Space- and Time-Efficient Data Structures for Massive Datasets

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08/03/2019

Evidence

The increase of data and, hence, information does **not** scale with technology.

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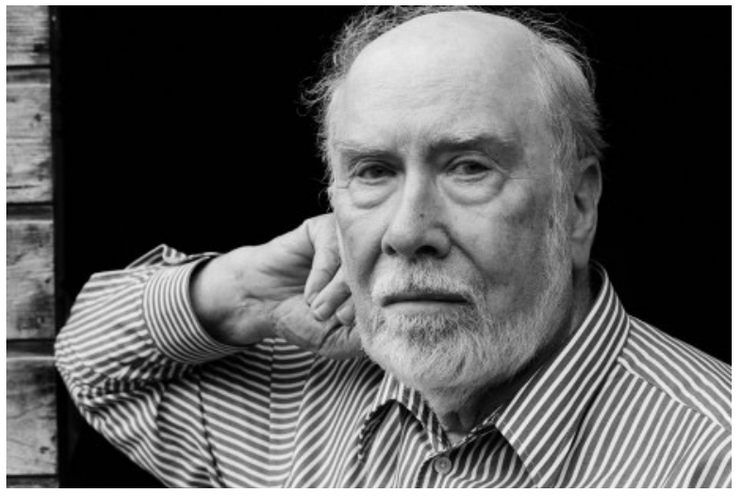

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Niklaus Wirth, A Plea for Lean Software

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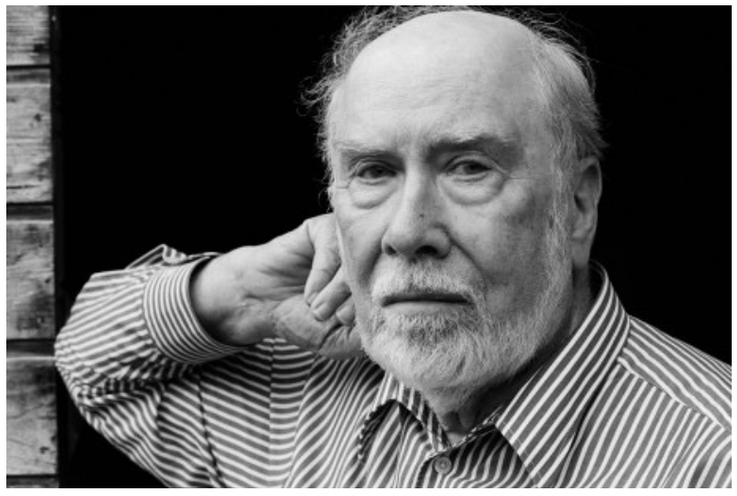

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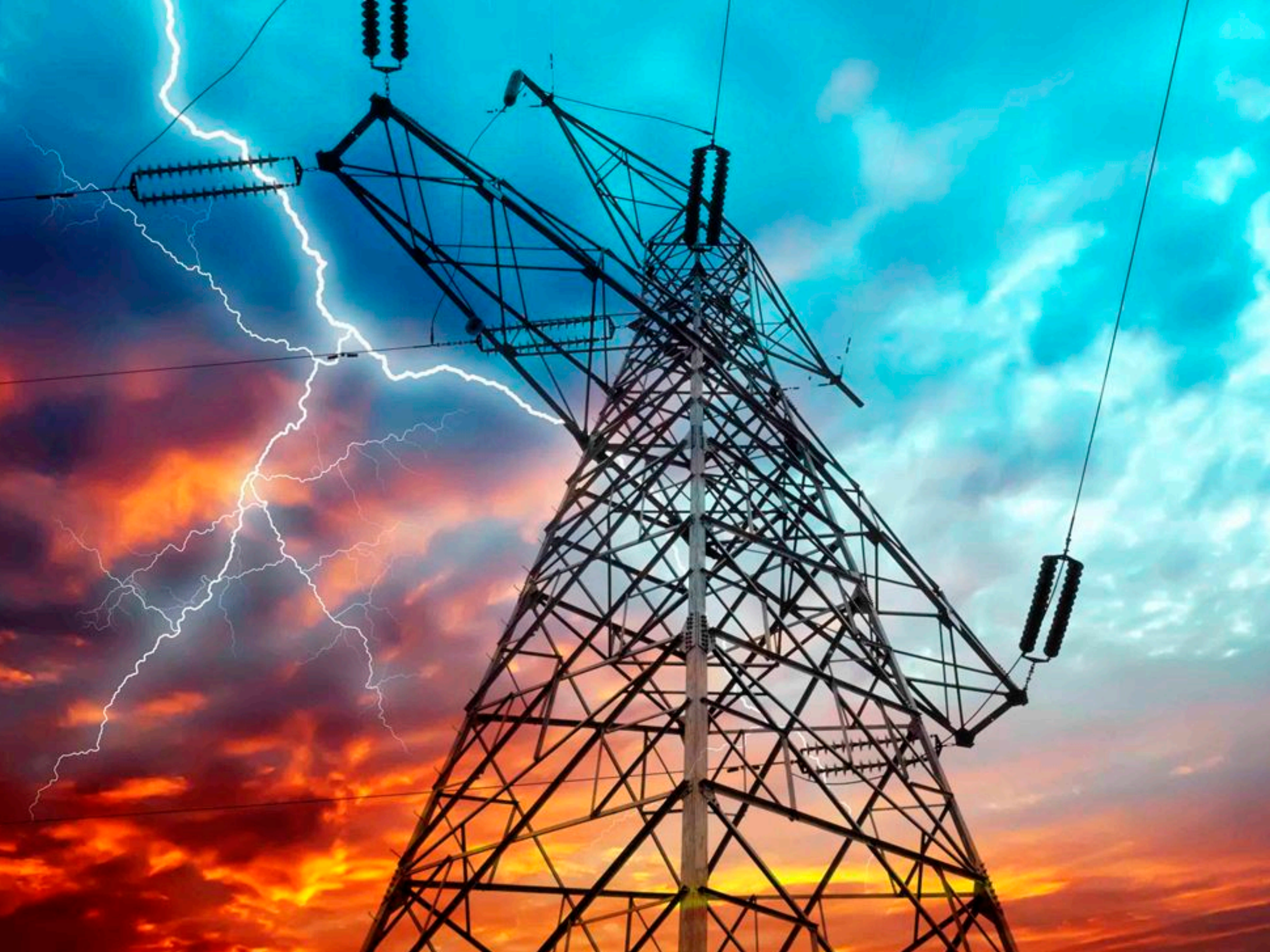
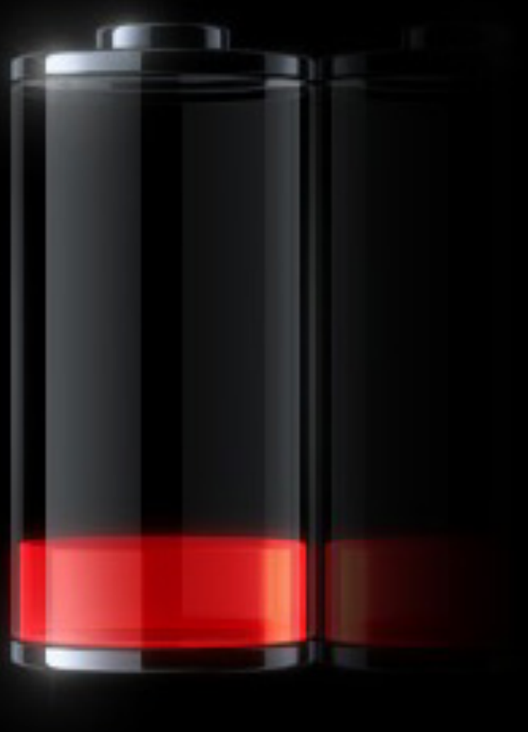
Even more relevant today!


Statistics
Progress
Growth & Decline
Statistical
Analysis
70%





Achieved results

Clustered Elias-Fano Indexes **Journal paper**

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Full paper, 34 pages, 2017.

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integer sequences

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short strings

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Problem 1

Consider a sorted integer sequence.

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How to represent it as a bit-vector where each original integer is uniquely-decodable, using **as few as possible** bits?

How to maintain **fast decompression speed**?

Ubiquity

Inverted indexes



Databases



E-Commerce


Graph compression



Semantic data


Geo-spatial data


Inverted indexes

The inverted index is the *de-facto* data structure at the basis of every large-scale retrieval system.

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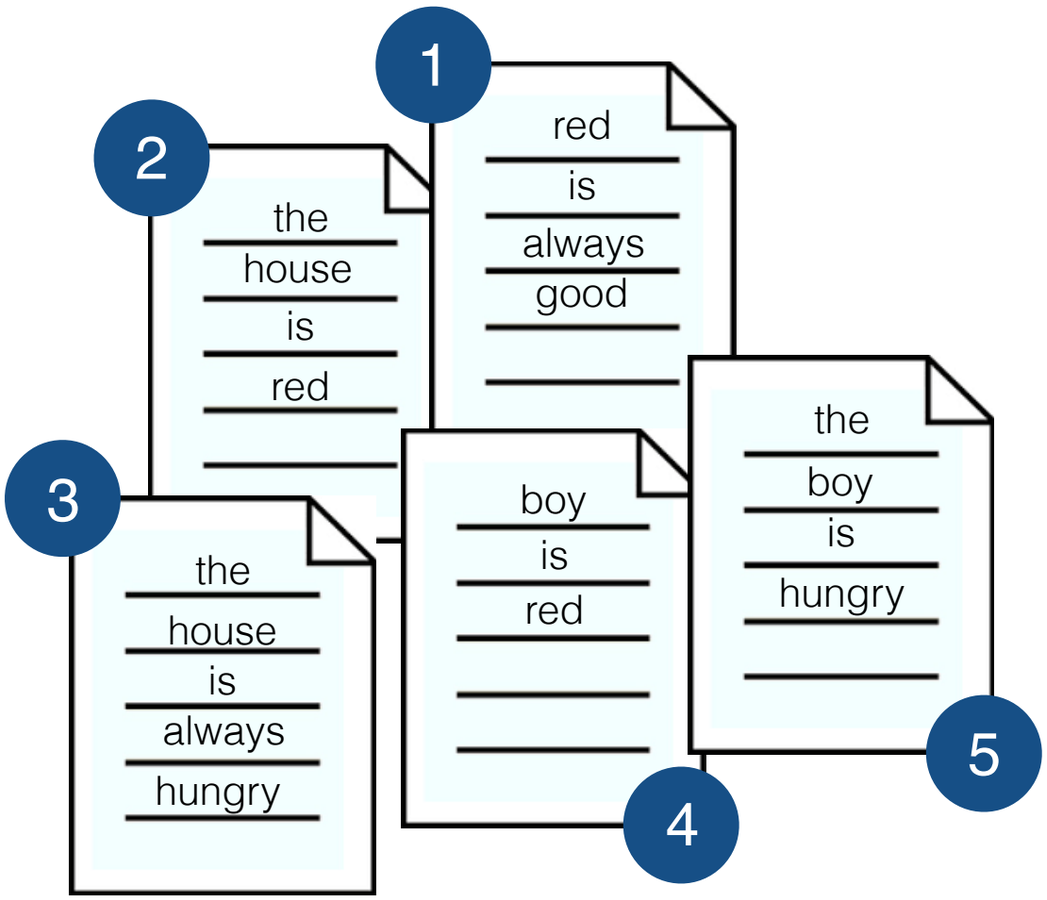

t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8

{always, boy, good, house, hungry, is, red, the}

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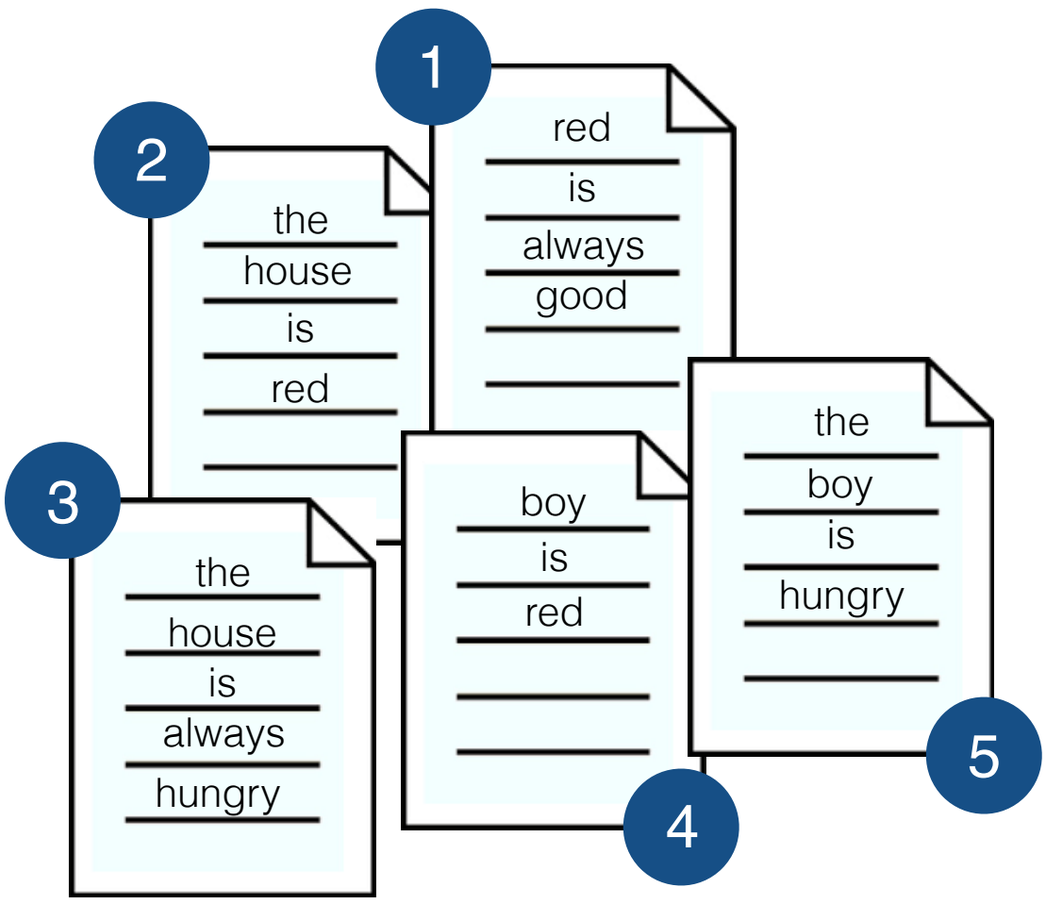

t_1 t_2 t_3 t_4 t_5 t_6 t_7 t_8

{always, boy, good, house, hungry, is, red, the}

$L_{t_1}=[1, 3]$
$L_{t_2}=[4, 5]$
$L_{t_3}=[1]$
$L_{t_4}=[2, 3]$
$L_{t_5}=[3, 5]$
$L_{t_6}=[1, 2, 3, 4, 5]$
$L_{t_7}=[1, 2, 4]$
$L_{t_8}=[2, 3, 5]$

Many solutions

Large research corpora describing different **space/time** trade-offs.

