Space and Time-Efficient Data Structures for Massive Datasets

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University of Pisa

17/10/2016


Statistics
Progress
Growth & Decline
Statistical
Analysis


Evidence

The increase of information
does **not** scale with technology.

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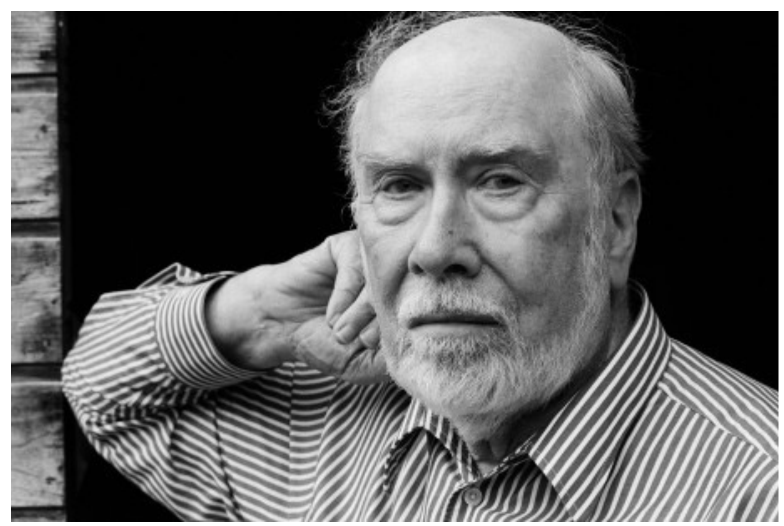
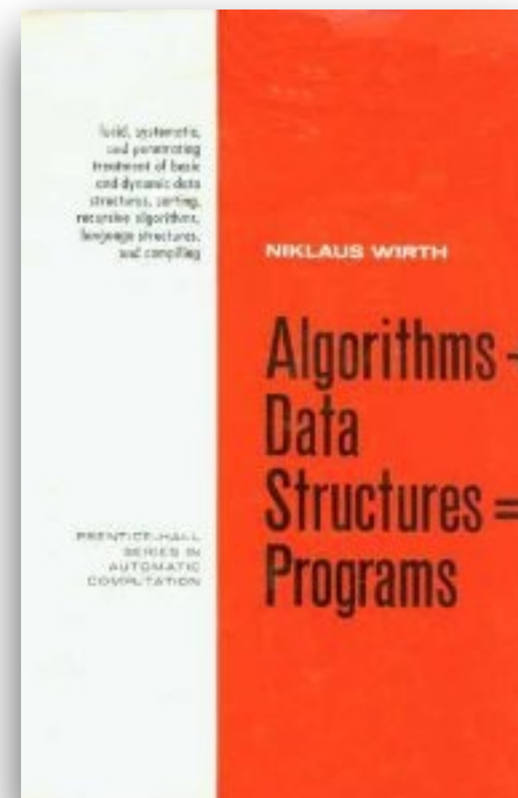




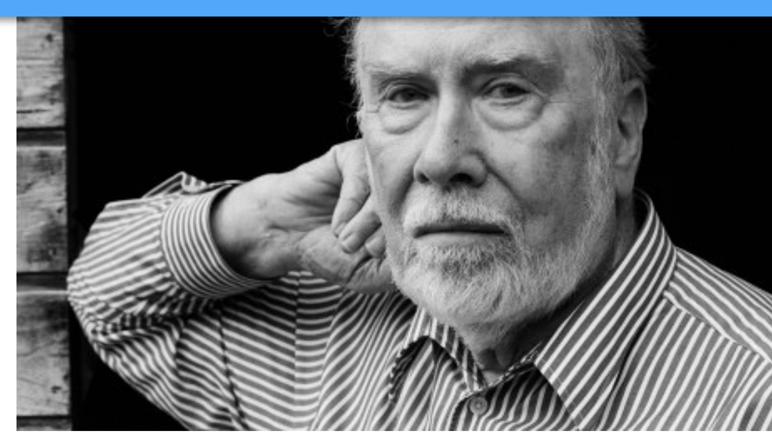
“Software is getting slower more rapidly than hardware becomes faster.”

Niklaus Wirth, A Plea for Lean Software

Evidence

The increase of information
does **not** scale with technology.

Even more relevant today!



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Scenario

Data Structures

PERFORMANCE

"how quickly a program does its work" - **faster** work

Algorithms

EFFICIENCY

"how much work is required by a program" - **less** work

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Data Compression

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Dichotomy Problem

Small VS Fast?

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Choose one.

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High Level Thesis

Data Structures + Data Compression → Faster Algorithms

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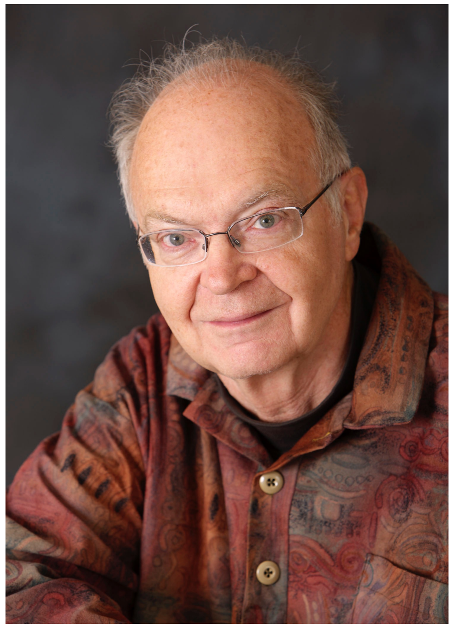
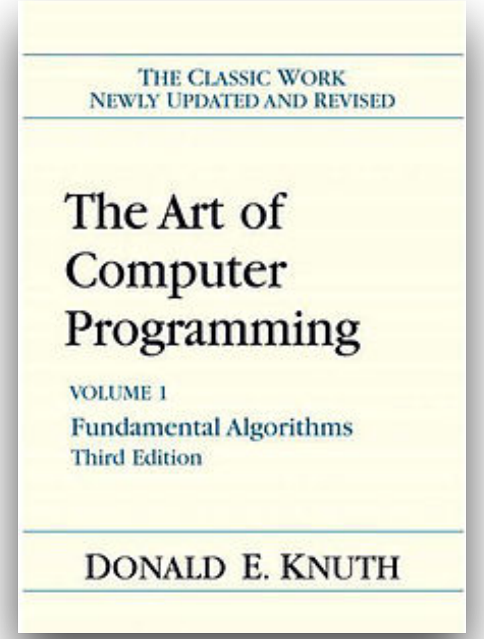
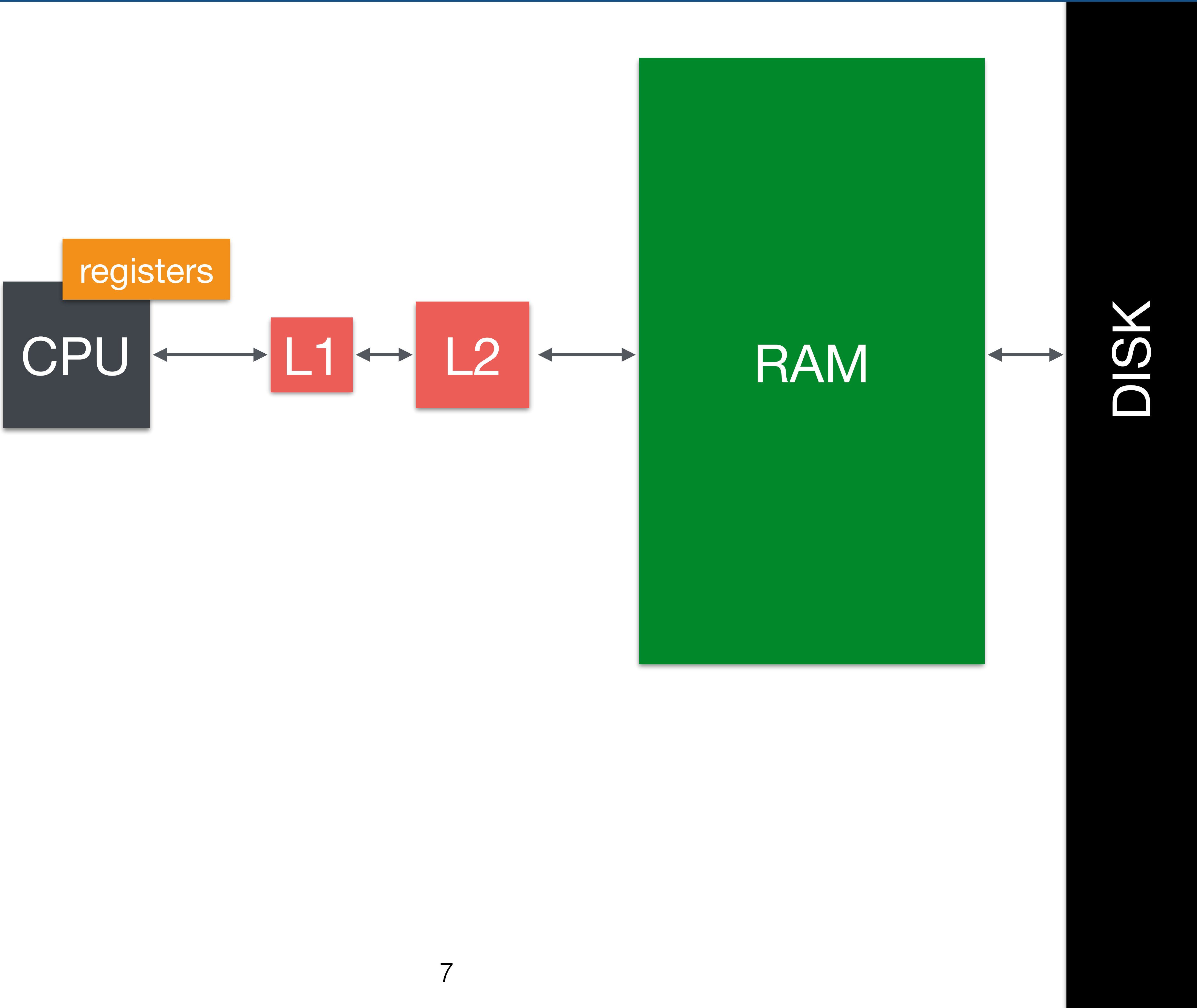




"Space optimization is closely related to time optimization in a disk memory."

Donald E. Knuth, The Art of Computer Programming

Hierarchical Memory Organisation



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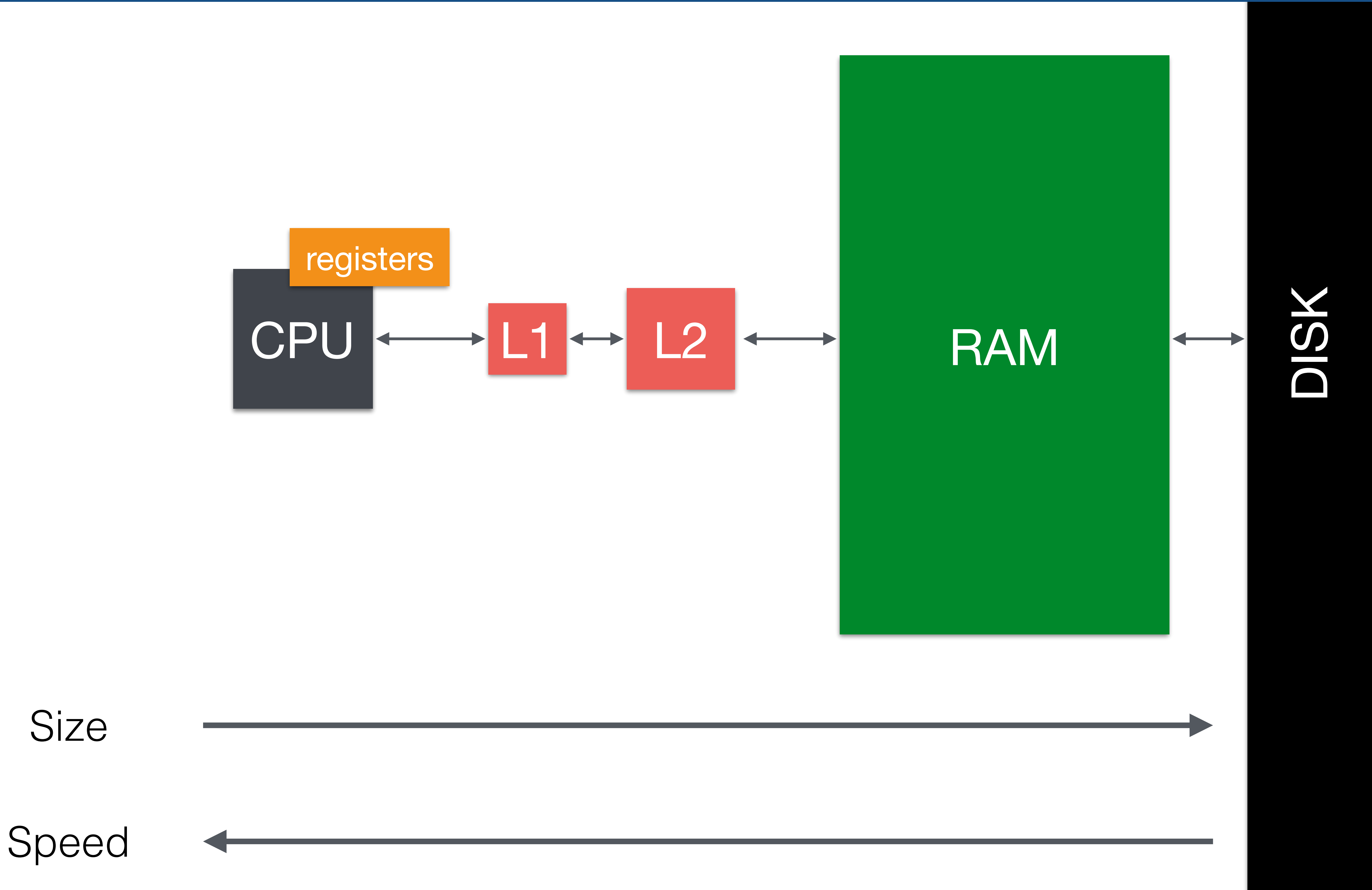

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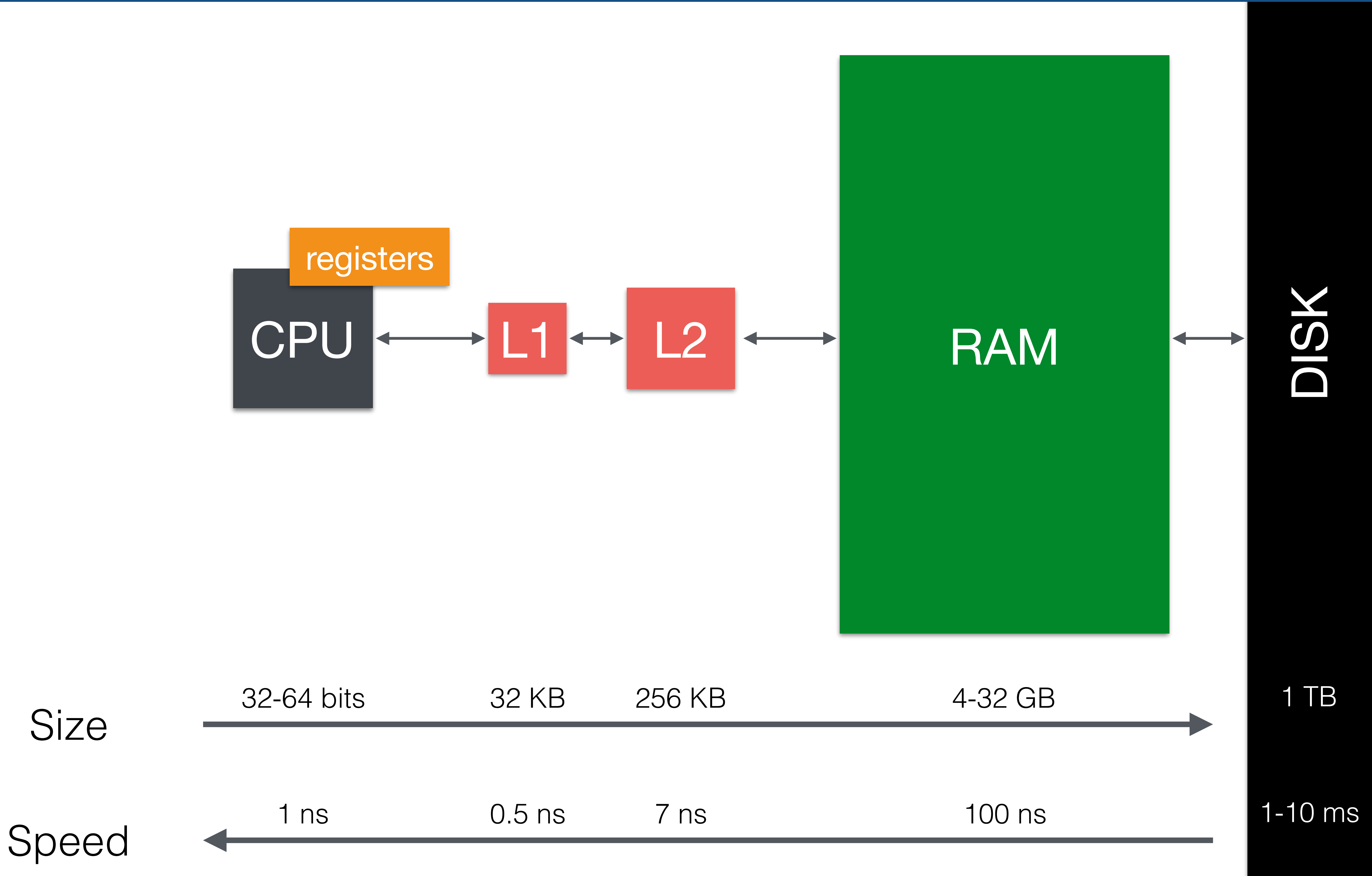

Numbers are taken from: https://gist.github.com/jboner/2841832

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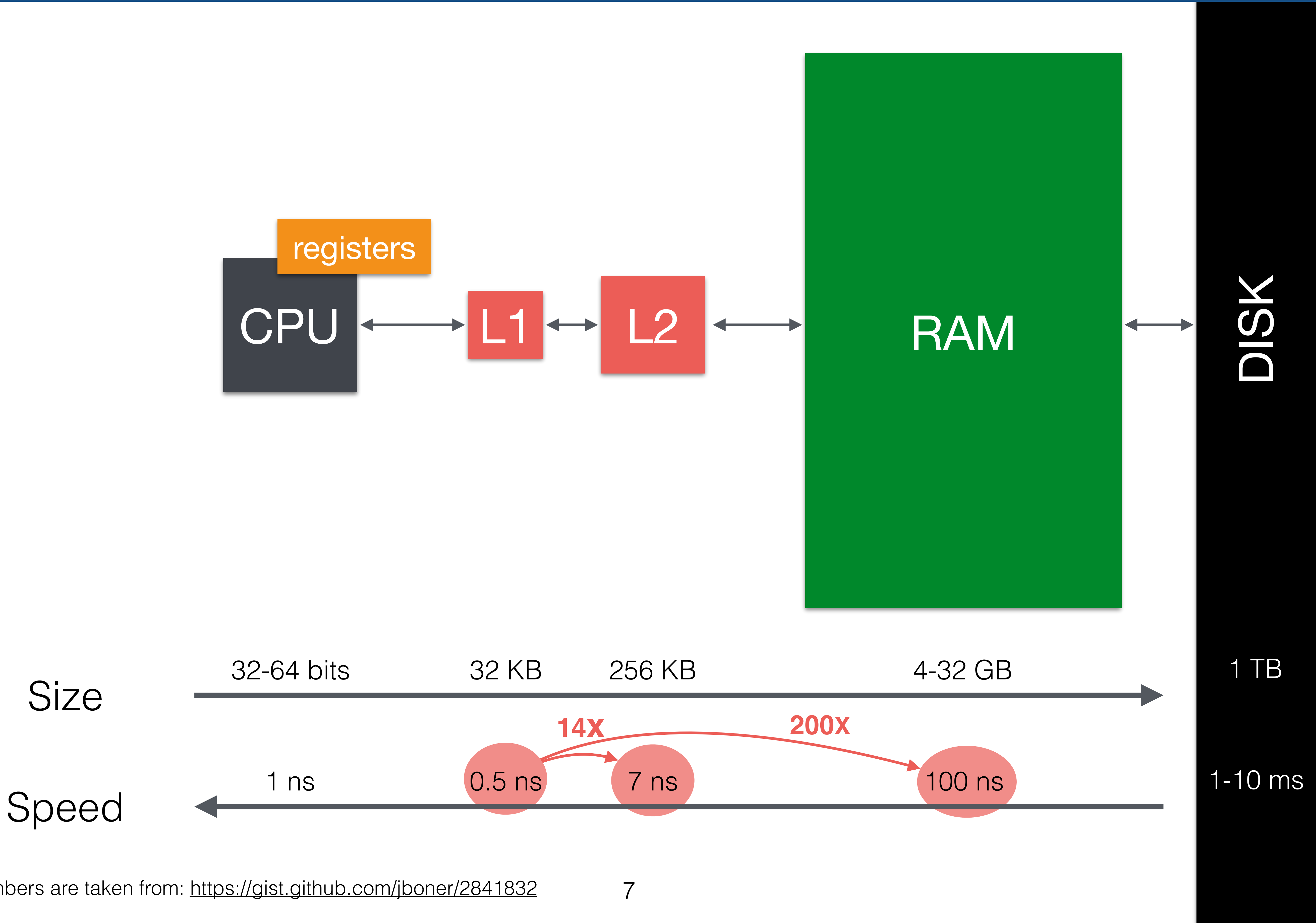

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Design **space-efficient** *ad-hoc* data structures, both from a theoretical *and* practical perspective, that support **fast data extraction**.

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Data compression & Fast Retrieval
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Mature algorithmic solutions
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Must exploit properties of the addressed problem.

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Why? Who cares?

Because engineered data structures are the ones the *boost* the *availability* and *wealth* of information around us.

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Prefer **cache-friendly** (non discontiguous) data structures.
Always.

Use std::vector.

Example: vector of strings VS string pool

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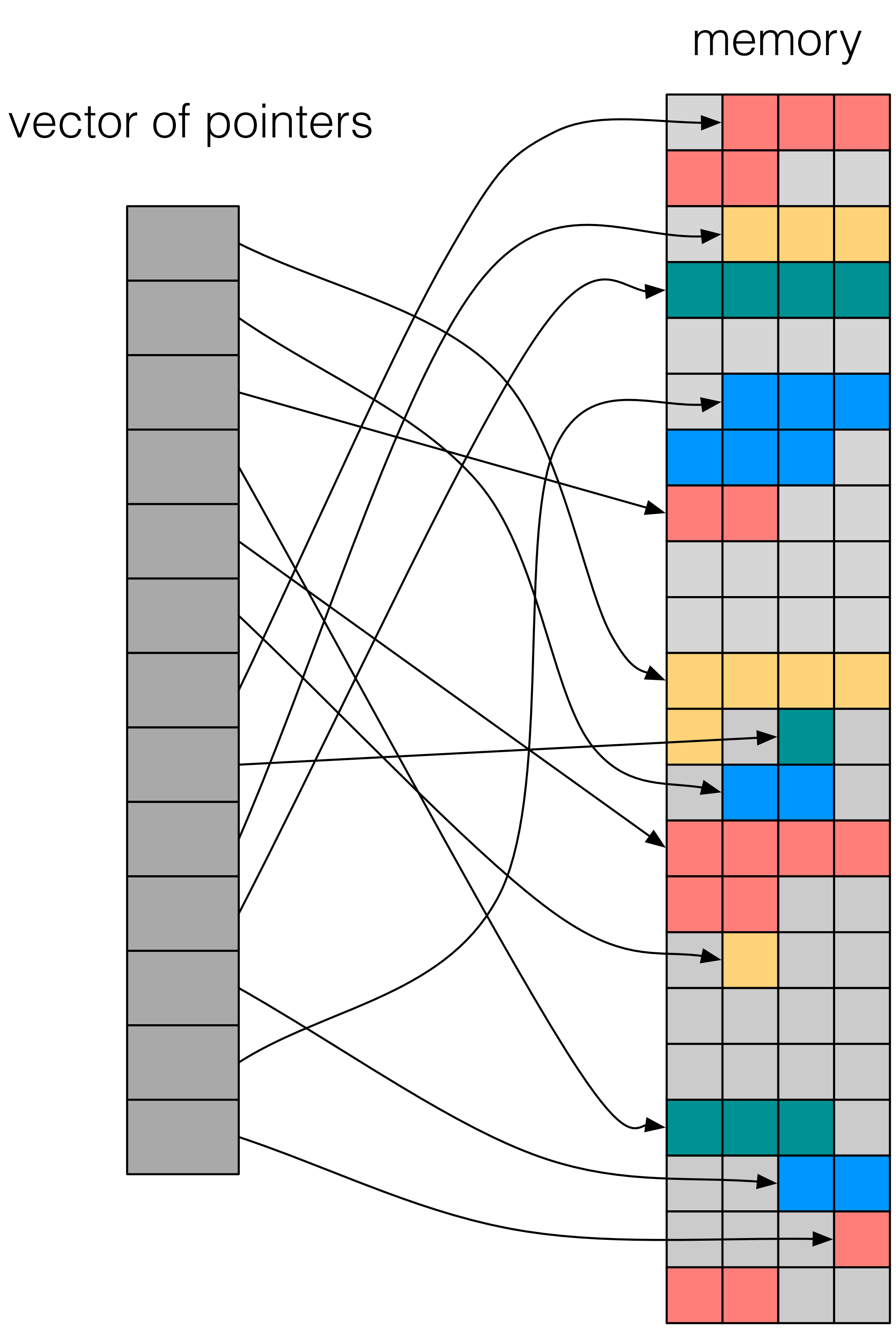

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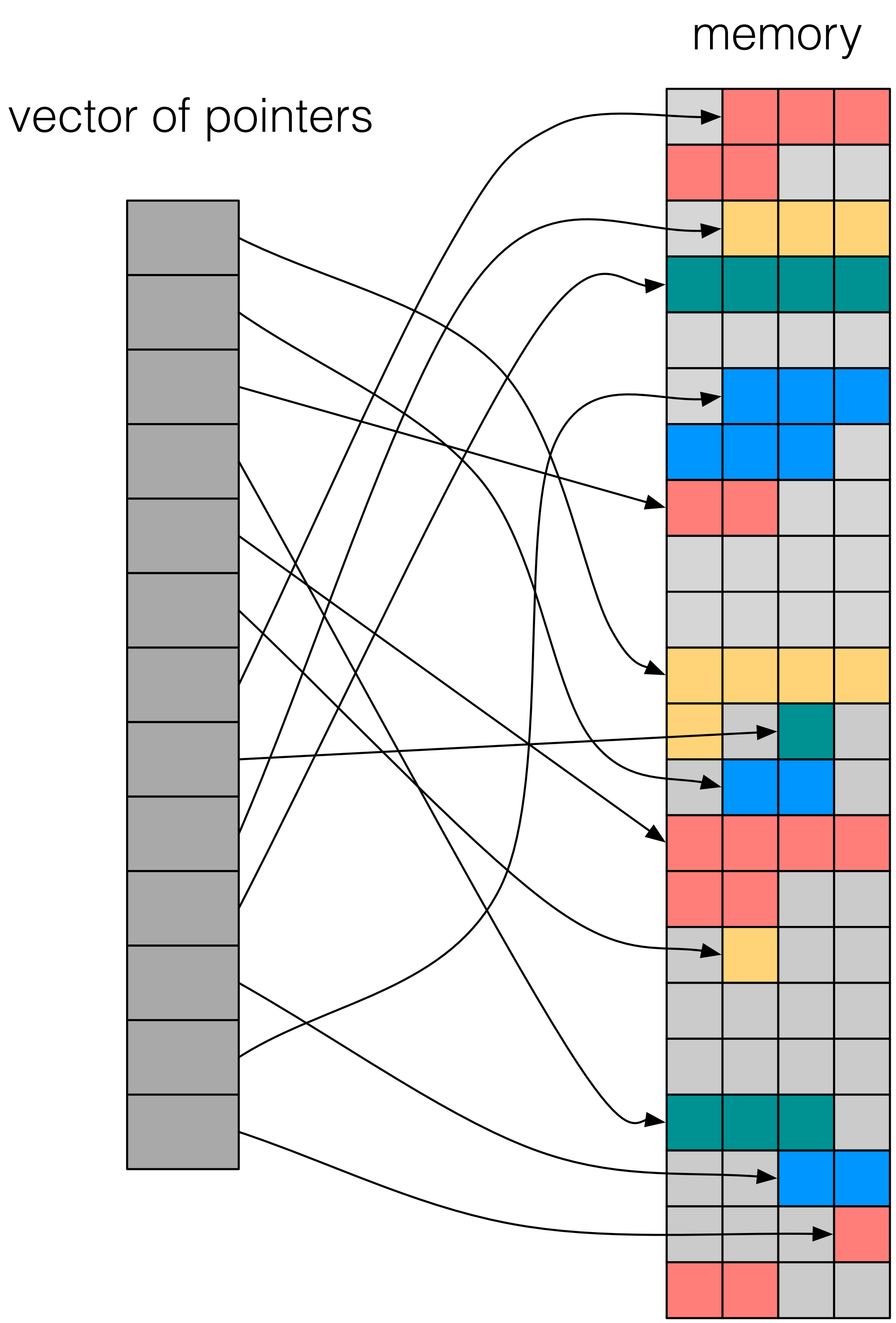

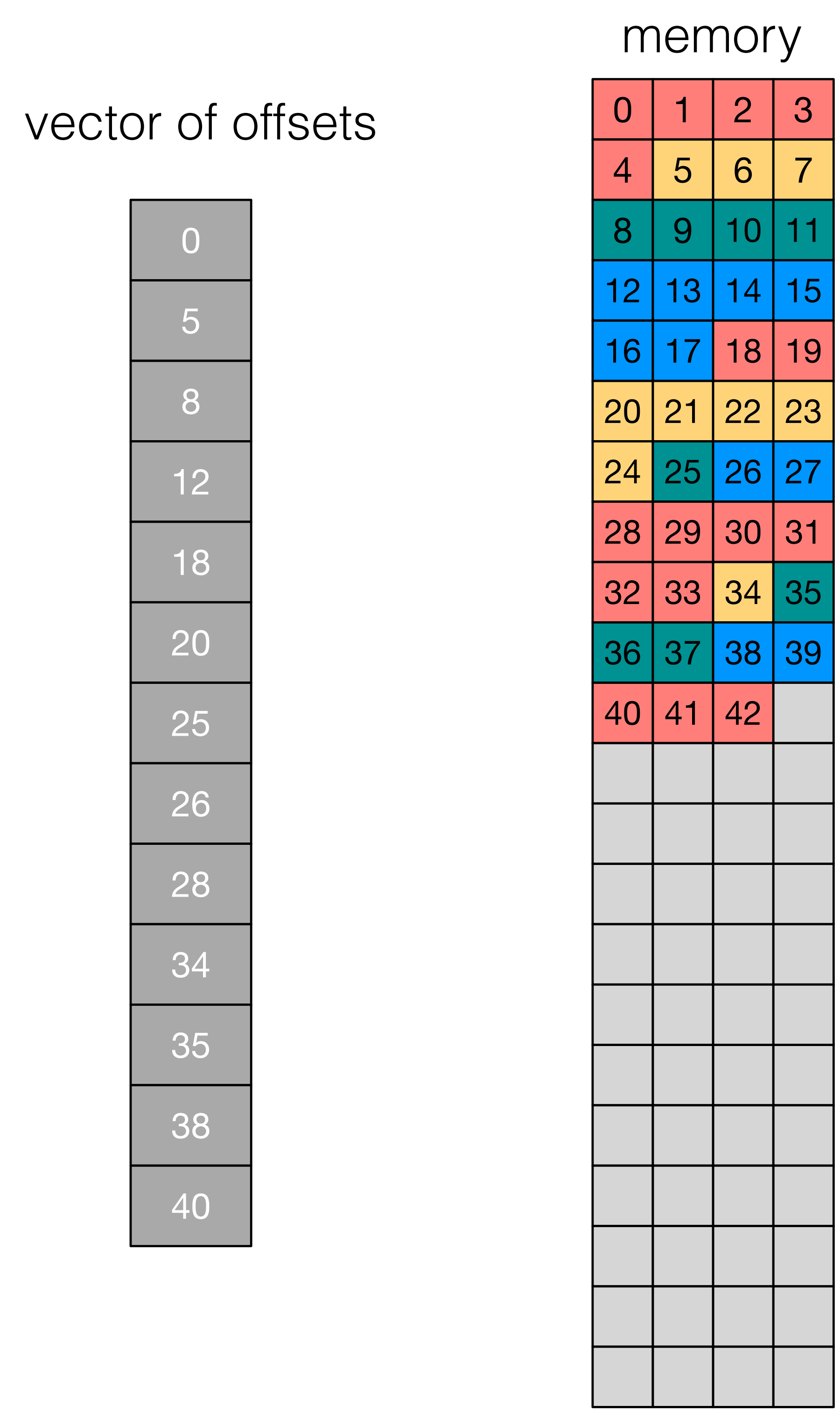

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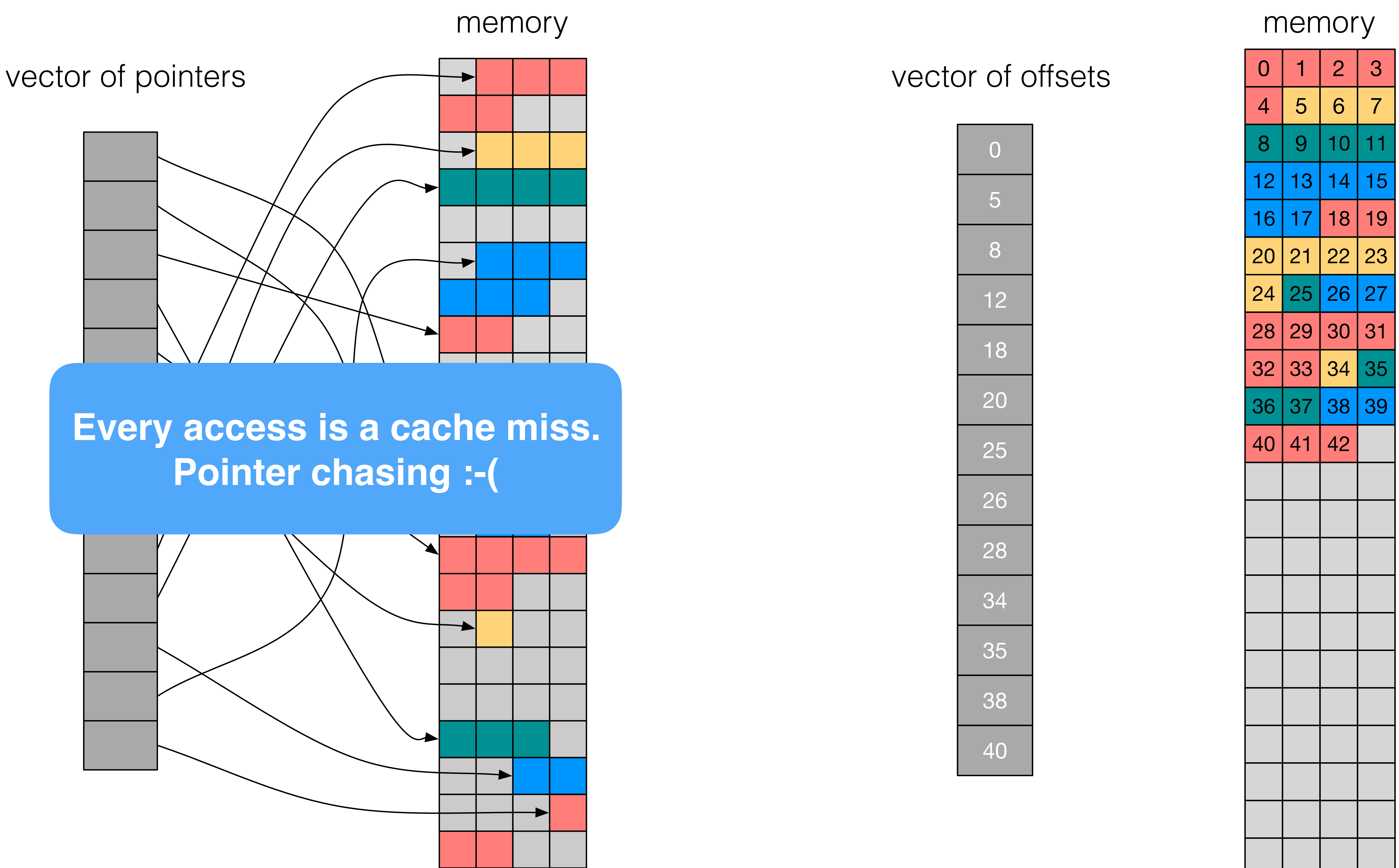

