# Algorithmic game theory

Laurea Magistrale in Computer Science

2024/25

Lecture 5

# Strategic dominance

Let $(N, (S_i)_{i \in N}, (u_i)_{i \in N})$ be a strategic game.

## Definition

(i) A strategy $x_i^* \in S_i$ is dominant (for player $i$) if it is a best response to every strategy profile $x_{-i} \in S_{-i}$, i.e., $x_i^* \in R_i(x_{-i})$ or equivalently

$$u_i(x_i^*, x_{-i}) \geq u_i(x_i, x_{-i}) \quad \text{for all } x_i \in S_i \text{ and all } x_{-i} \in S_{-i}.$$

(ii) A strategy $x_i^* \in S_i$ is strictly dominant (for player $i$) if it is the unique best response to every strategy profile $x_{-i} \in S_{-i}$, i.e., $R_i(x_{-i}) = \{x_i^*\}$ or equivalently

$$u_i(x_i^*, x_{-i}) > u_i(x_i, x_{-i}) \quad \text{for all } x_i \in S_i, x_i \neq x_i^*, \text{and all } x_{-i} \in S_{-i}.$$

Dominant strategies of a player are completely equivalent: same payoffs

If it exists, a strictly dominant strategy is unique

A (rational) player with a strictly dominant strategy is totally predictable

# Strategic dominance and equilibria

## Definition

A strategy profile $x^* \in S$ is [strictly] dominant if for each player $i$ the strategy $x_i^* \in S_i$ is [strictly] dominant.

## Proposition

*(i) A dominant strategy profile is a Nash equilibrium of the game.*

*(ii) A strictly dominant strategy profile is the unique Nash equilibrium of the game.*

Dominant strategy profiles are unlikely to exist

(none in the battle of sexes and Cournot duopoly)

Both reverse relationships do not hold

# Looking for bad strategies

| I/II | ① | ② | ③ | ④ |
|---|---|---|---|---|
| ① | (4,3) | (7,4) | (5,2) | (3,3) |
| ② | (1,5) | (5,7) | (2,1) | (2,5) |
| ③ | (3,4) | (4,3) | (4,7) | (6,2) |

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pairwise comparisons:

- ② always provides worse payoffs than ①: player I won't play it!
- ④ always provides worse payoffs than ②: player II won't play it!

# Looking for bad strategies: eliminate them

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# Looking for bad strategies: eliminate them and iterate

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iterate elimination

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| I/II | ② |
|---|---|
| ① | (7,4) |

Nash equilibrium

# Dominated strategies

## Definition

A strategy $\bar{x}_i \in S_i$ for player $i$ is strictly dominated if there exists another strategy $\hat{x}_i \in S_i$ for player $i$ such that

$$u_i(\hat{x}_i, x_{-i}) > u_i(\bar{x}_i, x_{-i}) \qquad \text{for all } x_{-i} \in S_{-i}.$$

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$\hat{x}_i$ dominates $\bar{x}_i$ but it is not necessarily a dominant strategy $\left(\hat{x}_i = \hat{x}_i(\bar{x}_i)\right)$

A rational player would never play a strictly dominated strategy

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③ and ③ are not strictly dominated

(③,③) is not a Nash equilibrium

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no strictly dominated strategies anymore

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Nash equilibrium

# Weakly dominated strategies

## Definition

A strategy $\bar{x}_i \in S_i$ for player $i$ is weakly dominated if there exists another strategy $\hat{x}_i \in S_i$ for player $i$ such that

$$u_i(\hat{x}_i, x_{-i}) \geq u_i(\bar{x}_i, x_{-i}) \qquad \text{for all } x_{-i} \in S_{-i},$$

and

$$u_i(\hat{x}_i, \tilde{x}_{-i}) > u_i(\bar{x}_i, \tilde{x}_{-i})$$

holds for at least one strategy profile $\tilde{x}_{-i} \in S_{-i}$.

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holds for at least one strategy profile $\tilde{x}_{-i} \in S_{-i}$.