- Elias' Gamma and Delta
- Variable-Byte Family
- Binary Interpolative Coding
- Simple Family
- PForDelta
- QMX
- Elias-Fano
- Partitioned Elias-Fano

~1970

2014

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Space

Binary Interpolative Coding

~**3X** smaller

Spectrum

Time

Variable-Byte Family

~**4.5X** faster

Key research questions

Space

Binary Interpolative Coding (BIC)

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← Spectrum →

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Key research questions

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Binary Interpolative Coding (BIC) ← Spectrum → **Variable-Byte (VByte) Family**

~**3X** smaller

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1 Is it possible to design an encoding that is **as small as BIC** and **much faster**?

Key research questions

Space ← Spectrum → **Time**

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~**3X** smaller

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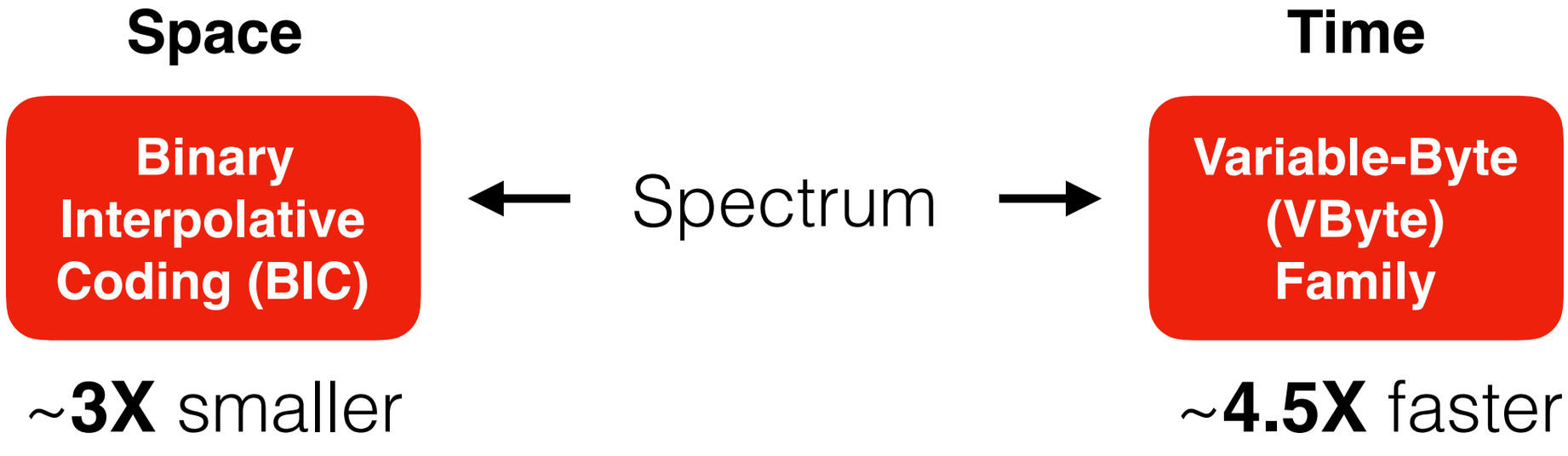
1 Is it possible to design an encoding that is **as small as BIC** and **much faster**?

2 Is it possible to design an encoding that is **as fast as VByte** and **much smaller**?

Key research questions



1. Is it possible to design an encoding that is **as small as BIC** and **much faster**?

2. Is it possible to design an encoding that is **as fast as VByte** and **much smaller**?

3. What about **both** objectives at the same time?!

Key research questions

Space ← Spectrum → **Time**

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~**3X** smaller

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TOIS 2017

Is it possible to design an encoding that is **as fast as VByte** and **much smaller**?

2

TKDE 2019

What about **both** objectives at the same time?!

3

WSDM 2019

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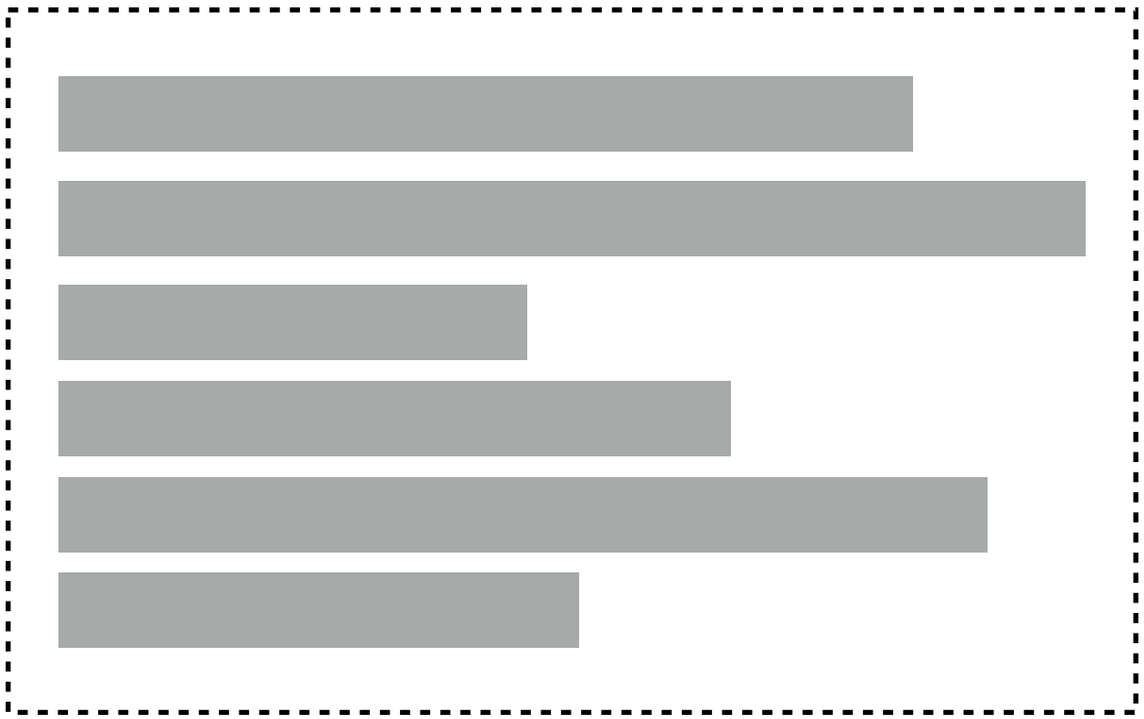

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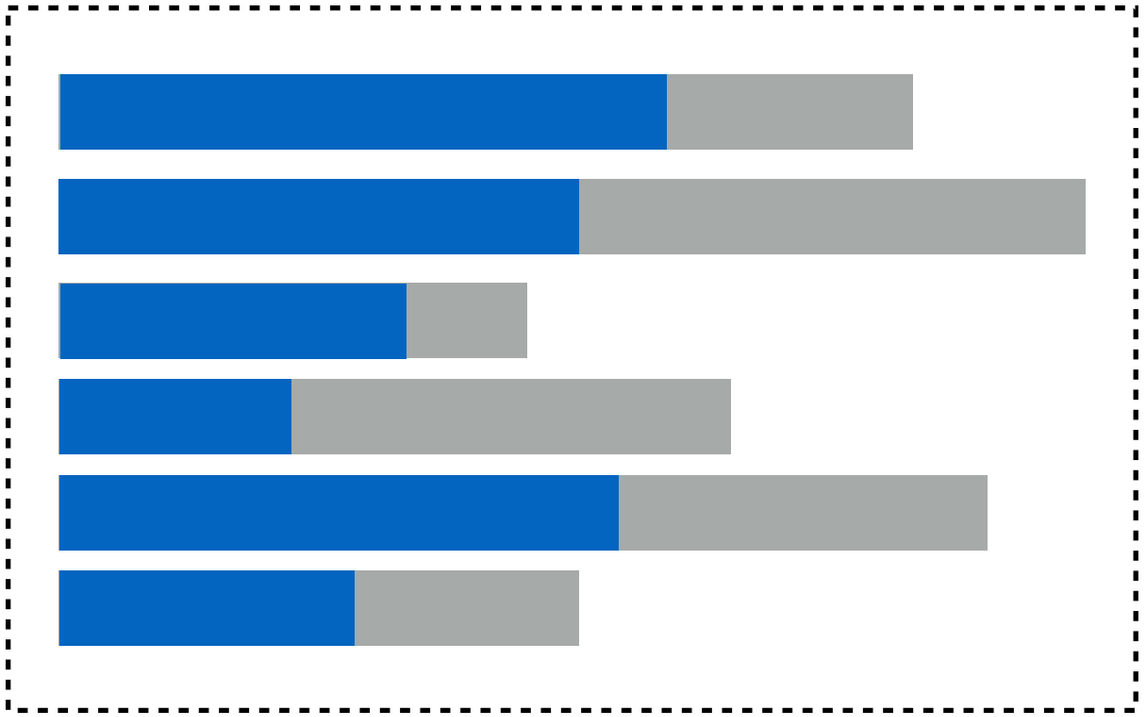

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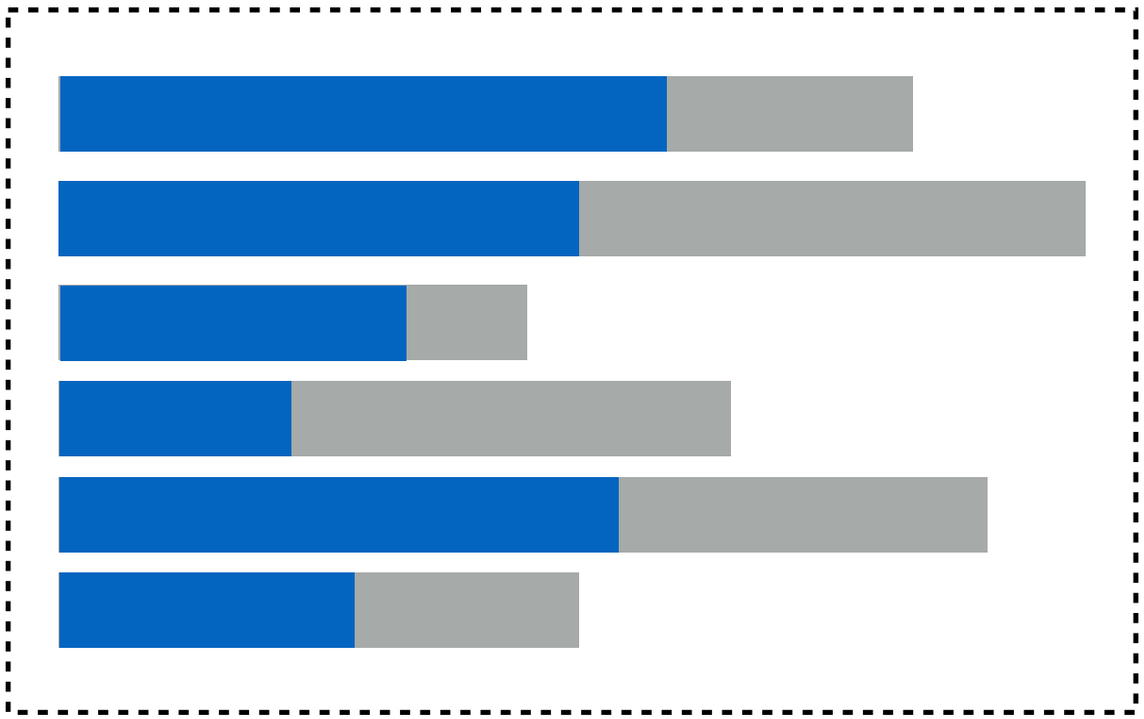

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Space

Always better than PEF (by up to **11%**) and better than BIC (by up to **6.25%**)

← Spectrum →

Time

Slightly slower than PEF (~20%) **Much faster** than BIC (2X)

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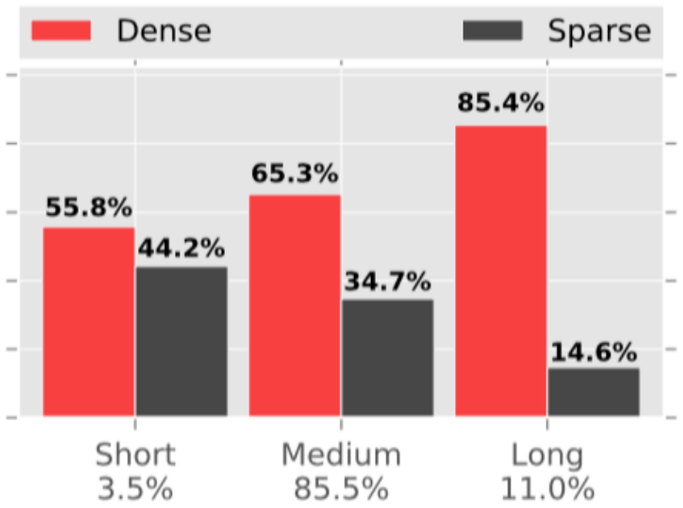
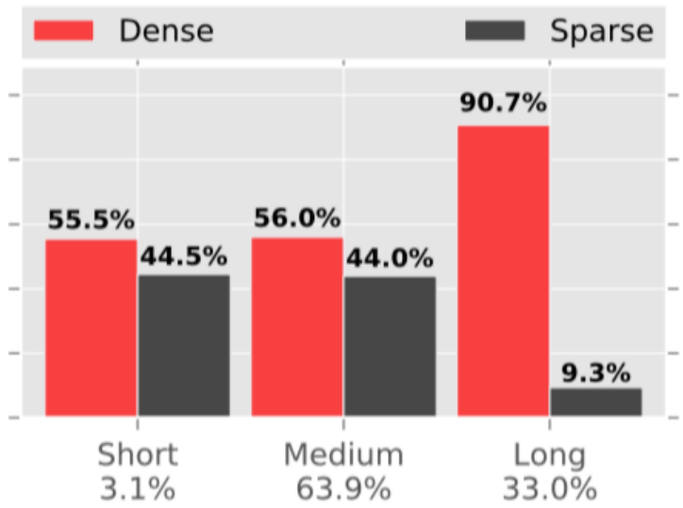