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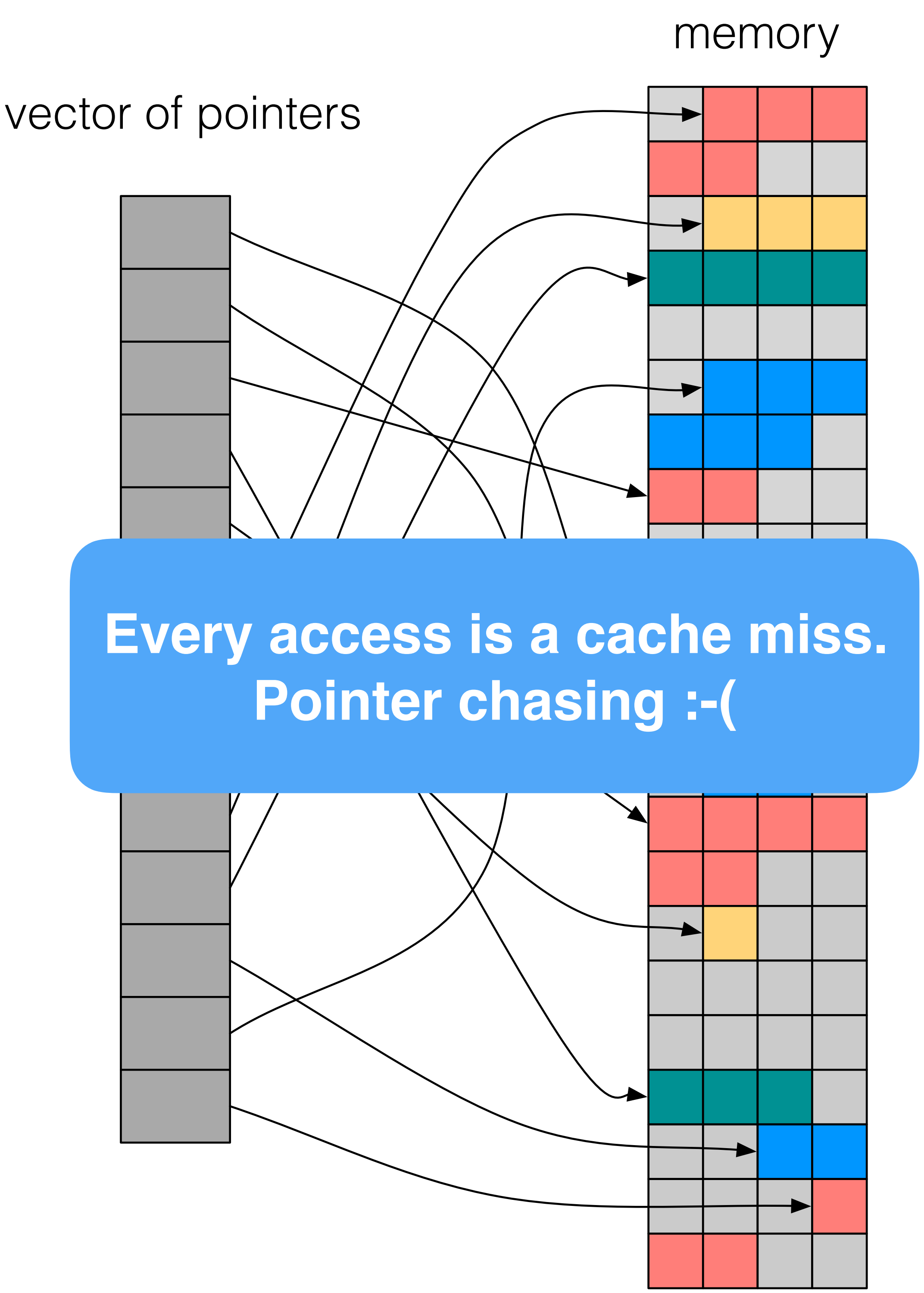

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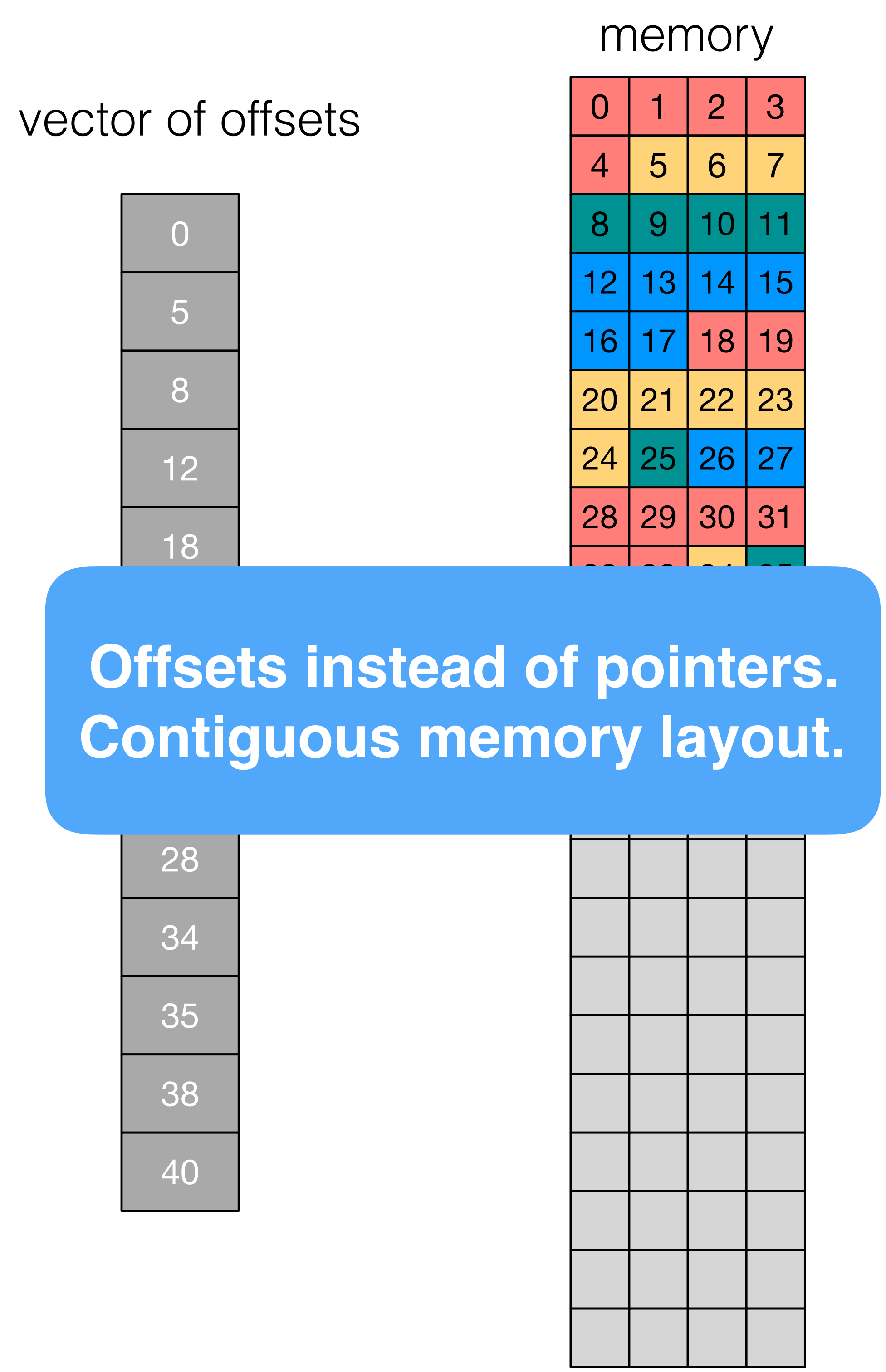

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Contiguous memory layout.

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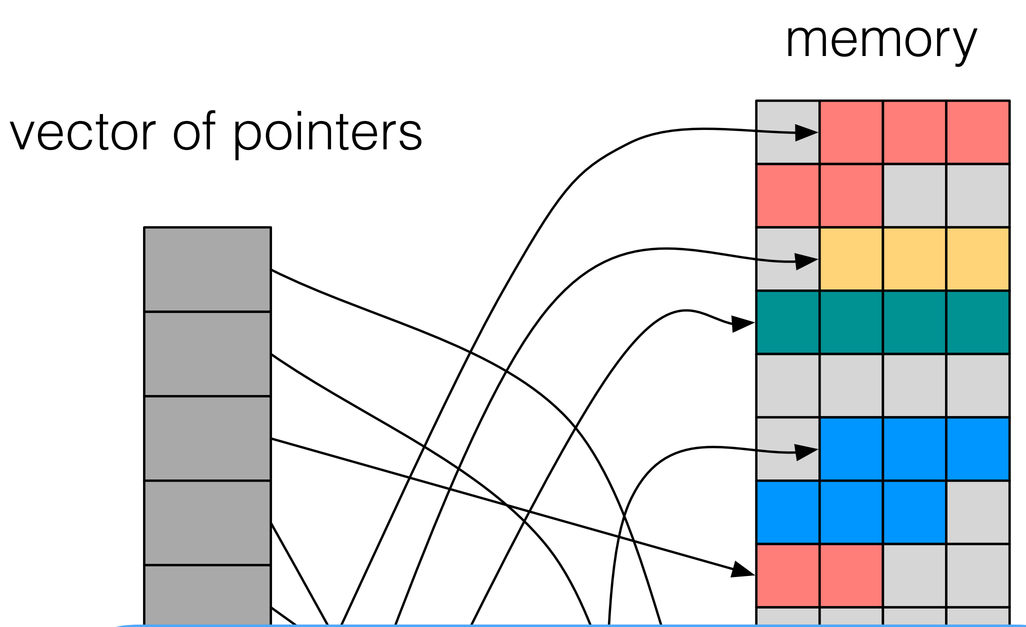

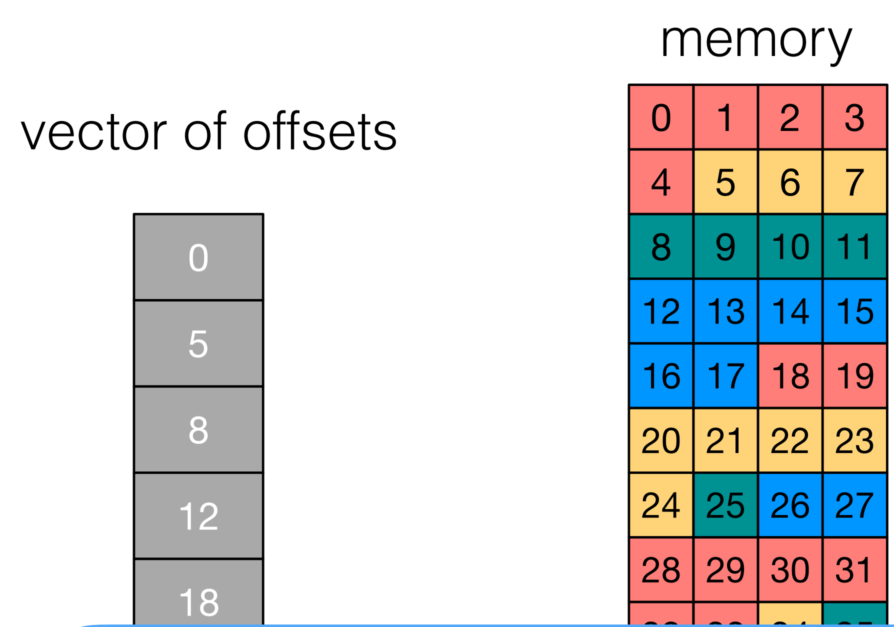

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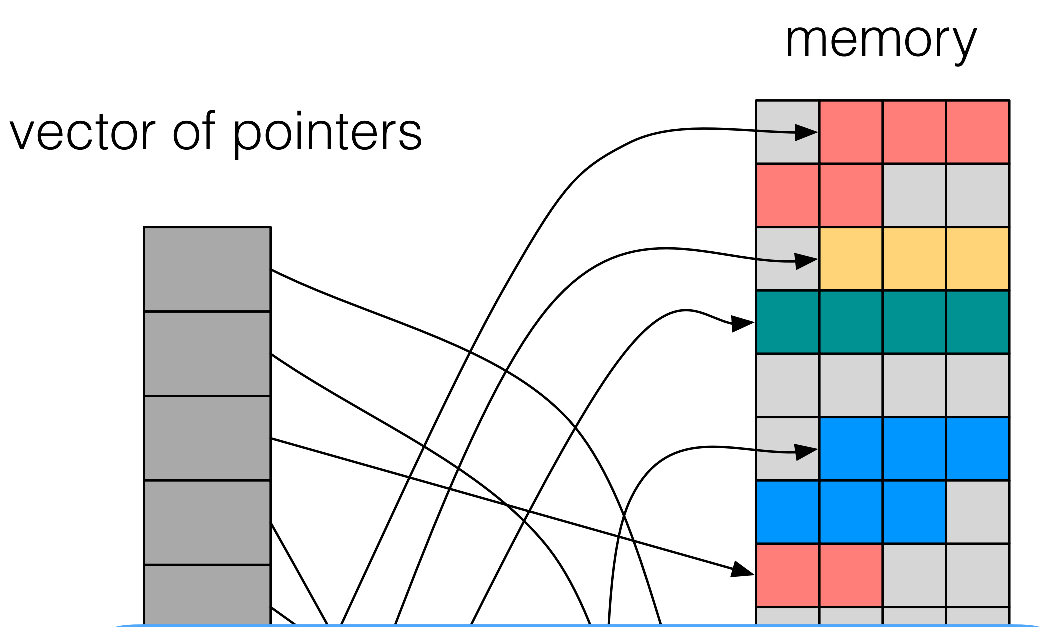

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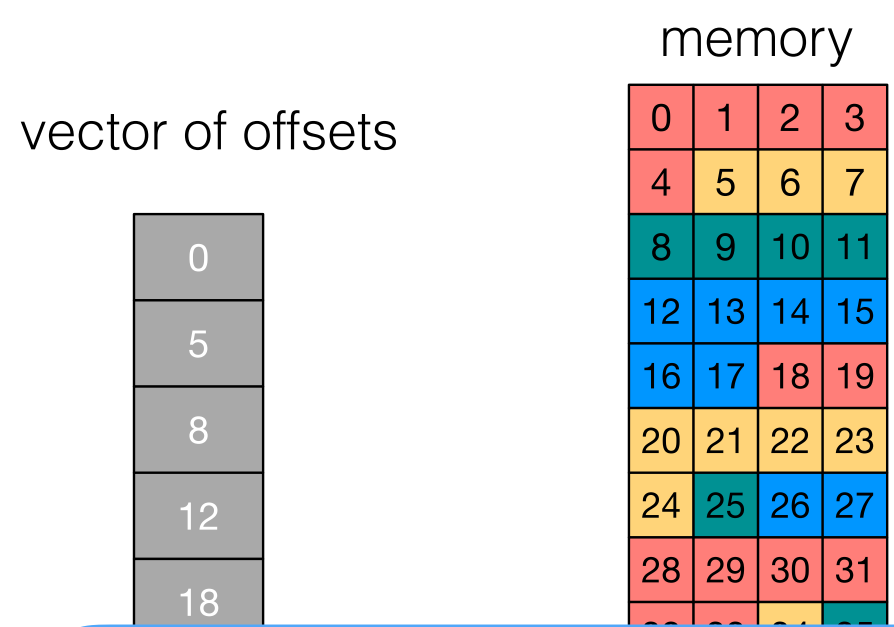

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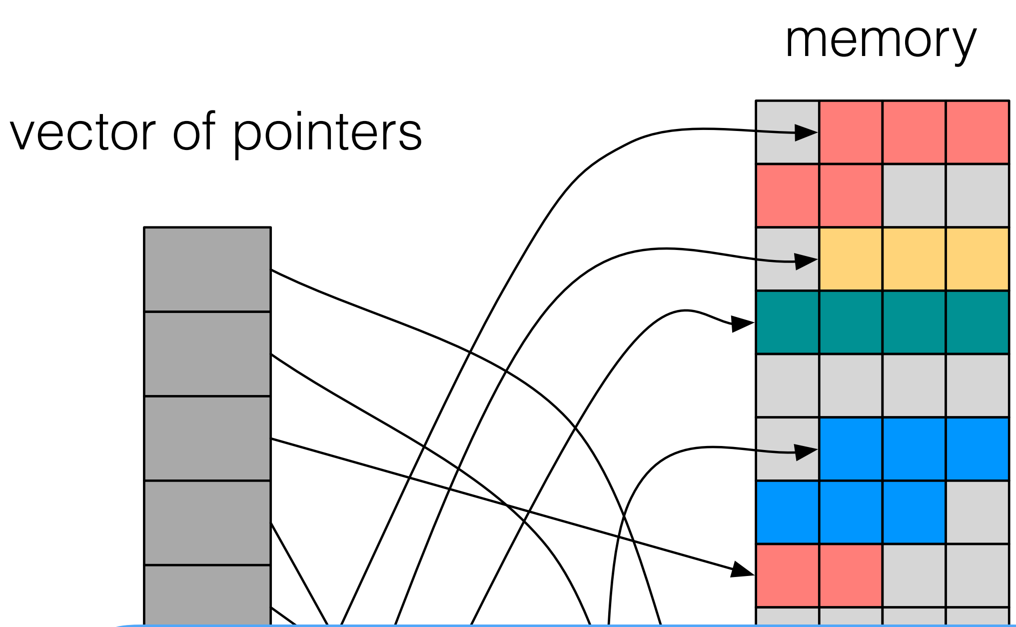

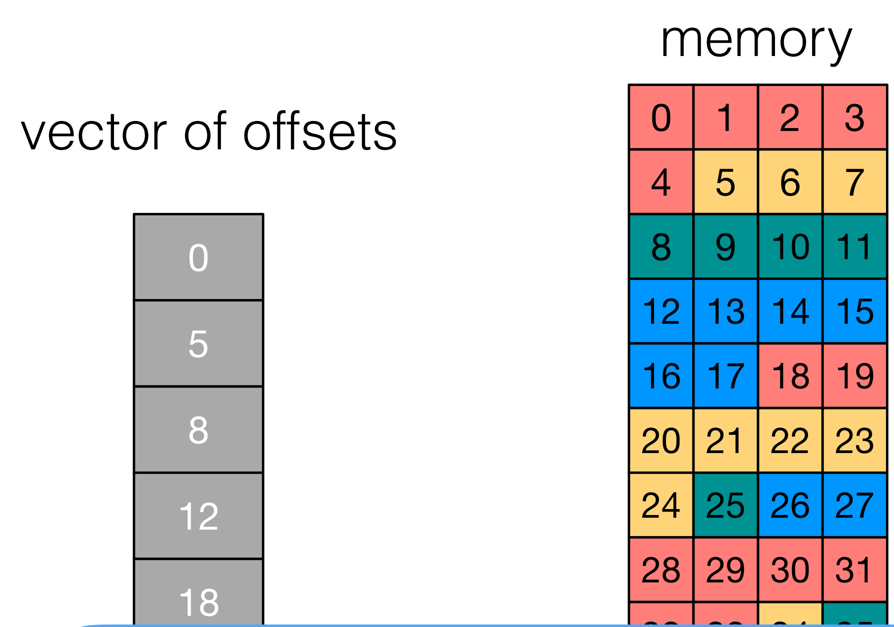

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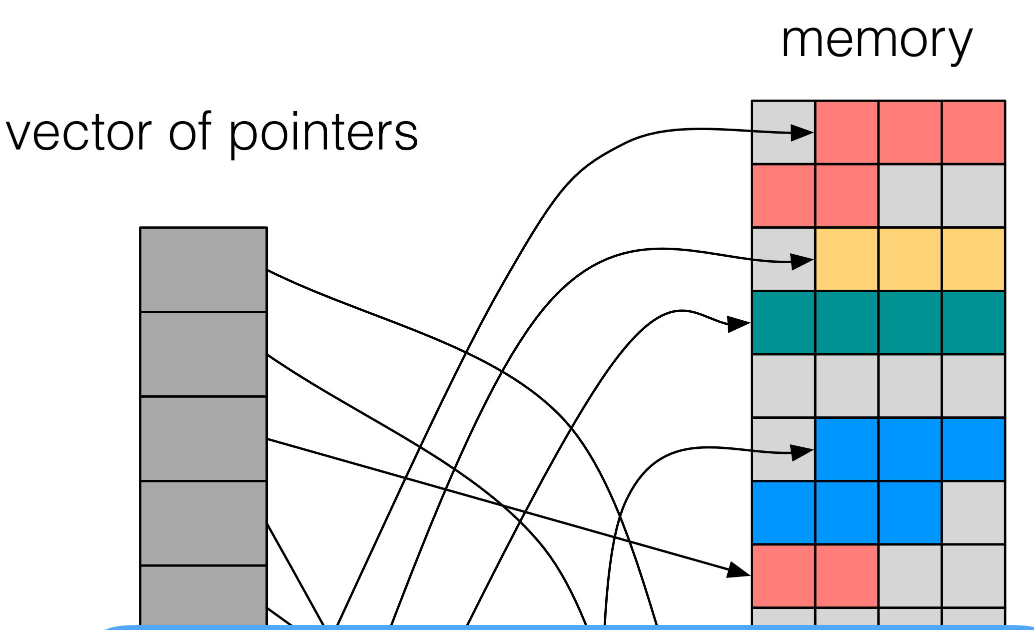

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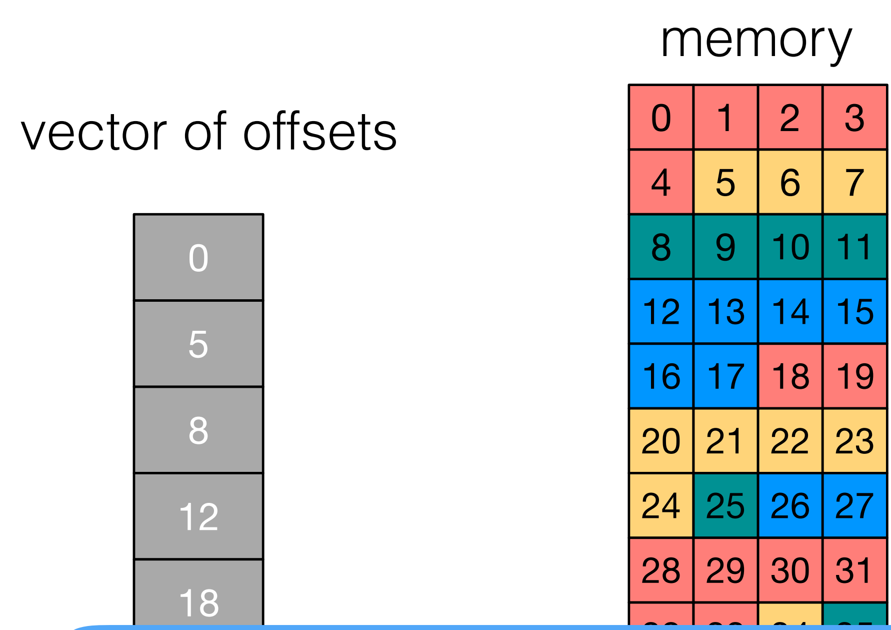

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~5.2GBs

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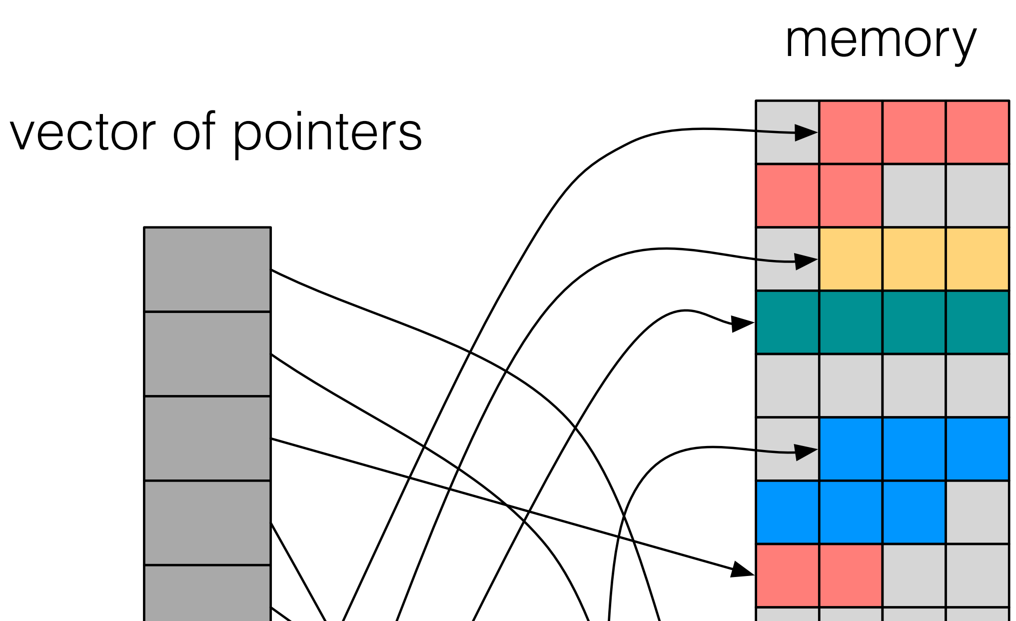

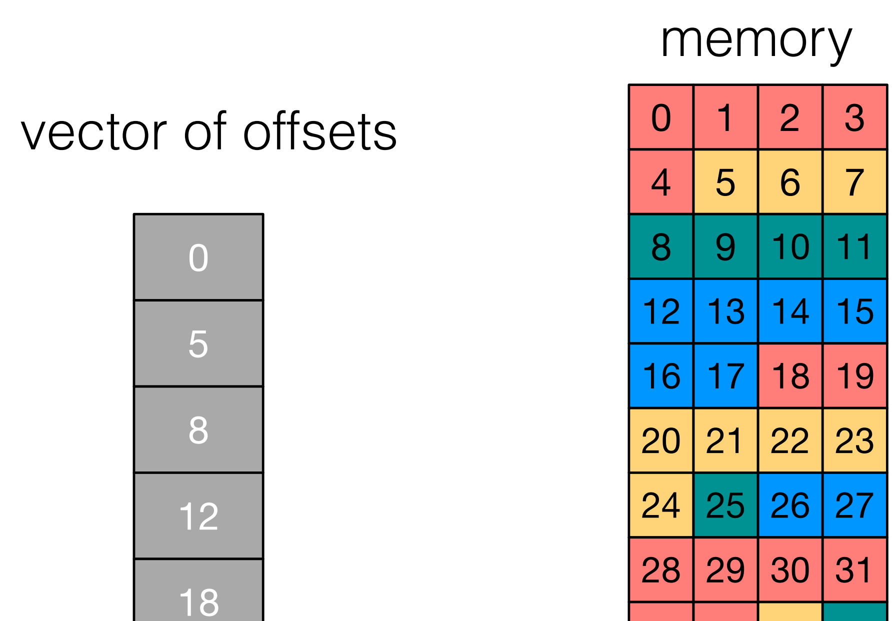

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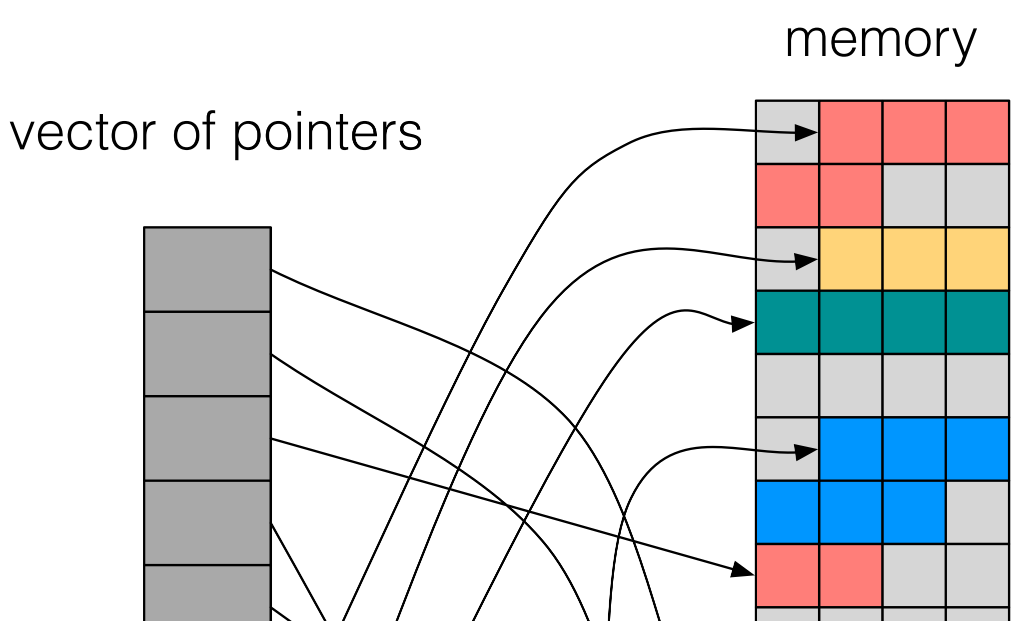

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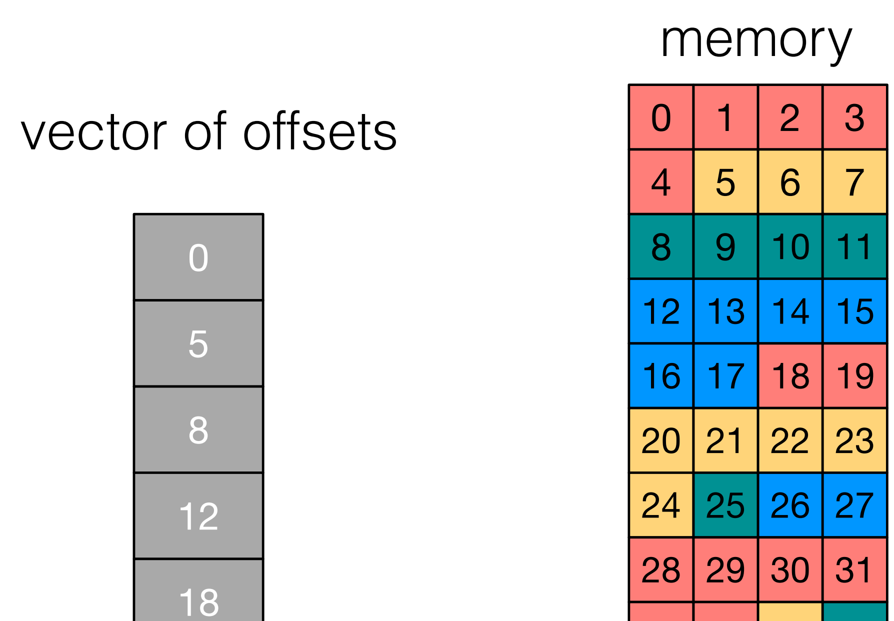

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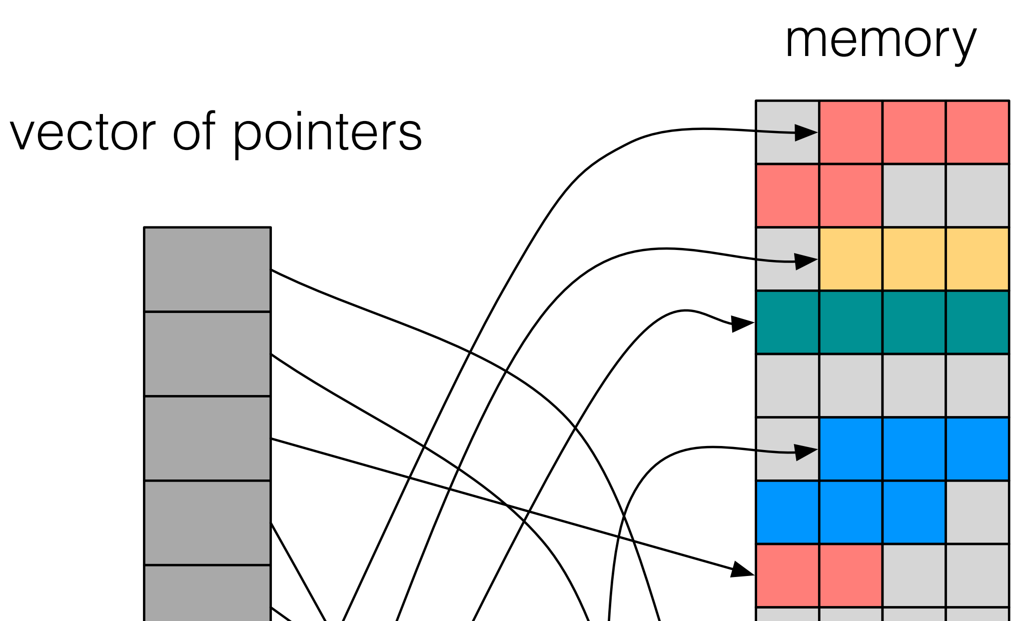

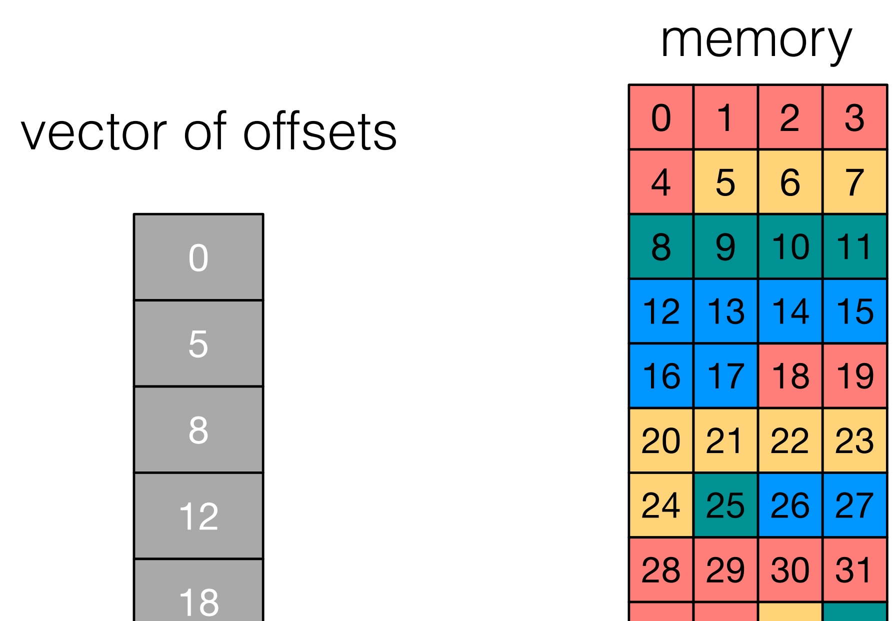

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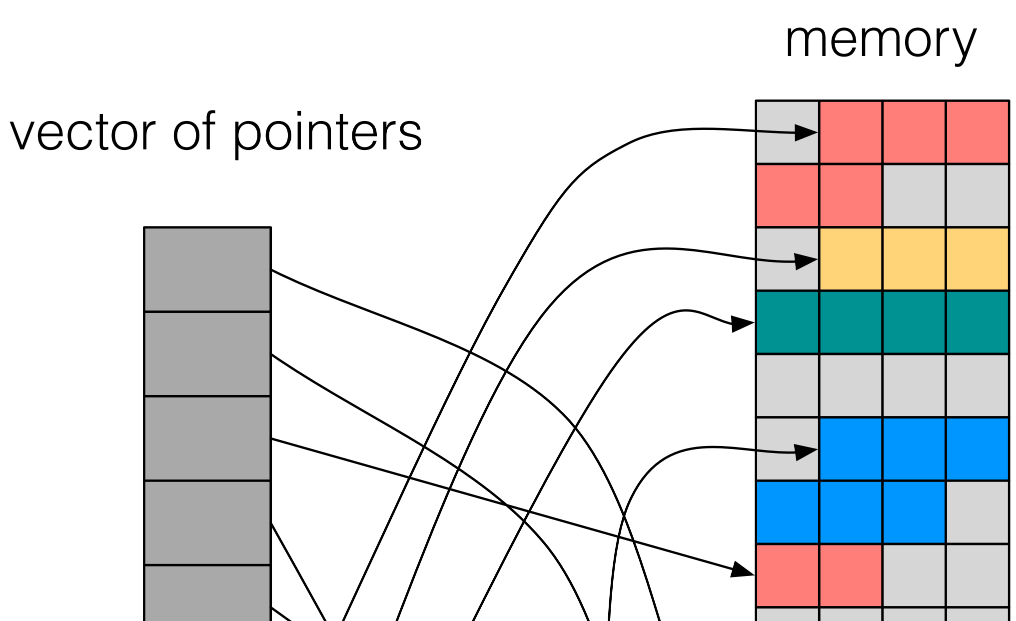

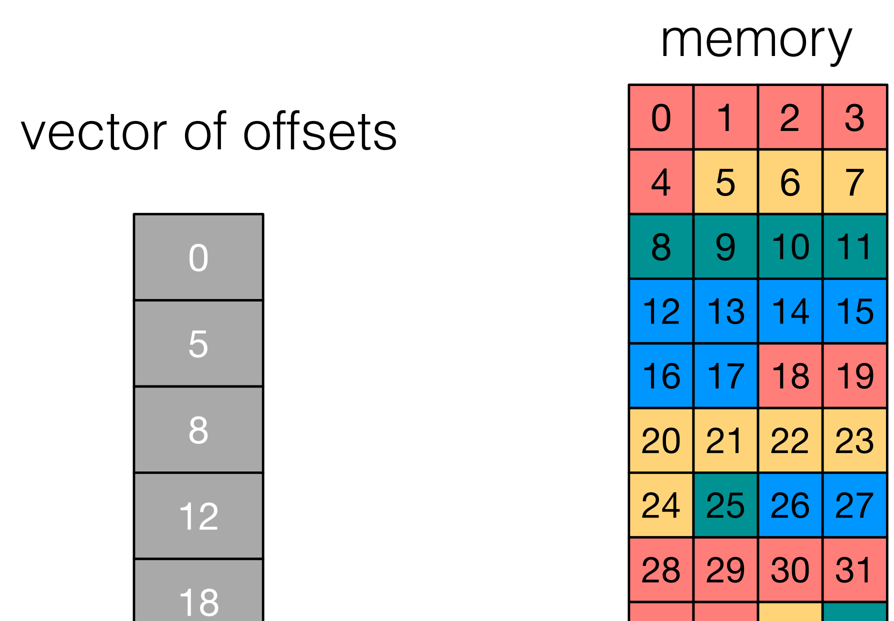

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X5

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Inverted Indexes

Given a *textual collection* D, each document can be seen as a (multi-)set of terms. The set of terms occurring in D is the *lexicon* T.

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The collection of all inverted lists $\{L_{t_1},\ldots,L_{t_T}\}$ is the inverted index.