A rational player would never play a weakly dominated strategy

$x^* \in S$ is a Nash equilibrium $\not\Longrightarrow$ $x_i^*$ is not weakly dominated for any $i \in N$

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③ is weakly dominated

(③,③) is a Nash equilibrium

# Equilibria by iterated elimination of dominated strategies

## Iterated elimination algorithms (IESDS - IEDS)

1. $S_i^0 = S_i \quad (i \in N),\ k = 0$
2. $G^k = (N, (S_i^k)_{i \in N}, (u_i)_{i \in N})$
   $S_i^{k+1} = S_i^k \setminus \{\textit{strictly/weakly dominated strategies for player } i \textit{ in } G^k\}$
3. $k = k + 1$ *and go back to 2*

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3. $k = k + 1$ *and go back to 2*

## Correctness of finite convergence

*Suppose that best responses always exist, i.e.,*

$$R_i(x_{-i}) \neq \emptyset \text{ for all } x_{-i} \in S_{-i} \text{ and all } i \in N.$$

*(i) If the iterated elimination of strictly dominated strategies leads to a unique $x^*$, then $x^*$ is the unique Nash equilibrium.*

*(ii) Suppose that the game is finite. If the iterated elimination of weakly dominated strategies leads to a unique $x^*$, then $x^*$ is a Nash equilibrium.*

# Restricted games and Nash equilibria

## Definition

$G' = (N, (T_i)_{i \in N}, (v_i)_{i \in N})$ is a restriction of $G = (N, (S_i)_{i \in N}, (u_i)_{i \in N})$ if each $i \in N$ satisfies

- $T_i \neq \emptyset$ and $T_i \subseteq S_i$
- $v_i(x) = u_i(x)$ for all $x \in T = T_1 \times ... \times T_n$

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## Theorem

*Let $T_i = S_i \setminus \{$strictly dominated strategies for player $i$ in $G\}$ for all $i \in N$.*

*(i) If $x^* \in S$ is a Nash equilibrium for $G$, then $x^* \in T$ and $x^*$ is a Nash equilibrium for $G'$.*

*(ii) If $x^* \in T$ is a Nash equilibrium for $G'$, then it is a Nash equilibrium for $G$ as well provided that best responses always exist for $G$.*

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## Theorem

*Let $T_i = S_i \setminus \{$weakly dominated strategies for player $i$ in $G\}$ for all $i \in N$.*

*(i) If $x^* \in S$ is a Nash equilibrium for $G$, then $x^* \in T$ and $x^*$ is a Nash equilibrium for $G'$. False!*

*(ii) If $x^* \in T$ is a Nash equilibrium for $G'$, then it is a Nash equilibrium for $G$ as well provided that best responses always exist for $G$. False!*

# Restricted games and Nash equilibria

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*(i) If $x^* \in S$ is a Nash equilibrium for $G$, then $x^* \in T$ and $x^*$ is a Nash equilibrium for $G'$.* *False!*

*(ii) If $x^* \in T$ is a Nash equilibrium for $G'$, then it is a Nash equilibrium for $G$ as well provided that the game is finite.*

# Rationalizability and "never best responses"

## Definition

(i) A strategy $x_i^* \in S_i$ is never best response if $x_i^* \notin R_i(x_{-i})$ for all $x_{-i} \in S_{-i}$ or equivalently

$$\text{for all } x_{-i} \in S_{-i} \text{ there exists } x_i \in S_i \text{ s.t. } u_i(x_i^*, x_{-i}) < u_i(x_i, x_{-i}).$$

$x_i \in S$ strictly dominated $\Longrightarrow x_i$ is never best response

(no relationship with weak dominance)

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$x_i \in S$ strictly dominated $\Longrightarrow$ $x_i$ is never best response

(no relationship with weak dominance)

## Theorem

*Let $T_i = S_i \setminus \{$never best response strategies for player $i$ in $G\}$ for all $i \in N$.*

*(i) If $x^* \in S$ is a Nash equilibrium for $G$, then $x^* \in T$ and $x^*$ is a Nash equilibrium for $G'$.*

*(ii) Suppose that best replies always exist for $G$. If $x^* \in T$ is a Nash equilibrium for $G'$, then it is a Nash equilbrium for $G$.*

# Equilibria by iterated elimination of never best responses

## Iterated elimination of never best responses (IENBR)

1. $S_i^0 = S_i \quad (i \in N)$, $k = 0$
2. $G^k = (N, (S_i^k)_{i \in N}, (u_i)_{i \in N})$
   $S_i^{k+1} = S_i^k \setminus \{\textit{never best response strategies for player } i \textit{ in } G^k\}$
3. $k = k + 1$ *and go back to 2*

## Correctness of finite convergence

*Suppose that best responses always exist. If the iterated elimination of never best responses strategies leads to a unique* $x^*$, *then* $x^*$ *is the unique Nash equilibrium.*

# Iterated elimination of never best responses at work

| I/II | ① | ② |
| --- | --- | --- |
| ① | (2,1) | (0,0) |
| ② | (0,2) | (2,0) |
| ③ | (1,1) | (1,2) |

- neither strictly nor weakly dominated strategies

# Iterated elimination of never best responses at work

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- neither strictly nor weakly dominated strategies
- ③ is never best response

eliminate and iterate

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- ② is never best response

| I/II | ① |
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# Iterated elimination of never best responses at work

