CFU (42 h)

Webpage: http://pages.di.unipi.it/bigi/dida/tmo.html

Instructor: Giancarlo Bigi (giancarlo.bigi@unipi.it)

Topics: nonlinear optimization/programming, equilibria (in finite dimension)

Prerequisites: linear algebra, basics of topology, multivariate calculus

Material: lecture notes + detailed references

Exam: oral test or seminar&report

Optimization

Nothing at all takes place in the universe in which some rule of maximum or minimum does not appear (L.Euler, 1744)

Our mathematical optimization framework

find minima and/or maxima of $f\colon \mathbb{R}^n \to \mathbb{R}$ over some $X \subseteq \mathbb{R}^n$

- $X = \mathbb{R}^n \longrightarrow$ *unconstrained optimization*
- X any (convex) subset of $\mathbb{R}^n \longrightarrow$ *constrained optimization*
- X explicitly described by algebraic inequalities and/or equalities

– f smooth and/or convex

(nonlinearities in f and/or in [the description of] X)

Optimization: a bit of history

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Optimization rooted in ancient times

problems of geometrical nature [Euclid (300bc), Apollonius (200bc) ....]

Calculus paved the way to pioneers

P.de Fermat (1629), G.W.von Liebniz (1684), I.Newton (1671)

L.Euler (1755), J.L.Lagrange (1797)

C.F.Gauss (1794), A.M.Legendre (1805), A.L.Cauchy (1847)

Insights from economics

diminishing returns [T.R.Malthus, D.Ricardo et al. (1815)] $\longrightarrow$ convexity

individual utility maximization [A.A.Cournot (1838), L.Walras (1874)] $\longrightarrow$ equilibria

First textbook in 1917 (by H.Hancock)

Springer encyclopedia in 2009: 4646 (two-column) pages!

Contents: a magic blend

Mixing (variable doses):

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Mixing (variable doses):

– Theory

- convex analysis
- optimality conditions
- duality

– Algorithms

- unconstrained optimization & nonlinear least squares
- constrained optimization
- equilibria

– Applications

a never ending list in so many areas
(physics, chemistry, statistics, engineering, transportation, computer graphics
biology, life sciences, logistics, finance, economics and social sciences, etc.)

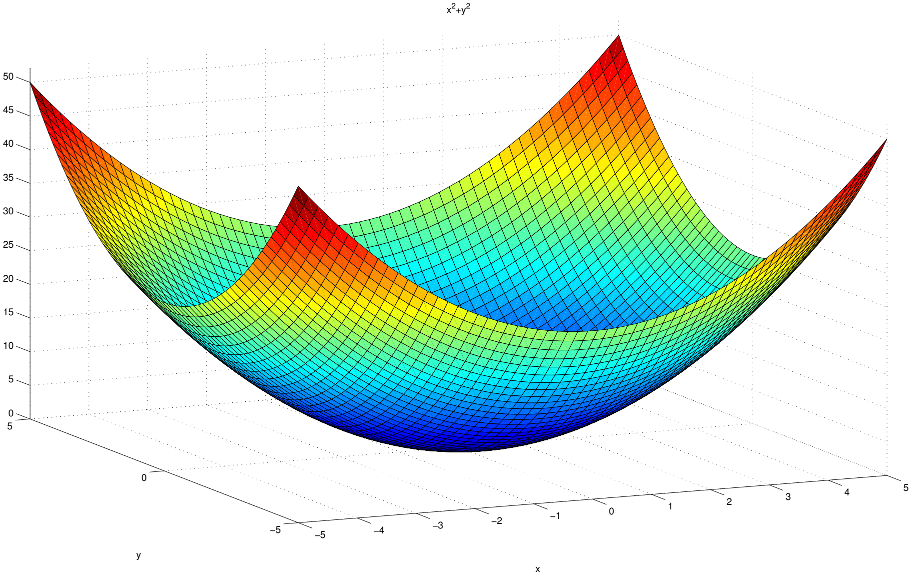
plus strong ties with numerical analysis, calculus of variations, control theory ...