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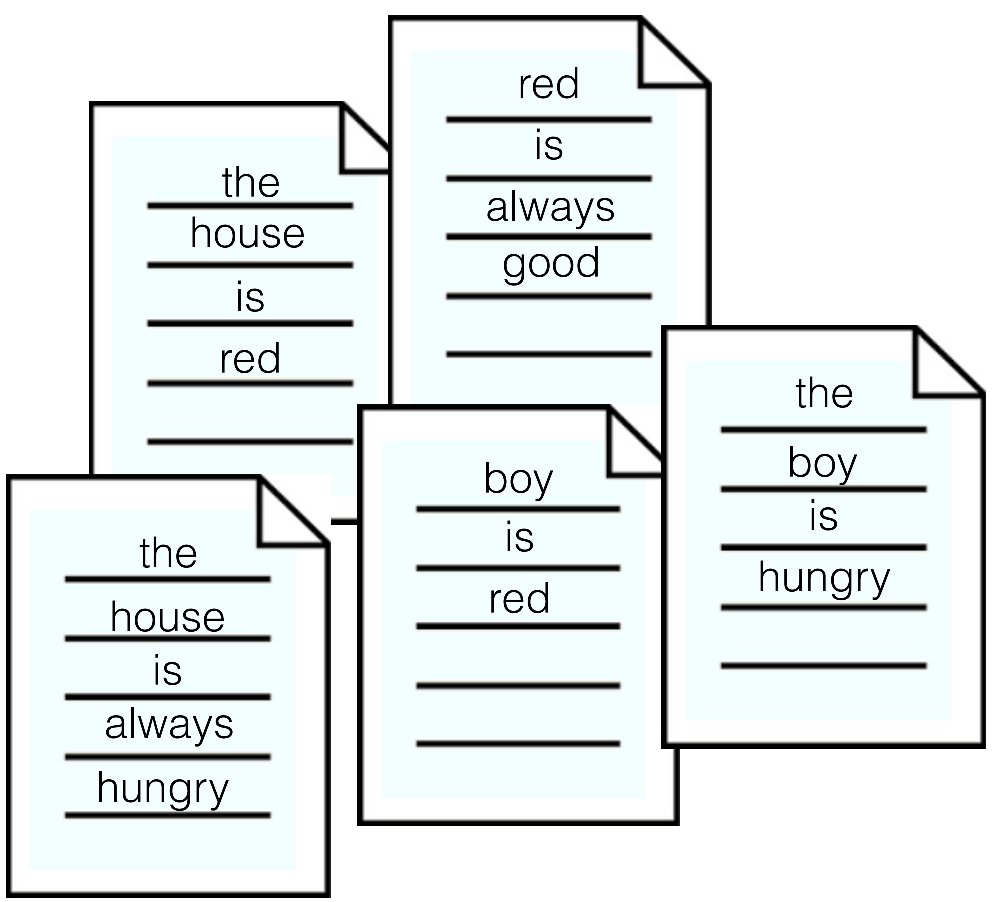

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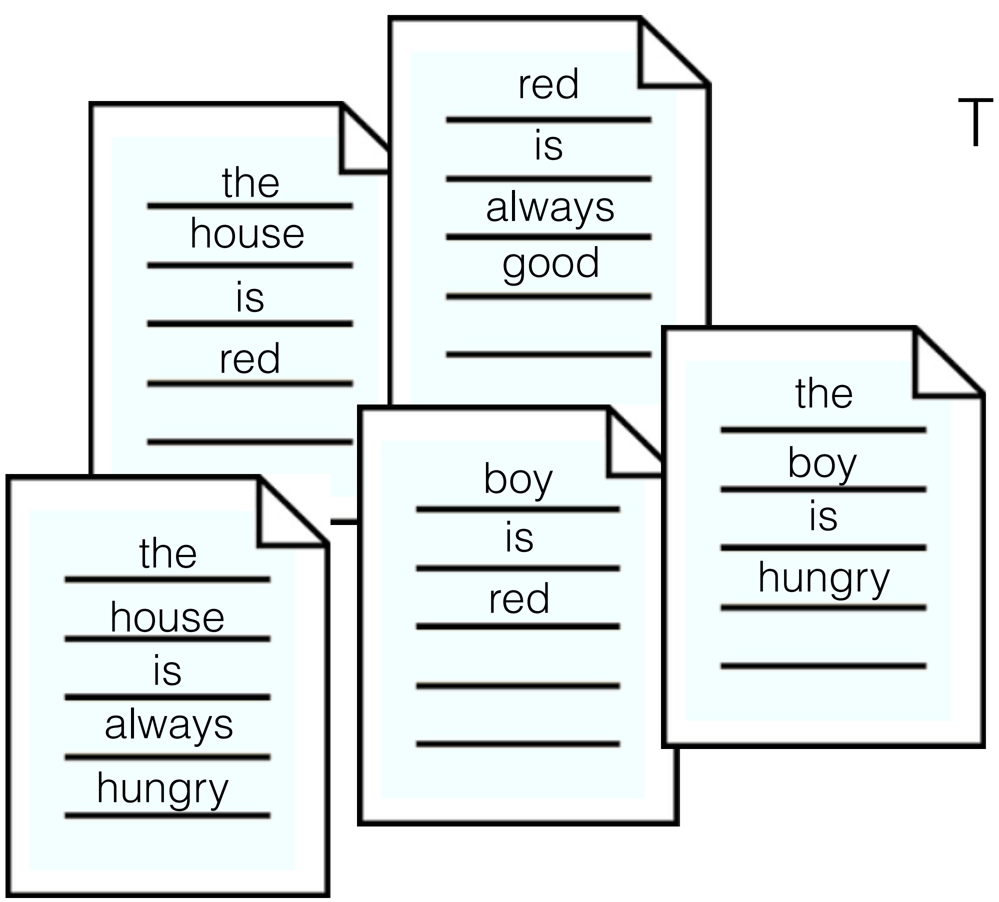

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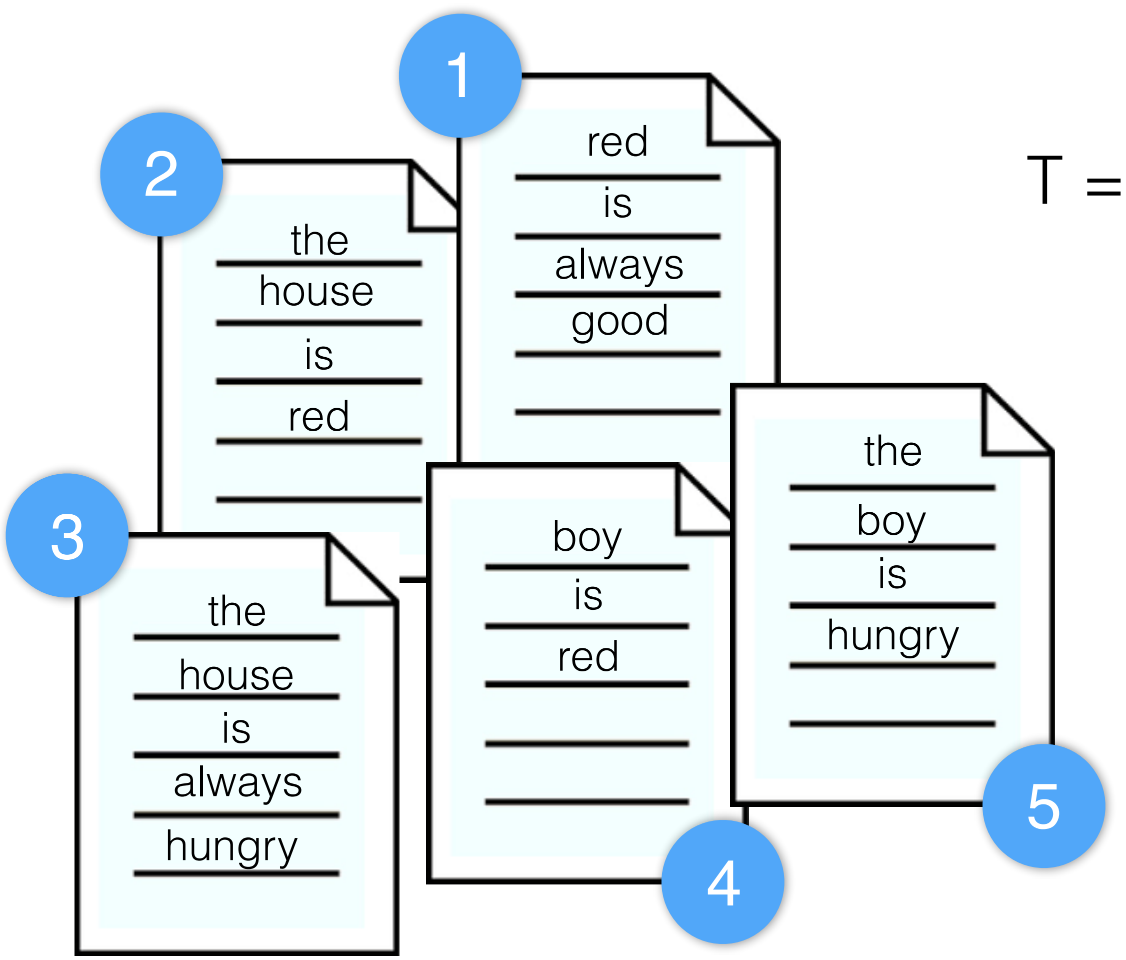

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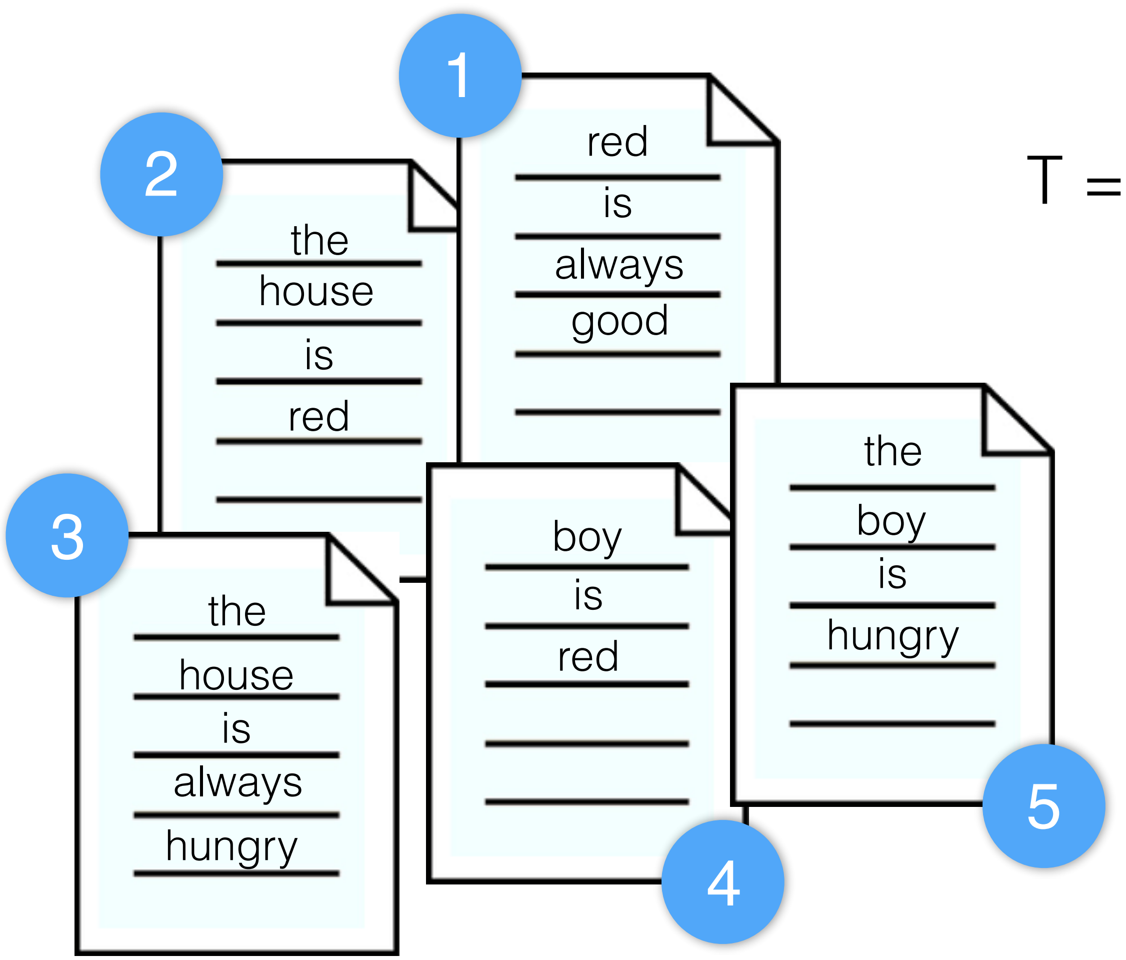

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$L_{t_2}=[4, 5]$
$L_{t_3}=[1]$
$L_{t_4}=[2, 3]$
$L_{t_5}=[3, 5]$
$L_{t_6}=[1, 2, 3, 4, 5]$
$L_{t_7}=[1, 2, 4]$
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Inverted Indexes

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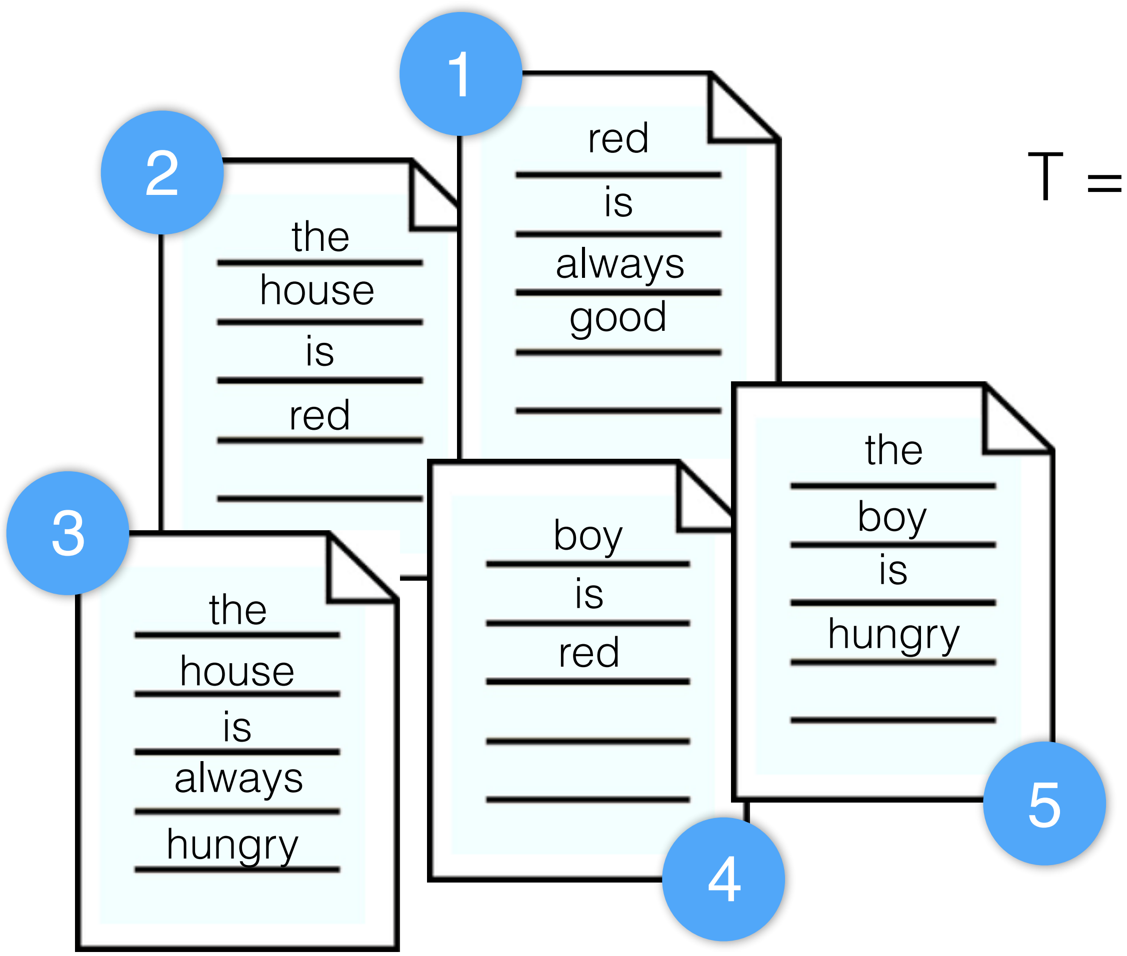

$$T = \{\overset{t_1}{\text{always}}, \overset{t_2}{\text{boy}}, \overset{t_3}{\text{good}}, \overset{t_4}{\text{house}}, \overset{t_5}{\text{hungry}}, \overset{t_6}{\text{is}}, \overset{t_7}{\text{red}}, \overset{t_8}{\text{the}}\}$$

$L_{t_1}=[1, 3]$
$L_{t_2}=[4, 5]$
$L_{t_3}=[1]$
$L_{t_4}=[2, 3]$
$L_{t_5}=[3, 5]$
$L_{t_6}=[1, 2, 3, 4, 5]$
$L_{t_7}=[1, 2, 4]$
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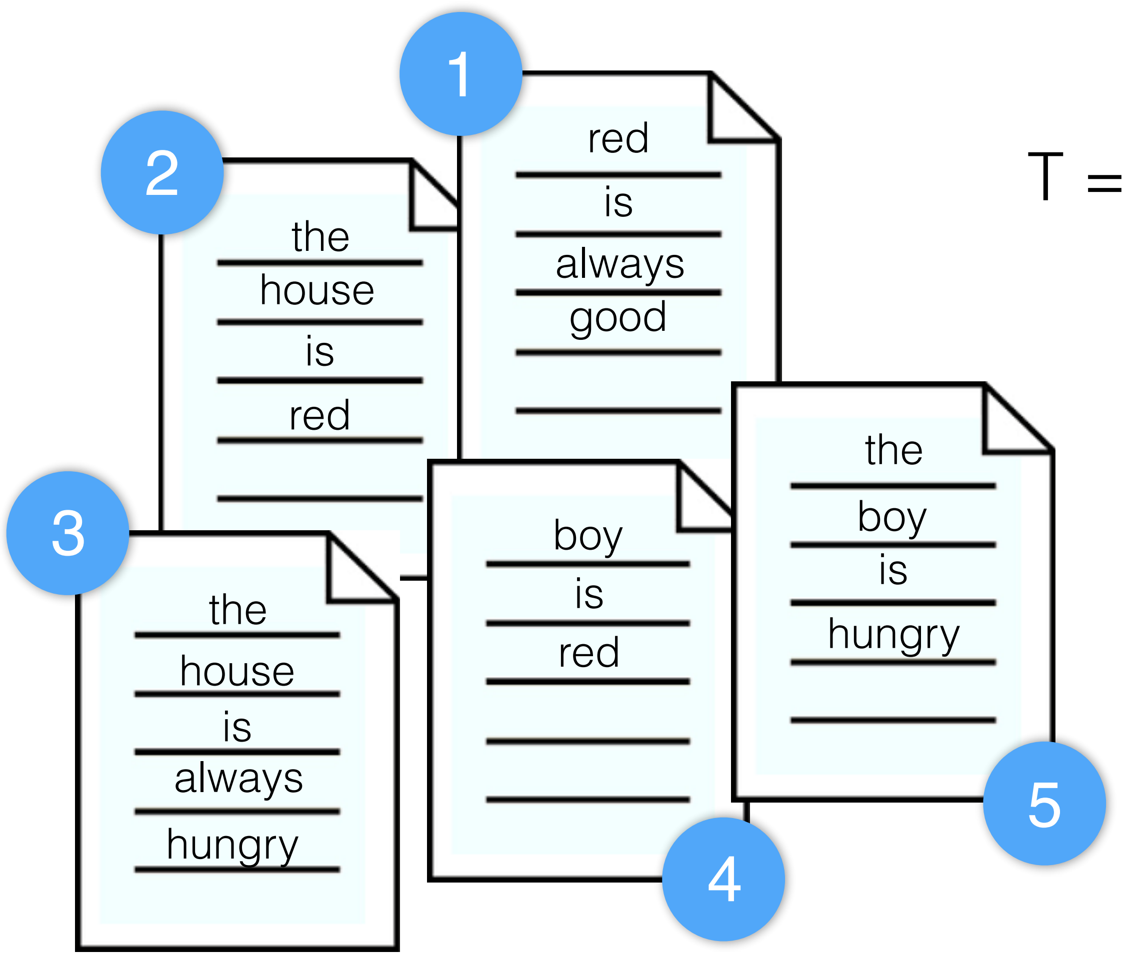

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Inverted Indexes 2

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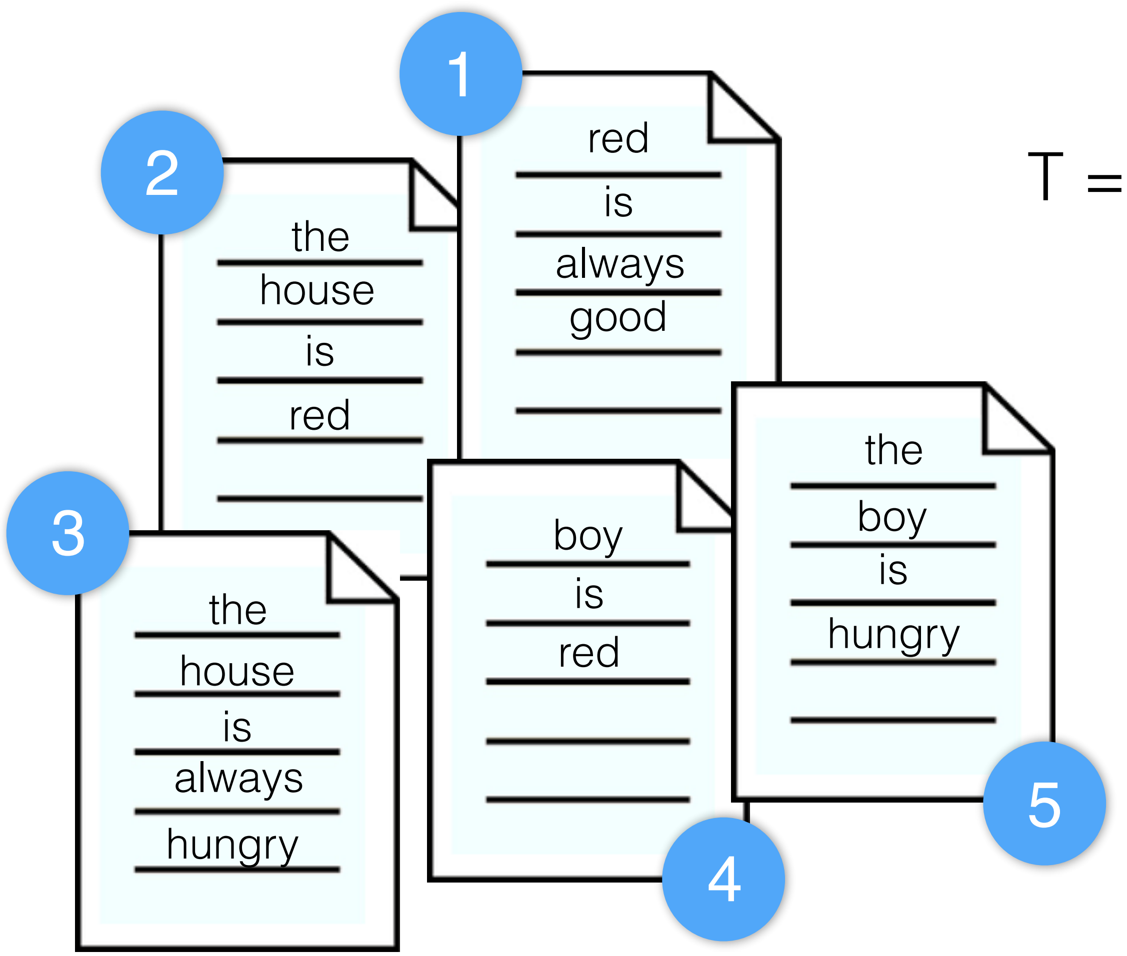

$$\begin{array}{cccccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 & t_7 & t_8 \\ \end{array}$$

$$T = \{\text{always, boy, good, house, hungry, is, red, the}\}$$

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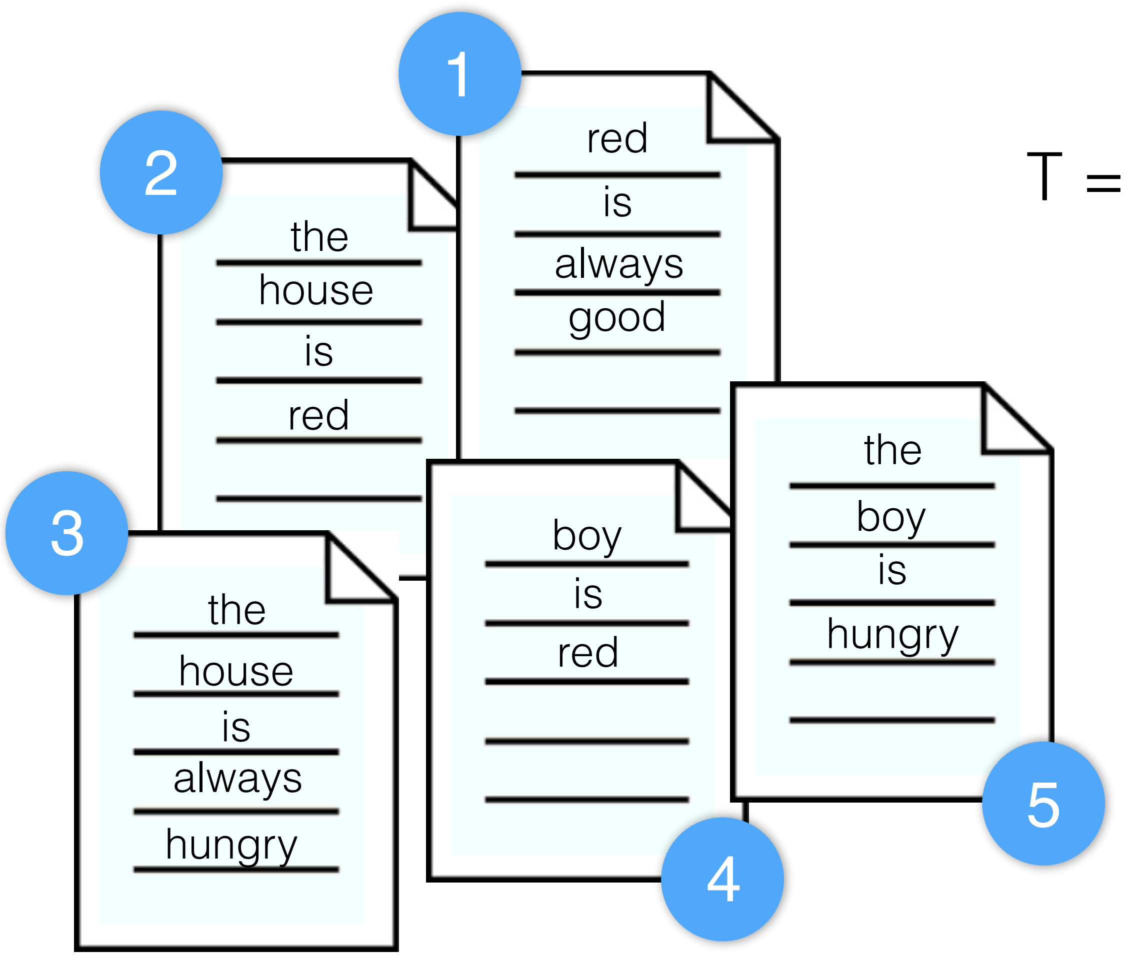

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posting lists intersection

General Problem

Consider a sequence S[0,n) of n *positive* and *monotonically increasing integers*, i.e., $S[i] \leq S[i+1]$ for $0 \leq i < n-1$, possibly repeated.

How to represent it as a *bit vector* in which each original integer is *self-delimited*, using as few as possible bits?

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https://blogs.dropbox.com/tech/2016/09/improving-the-performance-of-full-text-search/

Improving the performance of full-text search

Adam Faulkner | September 7, 2016

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For Firefly, Dropbox full text-search engine, speed has always been a priority.

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Consequence? Query time latencies deteriorate **from 250ms to 1s**.

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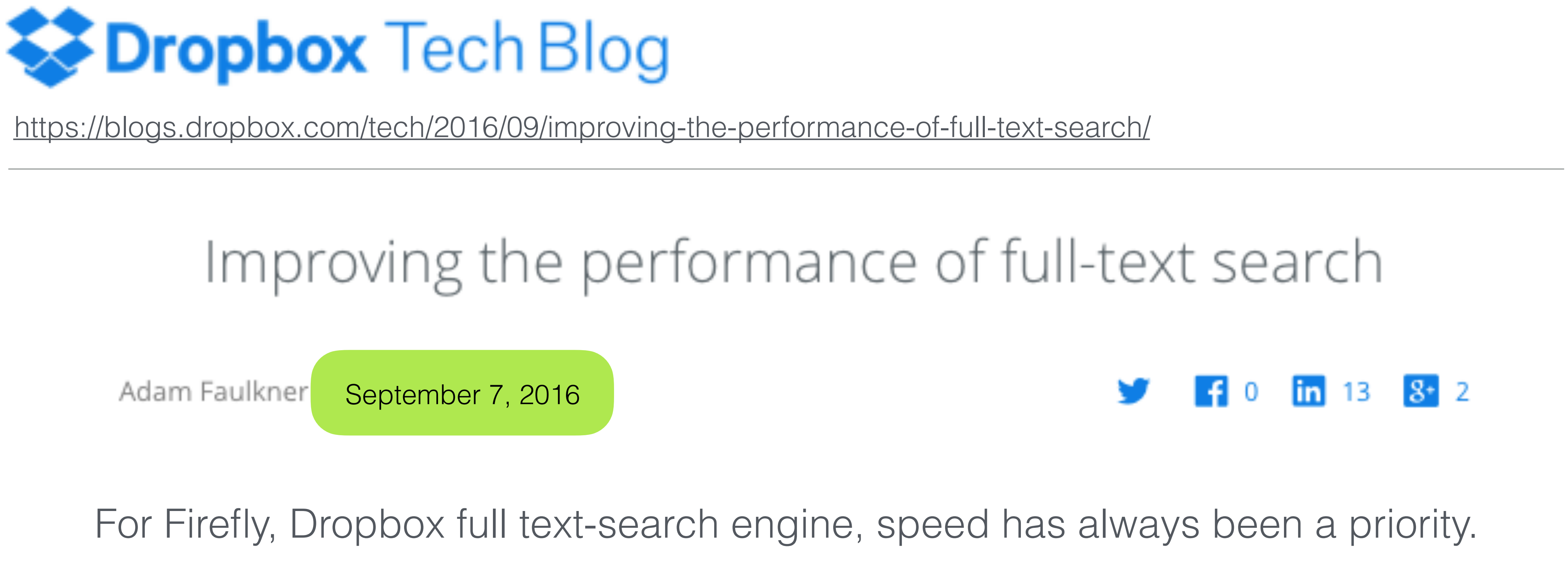

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Data Structures + Data Compression ➡ Faster Algorithms

Elias-Fano - Genesis

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Robert Fano
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Robert Fano. *On the number of bits required to implement an associative memory*. Memorandum 61, Computer Structures Group, MIT (1971).

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40 years later!

Sebastiano Vigna. *Quasi-Succinct Indices*.

In Proceedings of the 6-th ACM International Conference on Web Search and Data Mining (WSDM), 83-92 (2013).

Elias-Fano - Encoding example

3	1
4	2
7	3
13	4
14	5
15	6
21	7
43	8

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	3	1
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u =	43	8

Elias-Fano - Encoding example

2

0 0 0 0 1 1	3	1
0 0 0 1 0 0	4	2
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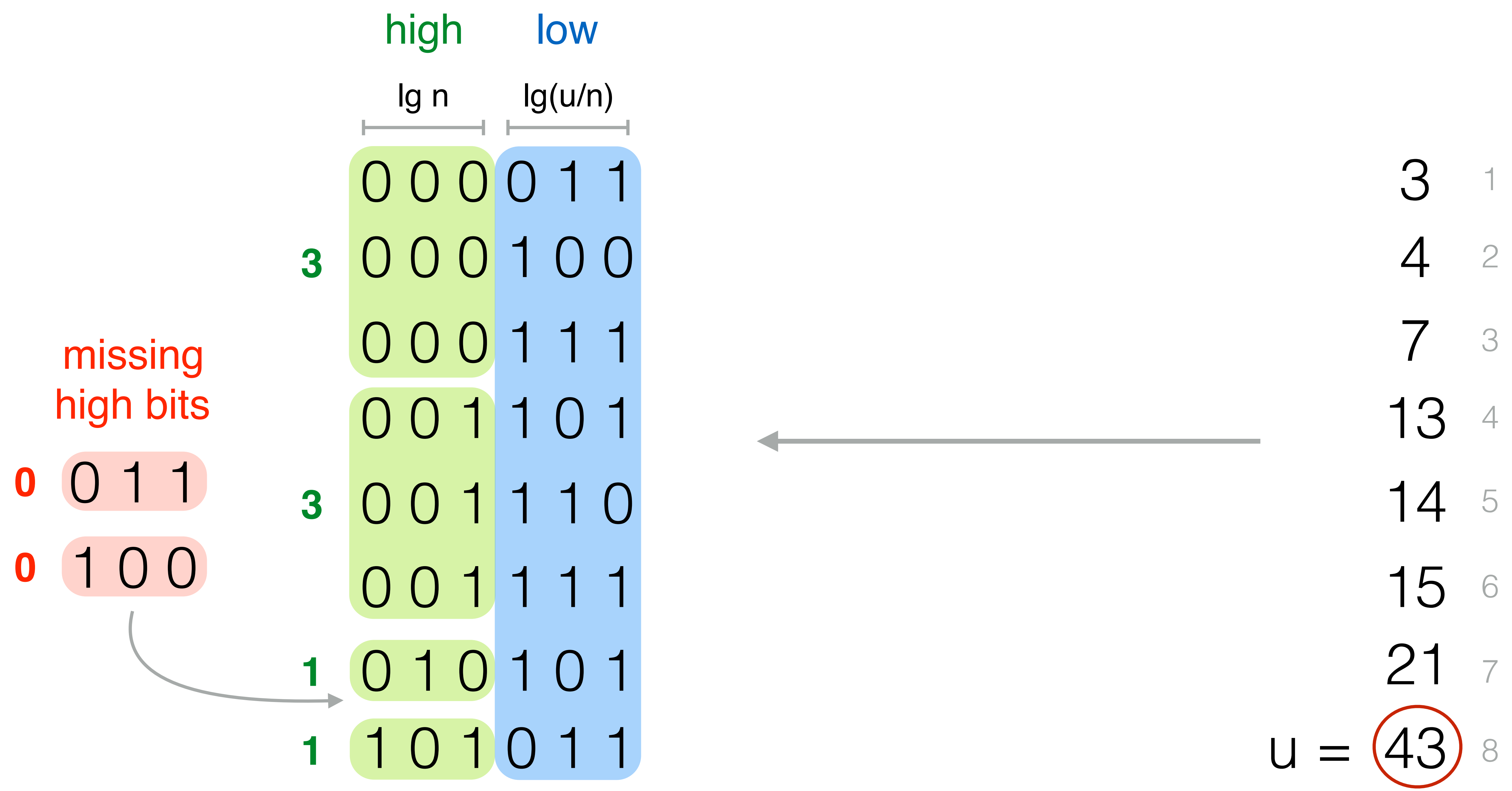

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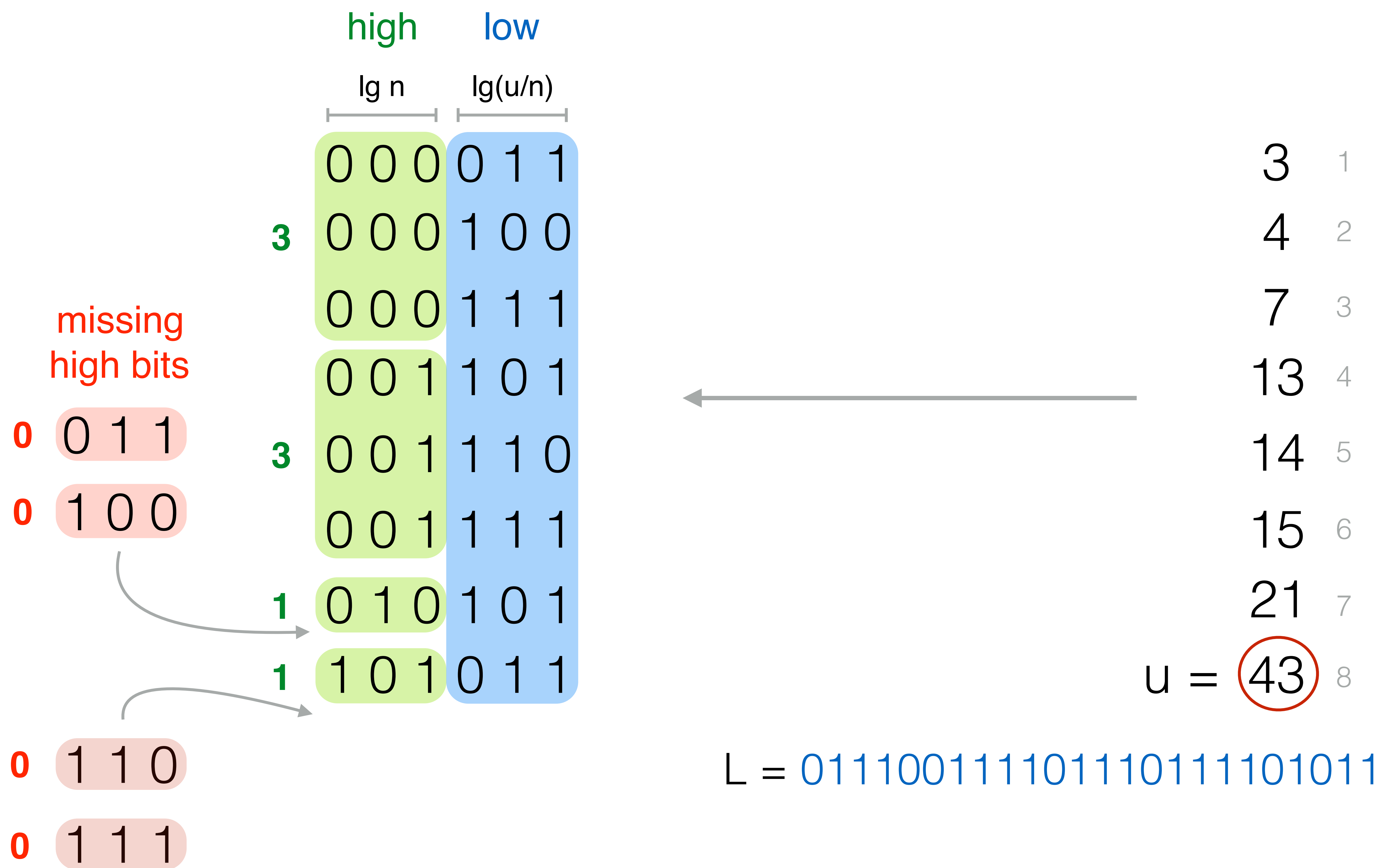