(a) Gov2



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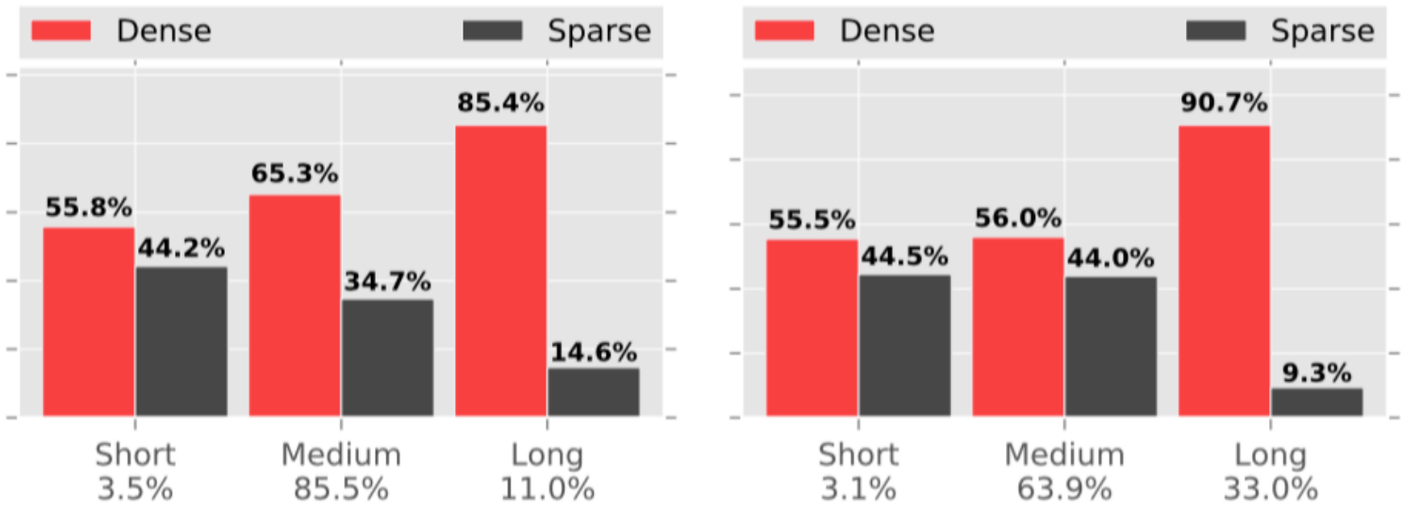

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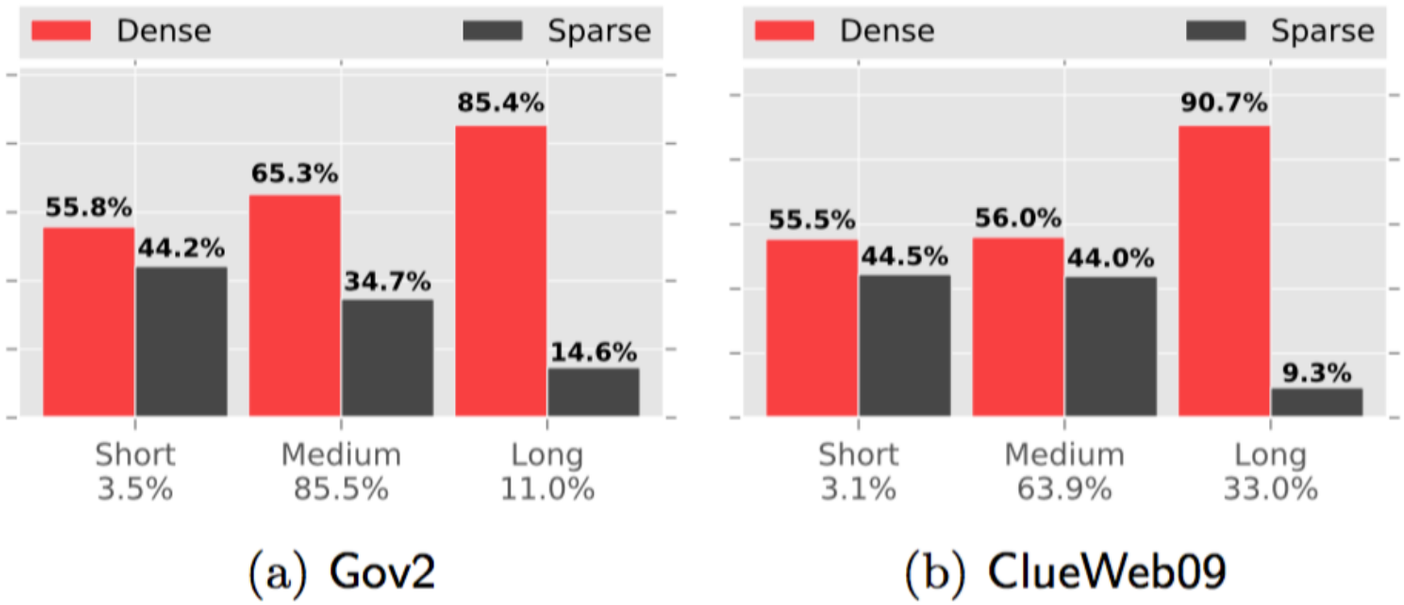

(a) Gov2 (b) ClueWeb09

Encode **dense** regions with unary codes, **sparse** regions with VByte.

Optimal partitioning in linear time and constant space.

Compression ratio improves by **2X**.

Query processing speed and sequential decoding (almost) **not affected**.

3 - Dictionary-based compression (WSDM 2019)

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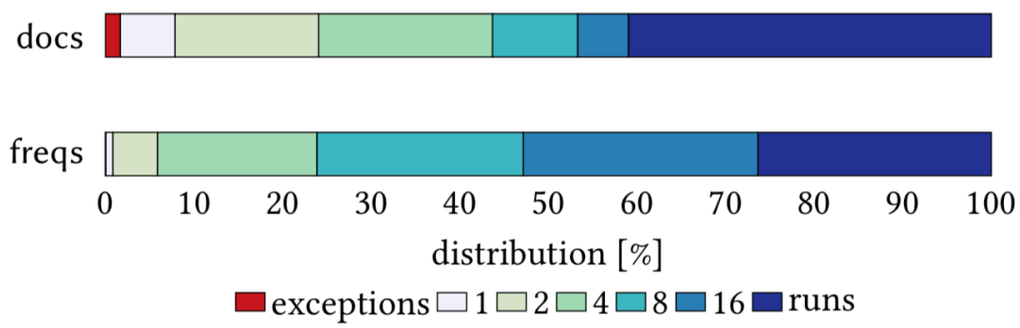

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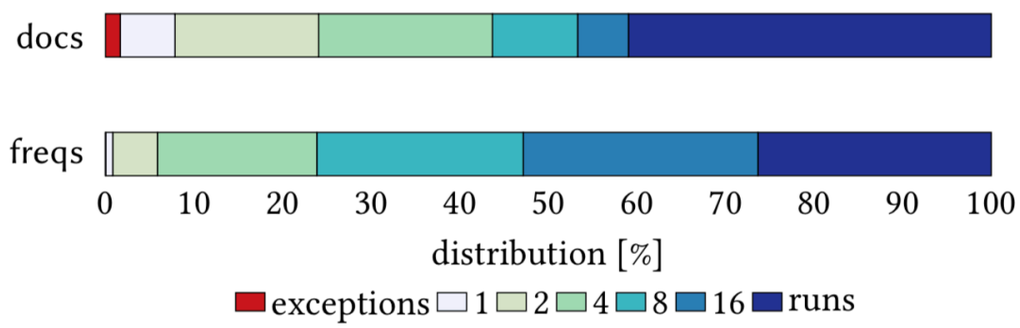
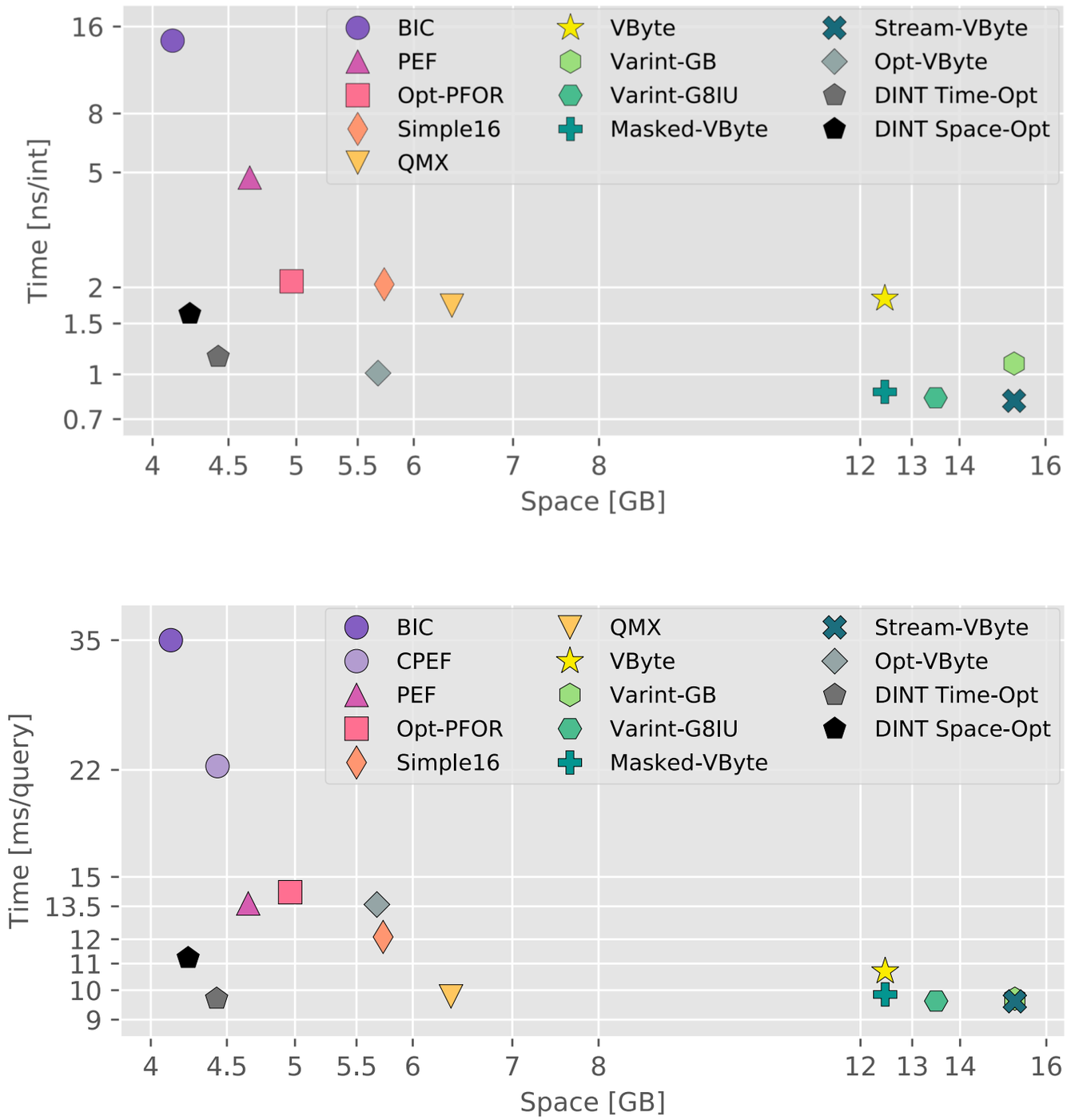


Close to the most space-efficient representation (~**7%** away from BIC).

Almost **as fast as** the fastest SIMD-ized decoders.

The bigger picture



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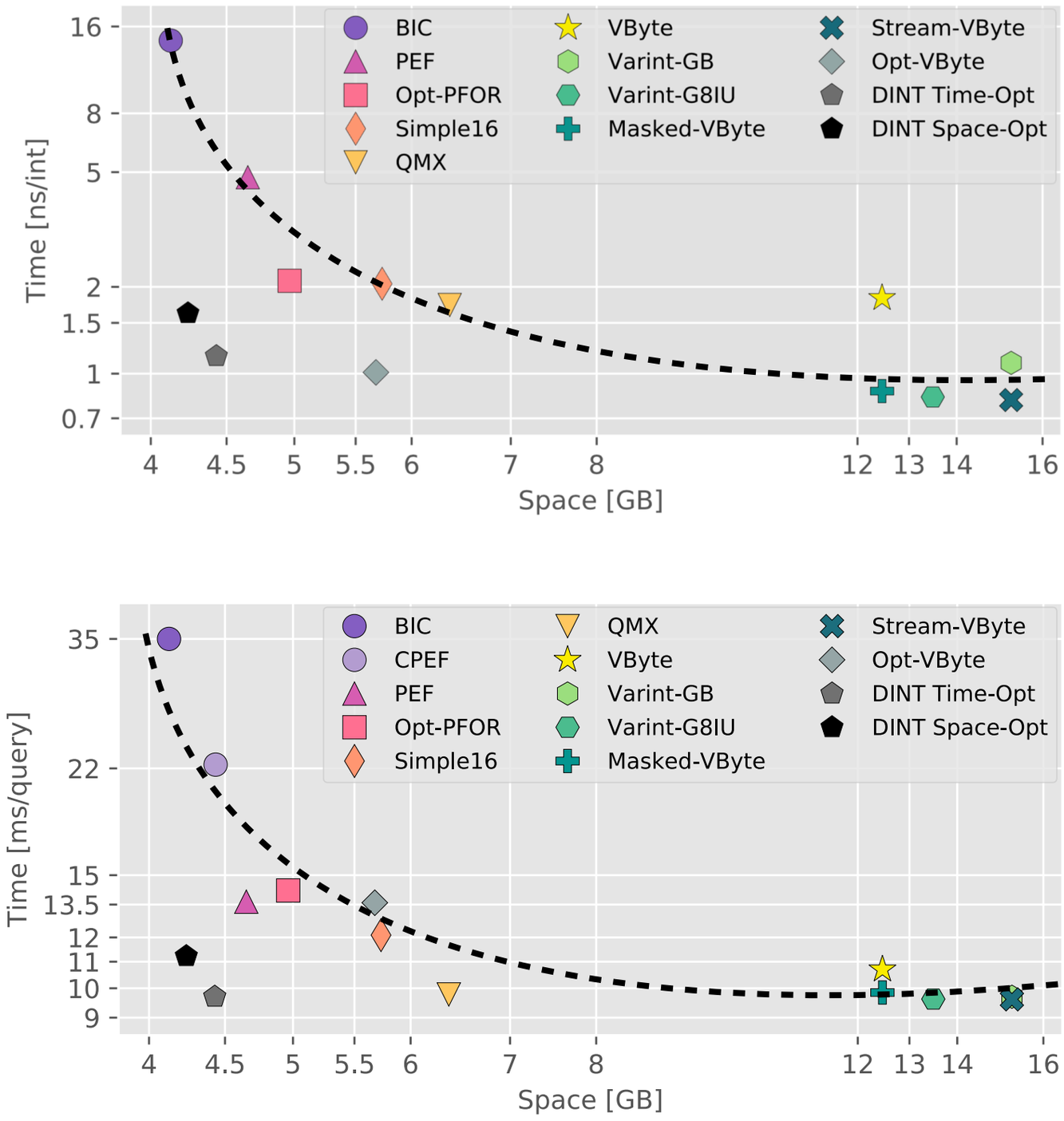