The great watershed in optimization

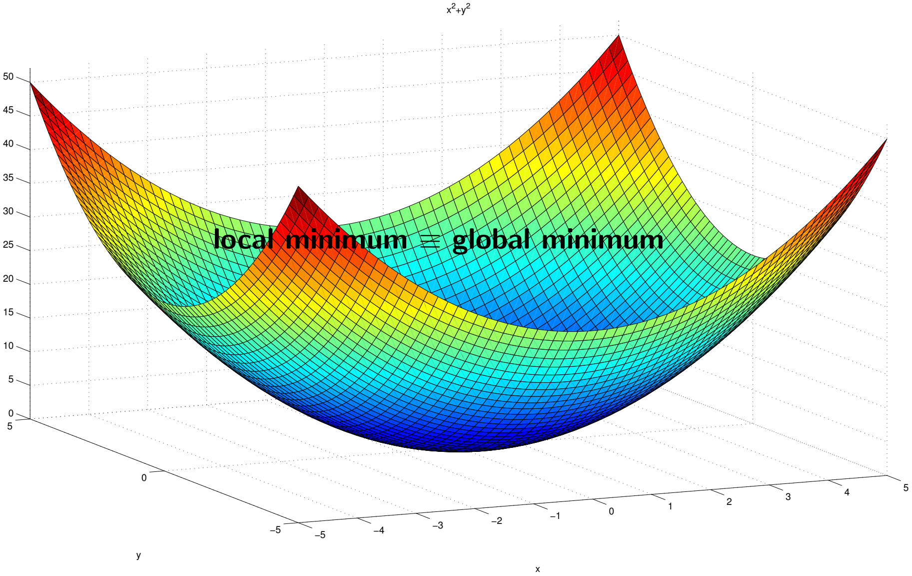
Infact the great watershed in optimization isn't between linearity and nonlinearity, but convexity and nonconvexity

(R.T.Rockafellar, SIAM Review 1993)

Convexity versus nonconvexity



Convexity versus nonconvexity



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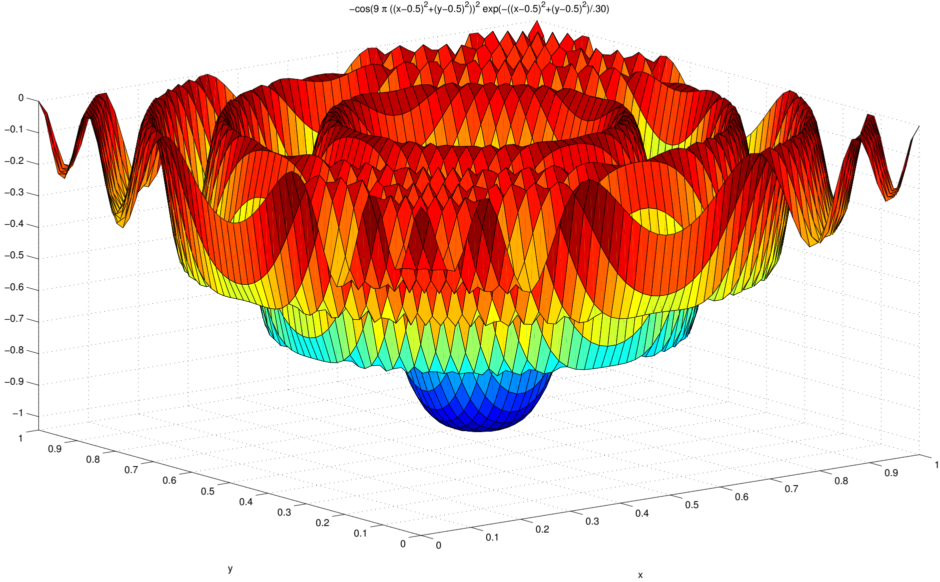

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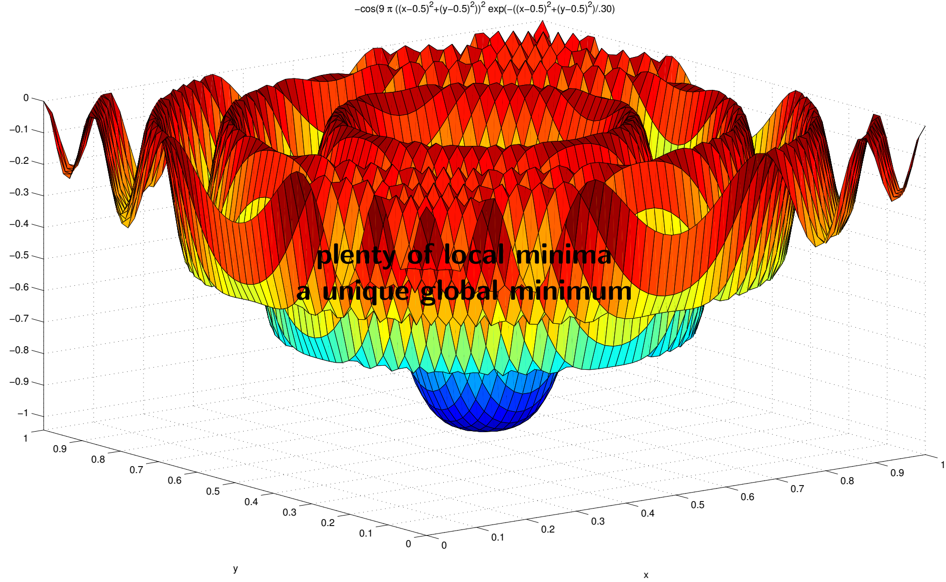