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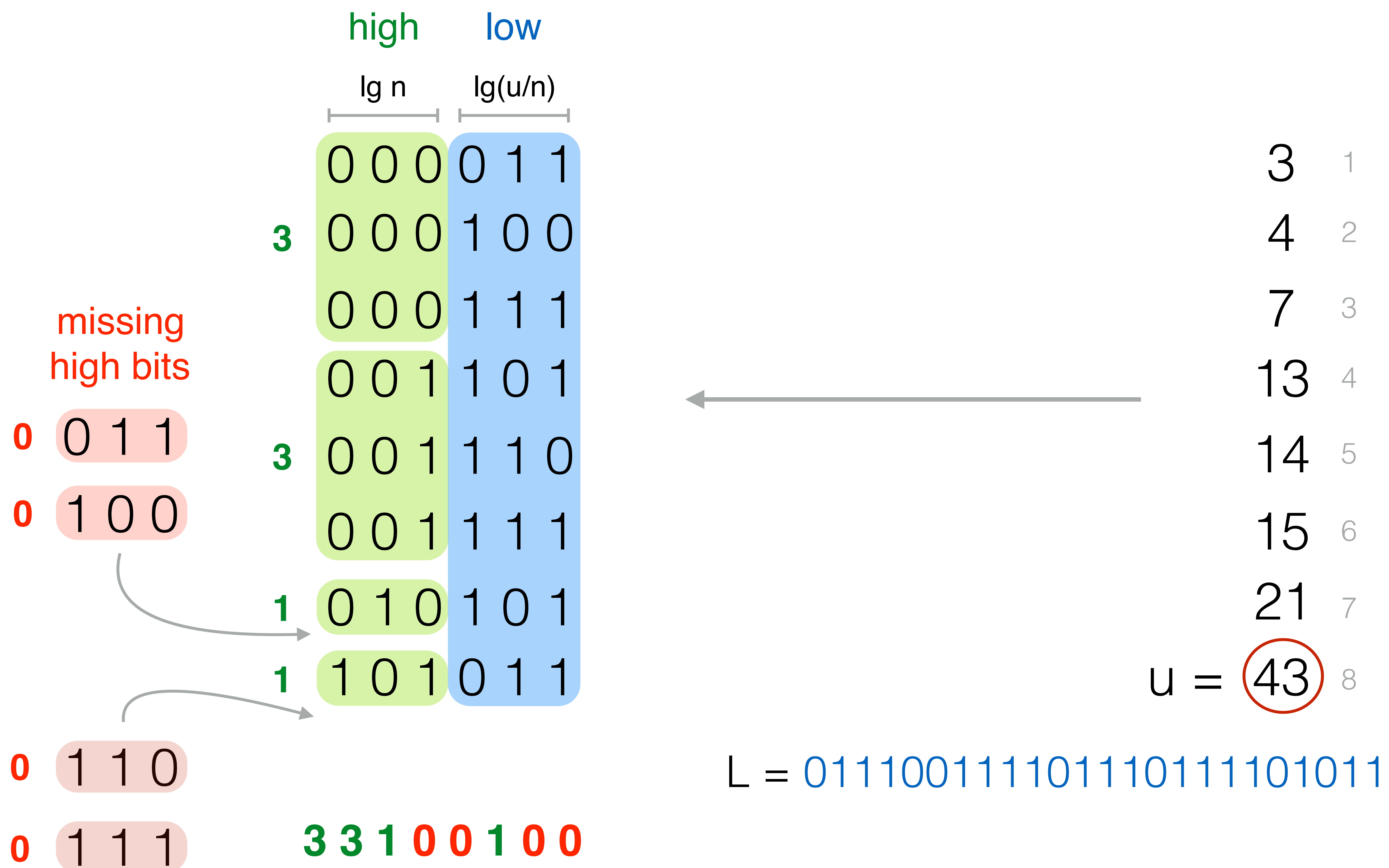

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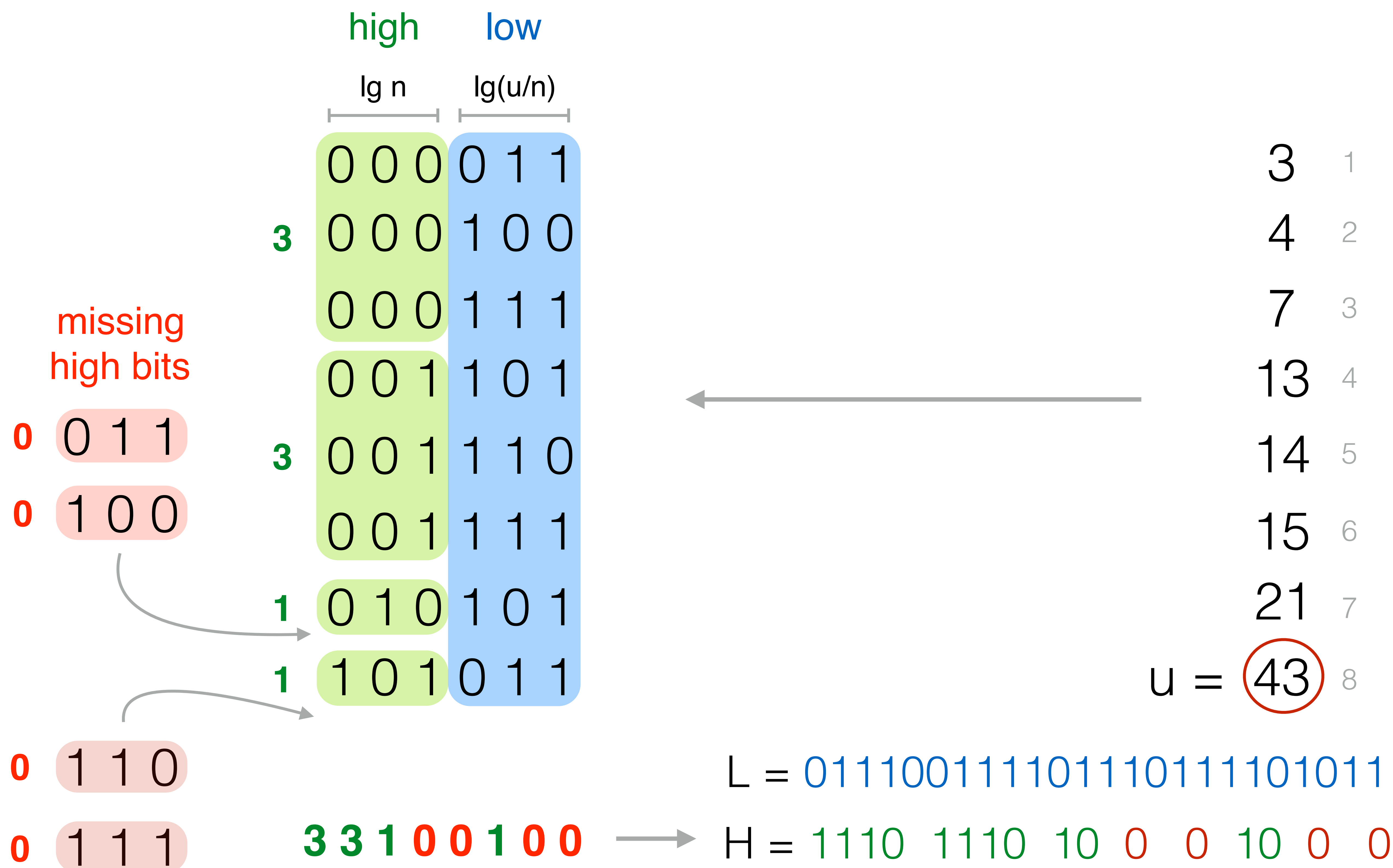

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3 3 1 0 0 1 0 0 → H = 1110 1110 10 0 0 10 0 0

Elias-Fano - Properties

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Clustered Elias-Fano Indexes

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No exploitation of redundancy.

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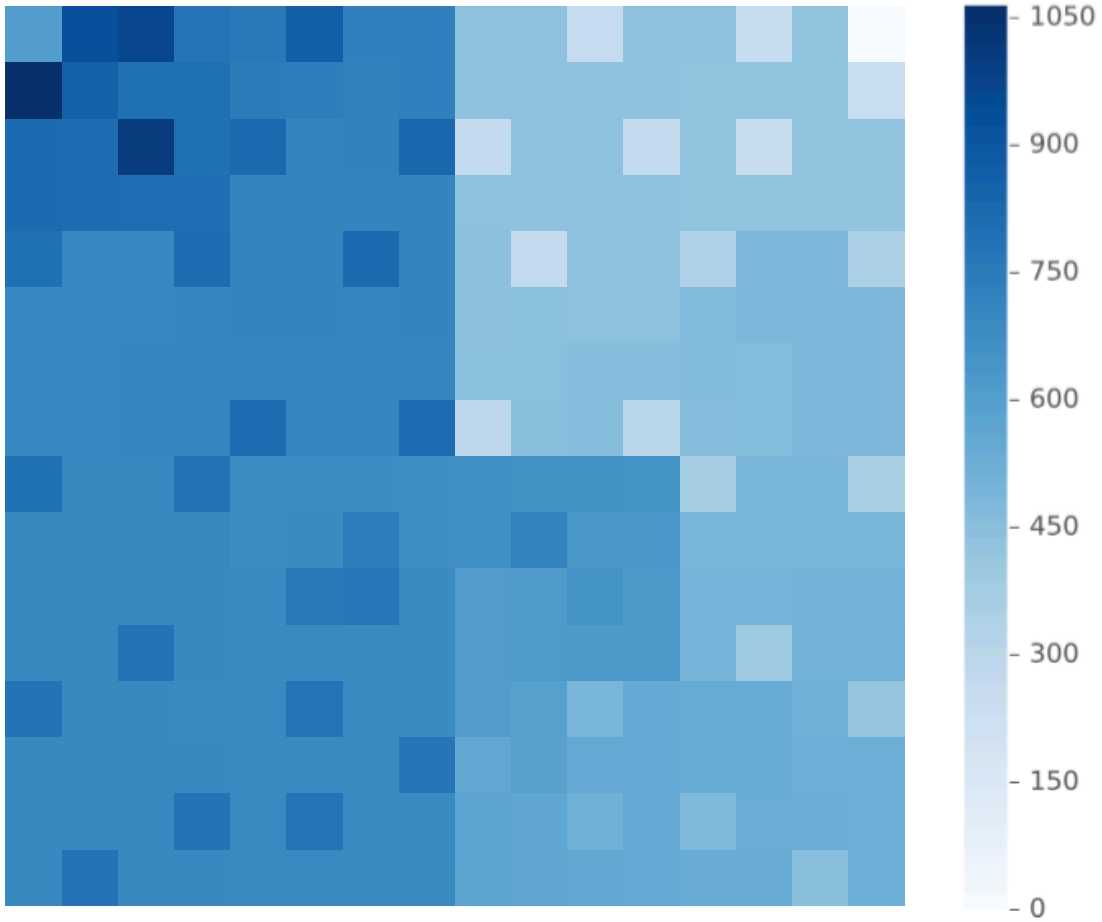

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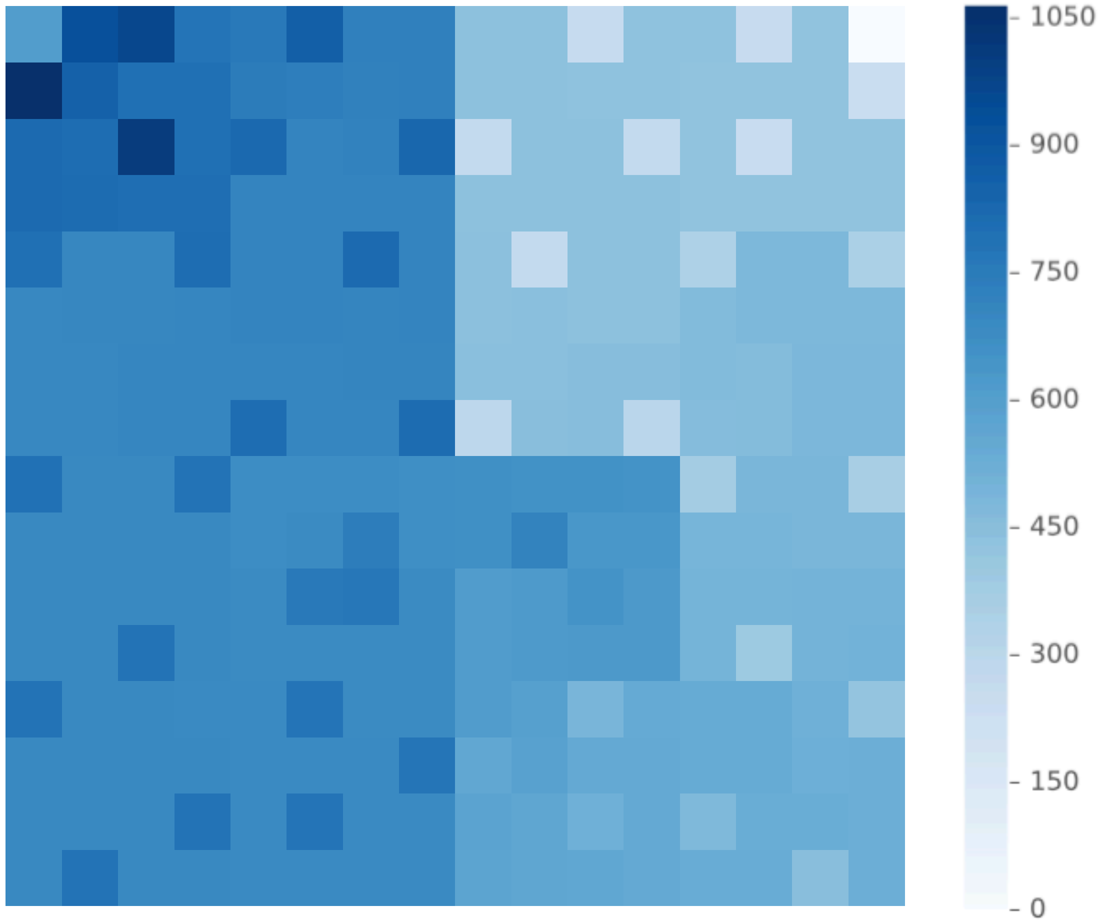
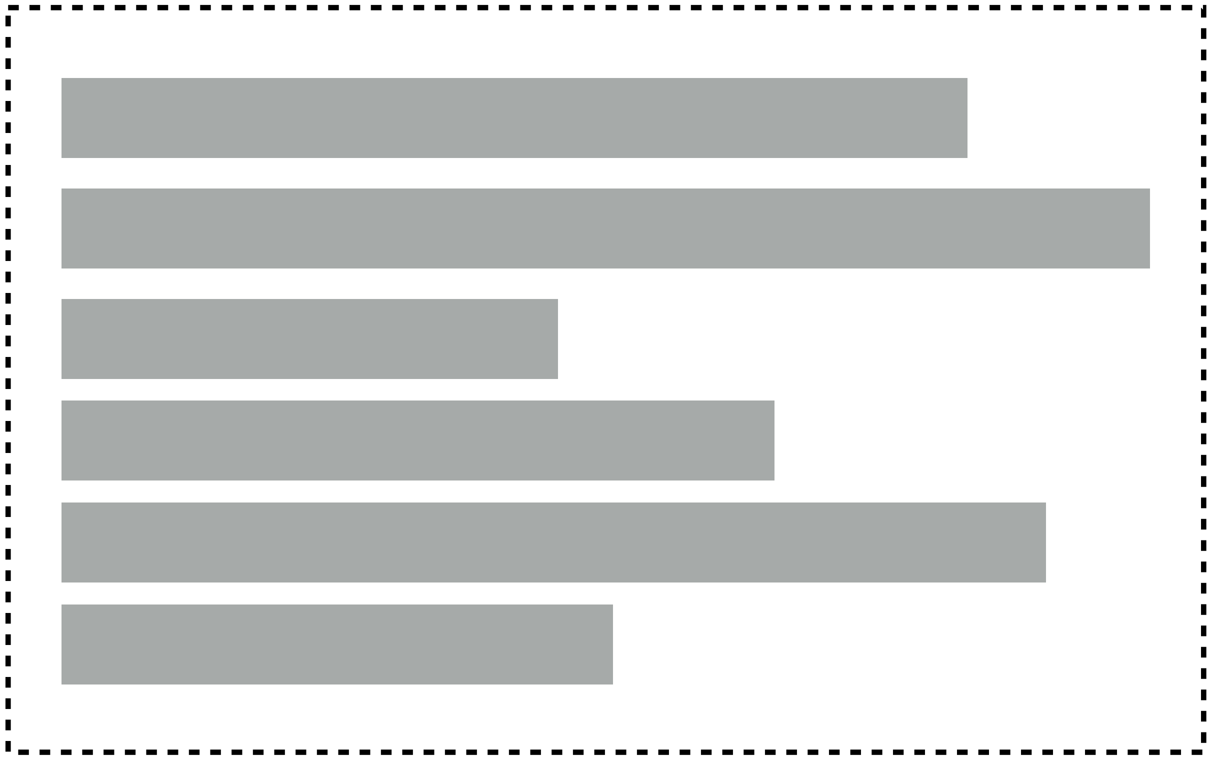


Idea: encode **clusters** of posting lists.

Clustered Elias-Fano Indexes

cluster of posting lists



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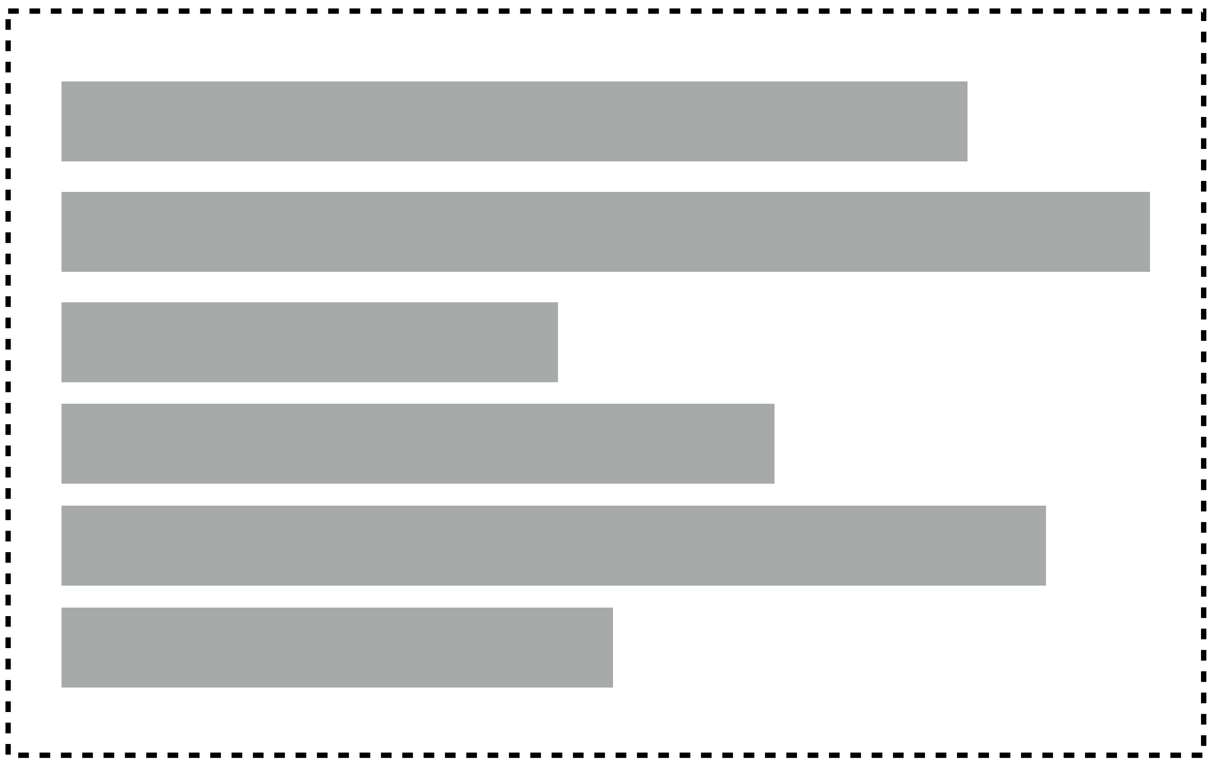


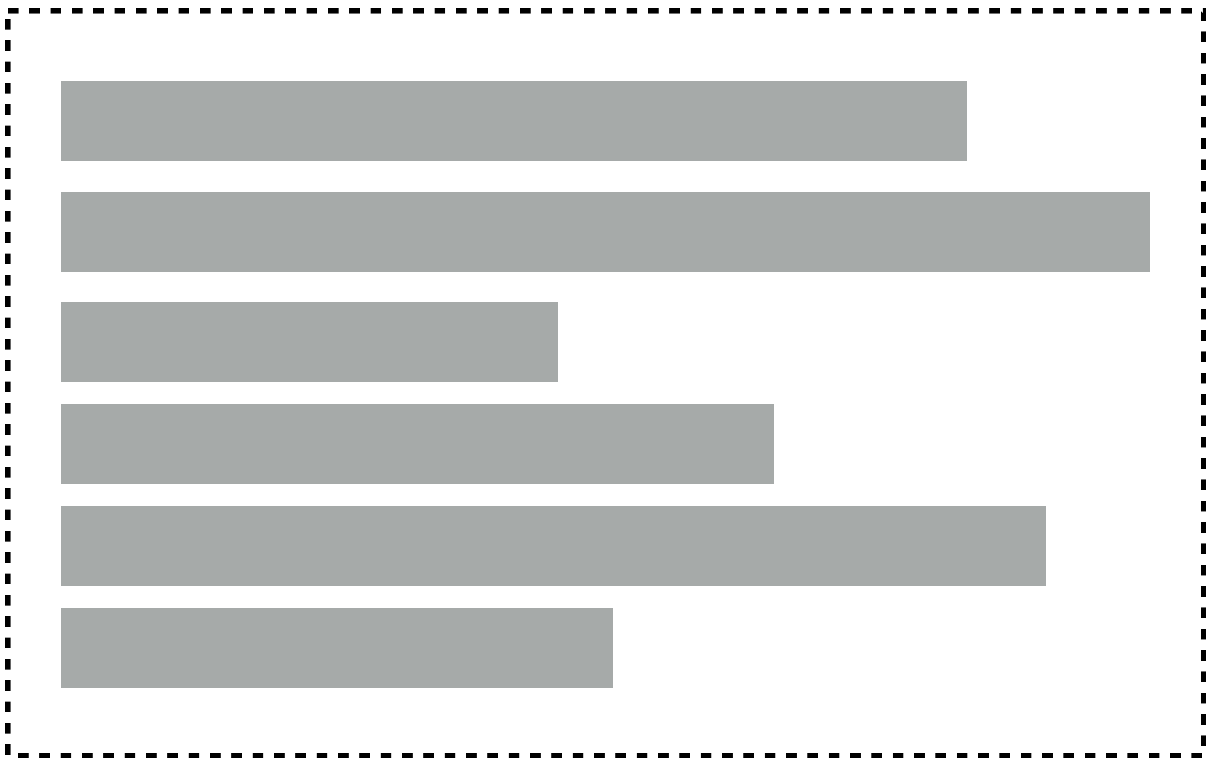
unbounded universe

lg u bits

Clustered Elias-Fano Indexes

Clustered Elias-Fano Indexes 2

cluster of posting lists



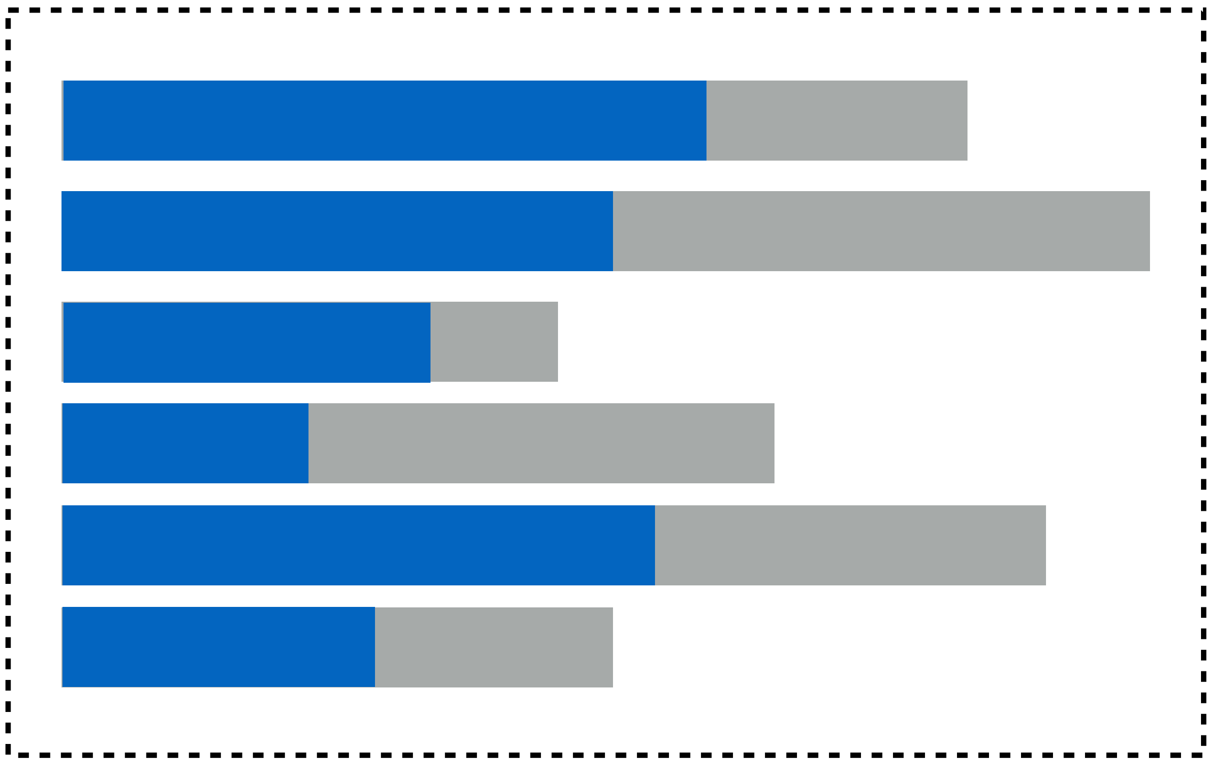
unbounded universe

lg u bits

reference list

R

cluster of posting lists



unbounded universe

lg u bits

reference list

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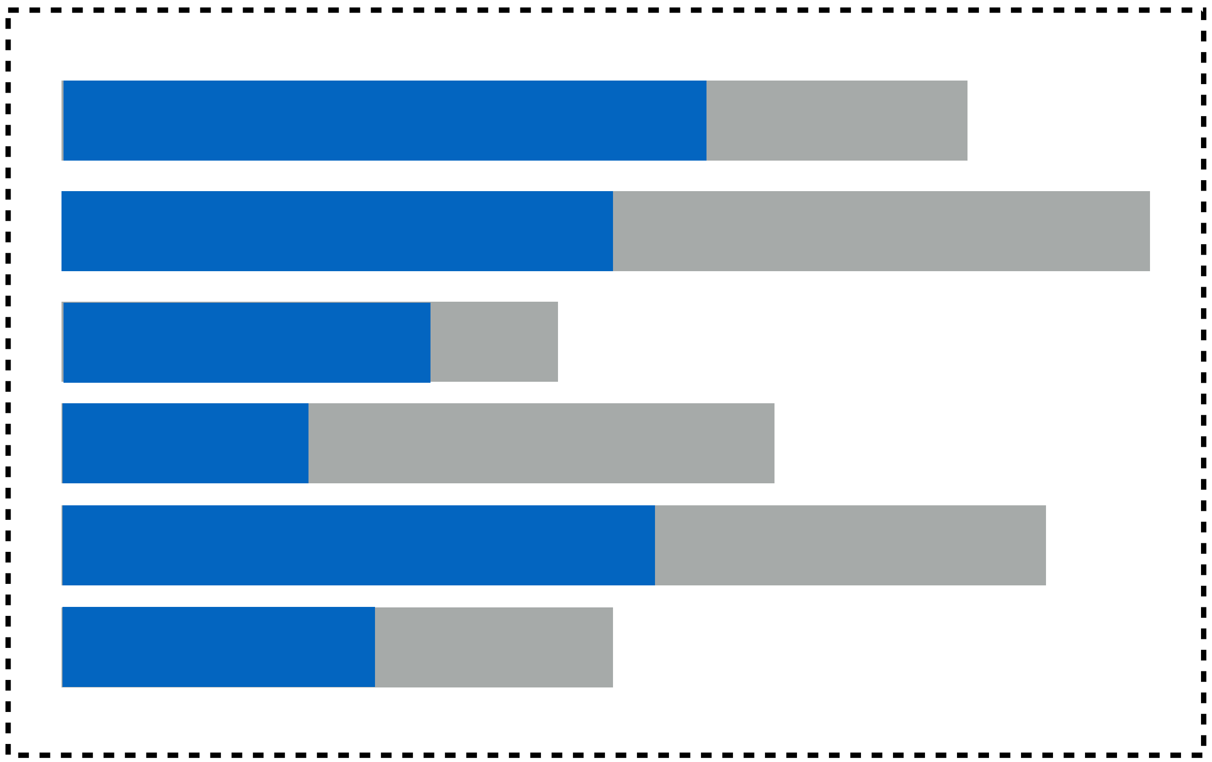


unbounded universe

lg u bits

VS

reference list

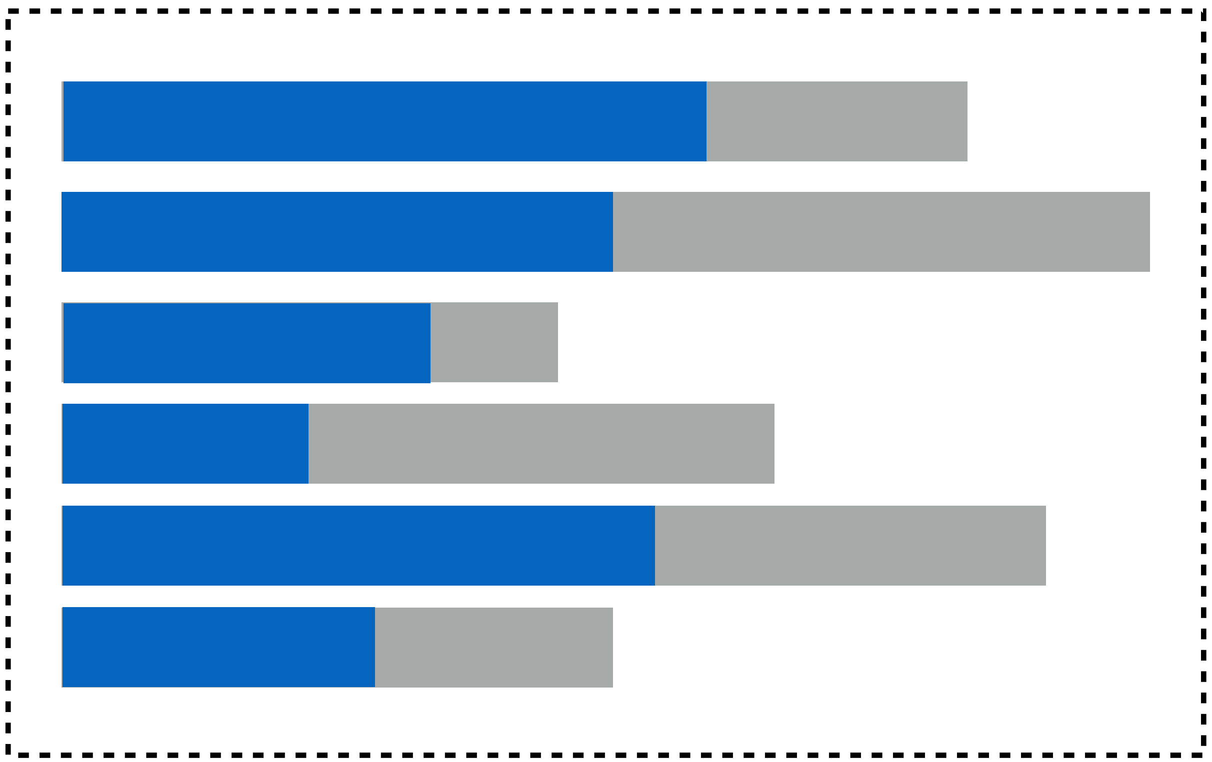
R

R << u

lg R bits

Clustered Elias-Fano Indexes

cluster of posting lists



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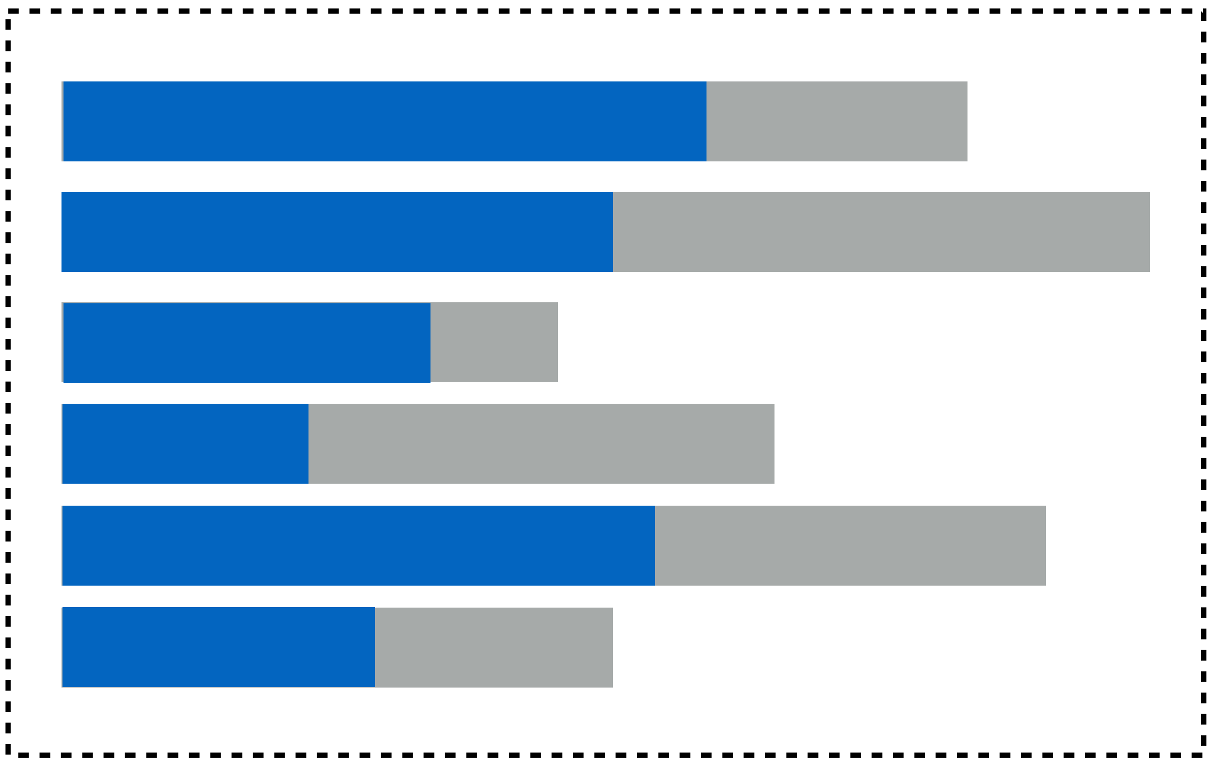
Problems

1. how to build clusters
2. how to synthesise the reference list

Clustered Elias-Fano Indexes 2

cluster of posting lists



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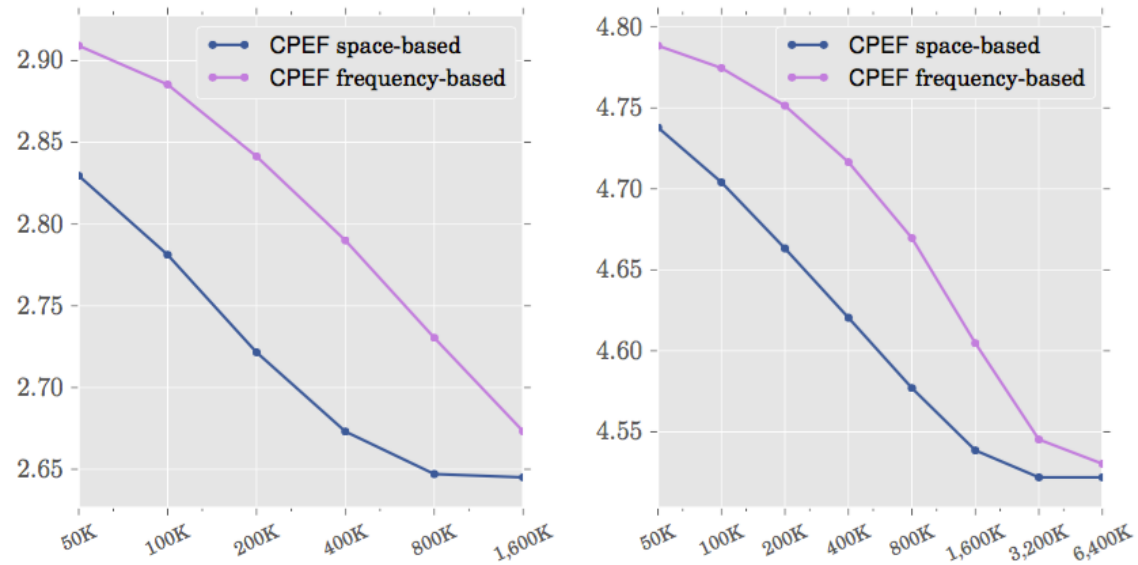
NP-hard problem
already for a simplified formulation.

Time VS Space tradeoffs by varying reference size



(a) Gov2 (b) ClueWeb09

Figure 2: Bits per posting of Gov2 and ClueWeb09 by varying the reference size.

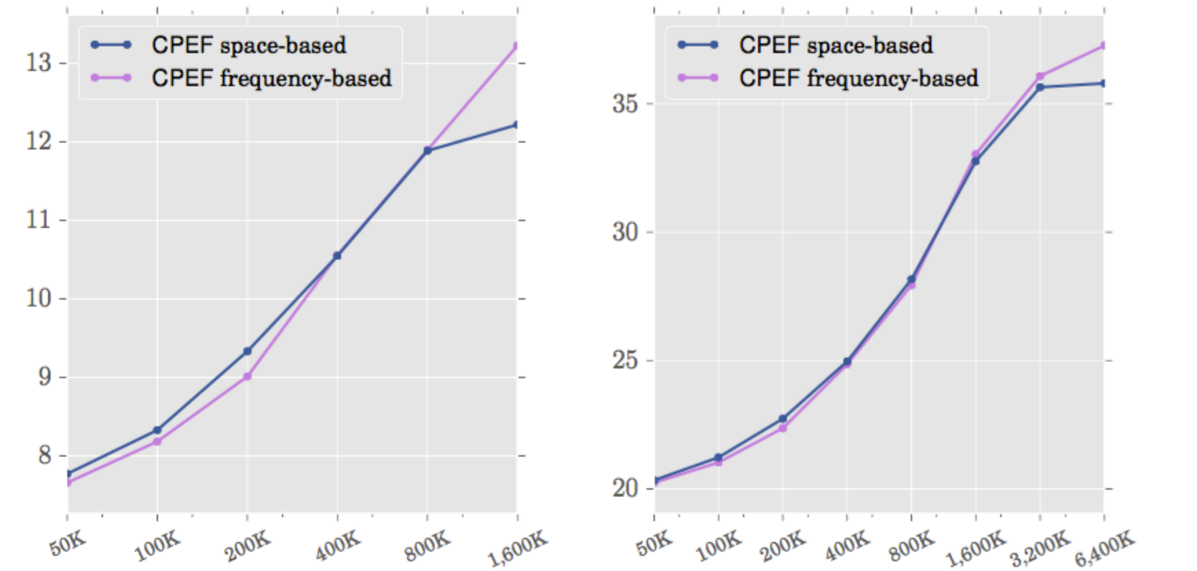


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Figure 3: Timings for AND queries by varying the reference size on Gov2 and ClueWeb09, using the query set TREC 06.