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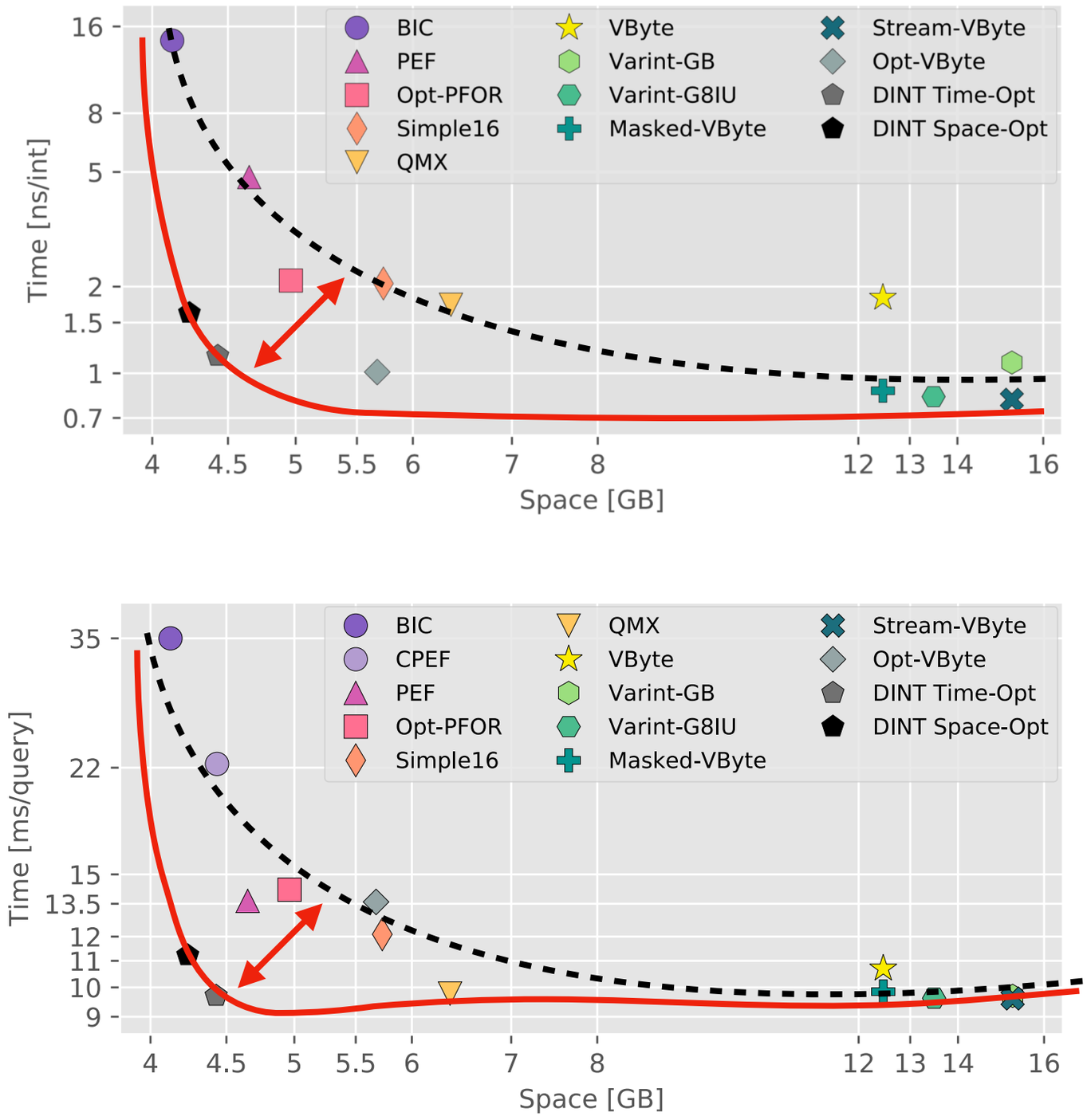

Problem 2

Consider a large text.

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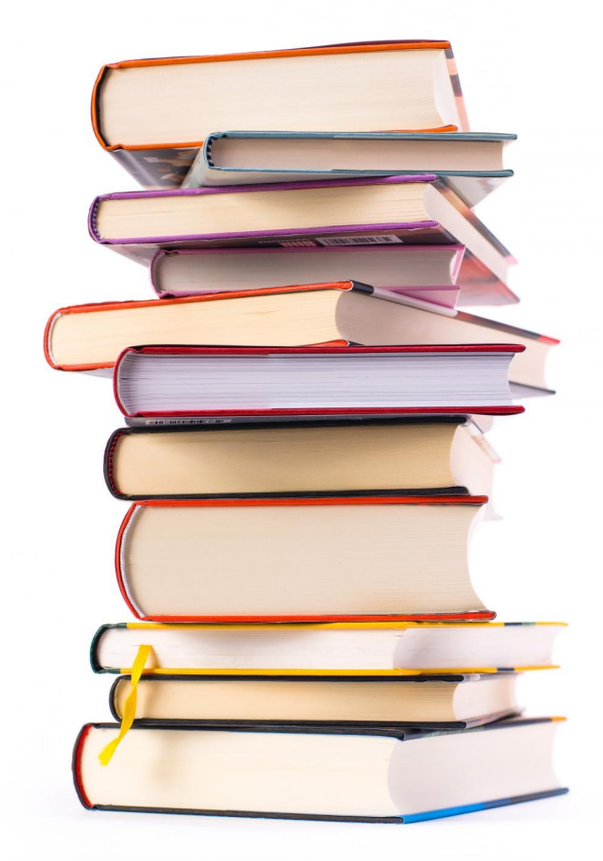
Consider a large text.

How to represent all its substrings of size $1 \leq k \leq N$ words for fixed N (e.g., N = 5), using **as few as possible** bits?

Fast **access** to individual N-grams?

How to **estimate** the probability of occurrence of the patterns under a given probability model?

Indexing



Google Books

~6% of the books ever published

N	number of N-grams
1	24,359,473
2	667,284,771
3	7,397,041,901
4	1,644,807,896
5	1,415,355,596

More than 11 billions of N-grams!

Context-based remapped tries (SIGIR 2017)

The number of words following a given context is **small**.

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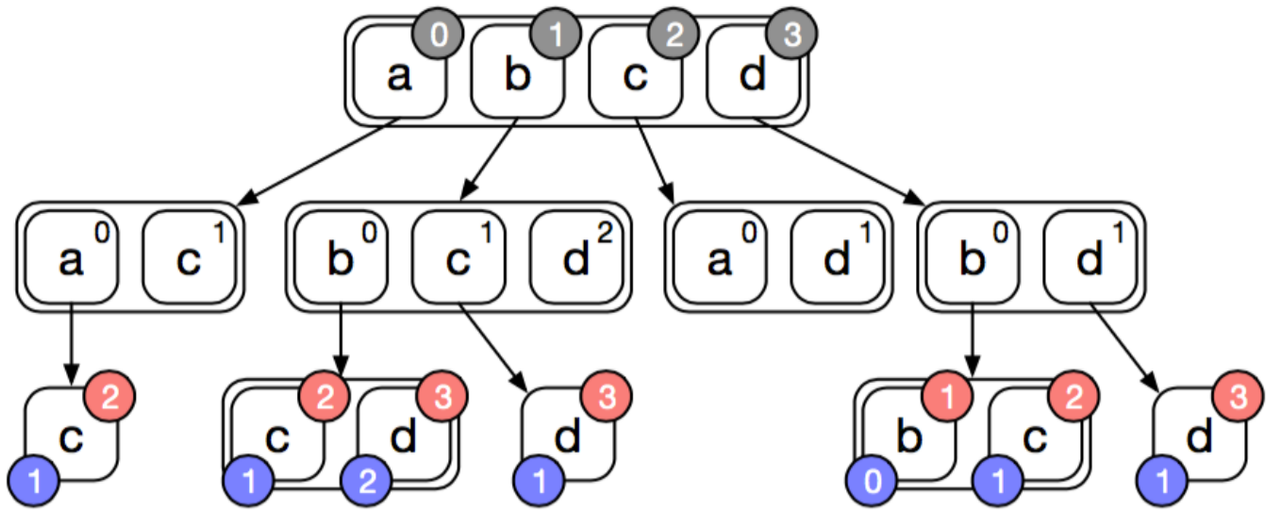
$k = 1$



Map a word ID to the **position** it takes within its *sibling* IDs (the IDs following a context of fixed length *k*).

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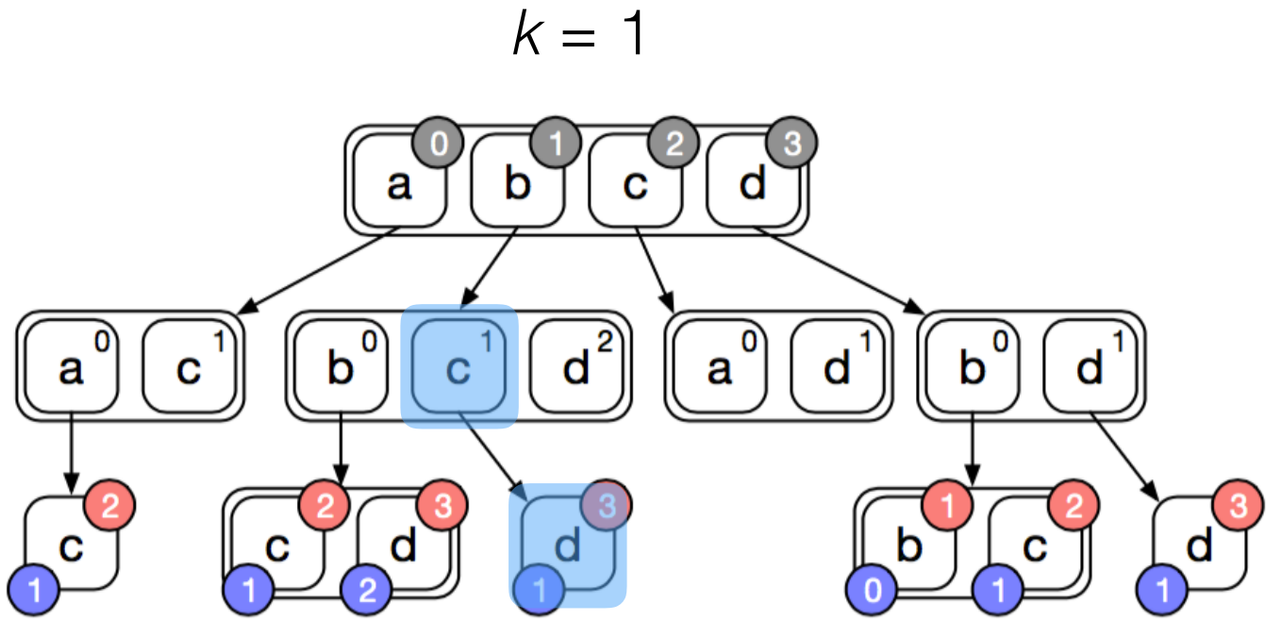

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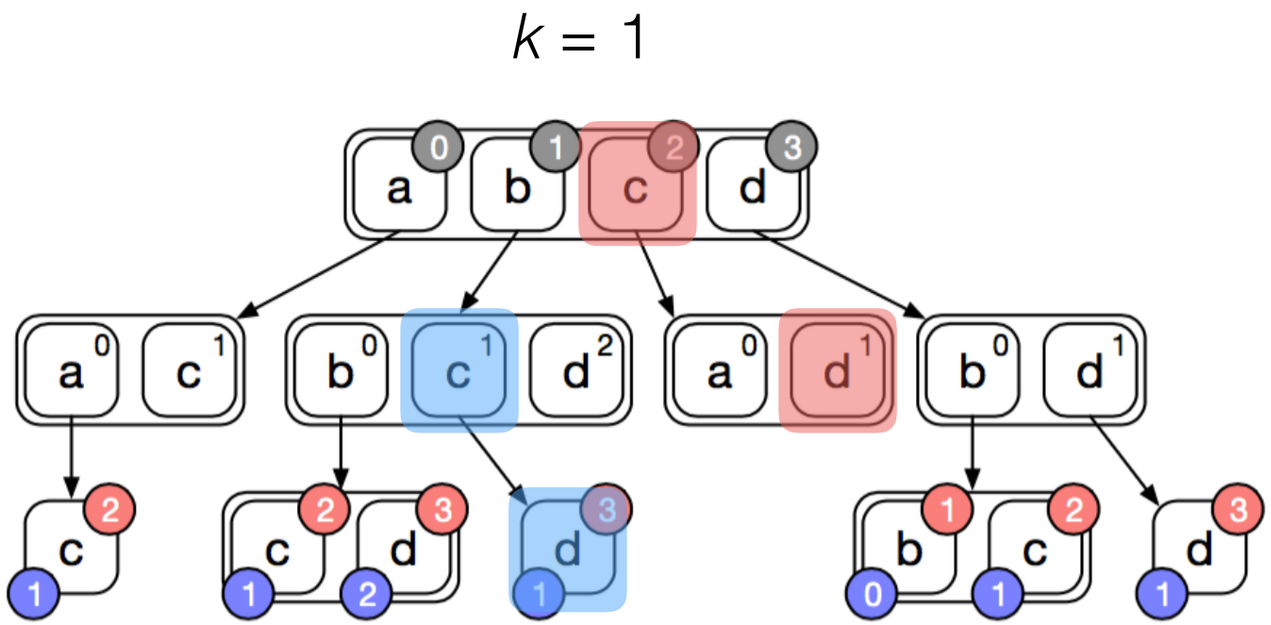

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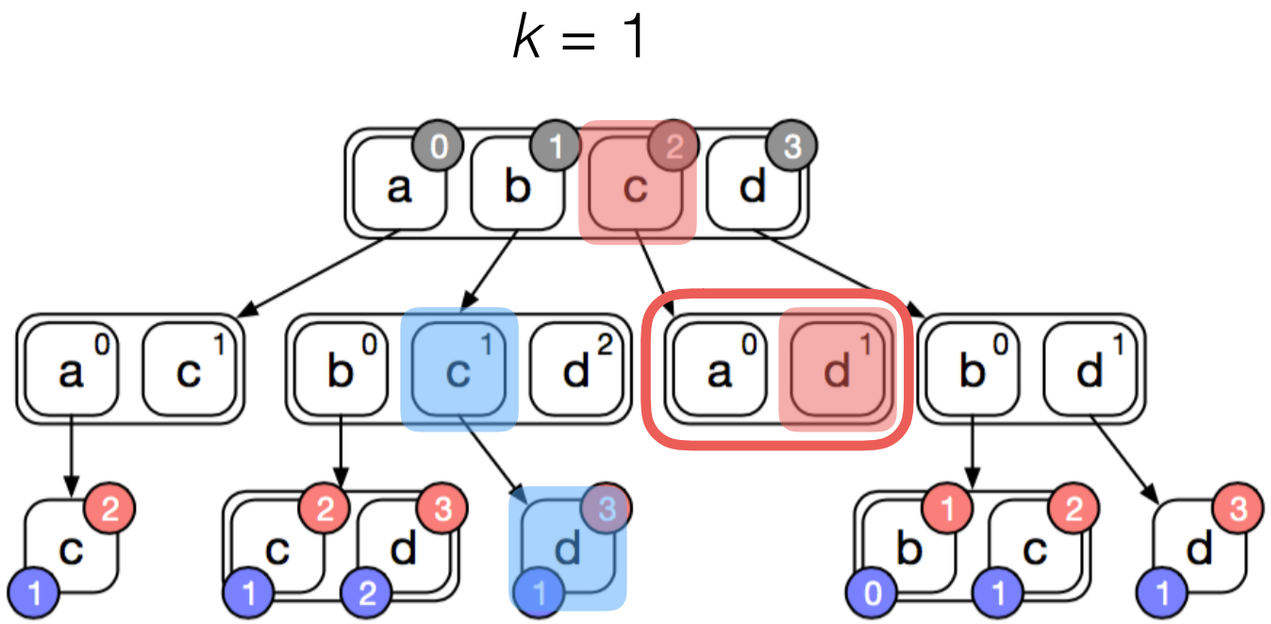