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|---|---|
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| ② | (0,2) |

| I/II | ① |
|---|---|
| ① | (2,1) |

Nash equilibrium

# A Hotelling game (with uncountably many strategies)

- An uniformly distributed population over a segment
- 2 shops to choose a location where to open
- customers will choose the closest shop
- utility = percentage of customers

# A Hotelling game (with uncountably many strategies)

- An uniformly distributed population over a segment [0,100]
- 2 shops to choose a location where to open $x_1, x_2 \in [0, 100]$
- customers will choose the closest shop
- utility $=$ percentage of customers

# A Hotelling game (with uncountably many strategies)

- An uniformly distributed population over a segment [0,100]
- 2 shops to choose a location where to open $x_1, x_2 \in [0, 100]$
- customers will choose the closest shop
- utility = percentage of customers $u_i(x_1, x_2) = \begin{cases} (x_1 + x_2)/2 & \text{if } x_i < x_{-i} \\ 100 - (x_1 + x_2)/2 & \text{if } x_i > x_{-i} \\ 50 & \text{if } x_i = x_{-i} \end{cases}$

# A Hotelling game (with uncountably many strategies)

- An uniformly distributed population over a segment [0,100]
- 2 shops to choose a location where to open $x_1, x_2 \in [0, 100]$
- customers will choose the closest shop
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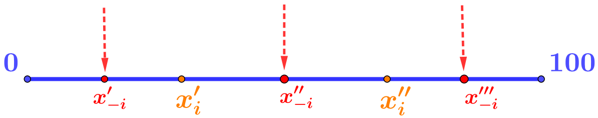
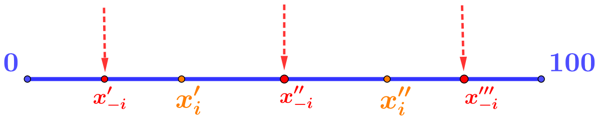



- $\bar{x}_i = 0$ and $\bar{x}_i = 100$ are strictly dominated by $\hat{x}_i = 50$
- no other strategy is strictly/weakly dominated

# A Hotelling game (with uncountably many strategies)

- An uniformly distributed population over a segment [0,100]
- 2 shops to choose a location where to open $x_1, x_2 \in [0, 100]$
- customers will choose the closest shop
- utility = percentage of customers $u_i(x_1, x_2) = \begin{cases} (x_1 + x_2)/2 & \text{if } x_i < x_{-i} \\ 100 - (x_1 + x_2)/2 & \text{if } x_i > x_{-i} \\ 50 & \text{if } x_i = x_{-i} \end{cases}$




* $\bar{x}_i = 0$ and $\bar{x}_i = 100$ are strictly dominated by $\hat{x}_i = 50$
* no other strategy is strictly/weakly dominated
* $x_i = 50$ is the unique best response to $x_{-i} = 50$
* no other strategy admits a best response

IENBR provides the unique equilibrium $(50, 50)$ in just one iteration

# A conservative approach to strategic games

security strategies

- compute the worst possible payoff for each fixed strategy
- choose a strategy maximizing of the above minima

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| ① | (5,2) | (7,4) | (5,3) | (3,3) | |
| ② | (1,5) | (5,7) | (2,4) | (2,5) | |
| ③ | (4,4) | (4,1) | (4,3) | (6,2) | |
| | | | | | |

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| min | 2 | 1 | 3 | 2 | |

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* (①,②) is the unique Nash equilibrium

Security strategies and equilibria can be meaningfully different

(Cornout duopoly: 0 is the security strategy for each firm)

# Security strategies: formal definition

Worst result for player $i$ while playing strategy $x_i \in S_i$

$$w_i(x_i) = \min\{u_i(x_i, x_{-i}) \, : \, x_{-i} \in S_{-i}\}$$

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Playing any

$$\bar{x}_i \in \arg\max\{w_i(x_i) \ : \ x_i \in S_i\},$$

player $i$ gets at least $v_i = \max\{w_i(x_i) \ : \ x_i \in S_i\}$

## Definition

Any such $\bar{x}_i$ is called a security strategy for player $i$

$v_i = \max\{w_i(x_i) \ : \ x_i \in S_i\}$ is called the security level of player $i$

# Security strategies: failure in rock-paper-scissors

Worst result for player $i$ while playing strategy $x_i \in S_i$

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| I/II | paper | scissors | rock | min |
|---|---|---|---|---|
| paper | (0,0) | (-1,1) | (1,-1) | -1 |
| scissors | (1,-1) | (0,0) | (-1,1) | -1 |
| rock | (-1,1) | (1,-1) | (0,0) | -1 |
| min | -1 | -1 | -1 | |

every strategy is a security strategy