	MIN	MID	MAX
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CPEF	2.78	2.72	2.65
BIC	2.80 (+0.53%)	2.80 (+2.74%)	2.80 (+5.63%)

(a) Gov2

	MIN	MID	MAX
PEF	4.80 (+2.13%)	4.80 (+3.98%)	4.80 (+6.25%)
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	BIC	55.1 (+159.7%)	55.1 (+120.8%)	55.1 (+54.7%)

(a) ClueWeb09

		MIN	MID	MAX
TREC 05	PEF	3.7 (−30.4%)	3.7 (−37.5%)	3.7 (−52.1%)
	CPEF	5.3	5.9	7.8
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TREC 06	PEF	6.1 (−27.4%)	6.1 (−35.2%)	6.1 (−49.1%)
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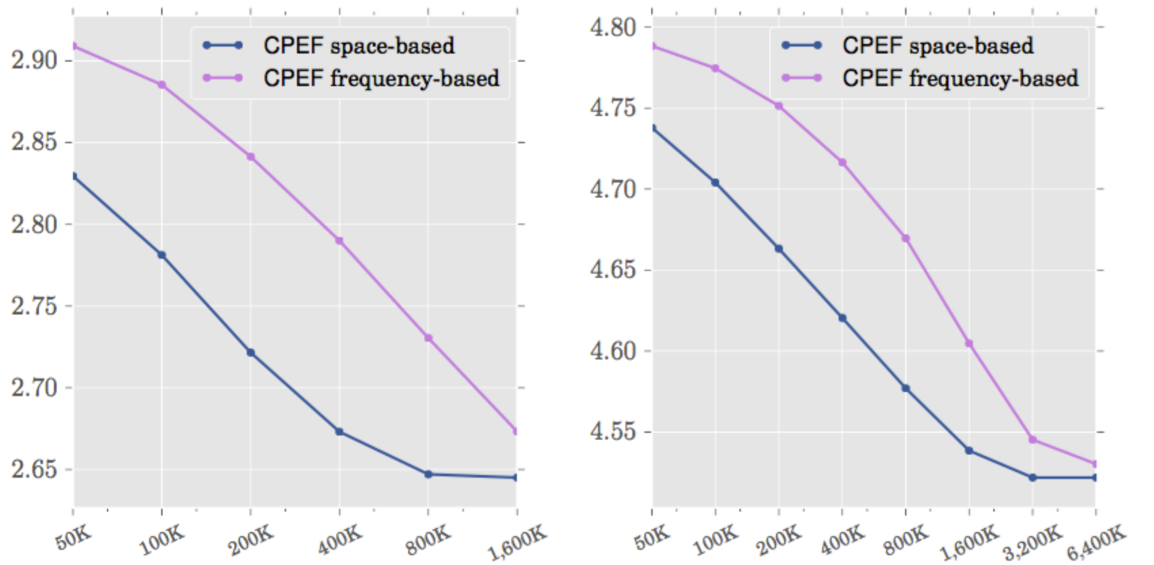
(b) Gov2

Table 3: Timings in milliseconds for AND queries on ClueWeb09 and Gov2, using query sets TREC 05 and TREC 05. In parentheses we show the relative percentage against CPEF.

Time VS Space tradeoffs by varying reference size



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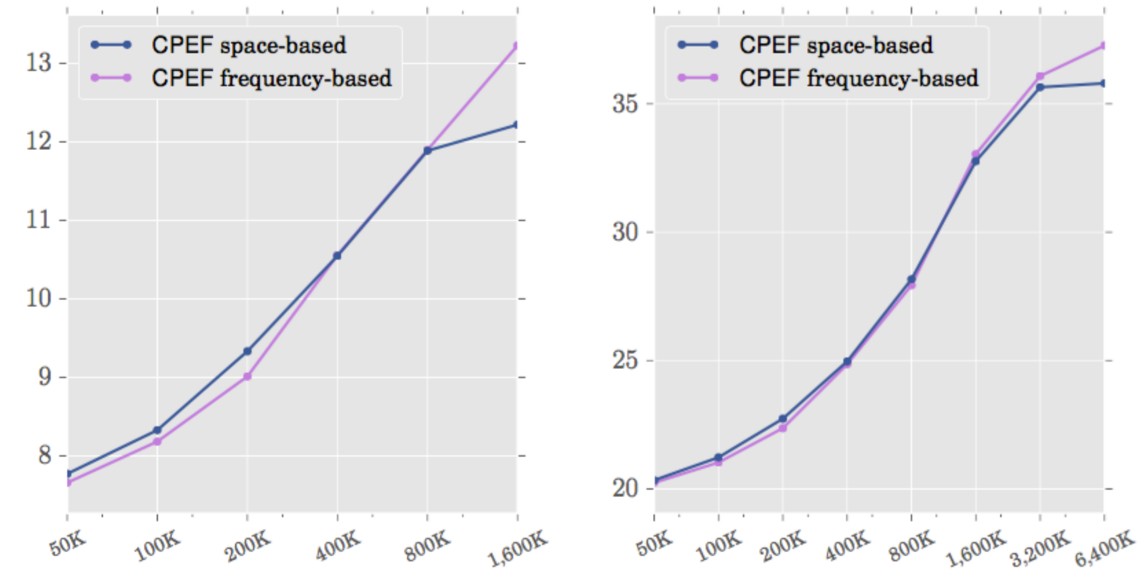

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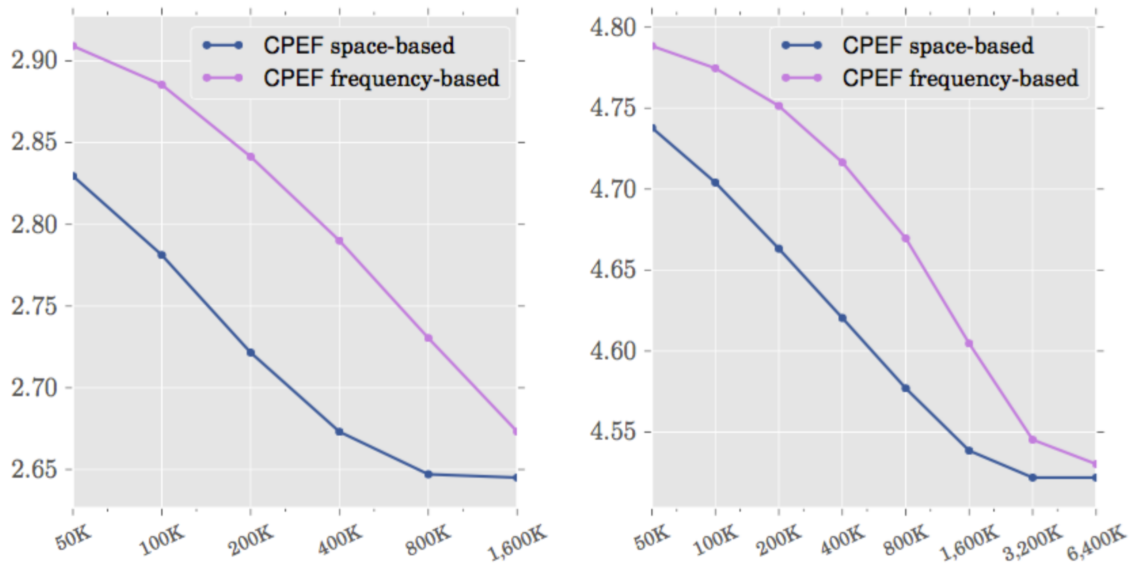

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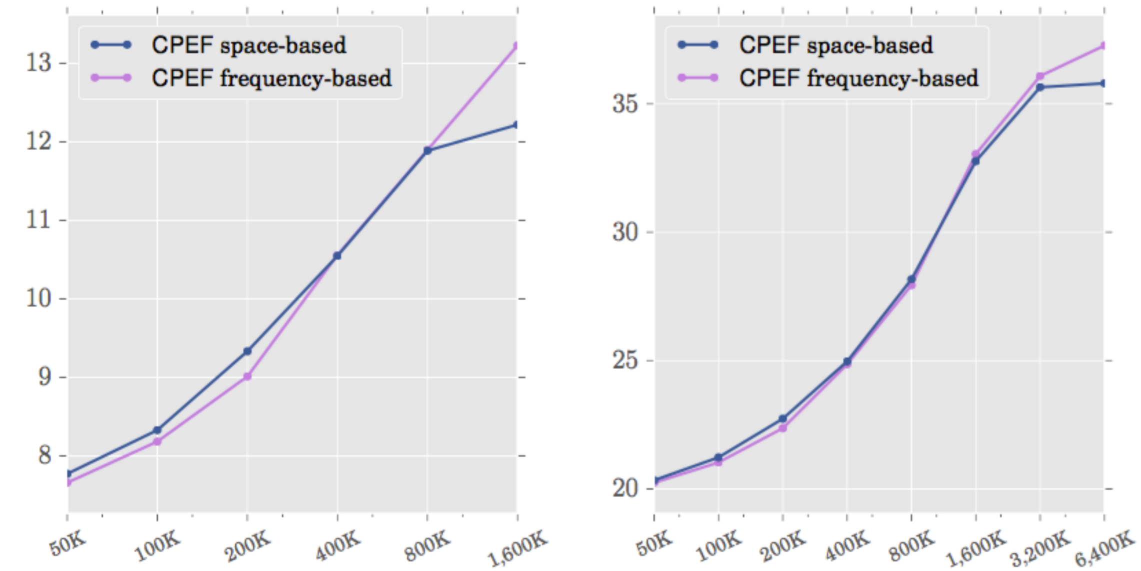

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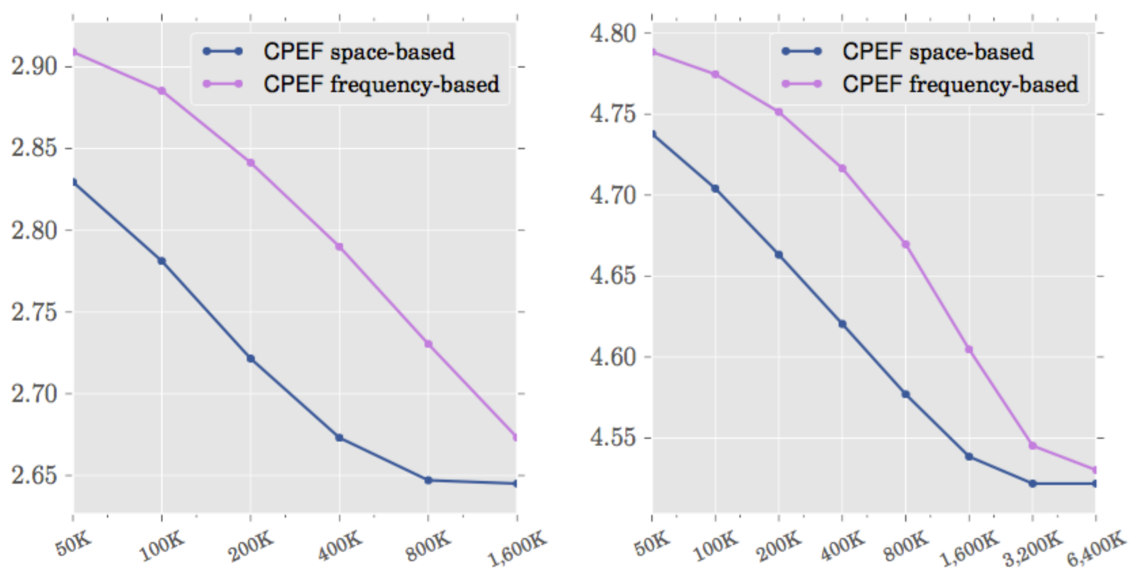

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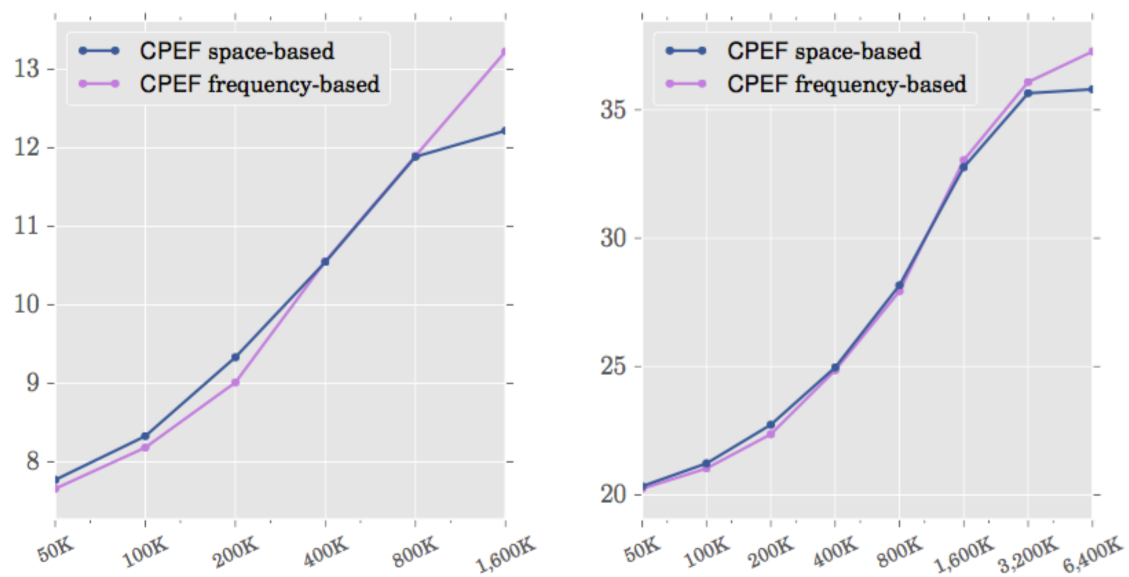

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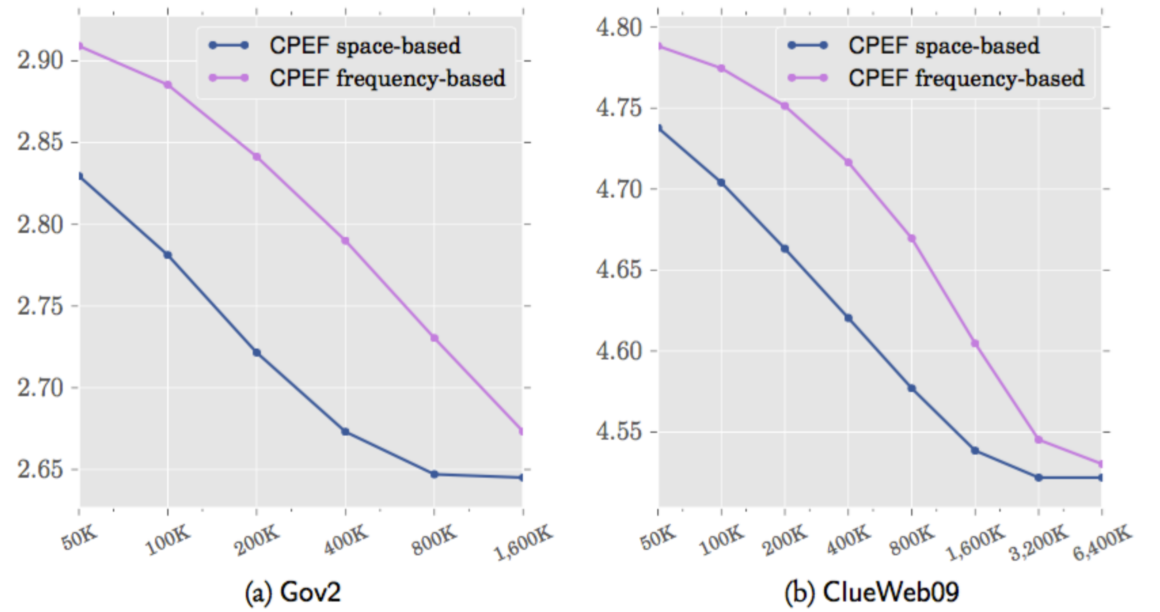

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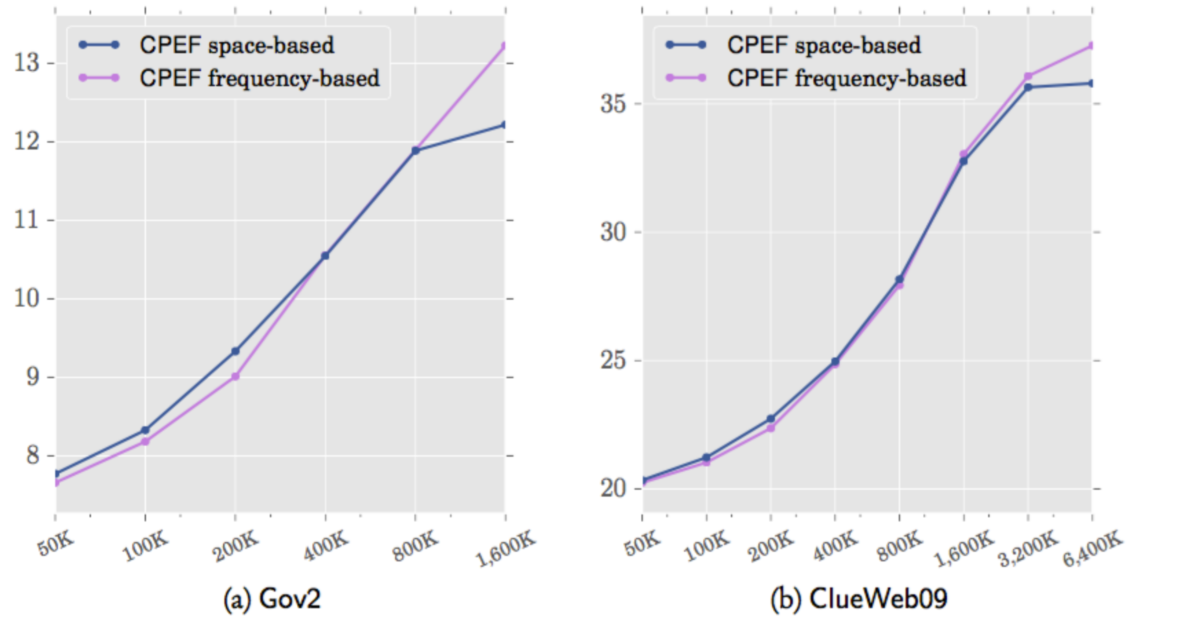

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Much faster than BIC (103% on average)
Slightly slower than PEF (20% on average)

Elias-Fano matches the
information theoretic minimum.

n lg(u/n) + 2n + o(n) bits

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- **O(1)** random access
- **O(lg(u/n))** predecessor/successor

Integer Data Structures

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Static succinct data structure.

NO dynamic updates.

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- Most of them take optimal time

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(or even worse)

The (general) Dictionary problem

The dynamic dictionary problem consists in representing a set S of n objects so that the following operations are supported.

- insert(x) inserts x in S
- delete(x) deletes x from S
- search(x) checks whether x belongs to S
- minimum() returns the minimum element of S
- maximum() returns the maximum element of S
- predecessor(x) returns $\max\{y \in S : y < x\}$
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The Dynamic List Representation problem

[Fredman and Saks, STC 1989]

Given a list S of n sorted integer, support the following operations

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under the assumption that $w \leq \lg^{\gamma} n$ for some γ.

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[Patrascu and Thorup, STC 2007]

Optimal space/time trade-off for a *static* data structure taking m = $n2^a w$ bits, where *a* is the number of bits necessary to represent the mean number of bits per integer, i.e., a = lg(m/n) – lg w

$$\Theta\left(\min\left\{\log_w n, \lg\frac{w-\lg n}{a}, \frac{\lg\frac{w}{a}}{\lg\left(\frac{a}{\lg n}\lg\frac{w}{a}\right)}, \frac{\lg\frac{w}{a}}{\lg\left(\lg\frac{w}{a}/\lg\frac{\lg n}{a}\right)}\right\}\right)$$

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Goals

Goals

$n \lg(u/n) + 2n + o(n)$ bits

Goals

n lg(u/n) + 2n + o(n) bits + **o(n)** bits

Goals

negligible redundancy!

n lg(u/n) + 2n + o(n) bits + **o(n)** bits

Goals

negligible redundancy!

n lg(u/n) + 2n + o(n) bits + **o(n)** bits

1. Extend the *static* Elias-Fano representation to support predecessor and successor queries in optimal worst-case O(lg lg n) time.

2. Maintain S in a *fully dynamic fashion*, supporting in optimal worst-case time all the operations defined in the Dynamic Dictionary and Dynamic List Representation problems.

- **optimal time/space trade-off for successor search** [Patrascu and Thorup, STC 2007]
- **y-fast tries** [Willard, IPL 1983]

▶ **Theorem 1.** *There exists a data structure representing an ordered set* $\mathcal{S}(n, u)$ *of* n *integers drawn from a polynomial universe of size* $u = n^\gamma$*, for any* $\gamma = \Theta(1)$*, that takes* $\mathsf{EF}(\mathcal{S}(n,u)) + o(n)$ *bits of space and supports* Access *in* $\mathcal{O}(1)$ *worst-case and* Predecessor/Successor *queries in optimal* $\mathcal{O}(\min\{1 + \log \frac{u}{n}, \log\log n\})$ *worst-case time.*

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Idea: divide the sequence into **blocks** and use a **y-fast trie** to index the blocks.

Results - Dynamic Elias-Fano

- **optimal time/space trade-off for successor search** [Patrascu and Thorup, STC 2007]
- **y-fast tries** [Willard, IPL 1983]
- **dynamic prefix-sum data structure** [Bille *et al.*, arXiv preprint 2015]

▶ **Lemma 4.** *The total order of the blocks of* $\mathcal{C}$ *can be maintained by using a data structure that takes* $\mathcal{O}(\text{polylog}\, n \cdot \log\log n)$ *bits of space and supports the following operations in* $\mathcal{O}(\log\log n)$ *worst-case time:* $\mathsf{Search}(x)$ *which returns a pointer to the block containing the integer* x*;* $\mathsf{Access}(i)$ *which returns the* i*-th integer of the total order;* $\mathsf{Insert}/\mathsf{Delete}$ *of a block.*

▶ **Theorem 3.** *There exists a data structure representing an ordered set* $\mathcal{S}(n, u)$ *of* n *integers drawn from a polynomial universe of size* $u = n^{\gamma}$*, for any* $\gamma = \Theta(1)$*, that takes* $\mathsf{EF}(\mathcal{S}(n, u)) + o(n)$ *bits of space and supports:* Access *in* $\mathcal{O}(\log n/\log\log n)$ *worst-case;* $\mathsf{Insert}/\mathsf{Delete}$ *in* $\mathcal{O}(\log n/\log\log n)$ *amortized;* $\mathsf{Minimum}/\mathsf{Maximum}$ *in* $\mathcal{O}(1)$ *and* $\mathsf{Predecessor}/\mathsf{Successor}$ *queries in* $\mathcal{O}(\min\{1 + \log \frac{u}{n}, \log\log n\})$ *worst-case time. These time bounds are optimal.*

Results - Dynamic Elias-Fano

- **optimal time/space trade-off for successor search** [Patrascu and Thorup, STC 2007]
- **y-fast tries** [Willard, IPL 1983]
- **dynamic prefix-sum data structure** [Bille *et al.*, arXiv preprint 2015]

▶ **Lemma 4.** *The total order of the blocks of $\mathcal{C}$ can be maintained by using a data structure that takes $\mathcal{O}(\text{polylog}\, n \cdot \log\log n)$ bits of space and supports the following operations in $\mathcal{O}(\log\log n)$ worst-case time:* Search(x) *which returns a pointer to the block containing the integer x;* Access(i) *which returns the i-th integer of the total order;* Insert/Delete *of a block.*

▶ **Theorem 3.** *There exists a data structure representing an ordered set $\mathcal{S}(n, u)$ of n integers drawn from a polynomial universe of size $u = n^\gamma$, for any $\gamma = \Theta(1)$, that takes* EF$(\mathcal{S}(n, u)) + o(n)$ *bits of space and supports:* Access *in $\mathcal{O}(\log n / \log\log n)$ worst-case;* Insert/Delete *in $\mathcal{O}(\log n / \log\log n)$ amortized;* Minimum/Maximum *in $\mathcal{O}(1)$ and* Predecessor/Successor *queries in $\mathcal{O}(\min\{1 + \log \frac{u}{n}, \log\log n\})$ worst-case time. These time bounds are optimal.*

Idea: use a 2-level indexing data structure.

- First level indexes **blocks** using a **y-fast trie** and the **dynamic prefix-sum** data structure by Bille *et al.*
- Second level indexes **mini blocks** using the data structure of the Lemma.

N-grams

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As an example, in Chapter 2, we will see two algorithms for sorting. The first, known as ***insertion sort***, takes time roughly equal to $c_1 n^2$ to sort n items, where c_1 is a constant that does not depend on n. That is, it takes time roughly proportional to n^2. The second, ***merge sort***, takes time roughly equal to $c_2 n \lg n$, where $\lg n$ stands for $\log_2 n$ and c_2 is another constant that also does not depend on n. Insertion sort typically has a smaller constant factor than merge sort, so that $c_1 < c_2$. We shall see that the constant factors can have far less of an impact on the running time than the dependence on the input size n. Let's write insertion sort's running time as $c_1 n \cdot n$ and merge sort's running time as $c_2 n \cdot \lg n$. Then we see that where insertion sort has a factor of n in its running time, merge sort has a factor of $\lg n$, which is much smaller. (For example, when $n = 1000$, $\lg n$ is approximately 10, and when n equals one million, $\lg n$ is approximately only 20.) Although insertion sort usually runs faster than merge sort for small input sizes, once the input size n becomes large enough, merge sort's advantage of $\lg n$ vs. n will more than compensate for the difference in constant factors. No matter how much smaller c_1 is than c_2, there will always be a crossover point beyond which merge sort is faster.

27

N = 1	**N = 2**
different	different algorithms
algorithms	algorithms devised
devised	devised to
⋮	

N-grams

Strings of at most N words.

N typically ranges from 1 to 5.

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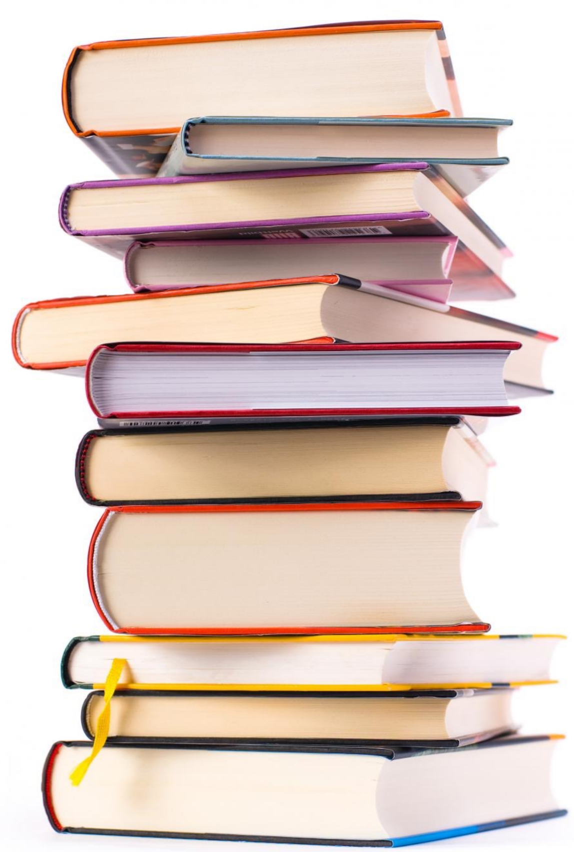
N = 1	**N = 2**
different	different algorithms
algorithms	algorithms devised
devised	devised to
⋮	⋮

N	number of grams
1	8761
2	38900
3	61516
4	70186
5	73187

N-grams

Strings of at most N words.

N typically ranges from 1 to 5.



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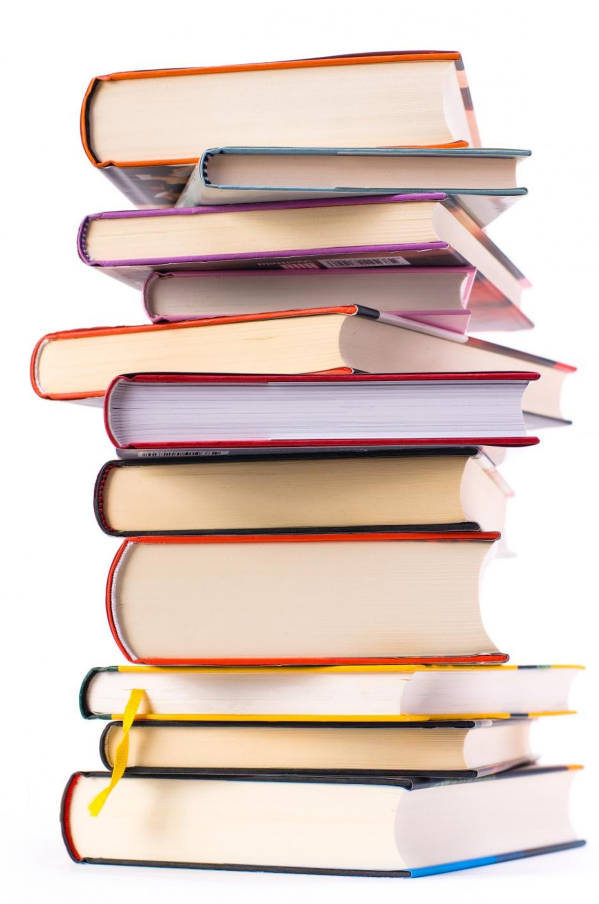

Google Books

~6% of the books ever published

N-grams

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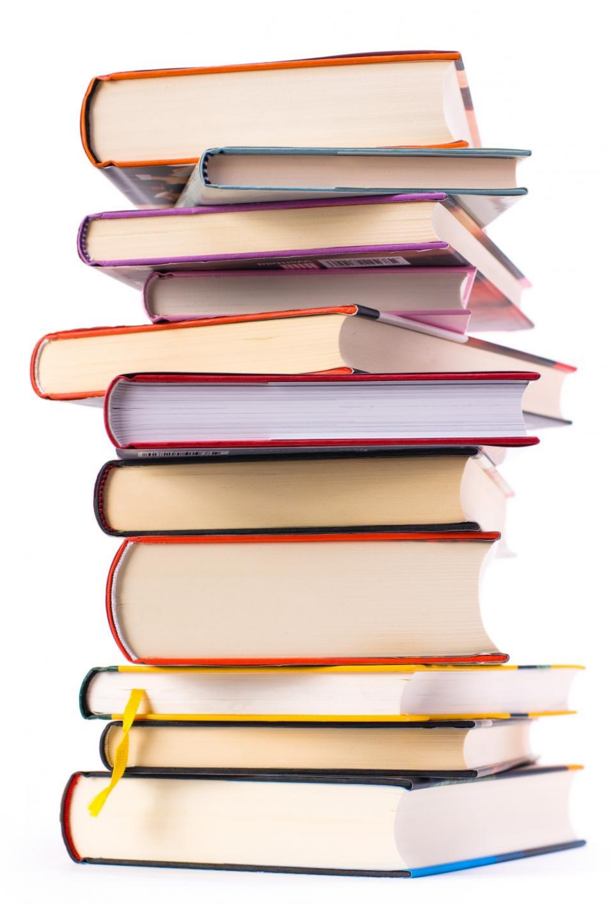

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N	number of grams
1	24,359,473
2	667,284,771
3	7,397,041,901
4	1,644,807,896
5	1,415,355,596

More than 11 billion grams.

N-grams - Why? 2

Word prediction.

N-grams - Why?

Word prediction.

space and time-efficient $\boxed{\quad ? \quad}$

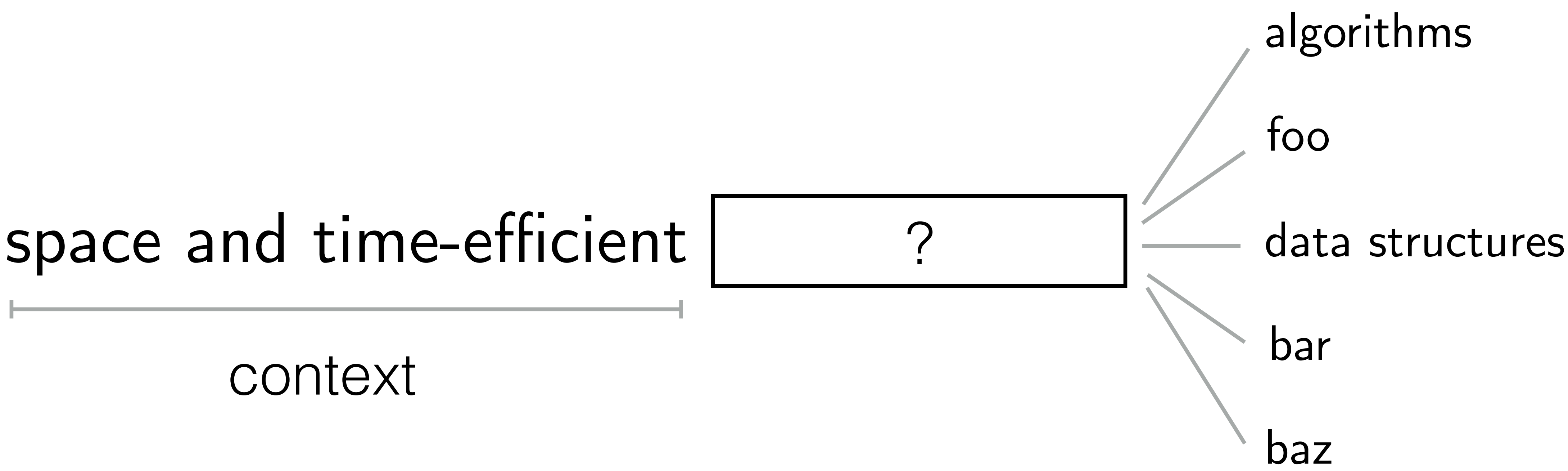
N-grams - Why?

Word prediction.

space and time-efficient [?]

context

Word prediction.



N-grams - Why? 2

Word prediction.

space and time-efficient ?

context

	frequency count
algorithms	**1214**
foo	**2**
data structures	**3647**
bar	**3**
baz	**1**

N-grams - Why?

Word prediction.

space and time-efficient ?

context

	frequency count
algorithms	**1214**
foo	**2**
data structures	**3647**
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baz	**1**

$$\mathcal{P}\left(\text{``data structures''} \mid \text{``space and time-efficient''}\right) \approx \frac{f\left(\text{``space and time-efficient data structures''}\right)}{f\left(\text{``space and time-efficient''}\right)}$$

N-grams - Who cares?

Google Research Blog

The latest news from Research at Google

All Our N-gram are Belong to You

Thursday, August 03, 2006

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word n-gram models for a variety of R&D projects, such as statistical machine translation, speech recognition, spelling correction, entity detection, information extraction, and others. While such models have usually been estimated from training corpora containing at most a few billion words, we have been harnessing the vast power of Google's datacenters and distributed processing infrastructure to process larger and larger training corpora. We found that there's no data like more data, and scaled up the size of our data by one order of magnitude, and then another, and then one more - resulting in a training corpus of *one trillion words* from public Web pages.

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What can I help you with?

What can I
help you with?

Siri

N-grams - Challenge

Store massive N-grams datasets such that given a pattern, we can return its frequency count at light speed.

N-grams - Challenge

Store massive N-grams datasets such that given a pattern, we can return its frequency count at light speed.

Efficient map.

N-grams - Challenge

Store massive N-grams datasets such that given a pattern, we can return its frequency count at light speed.

Efficient map.