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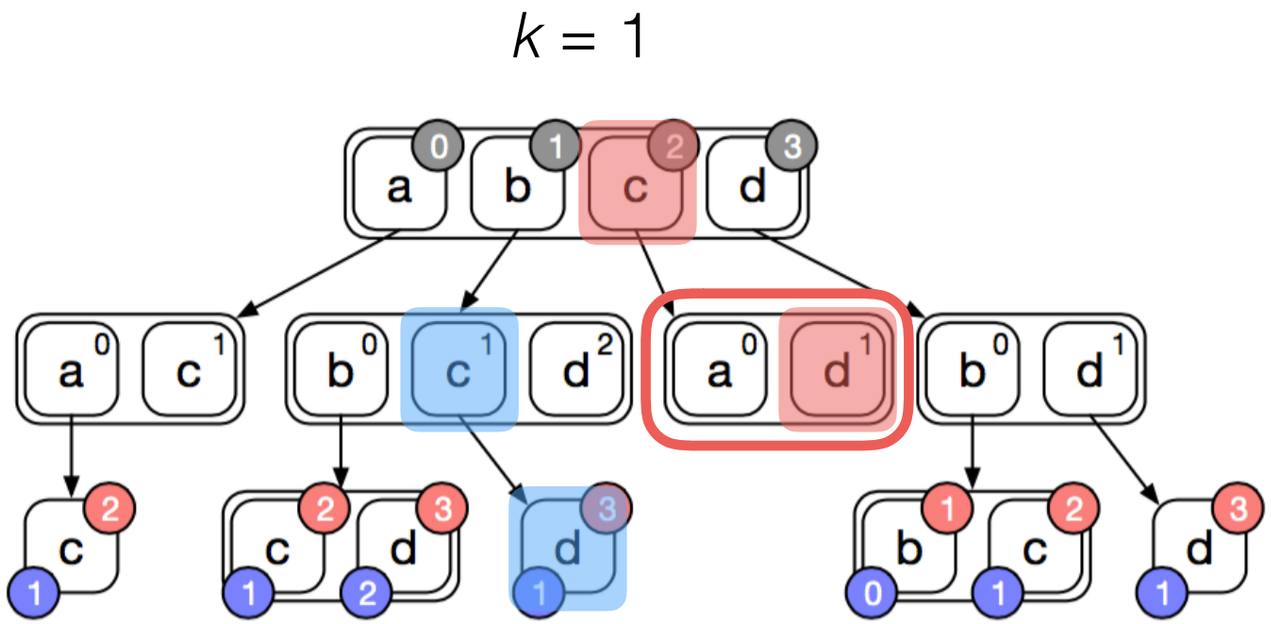

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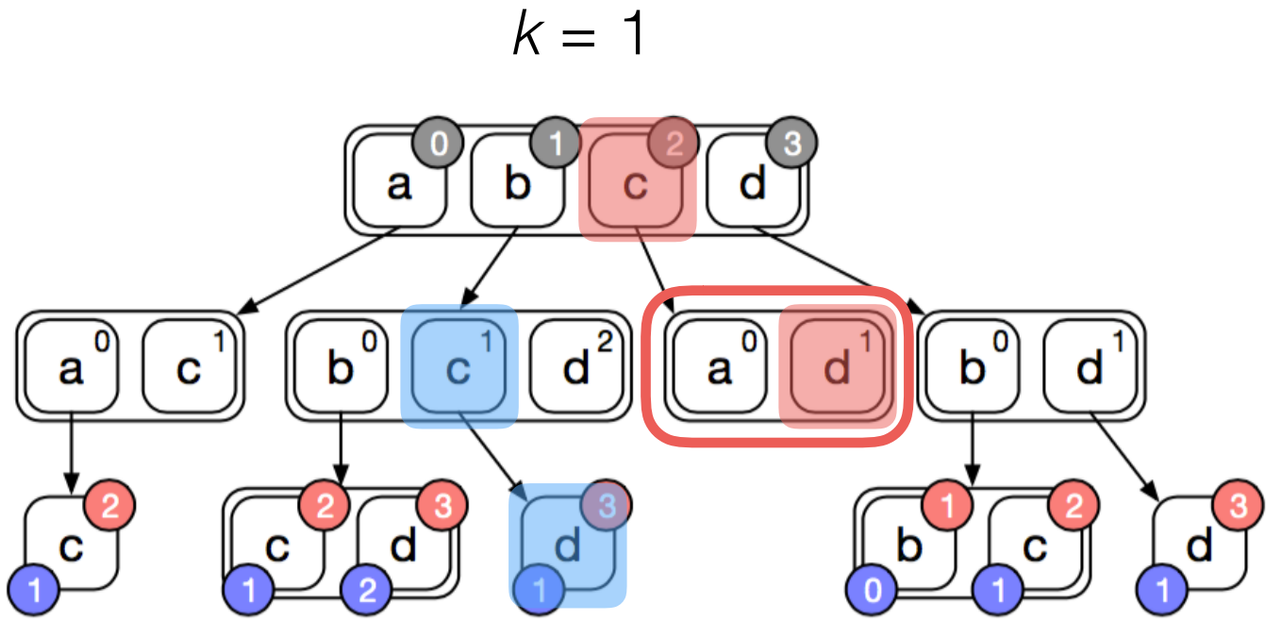

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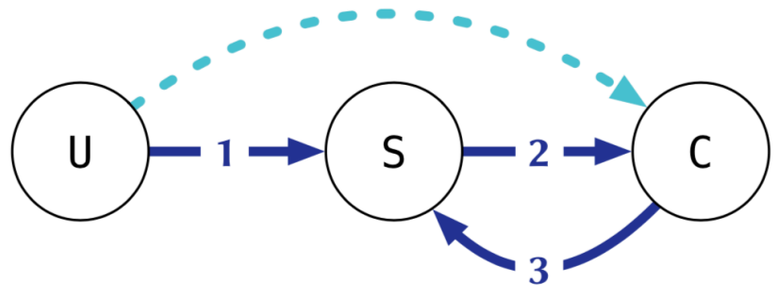
The (Elias-Fano) context-based remapped trie is **even smaller** than the most space-efficient competitors, that are lossy and with false-positives allowed, and up to **5X faster**.

Fast estimation in external memory (TOIS 2019)

To compute the modified Kneser-Ney probabilities of the N-grams, the fastest algorithm in the literature uses **3 sorting steps** in external memory.



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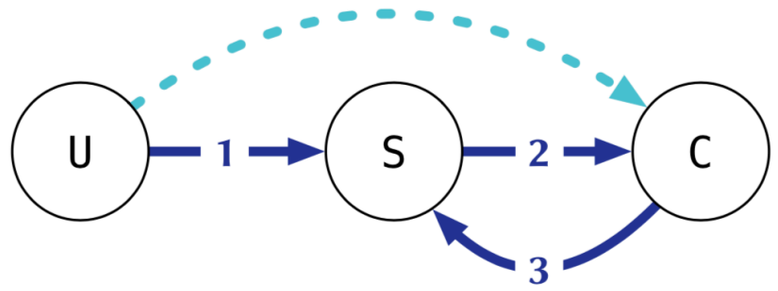

Computing the distinct left extensions.

1	2	3	4	5	6	7	8	9	10	11	12
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X
X	X	A	X	X	B	A	A	C	B	A	C

Suffix order

7	1	8	5	2	3	6	9	10	11	12	4
C	A	C	B	A	A	B	X	X	X	X	A
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
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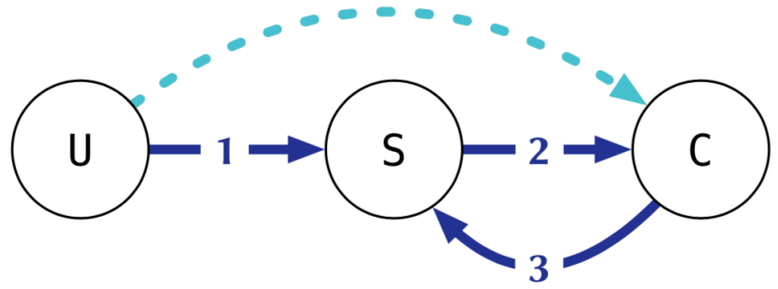

Computing the distinct left extensions.

1	2	3	4	5	6	7	8	9	10	11	12
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X
X	X	A	X	X	B	A	A	C	B	A	C

Suffix order

7	1	8	5	2	3	6	9	10	11	12	4
C	A	C	B	A	A	B	X	X	X	X	A
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X

Context order

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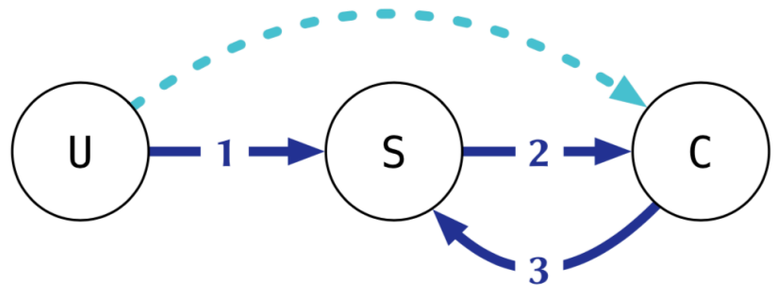

Computing the distinct left extensions.

1	2	3	4	5	6	7	8	9	10	11	12
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X
X	X	A	X	X	B	A	A	C	B	A	C

Suffix order

7	1	8	5	2	3	6	9	10	11	12	4
C	A	C	B	A	A	B	X	X	X	X	A
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X

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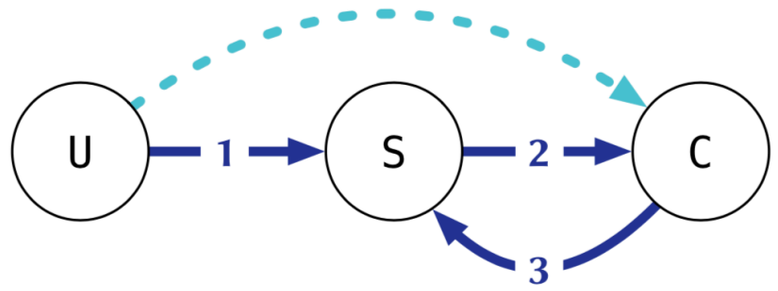

Computing the distinct left extensions.

1	2	3	4	5	6	7	8	9	10	11	12
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X
X	X	A	X	X	B	A	A	C	B	A	C

Suffix order

7	1	8	5	2	3	6	9	10	11	12	4
C	A	C	B	A	A	B	X	X	X	X	A
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X

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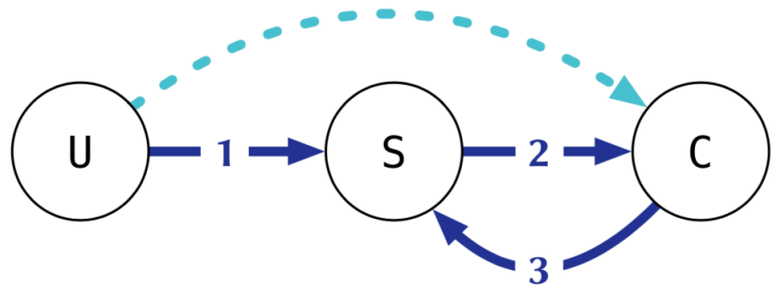
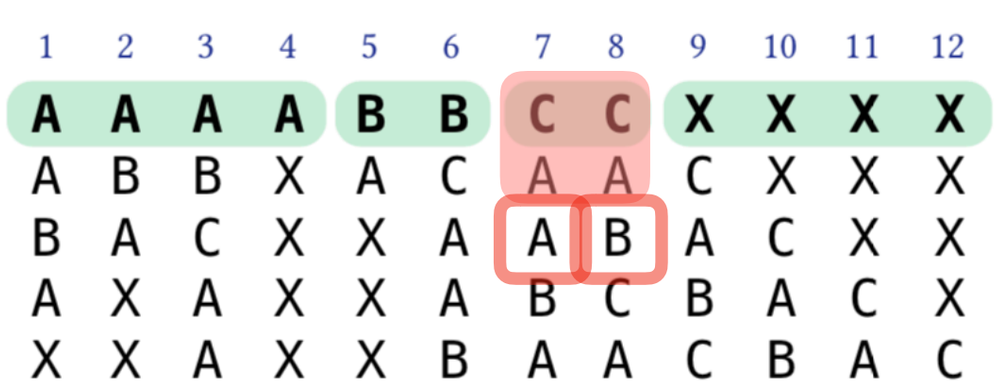
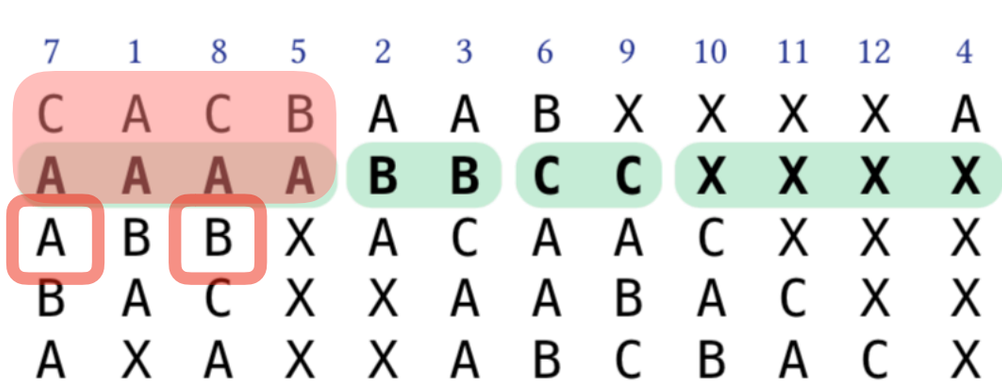


Computing the distinct left extensions.



Suffix order



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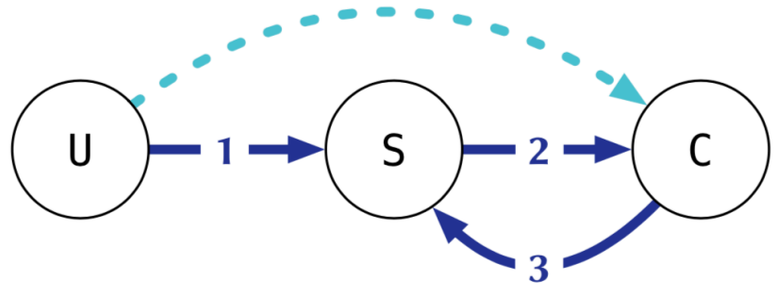

Computing the distinct left extensions.