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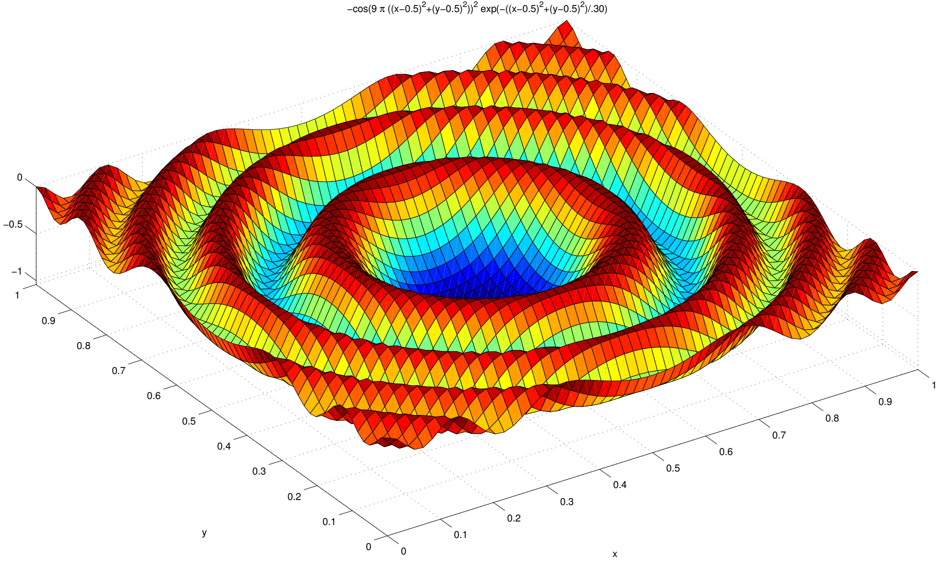


A sample of applications

- Transportation

  traffic over networks and road pricing

- Economics and finance

  oligopolistic and spatial price markets

  portfolio selection and risk management

  growth models and analysis of economic indicators

- Physics and chemistry

  configuration of molecules

  estimate of physical parameters

  laws of reflection

- Computer science

  point pattern matching

  supervised classification

- Information technology and telecommunications

  power allocation in radio systems

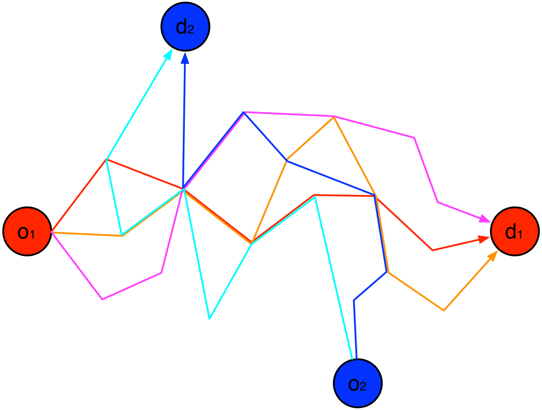
  cloud computing

Traffic over a transportation network

Predict the steady distribution of traffic over a network (N, A)

Origin–Destination pairs: $s \in OD \subseteq N \times N$ with a traffic demand $d_s > 0$

The travel time on each arc $a \in A$ depends on the total flow on a



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Optimization point of view

the management of the network is centralized

(a unique decision-maker)

$$\text{minimize} \begin{cases} \text{the total travel time} \\ \textit{or} \\ \text{the maximum travel time of } OD \text{ pairs} \\ \textit{or} \\ \dots\dots\dots \end{cases}$$

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Multi-agent point of view

no central management

each user aims at minimizing its own travel time
(the outcome for each one depends also on the choices of the others)

concepts of equilibrium come into play
[J.F.Nash (1950), G.J.Wardrop (1952)]