Data Structures **+** Data Compression ➡ Faster Algorithms

Open-addressing VS **Tries**

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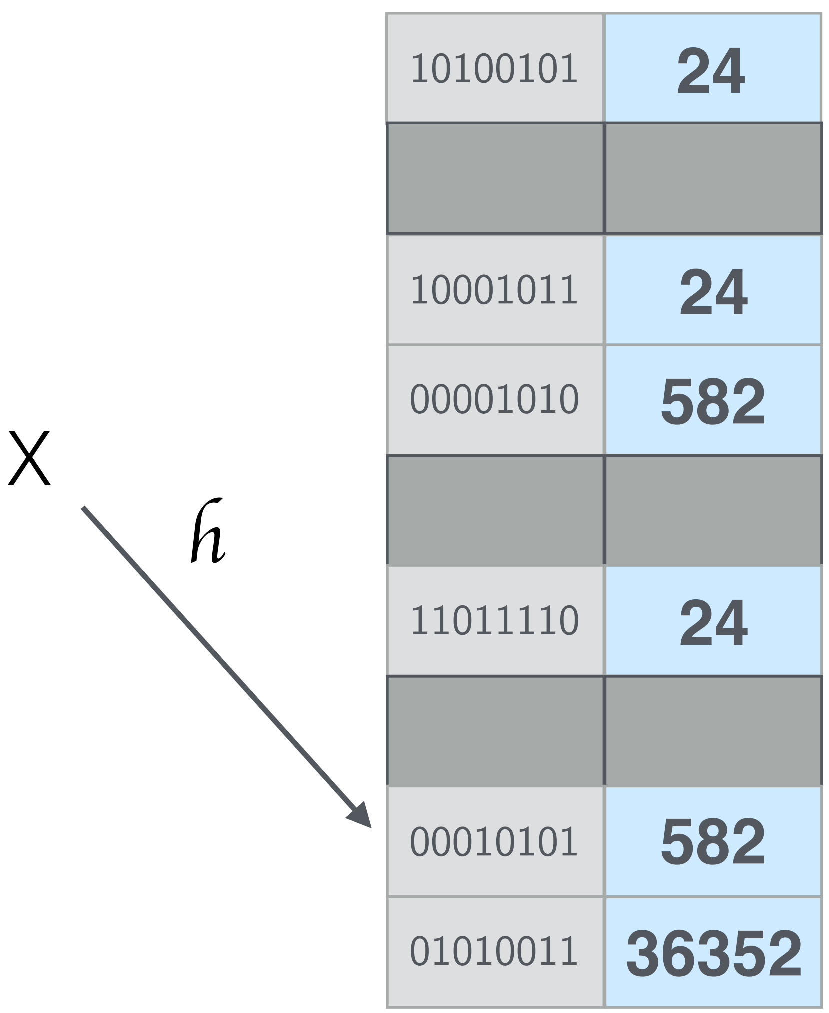
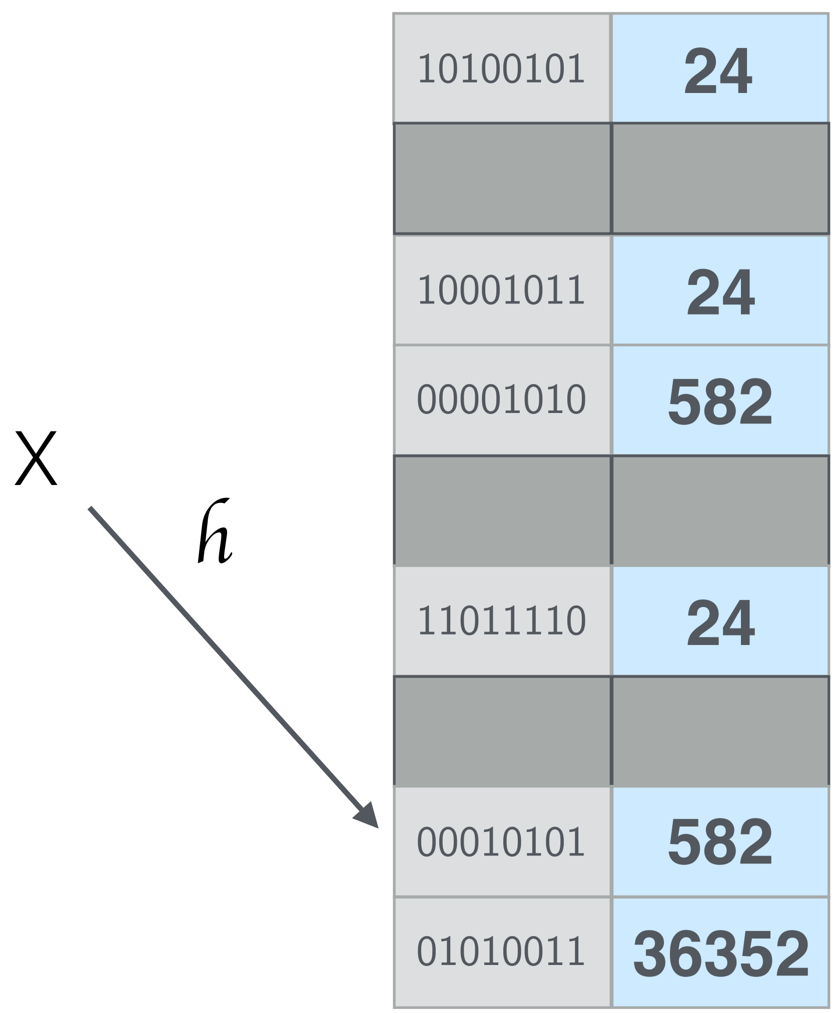
10100101	24
10001011	24
00001010	582
11011110	24
00010101	582
01010011	36352

Open-addressing VS **Tries**



Open-addressing VS Tries

+ time
- space



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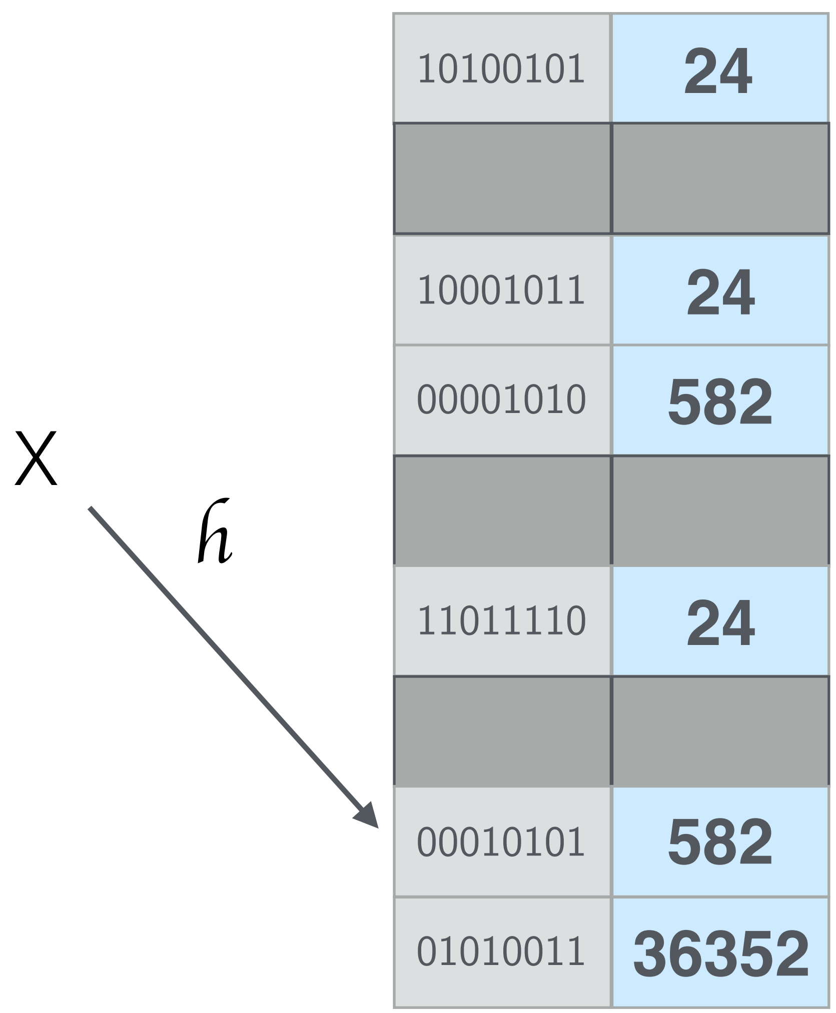

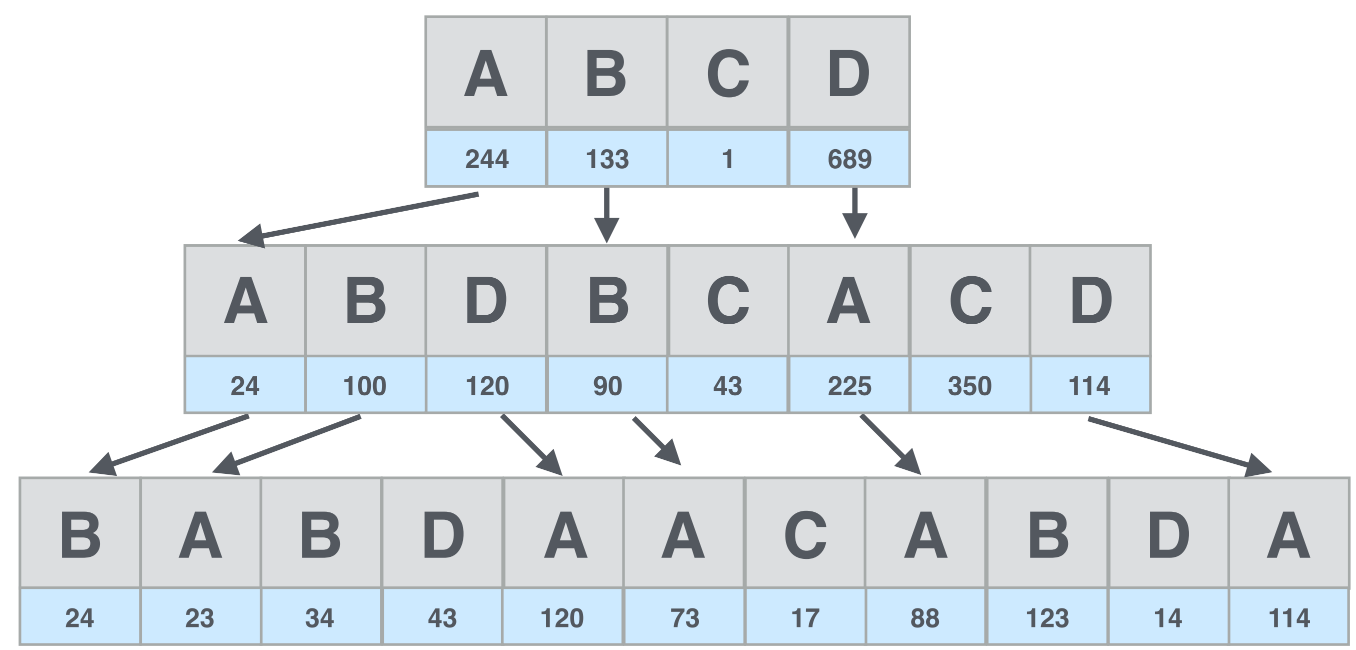

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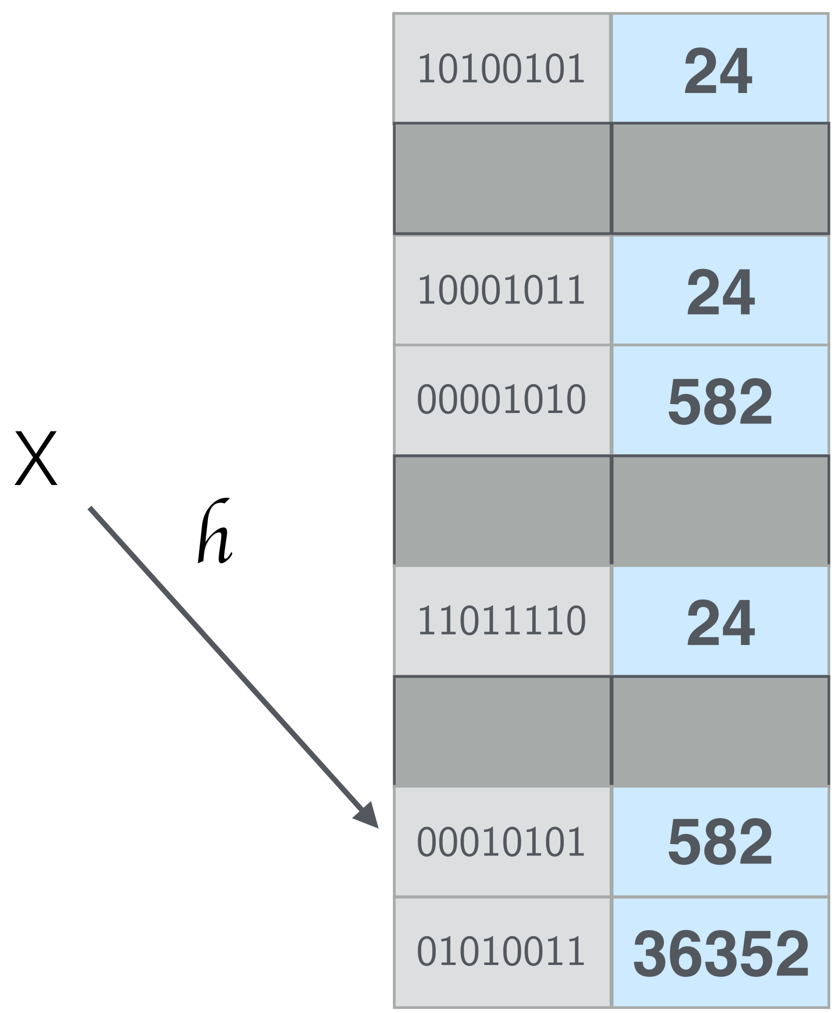

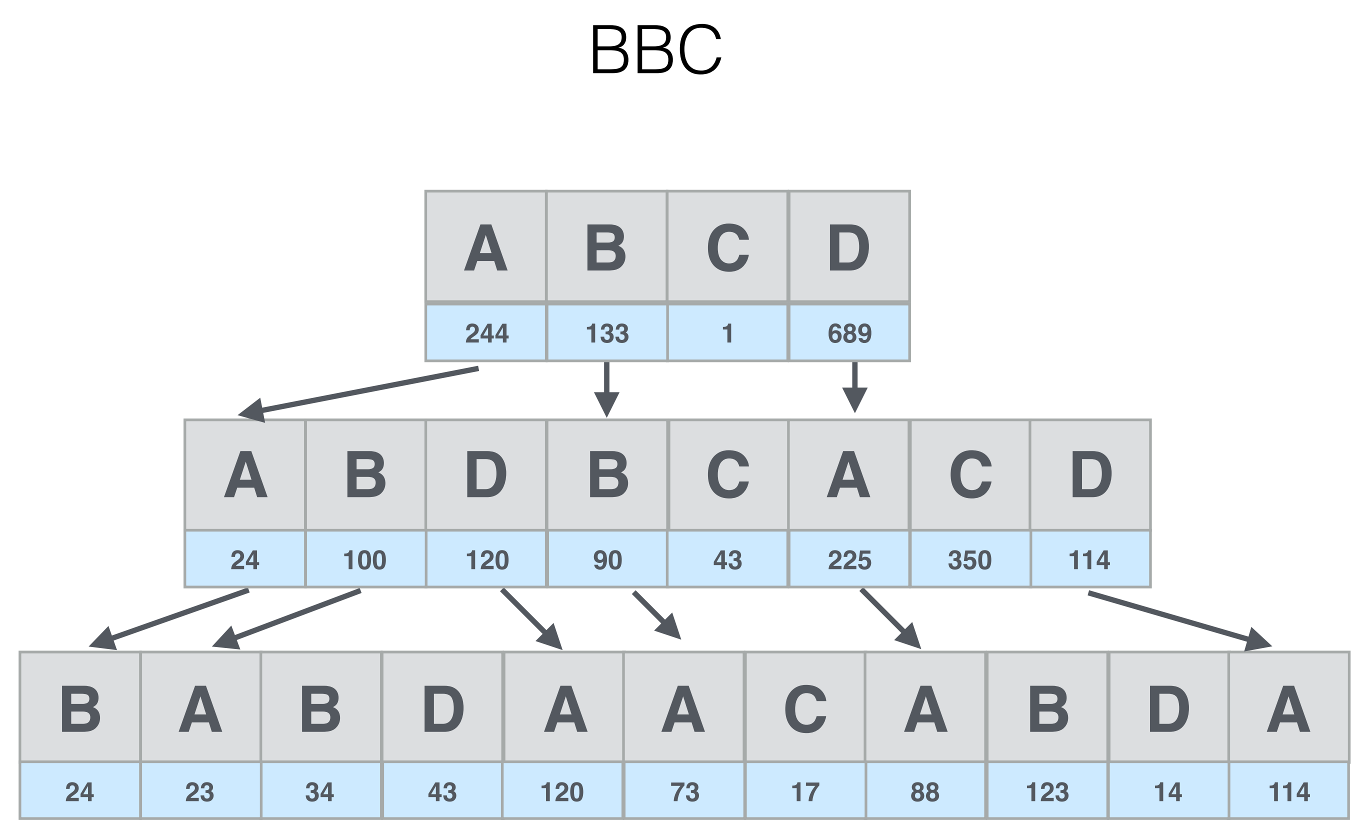

Open-addressing VS Tries

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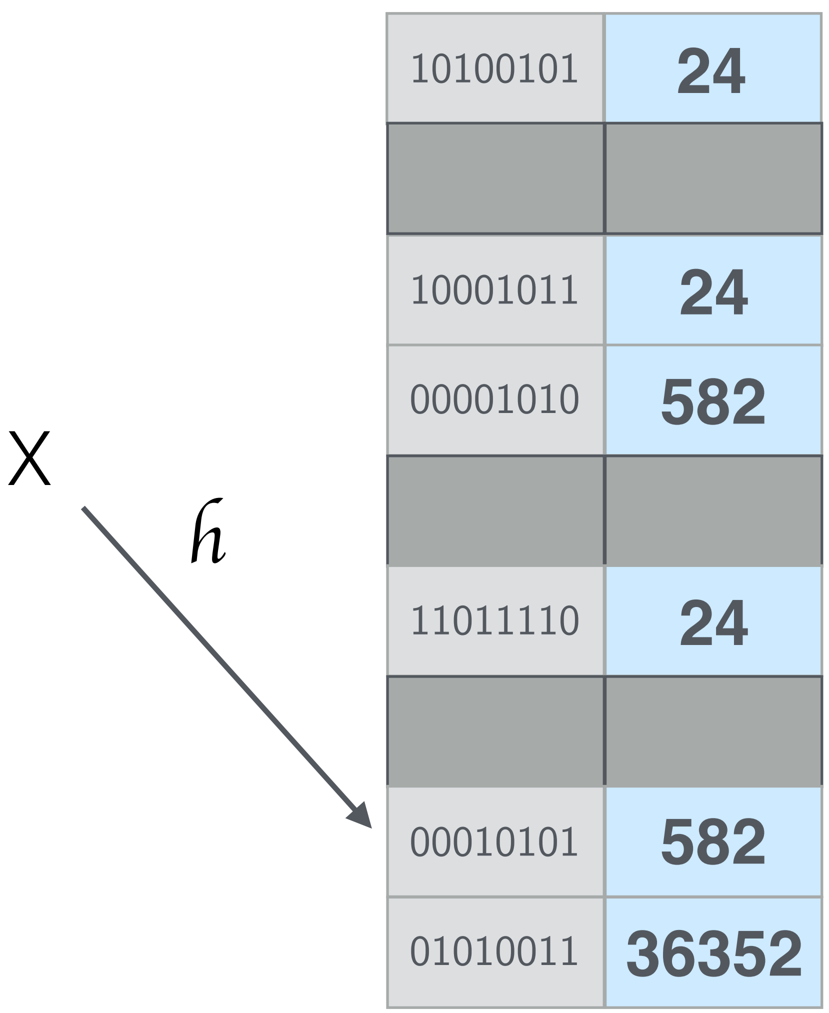

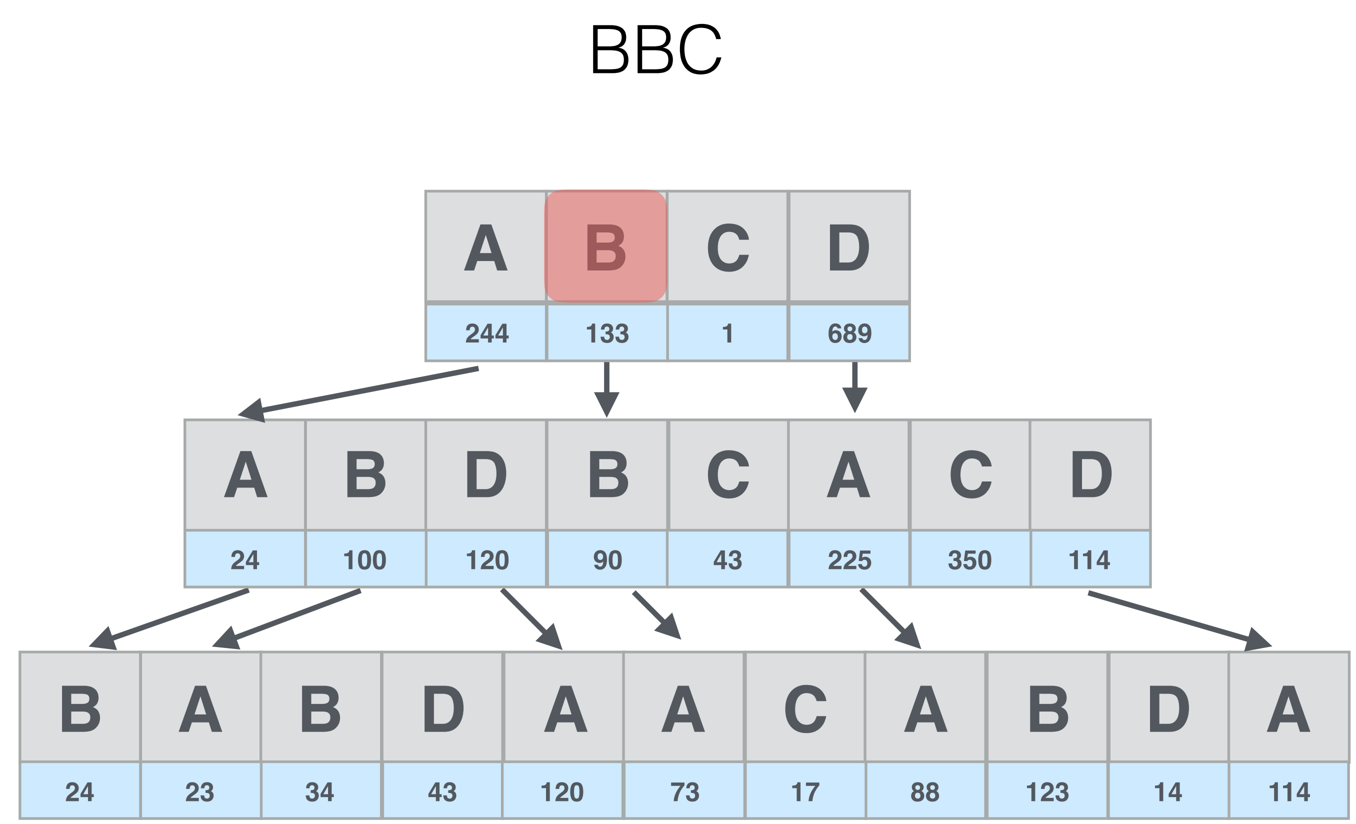

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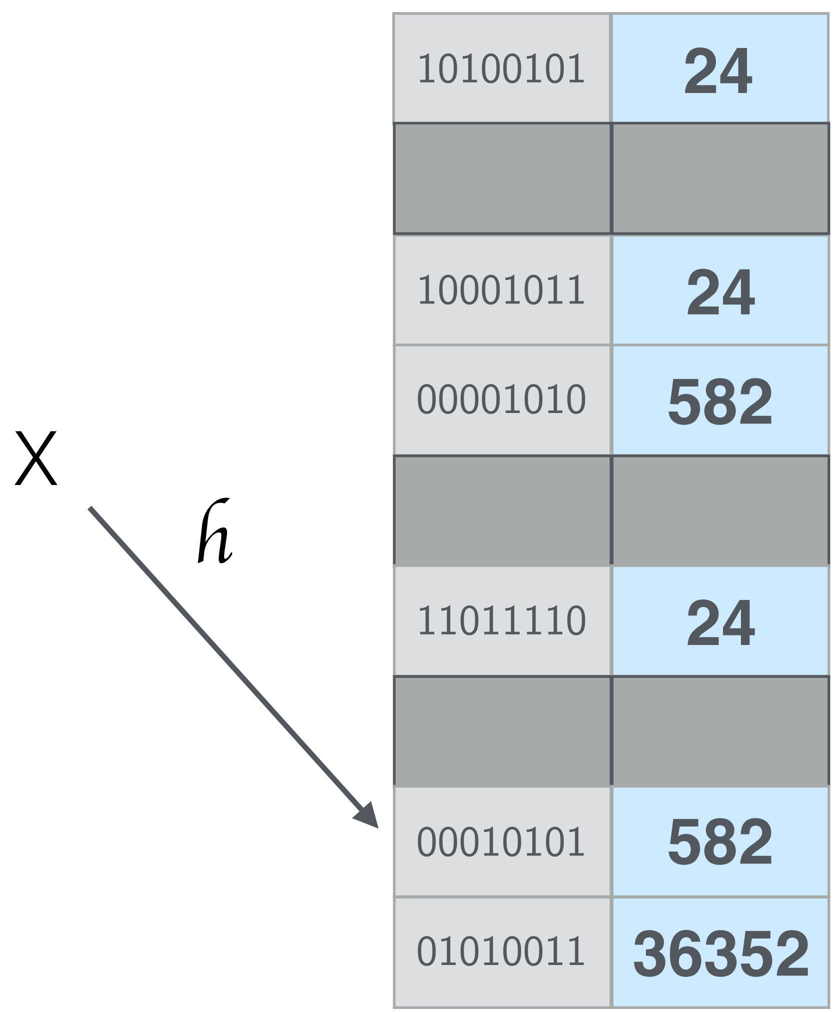

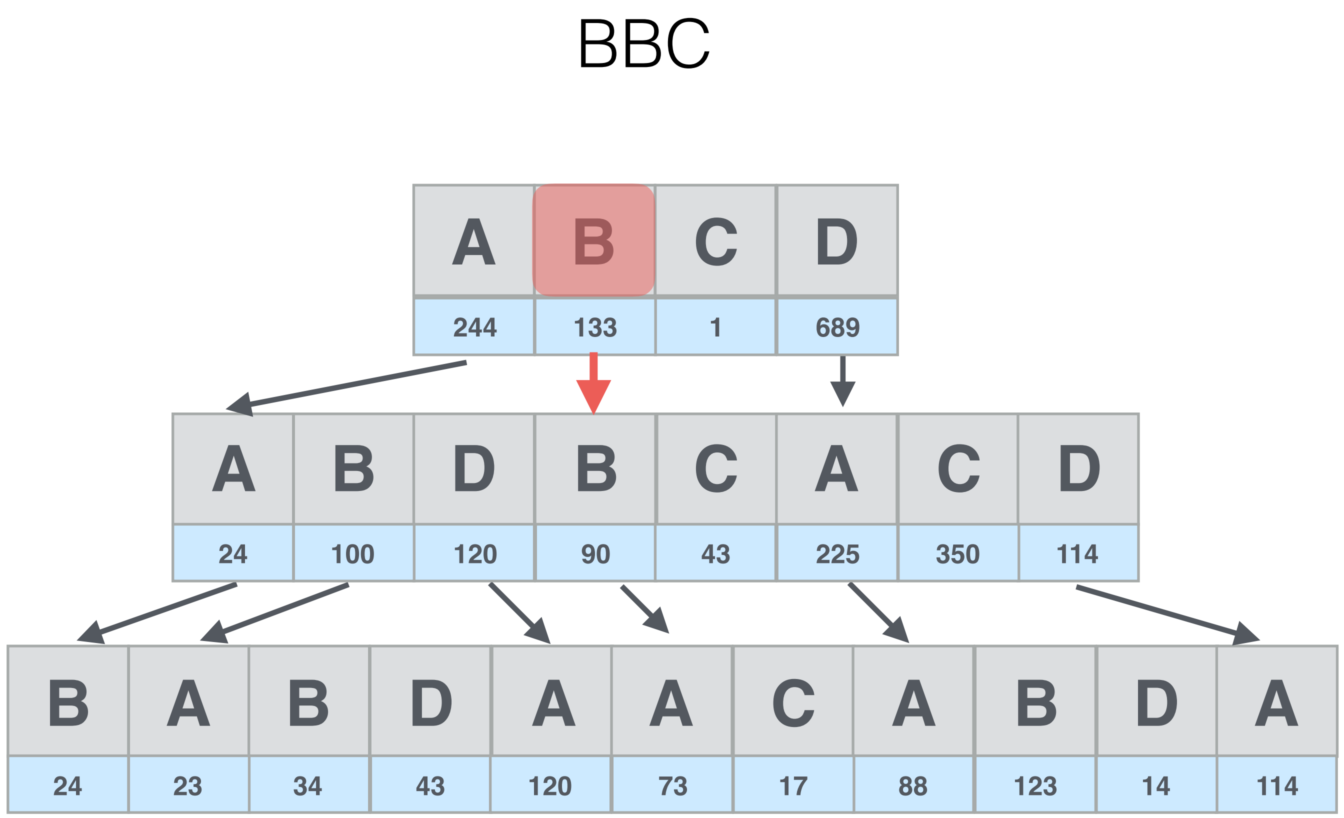

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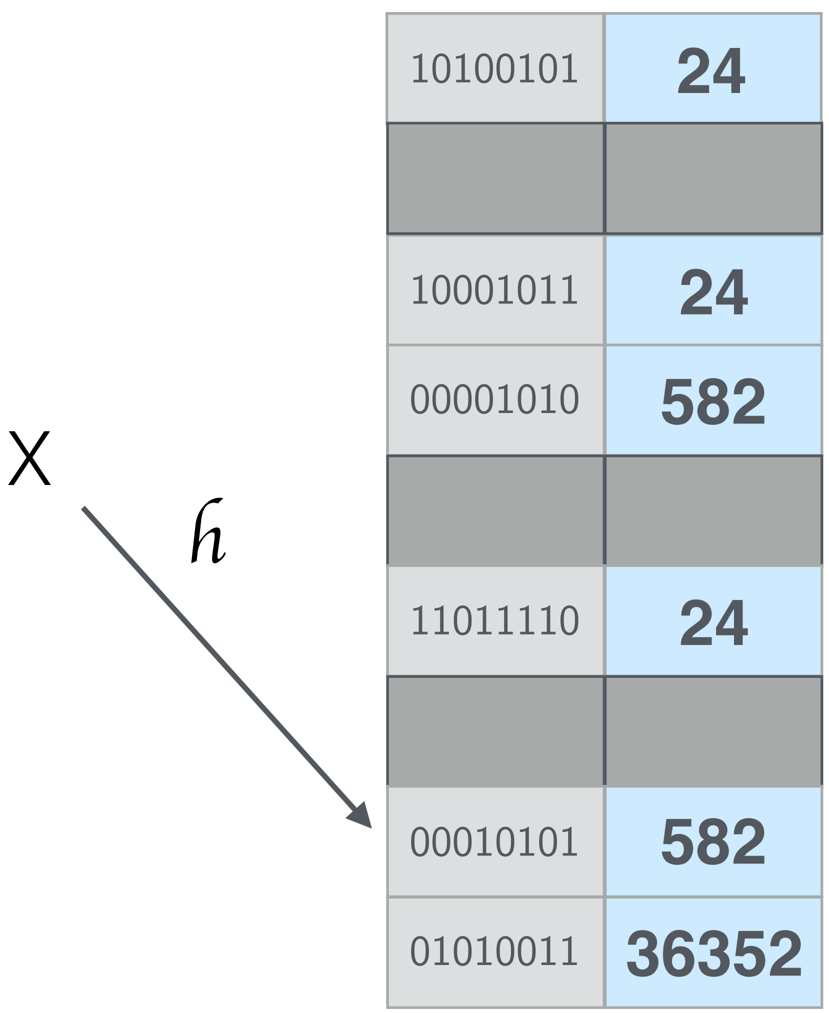

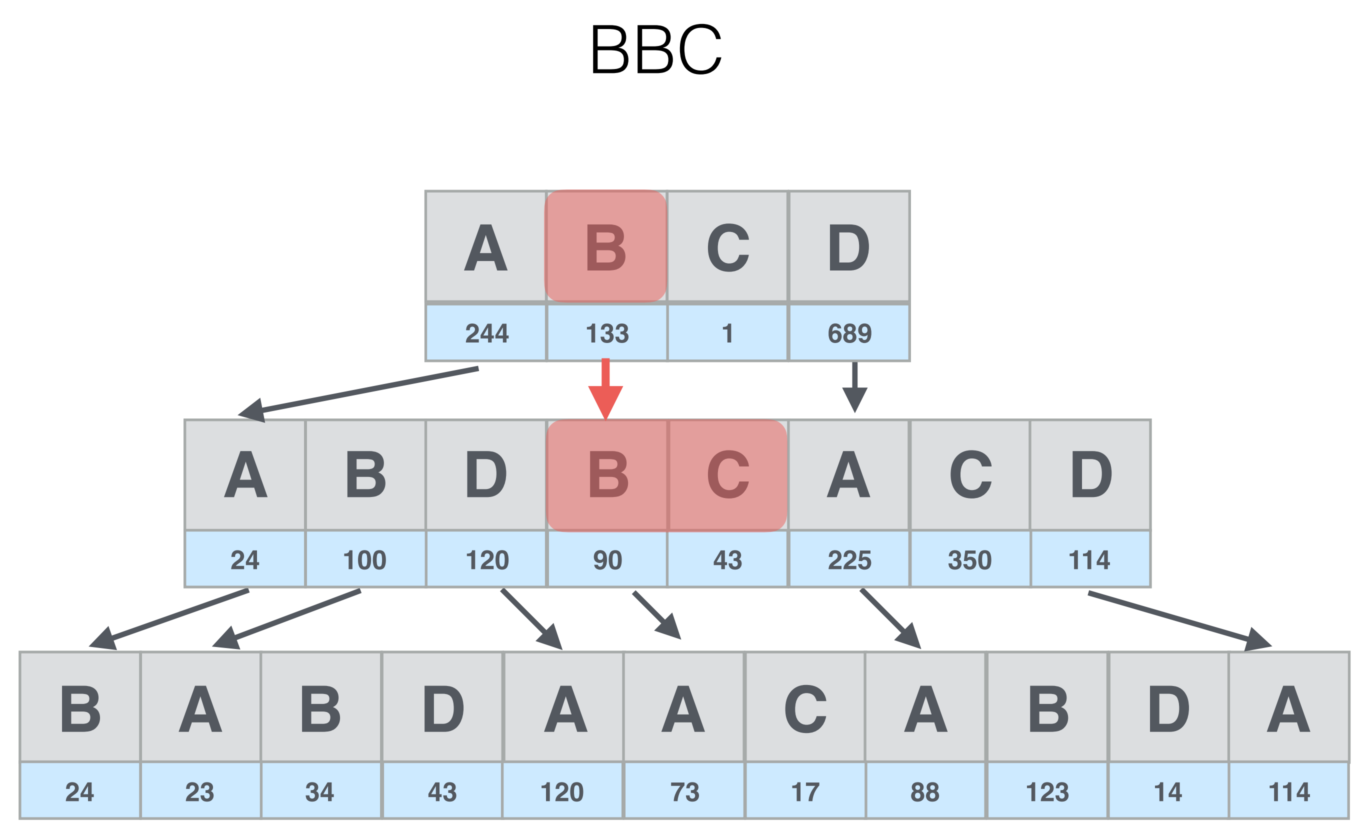

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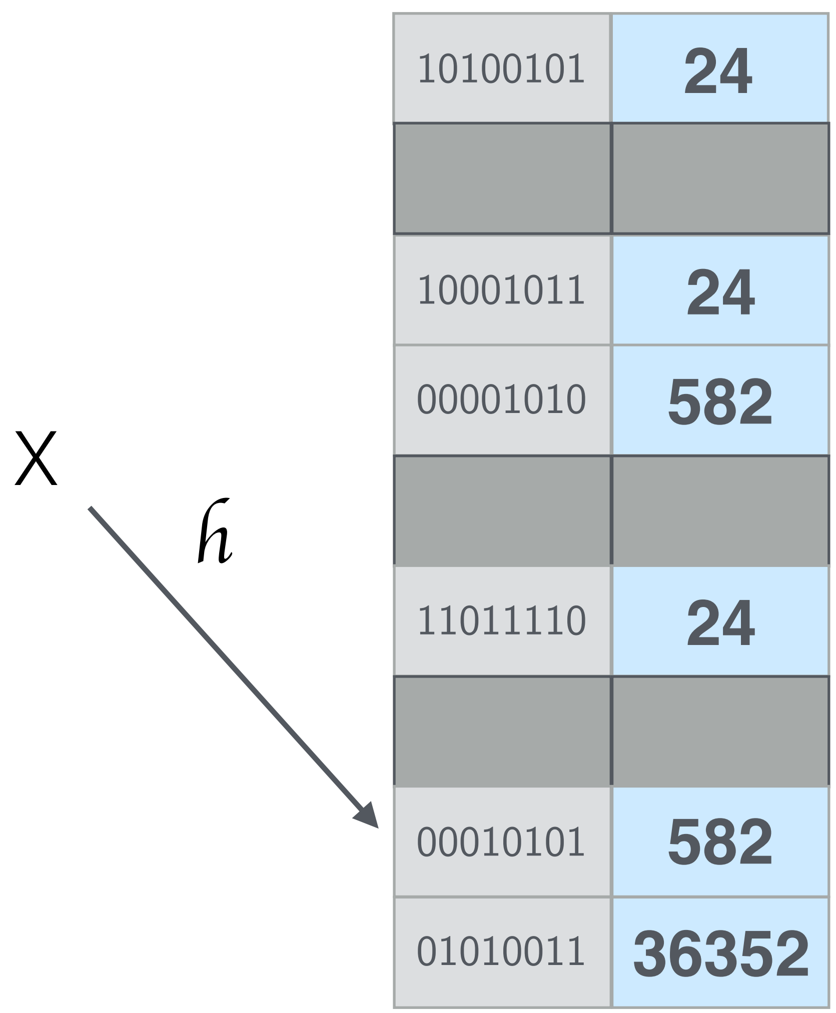
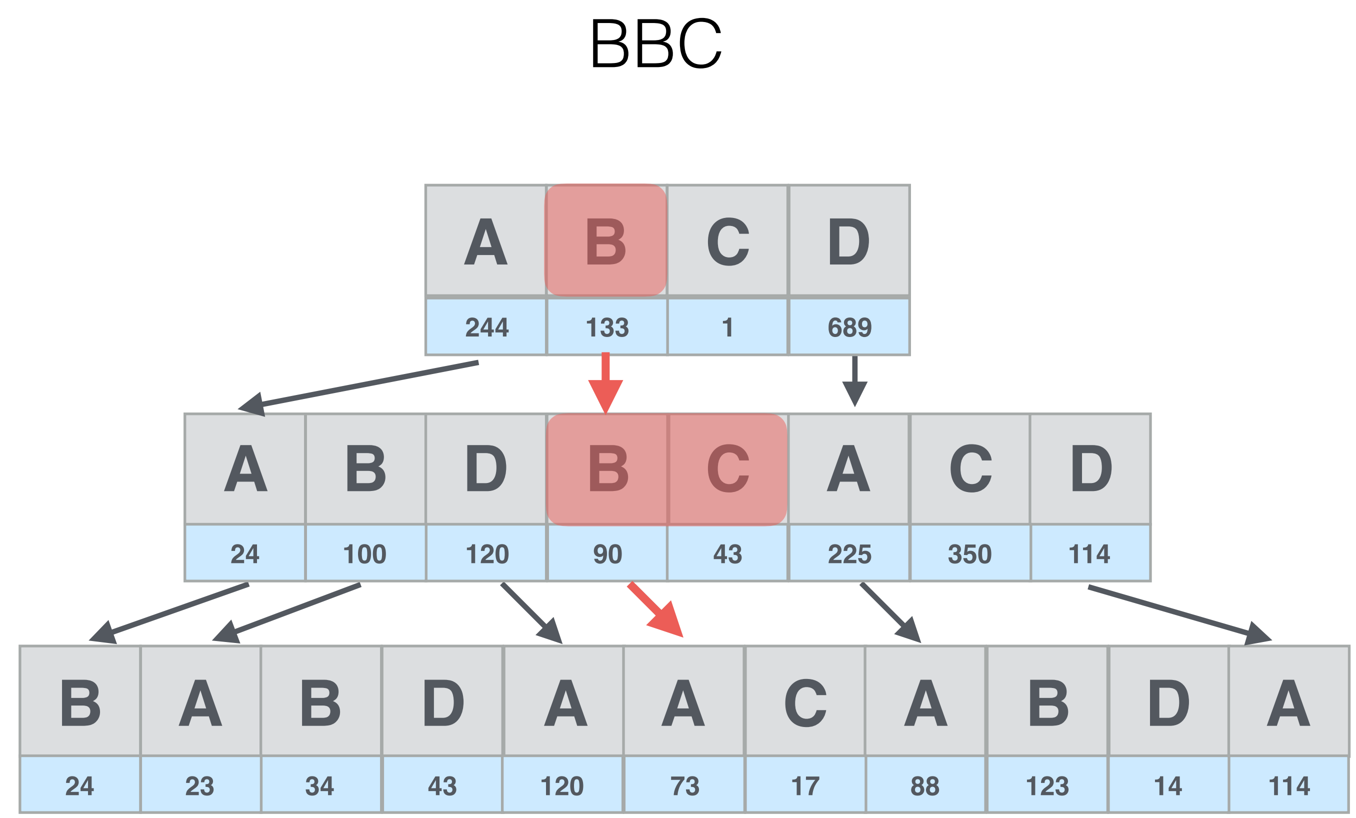
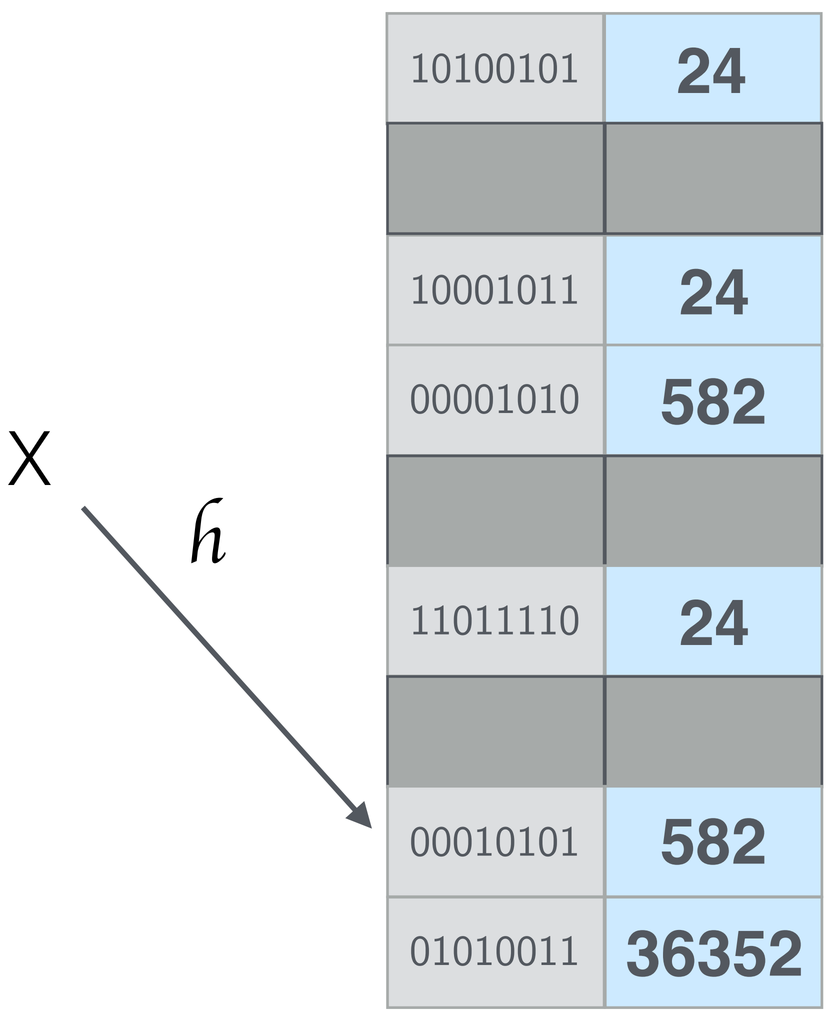