1	2	3	4	5	6	7	8	9	10	11	12
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X
X	X	A	X	X	B	A	A	C	B	A	C

Suffix order

7	1	8	5	2	3	6	9	10	11	12	4
C	A	C	B	A	A	B	X	X	X	X	A
A	**A**	**A**	**A**	**B**	**B**	**C**	**C**	**X**	**X**	**X**	**X**
A	B	B	X	A	C	A	A	C	X	X	X
B	A	C	X	X	A	A	B	A	C	X	X
A	X	A	X	X	A	B	C	B	A	C	X

Context order

Using a scan of the block and O(|V|) space.

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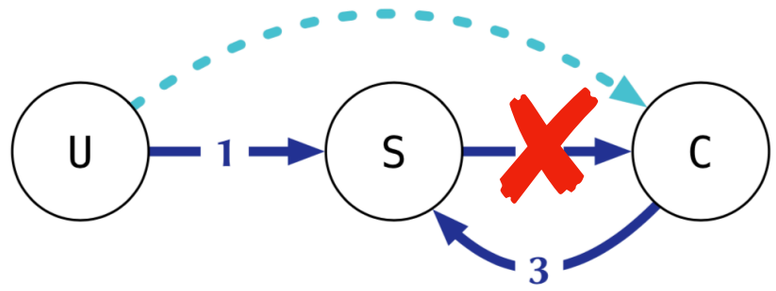

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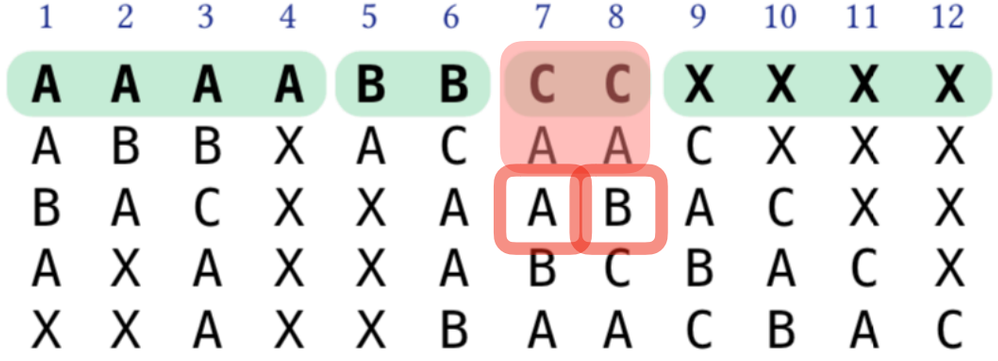

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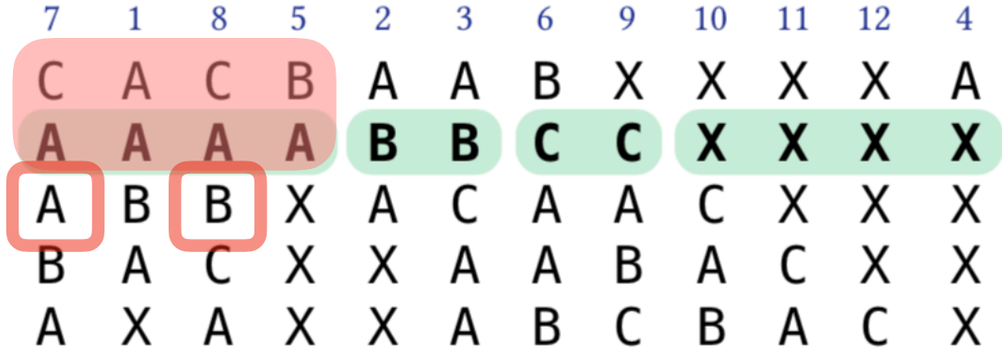

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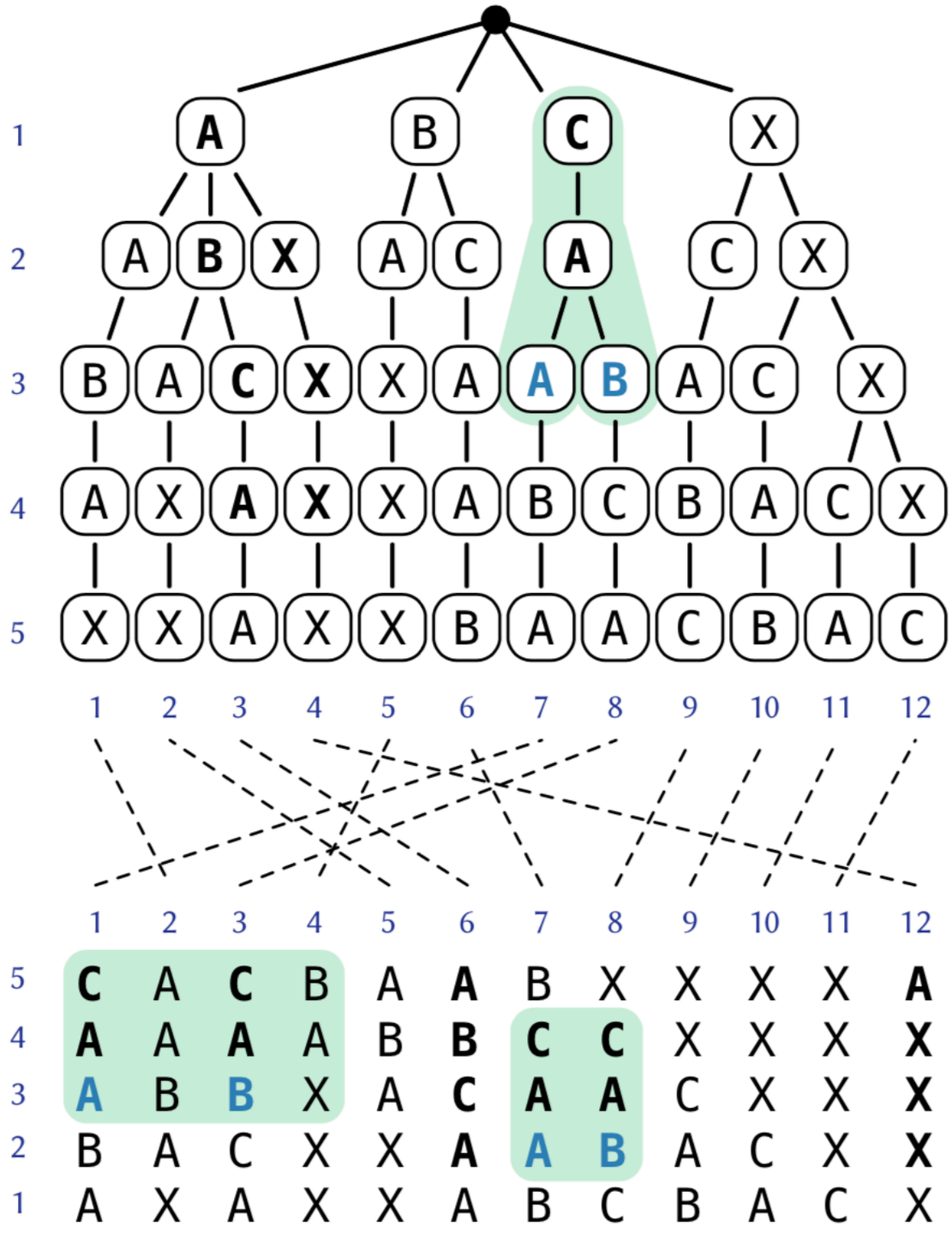

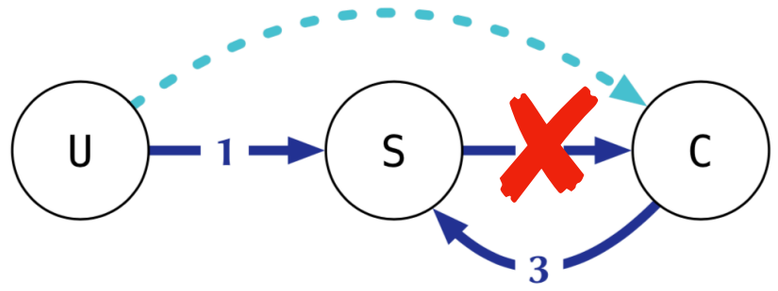

Rebuilding the last level of the trie.

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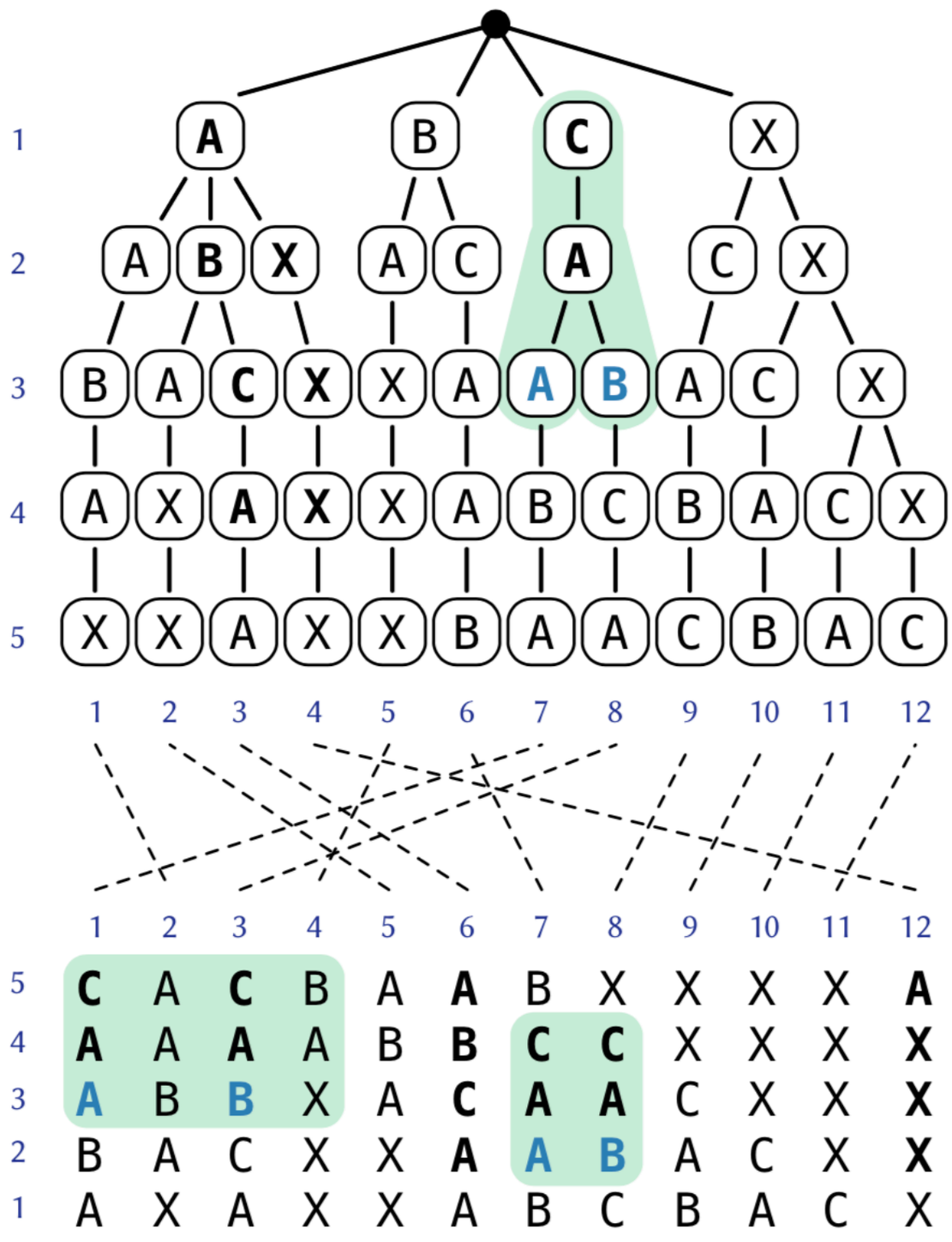
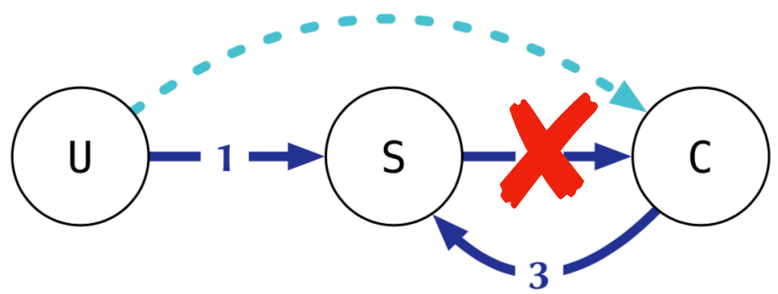
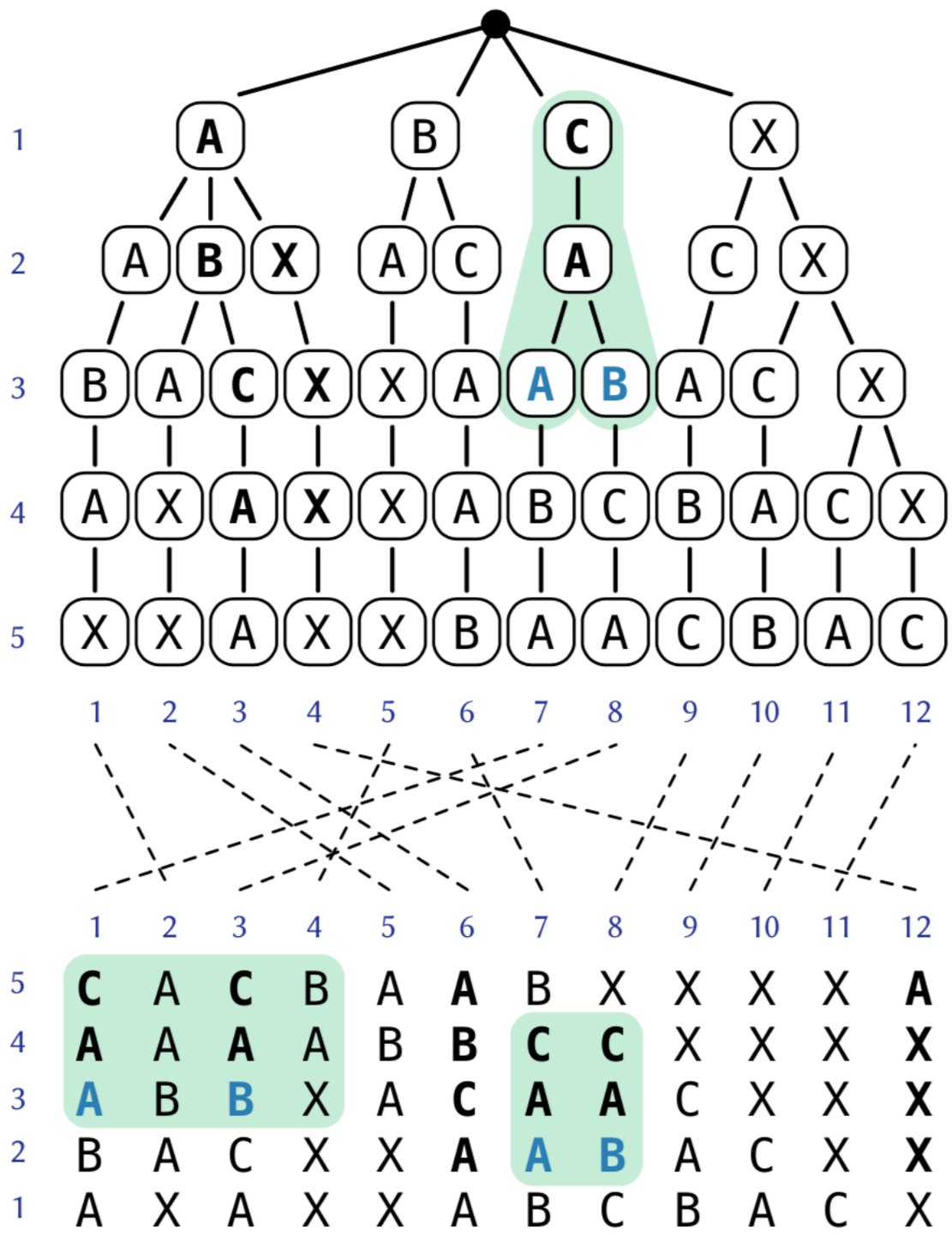


