$$h_i(\sigma_i, \sigma_{-i}) = \sum_{x \in S} u_i(x_i, x_{-i})\, \sigma_i(x_i)\, \sigma_{-i}(x_{-i})$$

$$\sigma^* = (\sigma_1^*, \ldots, \sigma_n^*) \qquad \sigma_i^* = \sigma_{x_i^*} \quad \to \quad \sigma_i^*(x) = \begin{cases} 1 & \text{if } x = x_i^* \\ 0 & \text{otherwise} \end{cases}$$

x^* Nash eq $\to$ $u_i(x_i^*, x_{-i}^*) \geq u_i(x_i, x_{-i}^*) \quad \forall x_i \in S_i$

$$h_i(\sigma_i, \sigma_{-i}^*) = \sum_{x \in S} u_i(x_i, x_{-i})\, \sigma_i(x_i)\, \sigma_{-i}^*(x_{-i}) =$$

$$= \sum_{x_i \in S_i} \Big(\sum_{x_{-i} \in S_{-i}} u_i(x_i, x_{-i})\, \sigma_{-i}^*(x_{-i}) \Big)\, \sigma_i(x_i) =$$

$$= \sum_{x_i \in S_i} u_i(x_i, x_{-i}^*)\, \sigma_i(x_i)$$

$$\leq \Big(\sum_{x_i \in S_i} \sigma_i(x_i) \Big)\, u_i(x_i^*, u_{-i}^*)$$

$$= u_i(x_i^*, x_{-i}^*) = h_i(\sigma_i^*, \sigma_{-i}^*)$$

$$\downarrow$$

σ^* Nash eq.