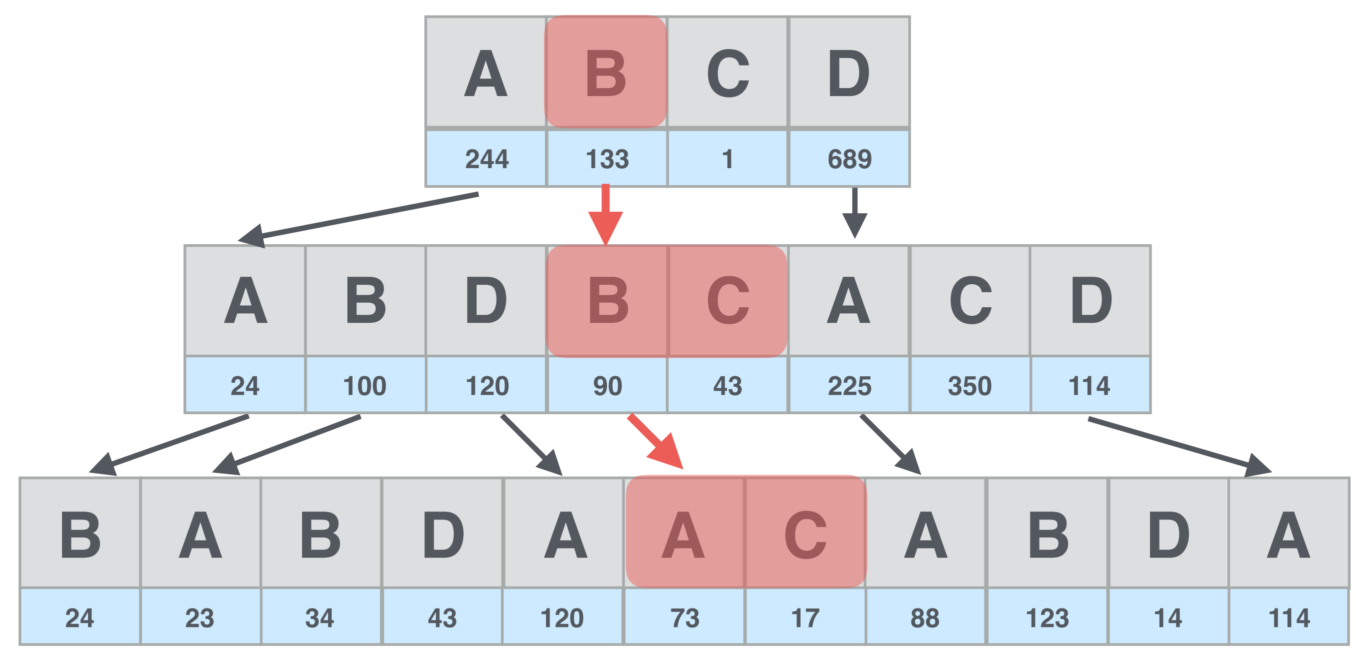


Open-addressing VS Tries

+ time
- space



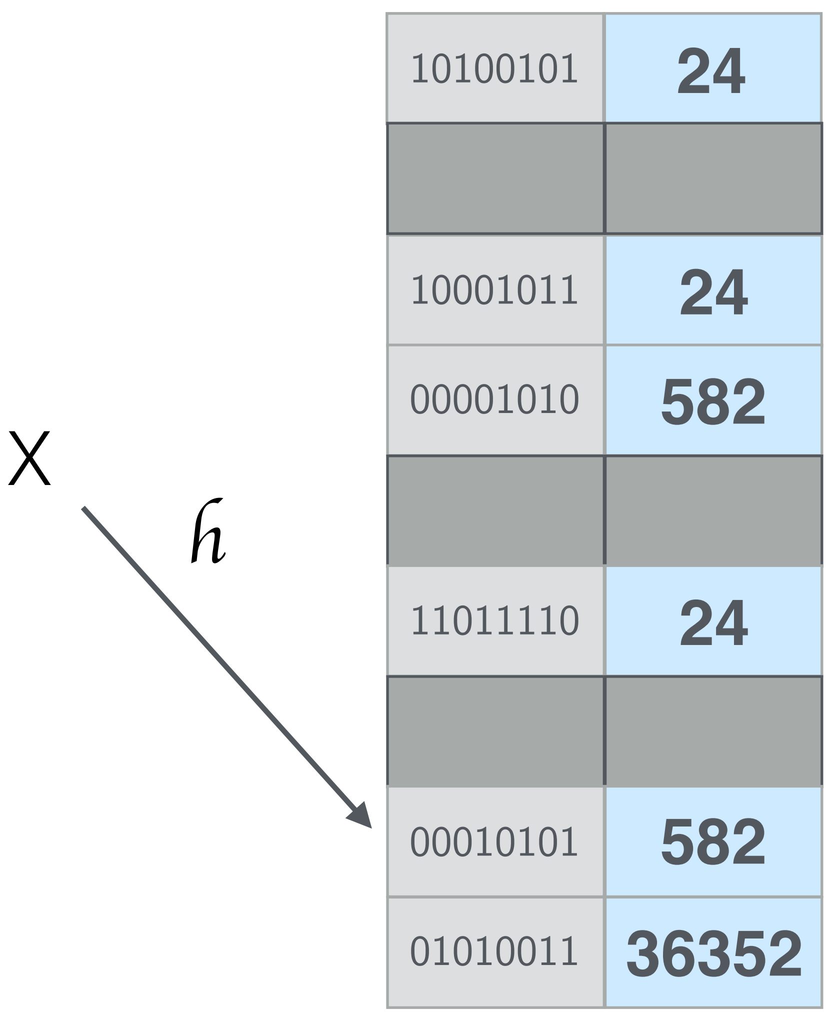
BBC

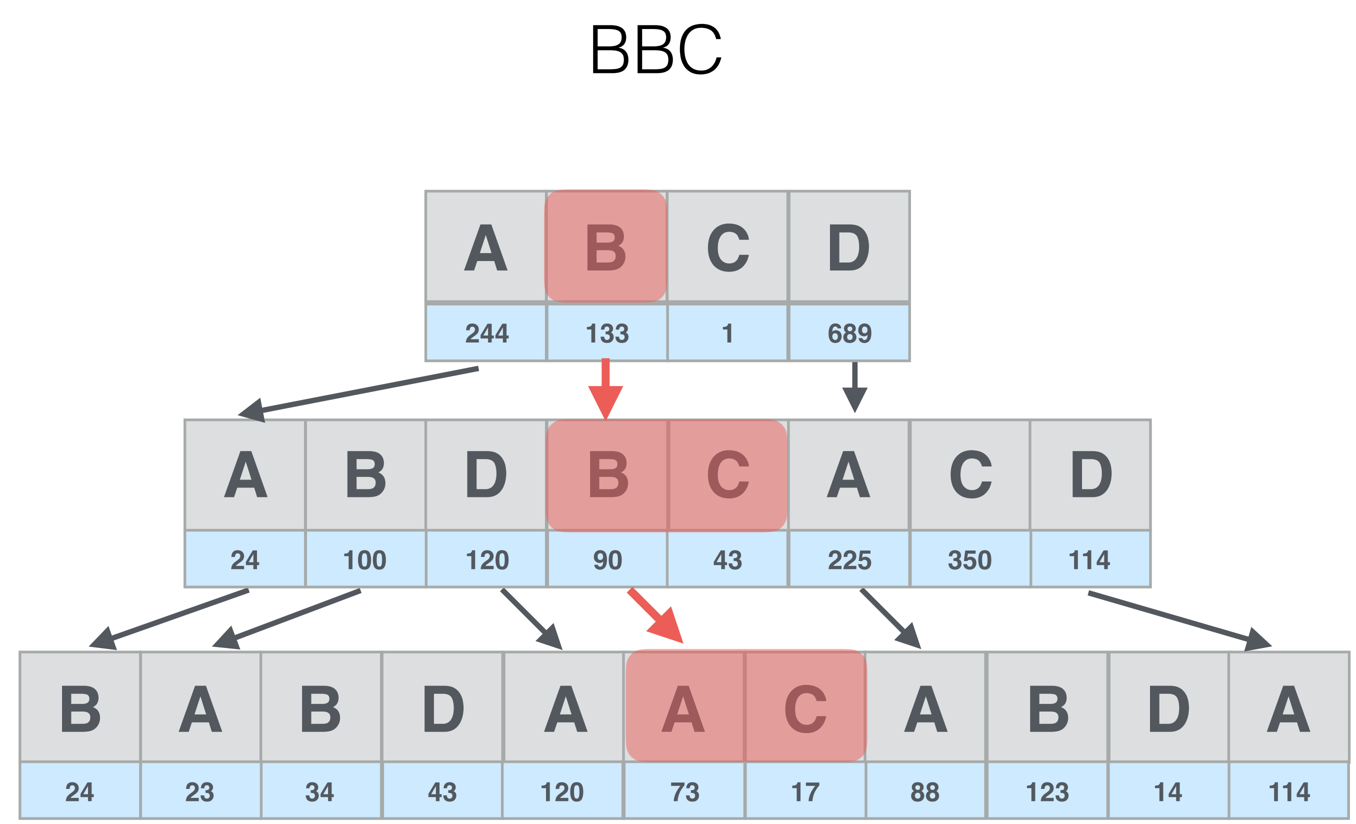


N-grams - Data structures

Open-addressing VS **Tries**

Open-addressing	Tries
+ time - space	+ space - time

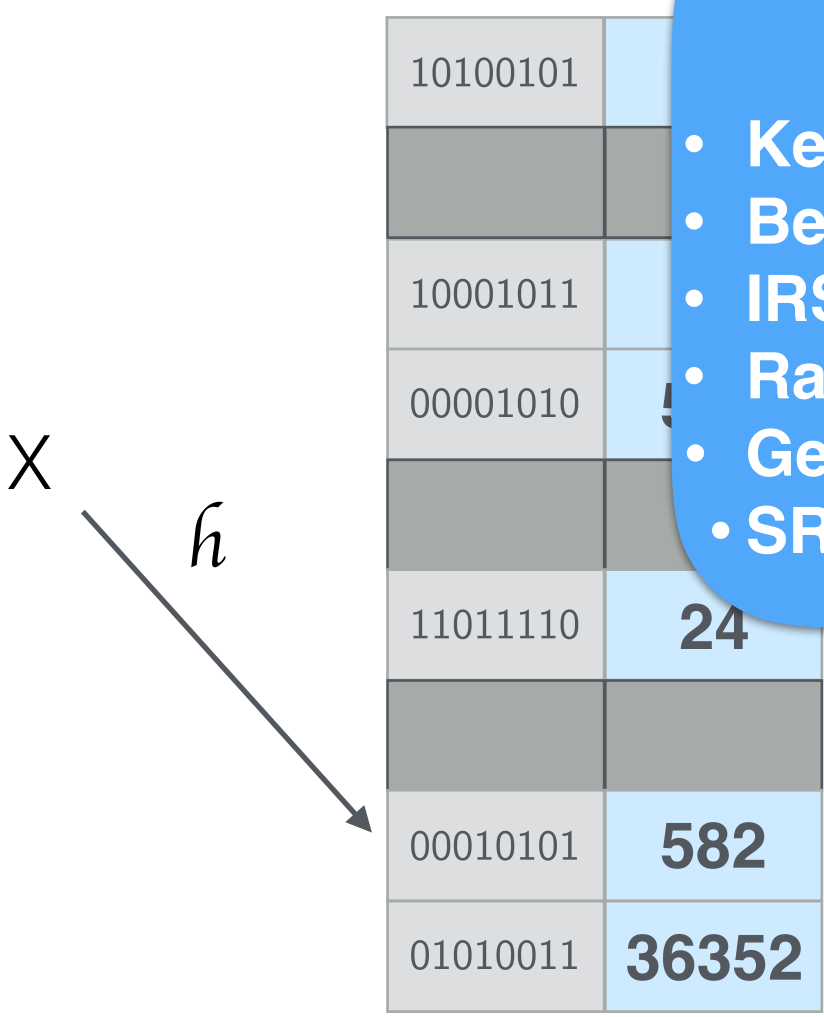


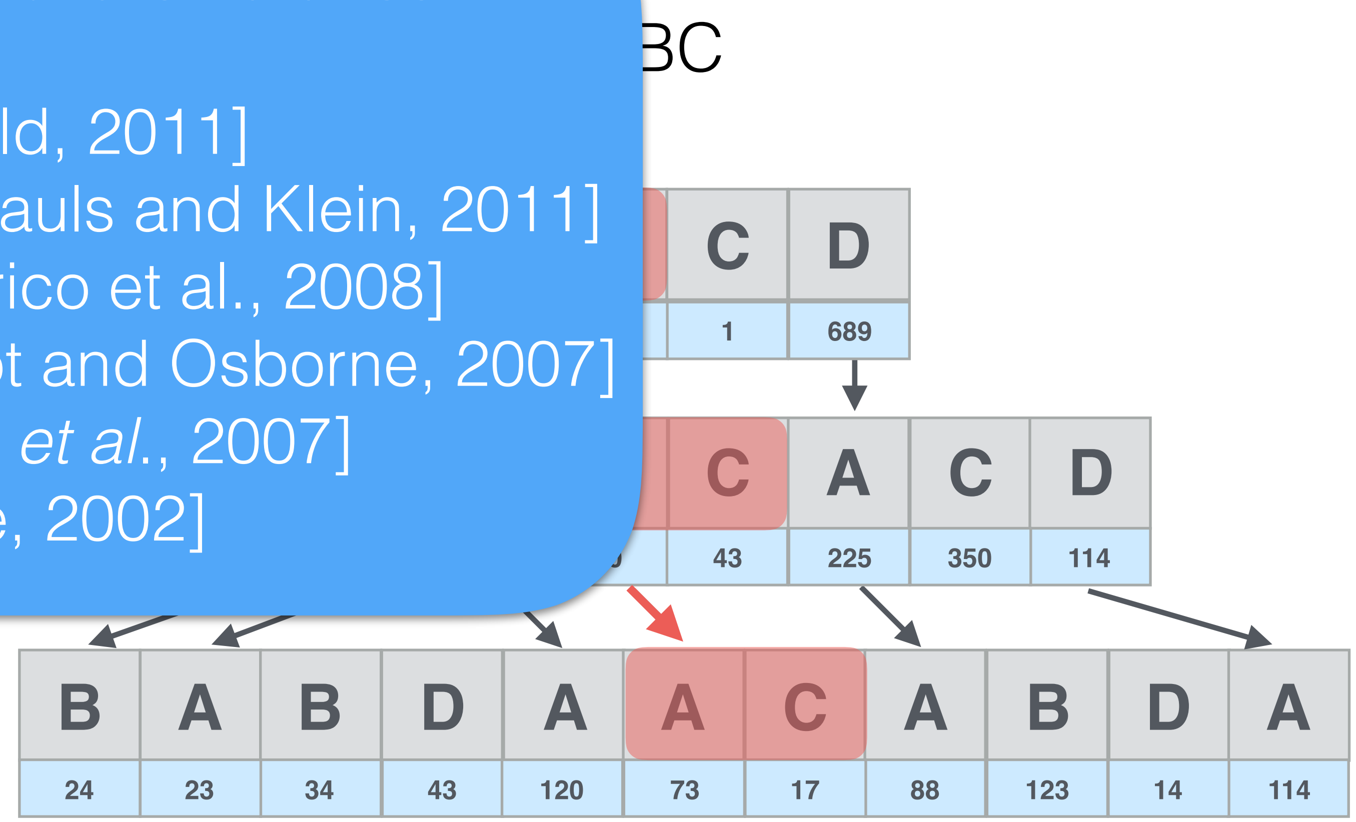


Open-addressing VS Tries

+ time
- space

+ space
- time

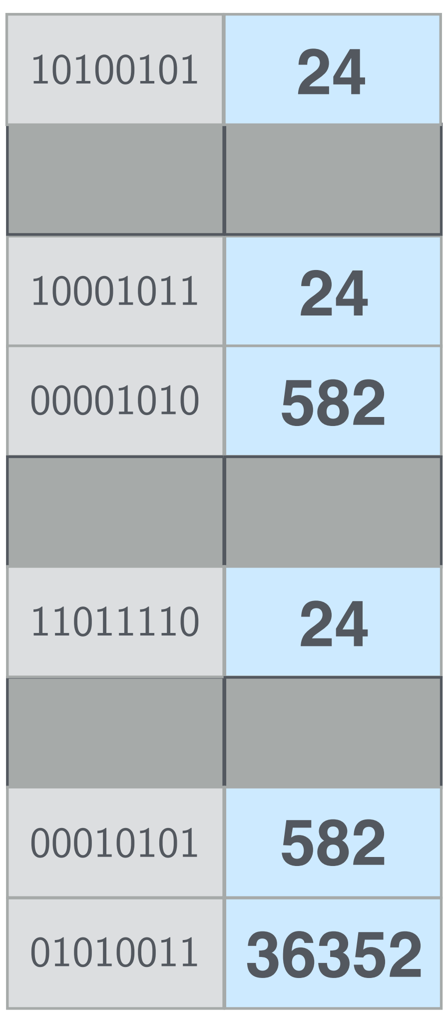


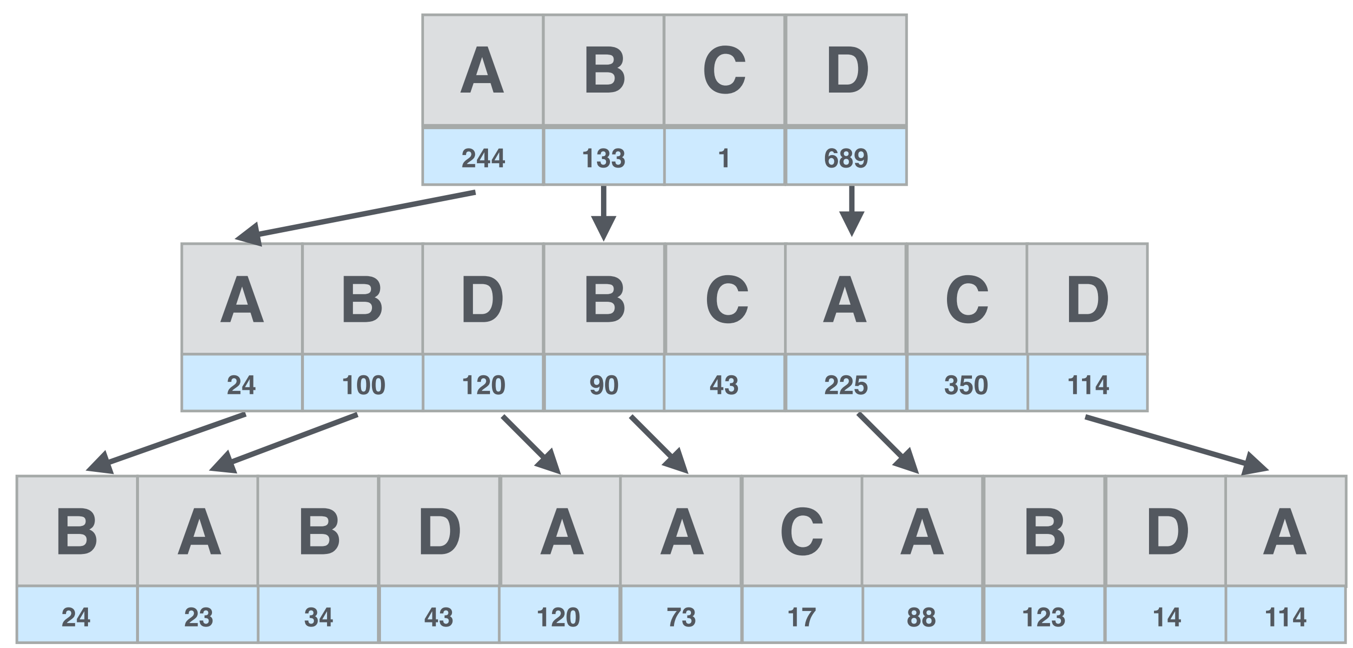


Several software libraries

- **KenLM** [Heafield, 2011]
- **BerkeleyLM** [Pauls and Klein, 2011]
- **IRSTLM** [Federico et al., 2008]
- **RandLM** [Talbot and Osborne, 2007]
- **Get1T** [Hawker *et al.*, 2007]
- **SRILM** [Stolcke, 2002]

Hash-based VS **Trie-based**





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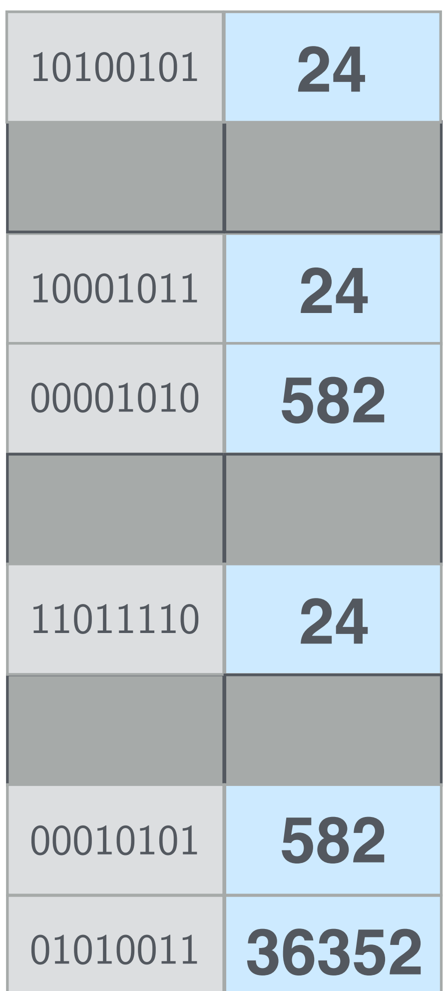

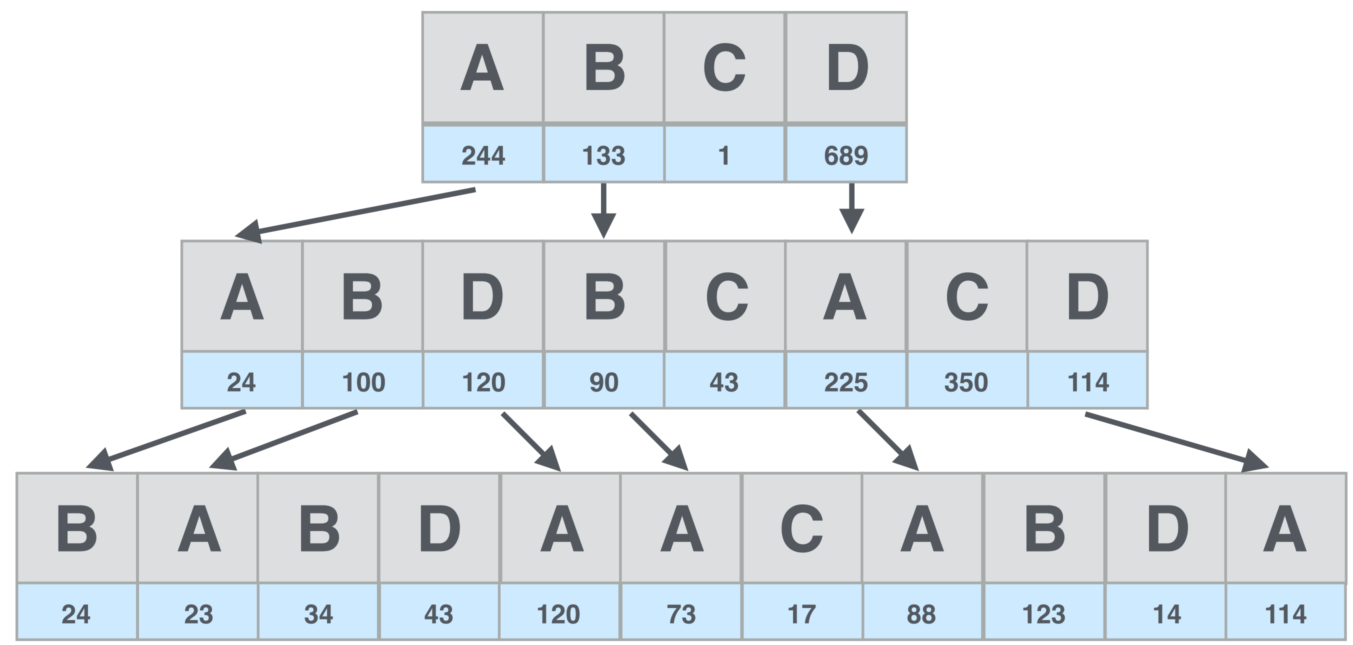

Hash-based VS Trie-based

Open addressing?

Data structure is *static*.

Minimal Perfect Hashing.

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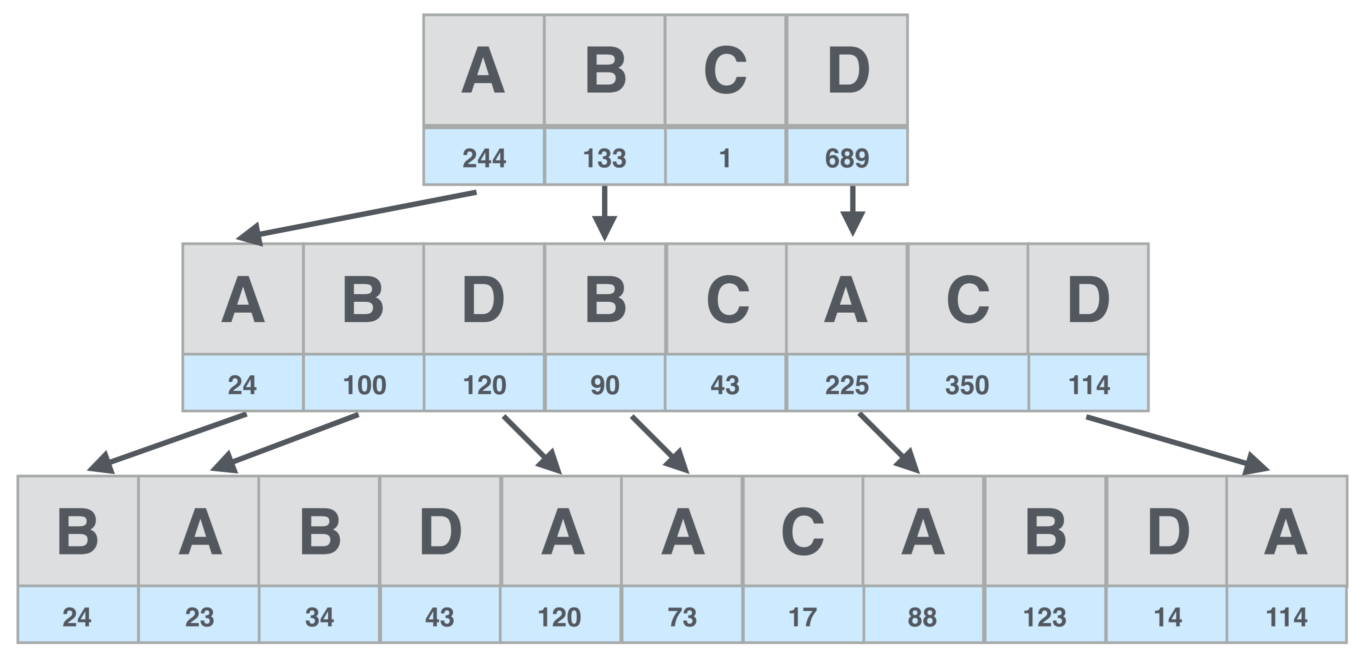
11011110	24
00010101	582
01010011	36352



`tongrams` - Tons of *N*-Grams

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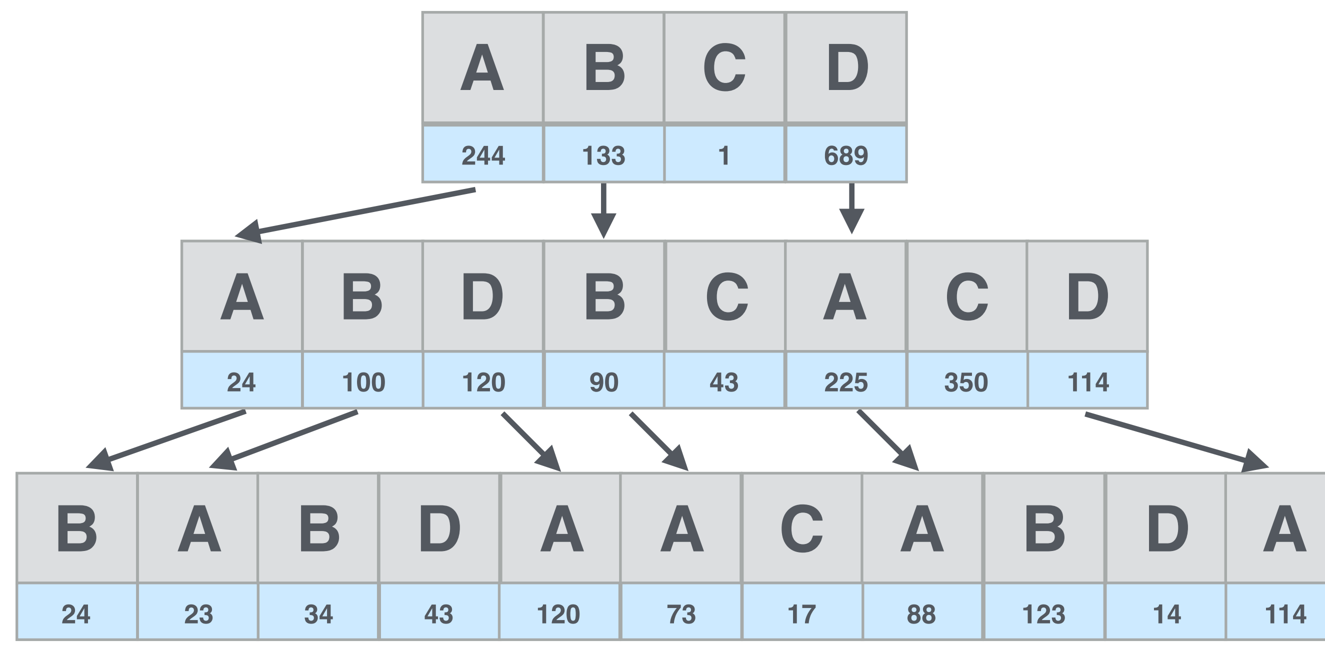

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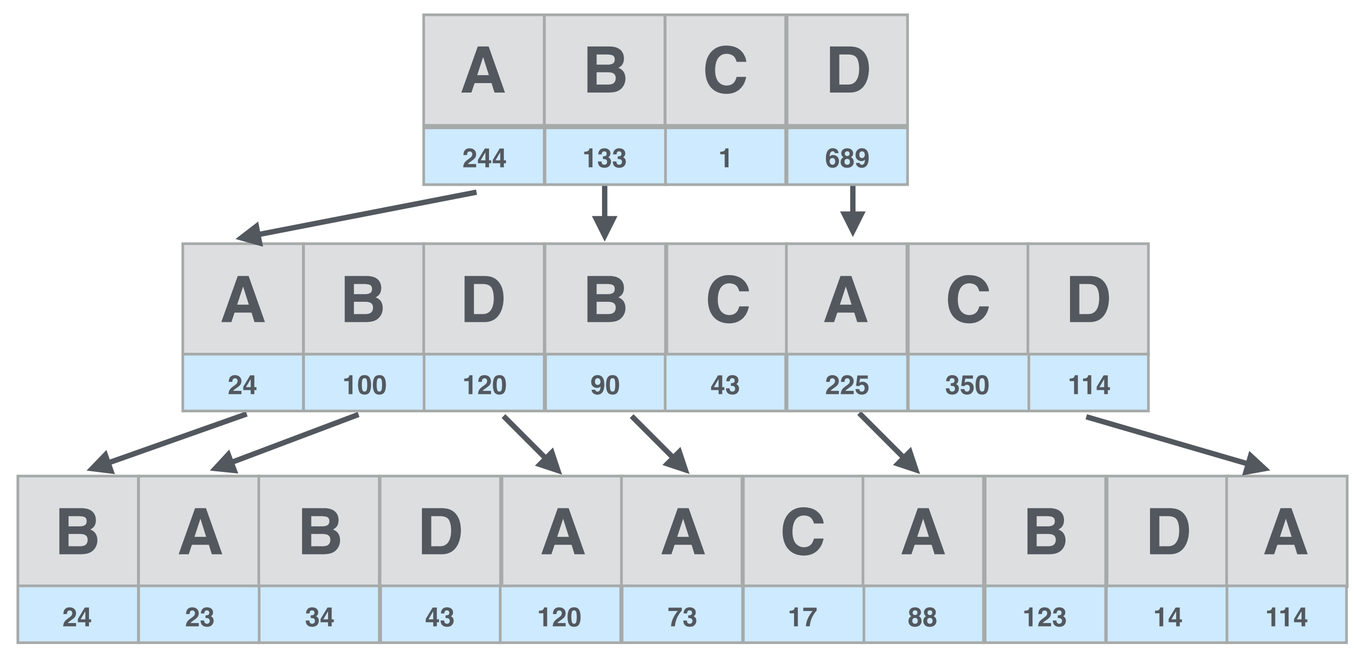