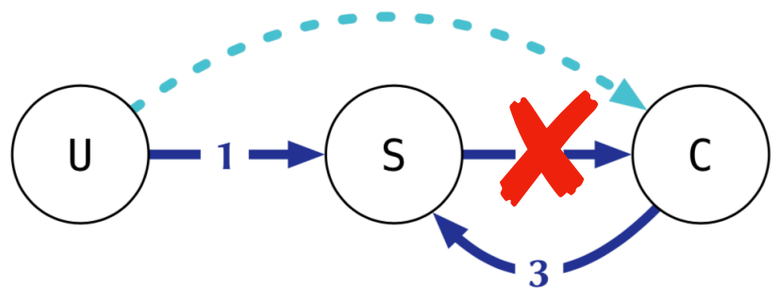


Rebuilding the last level of the trie.

A	4
B	2
C	2
X	4

Fast estimation in external memory (TOIS 2019)





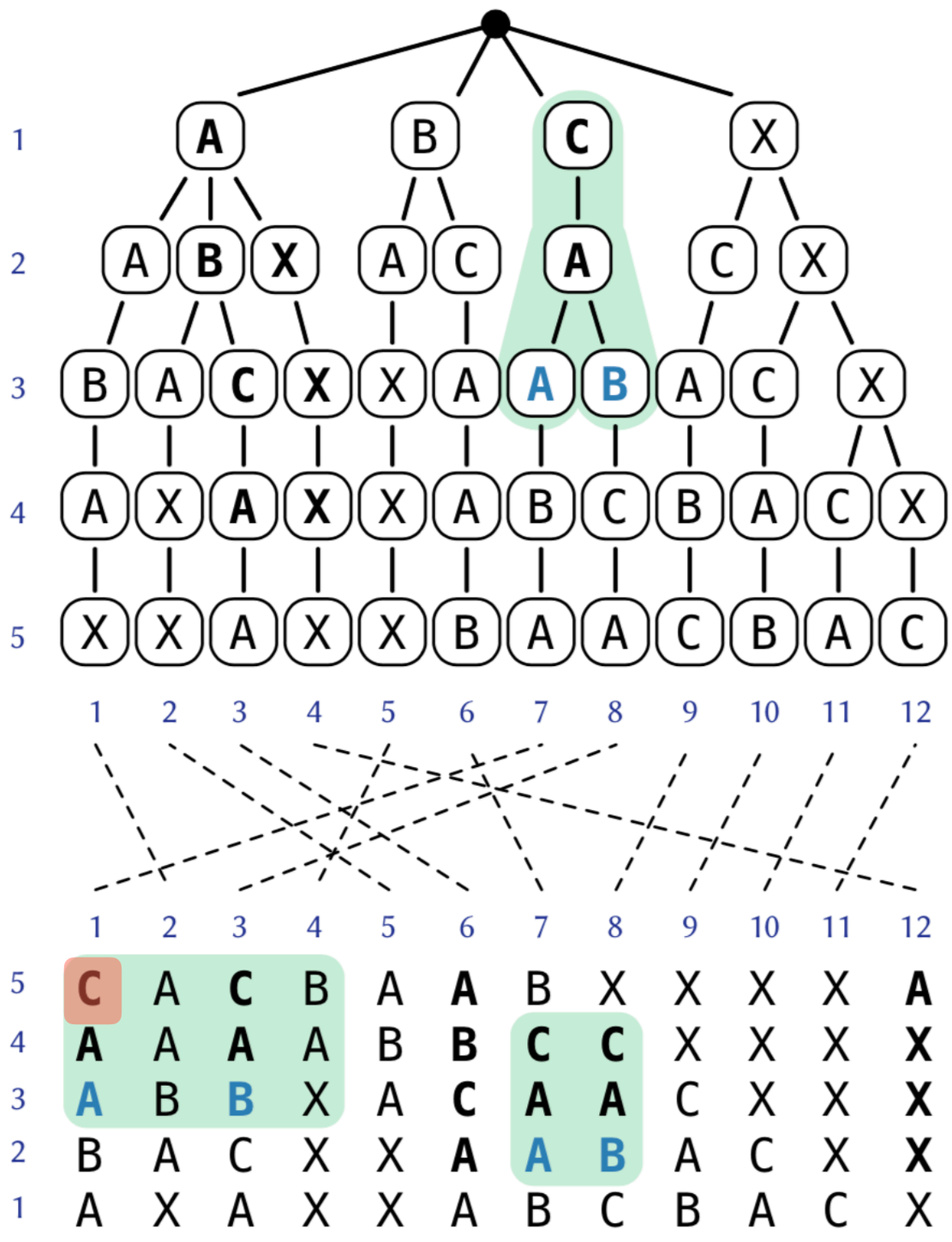
Rebuilding the last level of the trie.

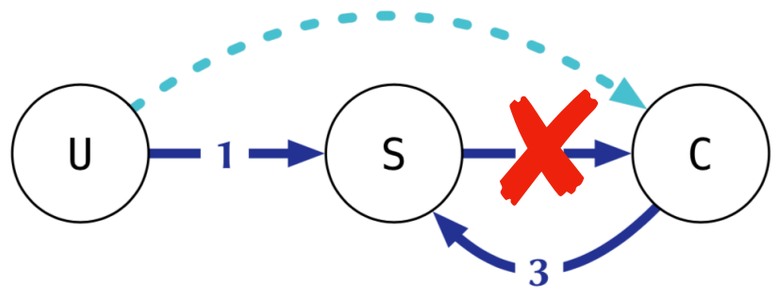
A	4
B	2
C	2
X	4

→

A	1
B	5
C	7
X	9

Fast estimation in external memory (TOIS 2019)





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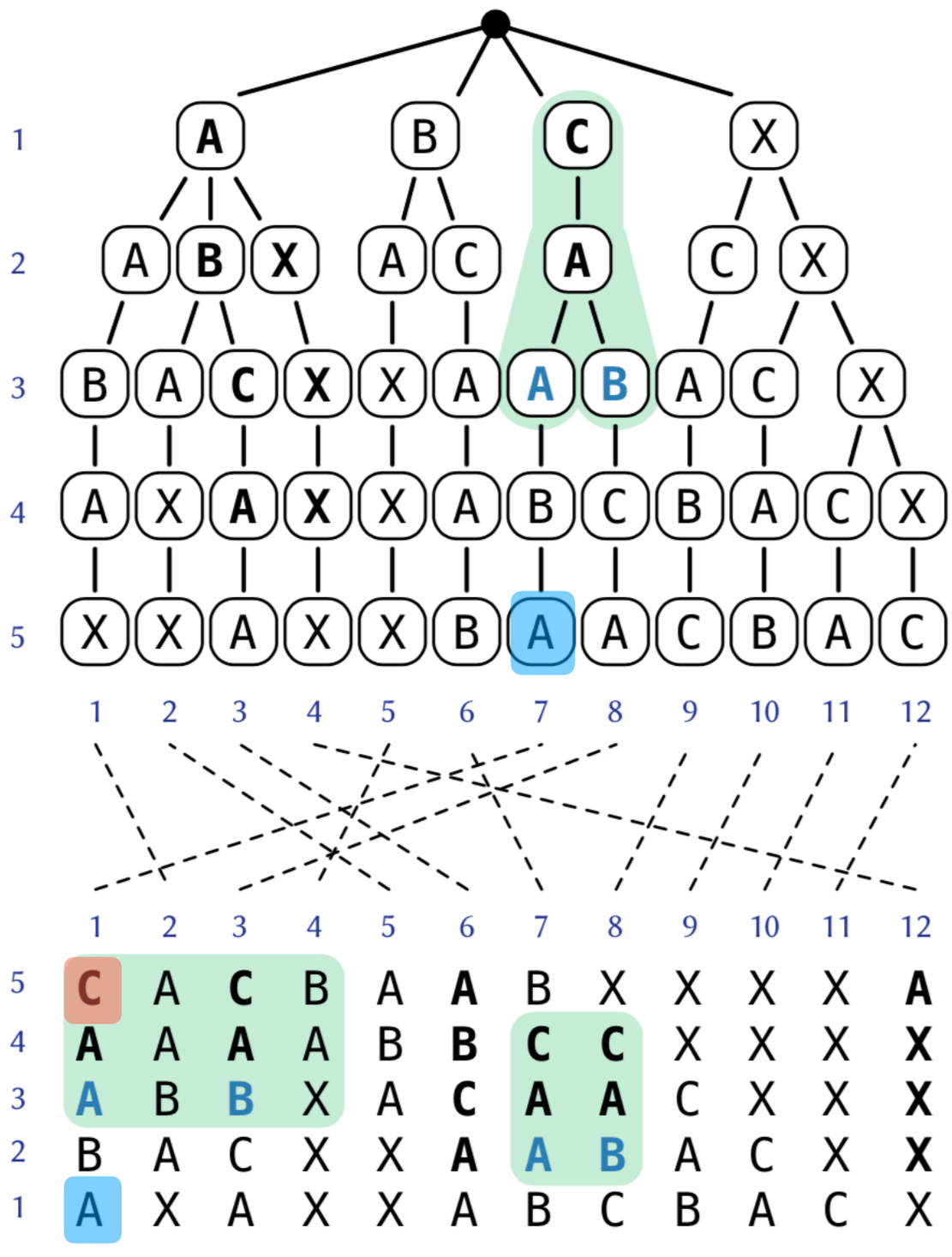

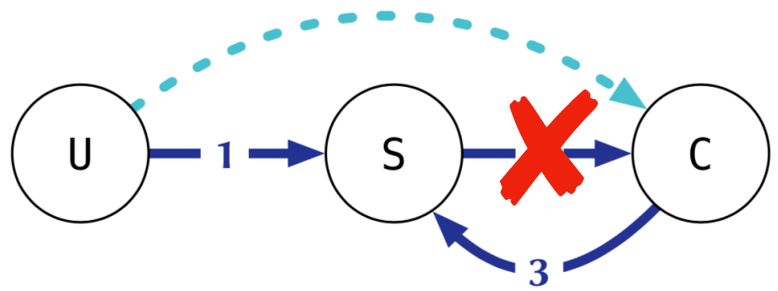

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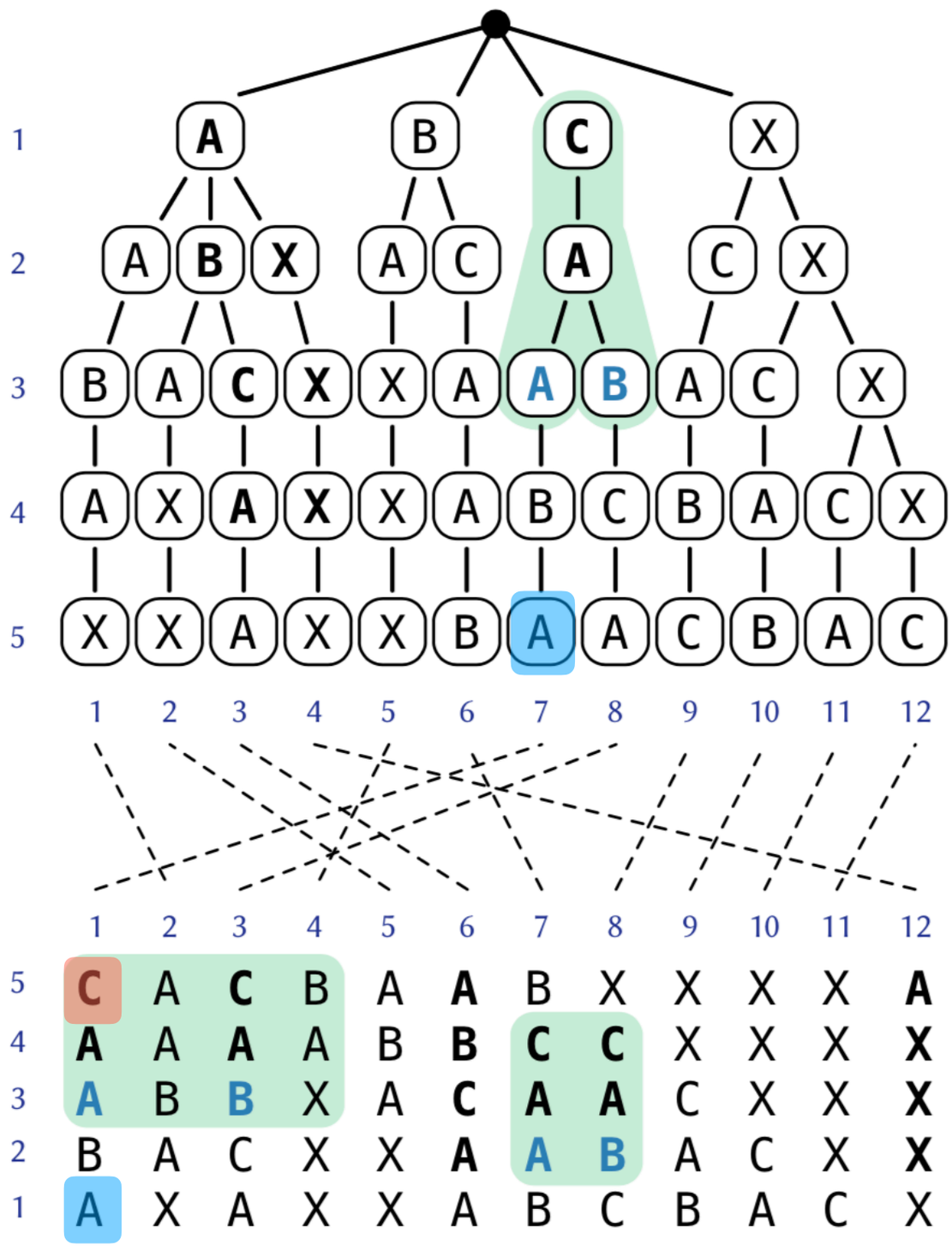