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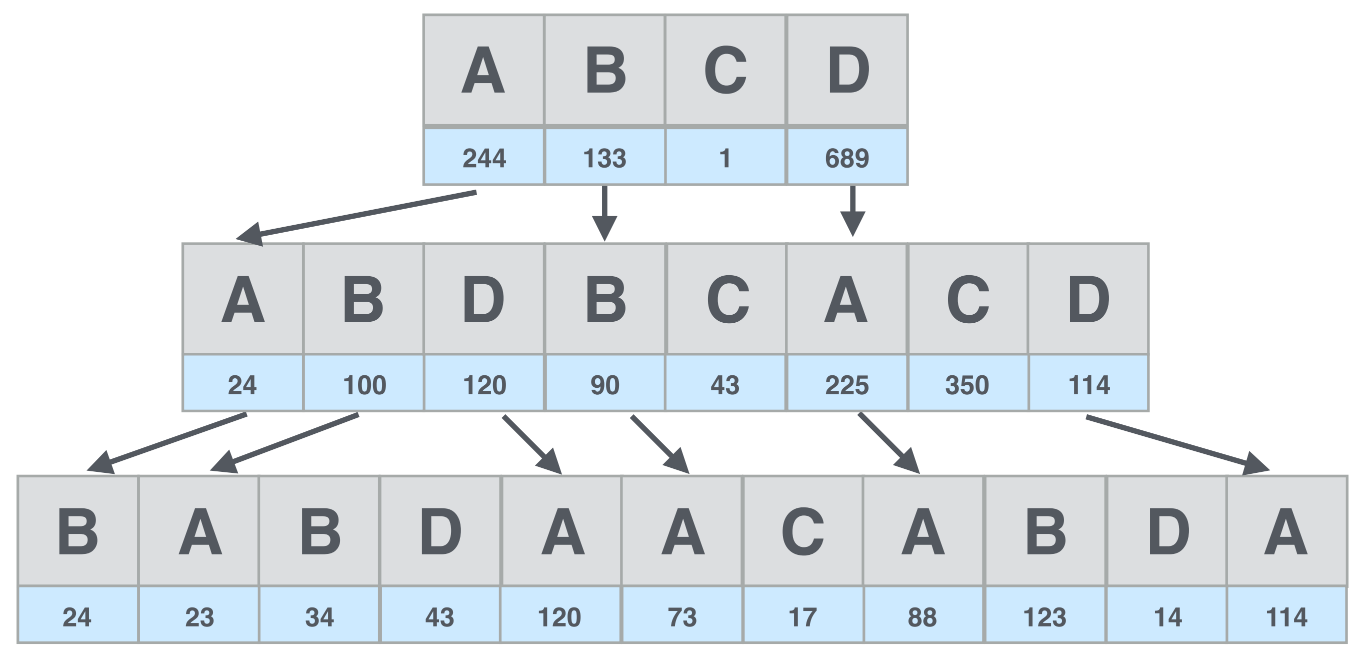

tongrams - Tons of *N*-Grams 1

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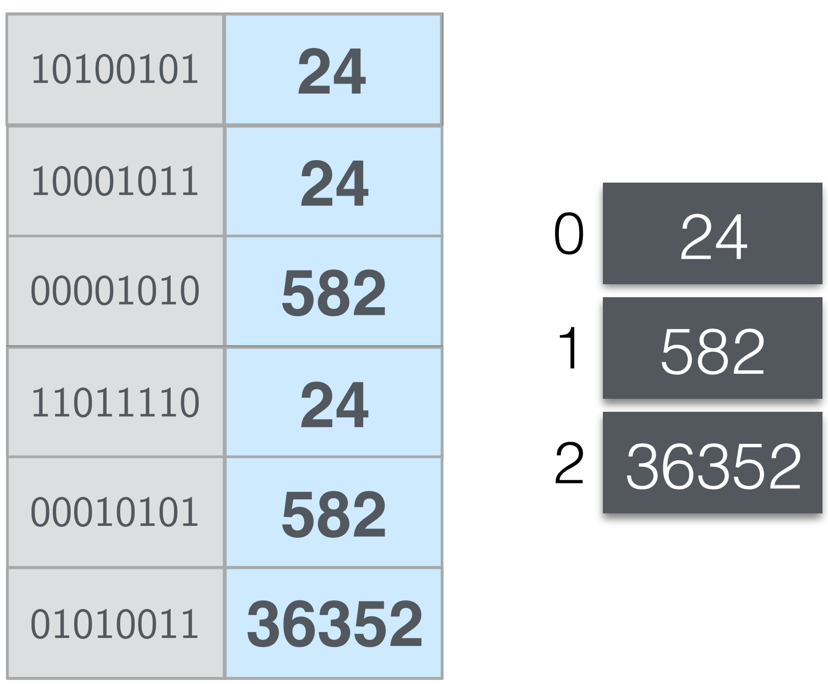

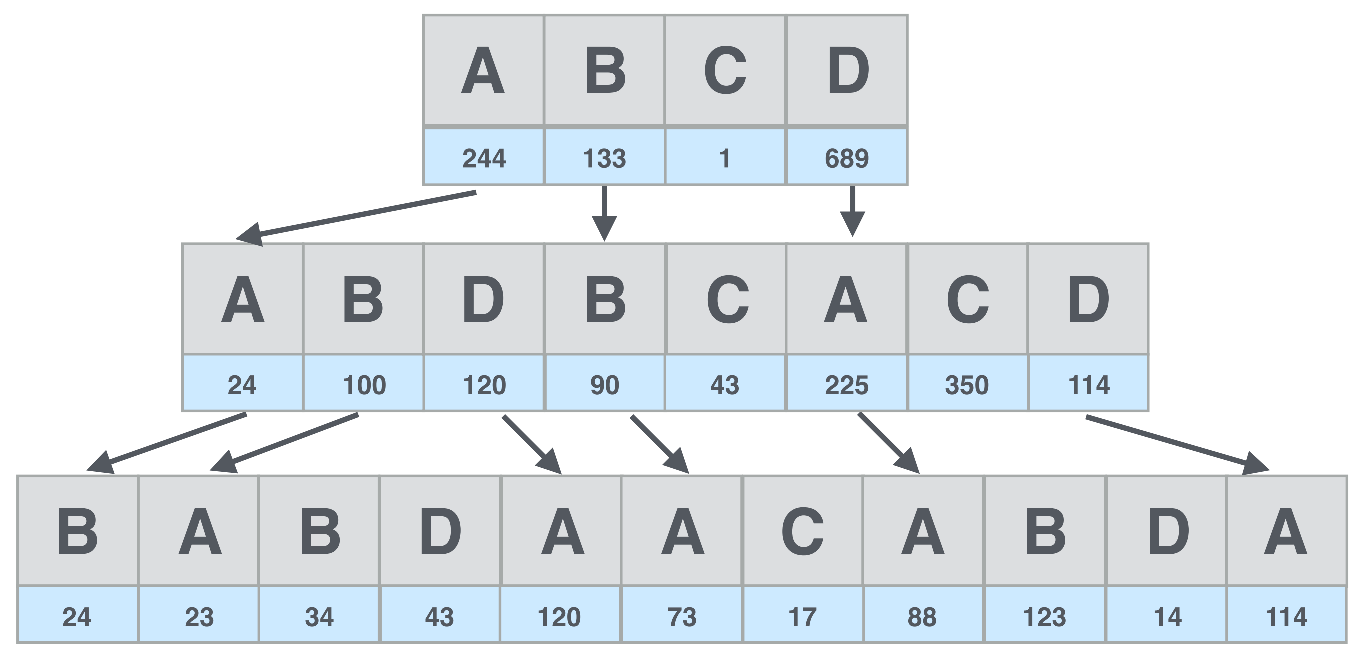

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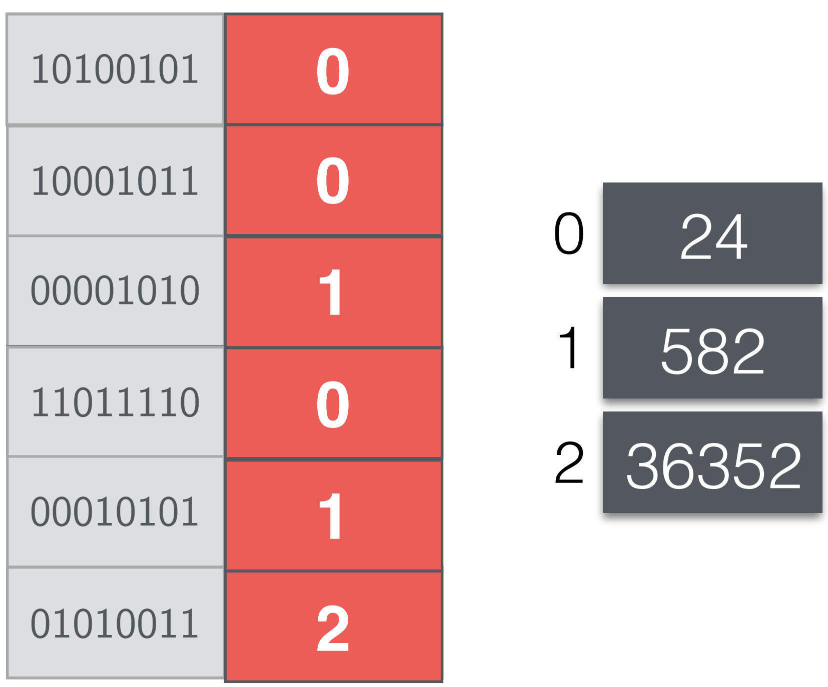

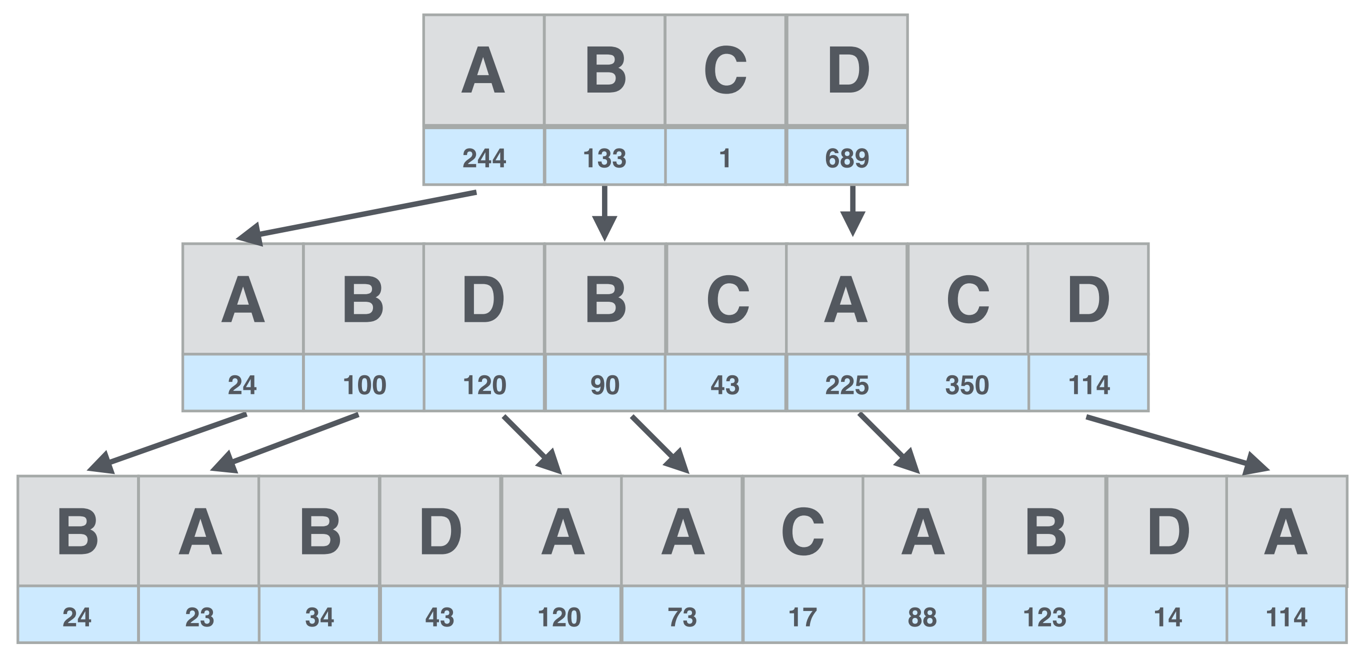

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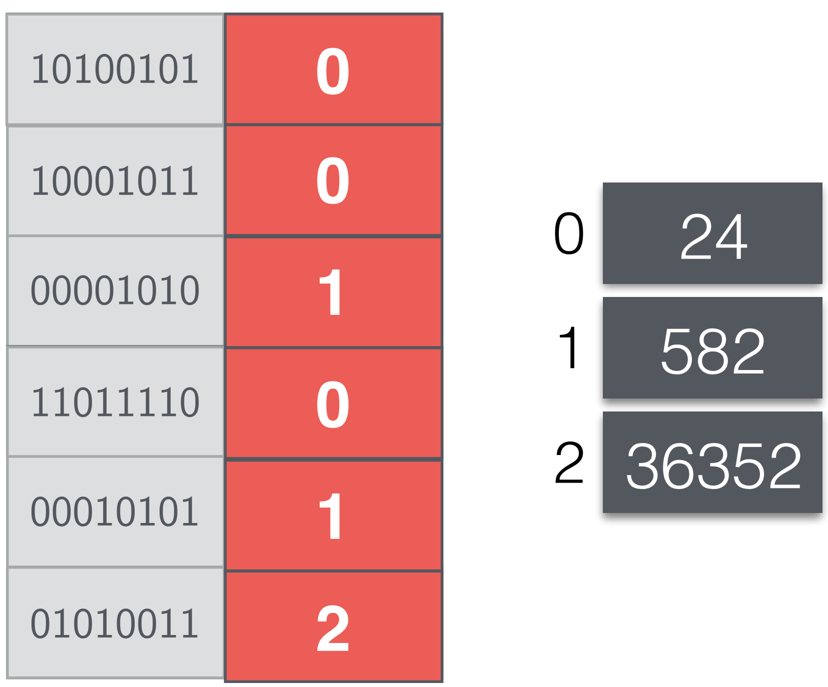

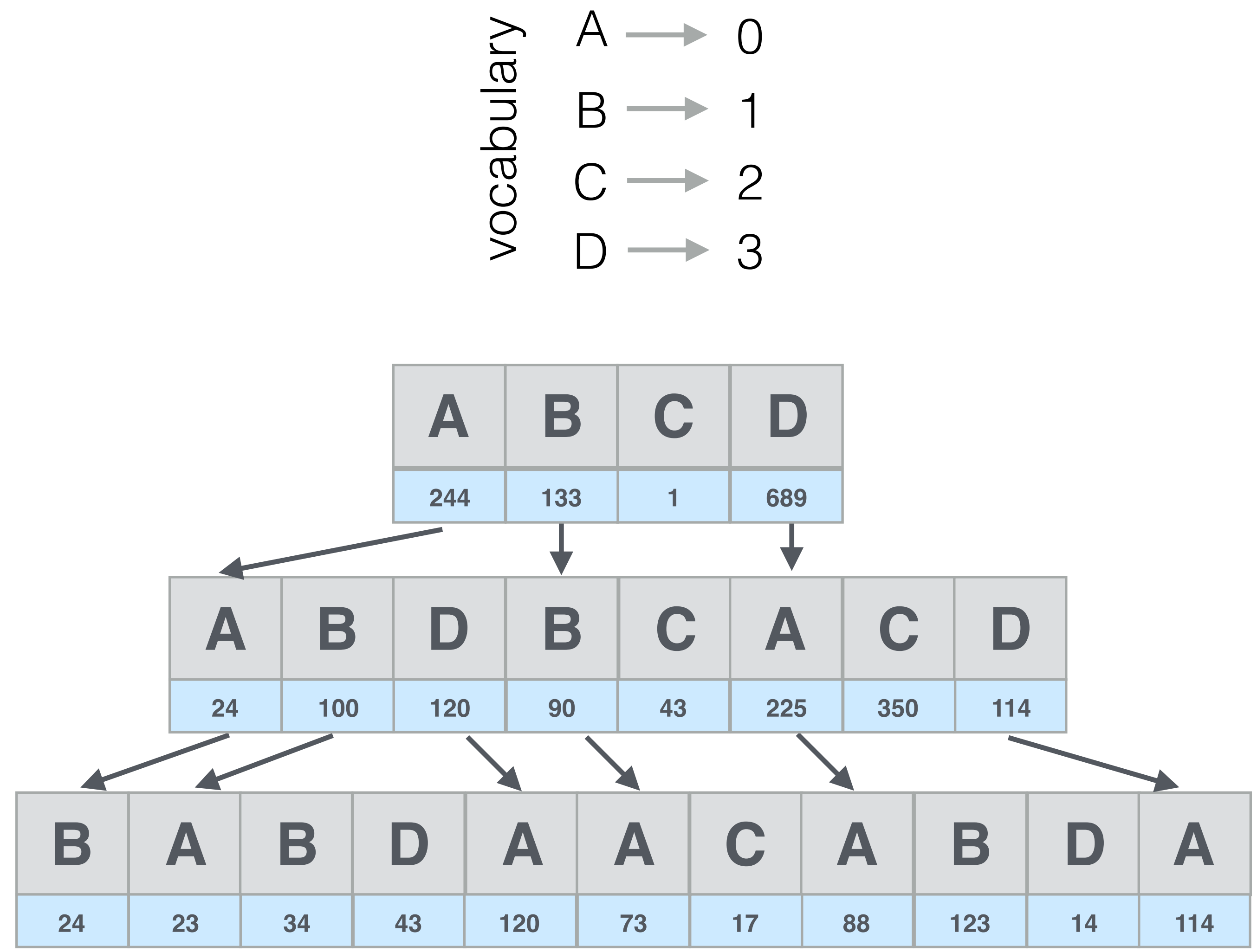

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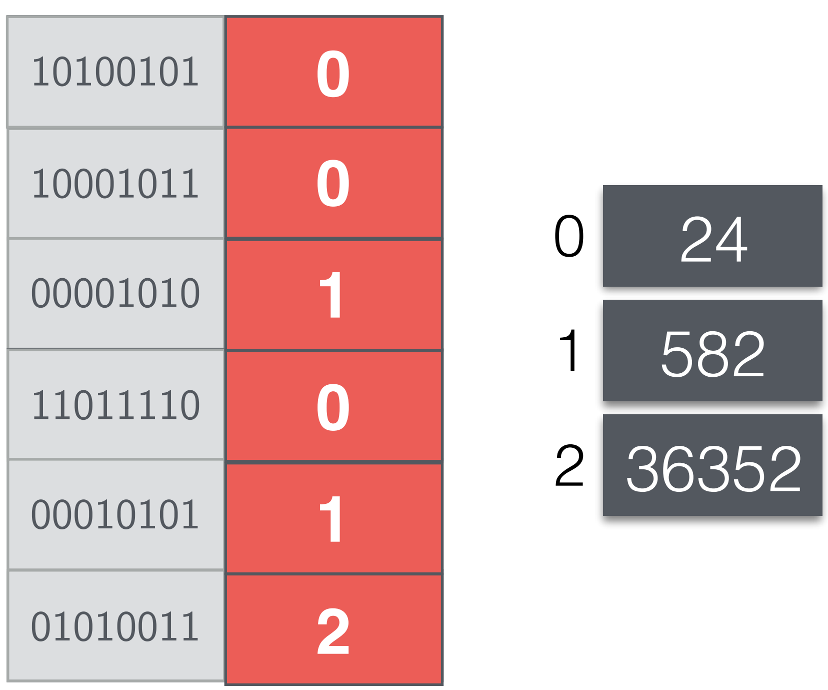

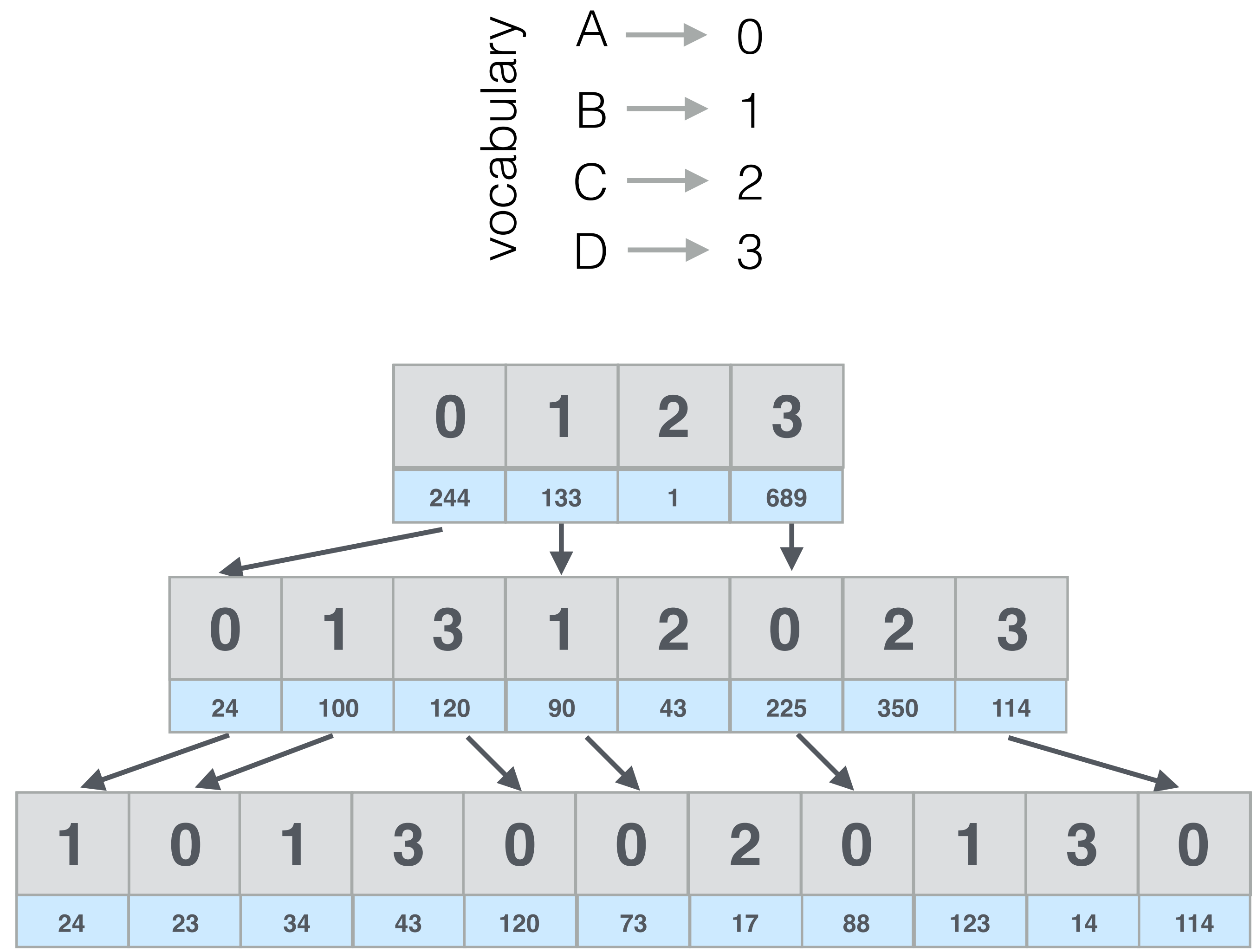

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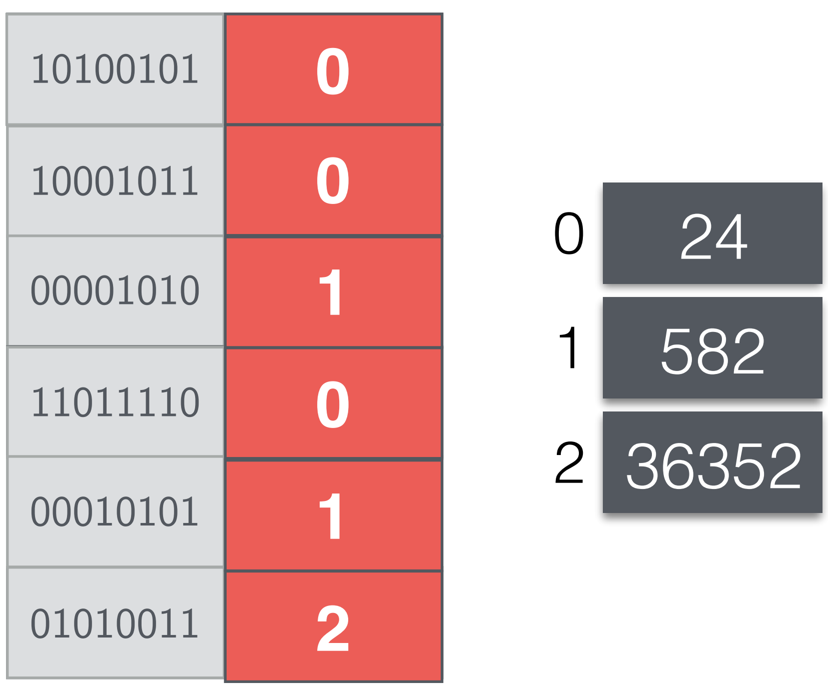
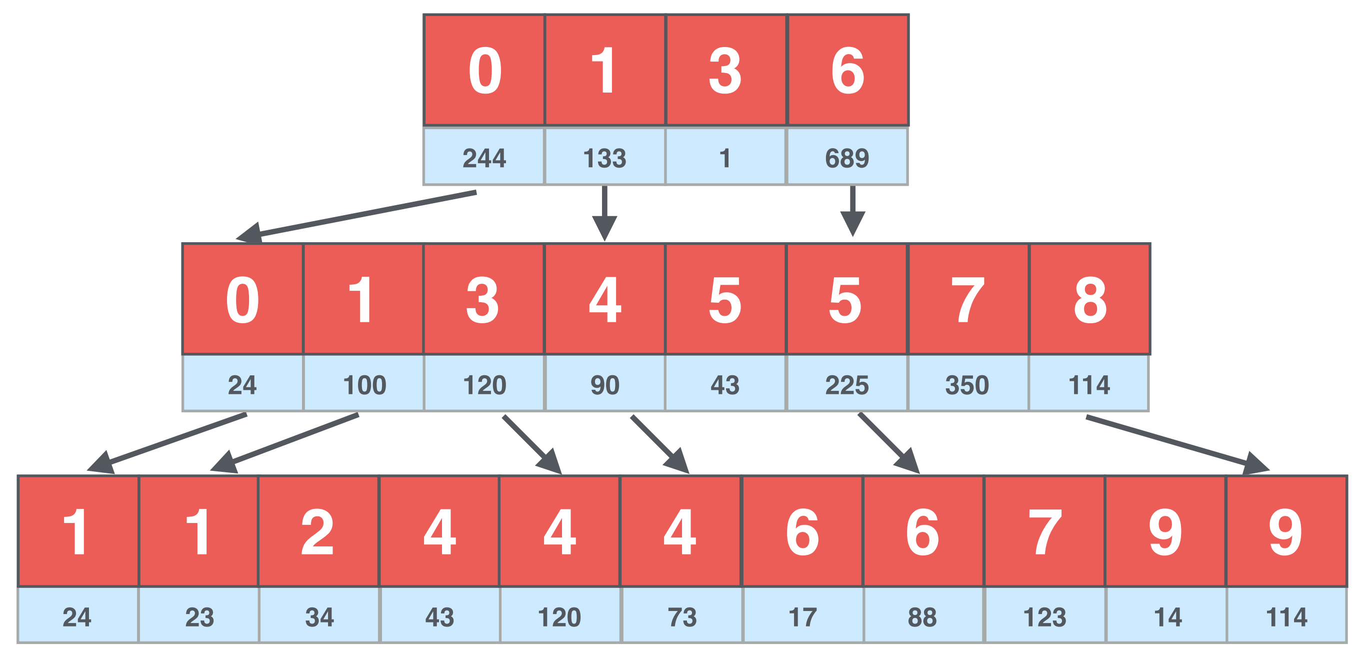


vocabulary
A → 0
B → 1
C → 2
D → 3



Hash-based

VS

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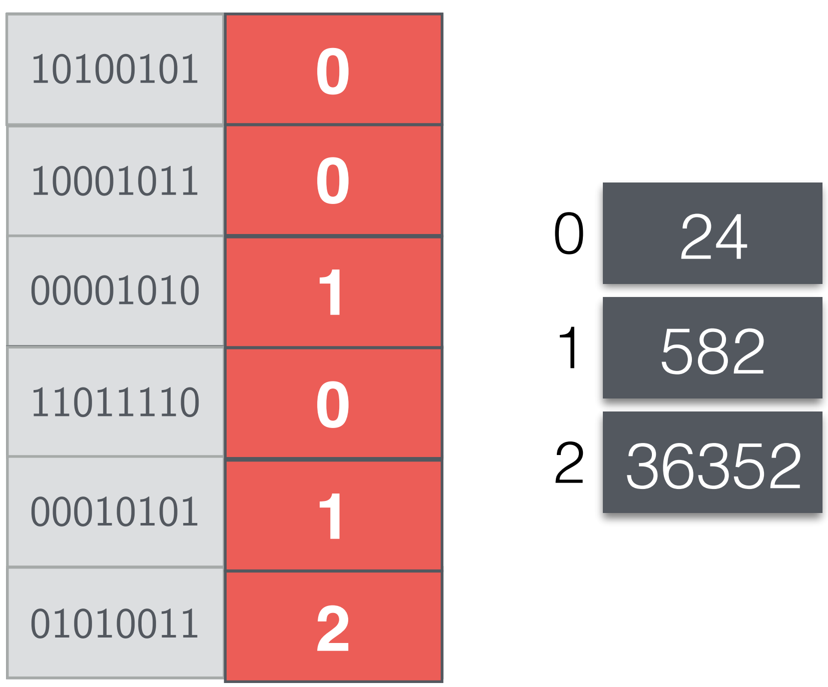

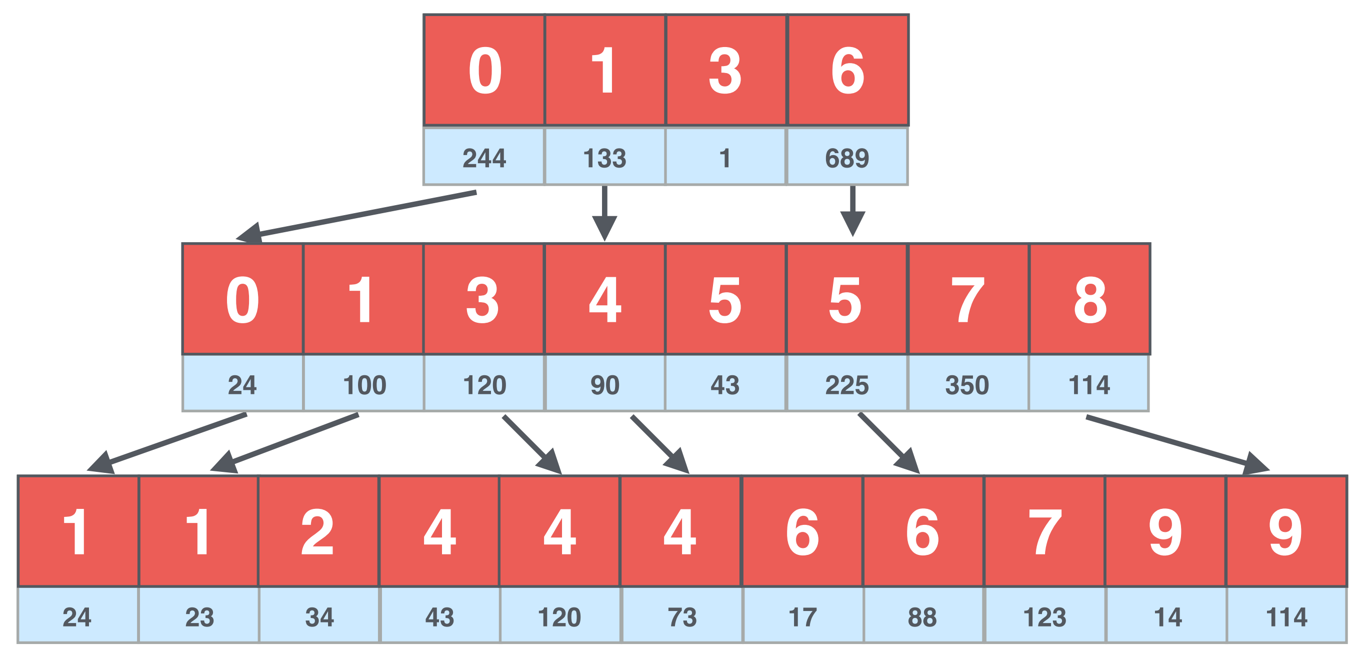

Encode each level
with **Elias-Fano**.

Hash-based

VS

Trie-based

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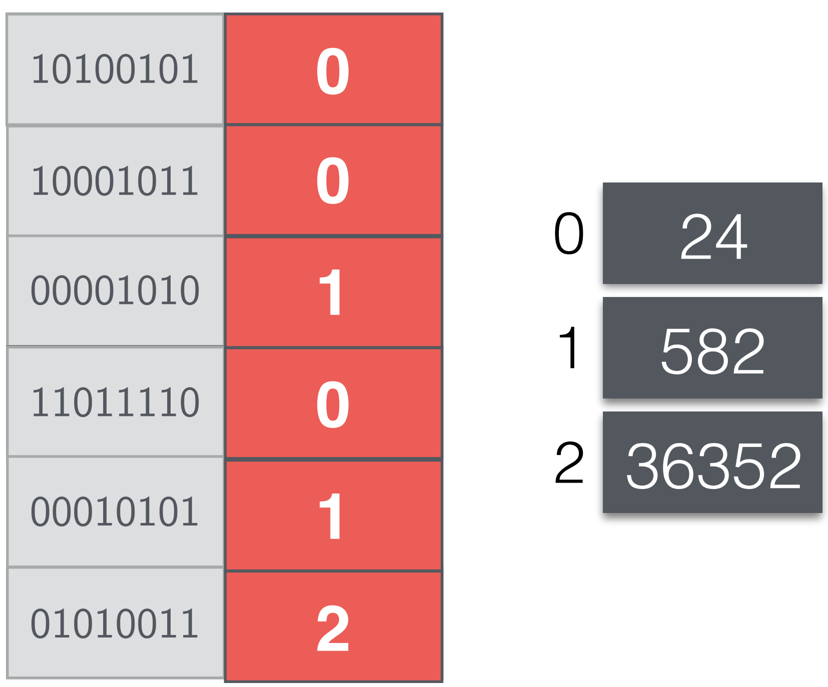
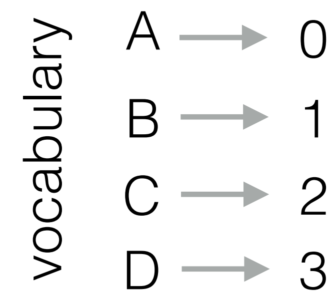
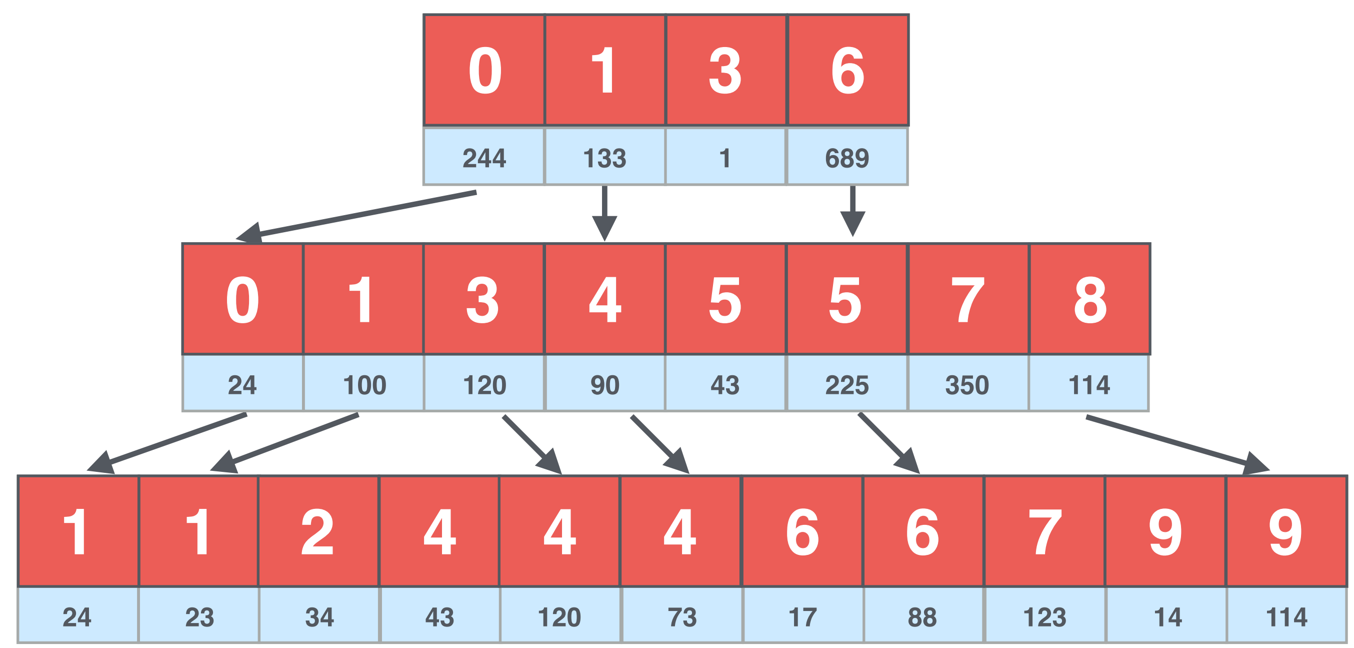
Data structure is *static*.
Minimal Perfect Hashing.







Encode each level
with **Elias-Fano**.

Random access.

tongrams - Preliminary results

n	Number of n-grams	Maximum frequency count	Unique frequency counts	$\lceil \lg \rceil$ of unique frequency counts
1	24,359,472	468,491,999,592	246,588	18
2	5,089,239	155,178,163	44,822	16
3	52,635,338	102,329,901	71,690	17
4	11,149,161	6,401,274	21,127	15
5	8,261,975	958,556	12,171	14
Total	101,495,185	468,491,999,592	266,760	19

Table 4: Basic statistics for the GoogleWeb1T subset.

		Total space in GBs	Bytes per gram		Lookup time [μs]	
HASH	KenLM	2.570	27.19		0.248	
	sxlm	**1.012**	**10.43**	(−61.64%)	**0.242**	(−2.42%)
TRIE	KenLM	1.829	21.5		1.272	
	sxlm	**0.541**	**5.7**	(−73.34%)	**1.229**	(−3.38%)

Table 5: Bytes per grams and average lookup time in μs for the GoogleWeb1T subset.

tongrams - Preliminary results 2

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(Some) Future Research Problems

Dynamic Inverted Indexes.

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Classic solution: use two indexes.
One is big and **cold**; the other is small and **hot**.
Merge them periodically.

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Compressed B-trees.

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Problem: maintain a dictionary on disk.

Motivations: databases and file-systems.

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Michael Bender

Stony Brook University

Martin Farach-Colton

Rutgers University

Bradley Kuszmaul

MIT Laboratory for Computer Science

Tokutek®

(Some) Future Research Problems

Fast Successor for IP-lookup.

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Successor search is what routers do for every incoming packet.

Hence, the most run algorithm in the world.

Time *and* space efficiency is crucial.

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0 0 0 0 1 1
0 0 0 1 0 0
0 0 0 1 1 1
0 0 1 1 0 1
0 0 1 1 1 0
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1110111010001000

Build an index on zeros.

0 0 0	0 1 1
0 0 0	1 0 0
0 0 0	1 1 1
0 0 1	1 0 1
0 0 1	1 1 0
0 0 1	1 1 1
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$$\mathbf{p = select_0(h_x) - h_x}$$

0 0 0	0 1 1
0 0 0	1 0 0
0 0 0	1 1 1
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0 0 0	0 1 1
0 0 0	1 0 0
0 0 0	1 1 1
0 0 1	1 0 1
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Thanks for your attention, time, patience!

Any questions?