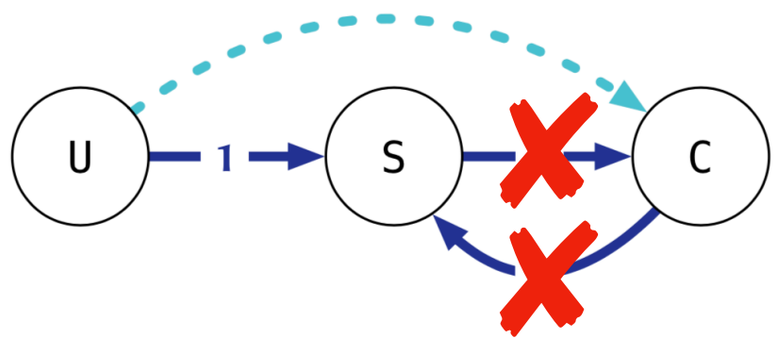

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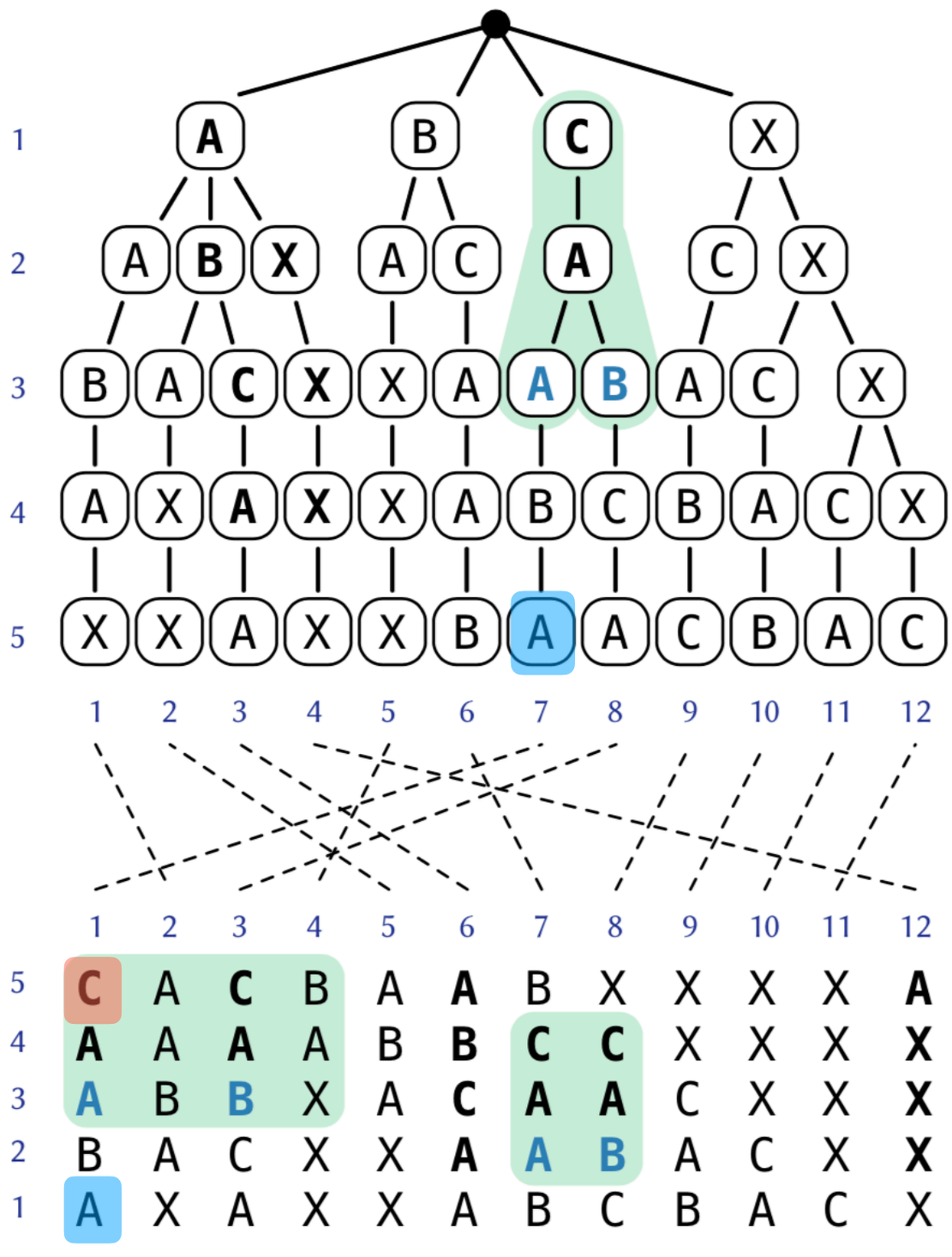

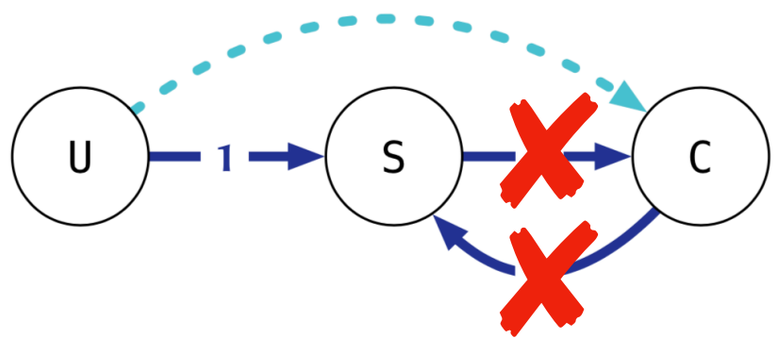

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X	9

Estimation runs 4.5X faster with billions of strings.

Take-home messages

- **Efficiency to deliver better services by using less resources. Impact is far reaching and implies substantial economic gains.**

- **Compression is mandatory if your data are “big”.**

- **Experiments are primary: design driven by numbers.**

Thanks for your attention,
time, patience!

Any questions?

High level thesis

Data Structures + Data Compression = Fast Algorithms

Design **space-efficient** *ad-hoc* data structures,
both from a theoretical *and* practical perspective,
that support **fast data extraction**.

Next word prediction

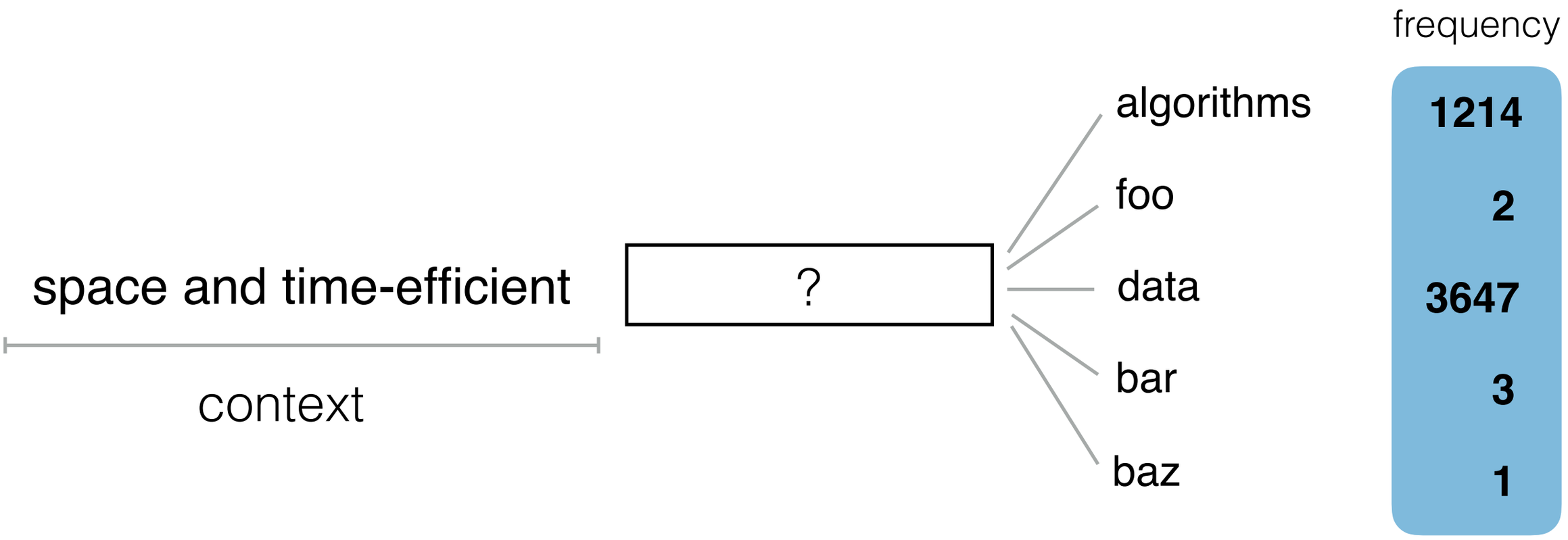
Next word prediction

space and time-efficient

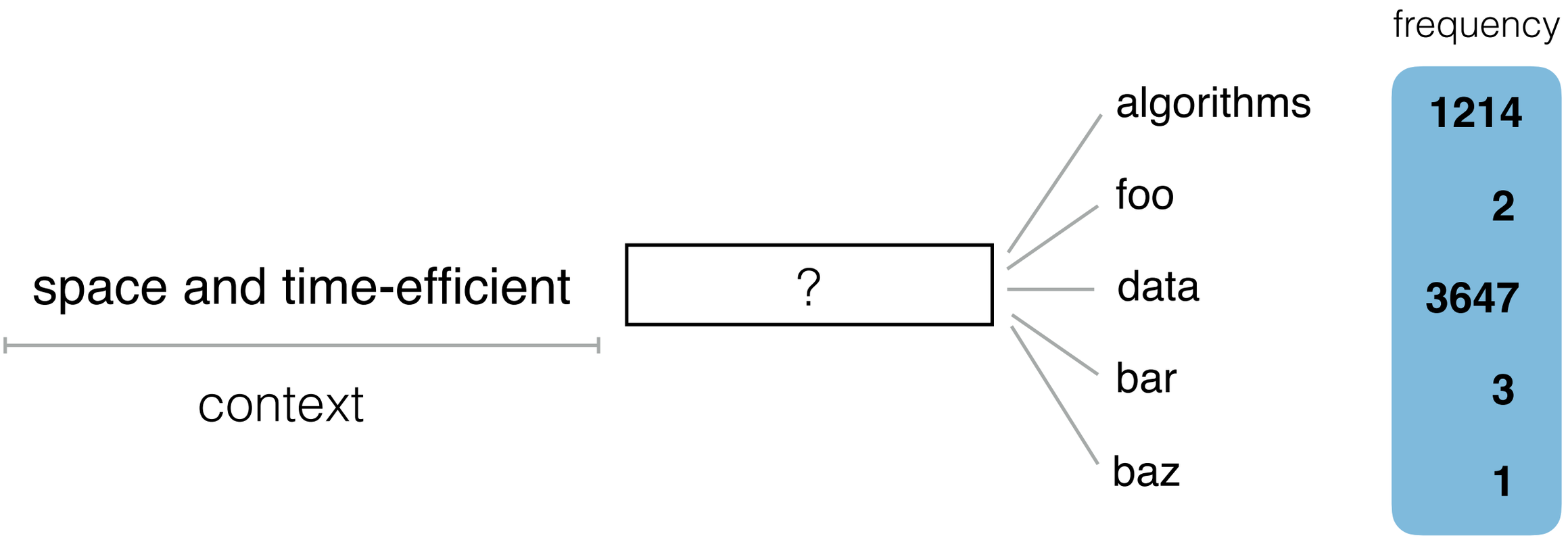
?

context

Next word prediction

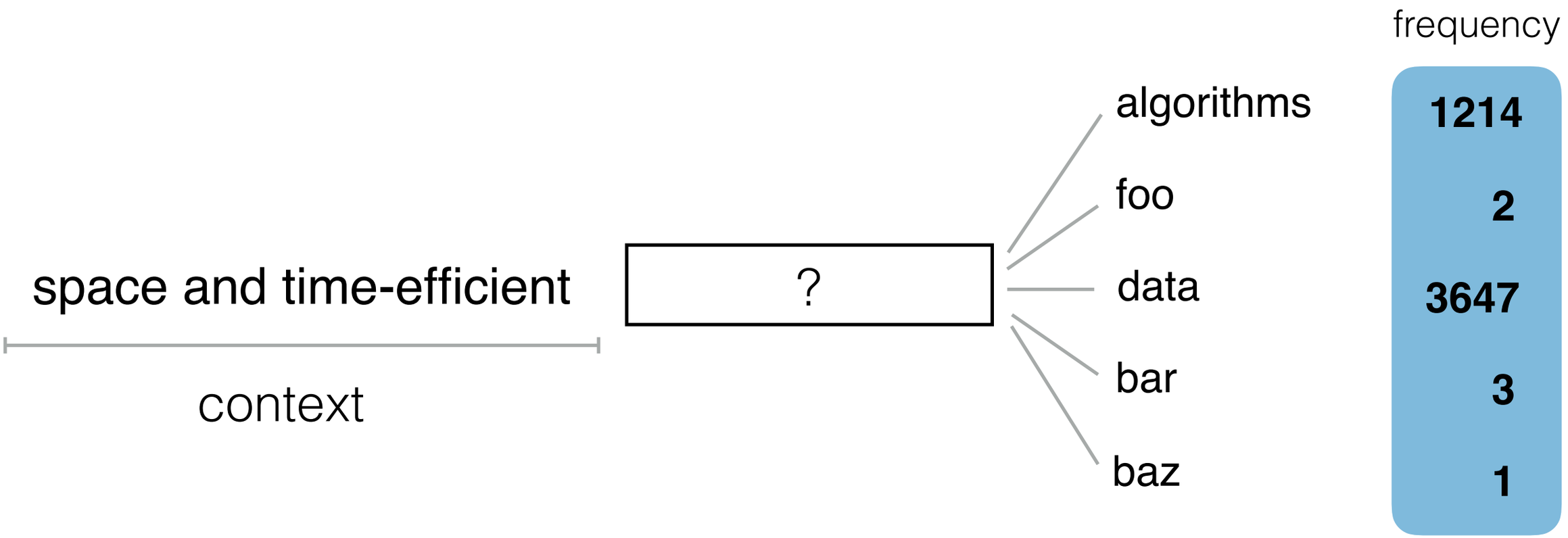


Next word prediction



$$\mathbf{P}(\text{“data” | “space and time-efficient”}) \approx \frac{\mathbf{f}(\text{“space and time-efficient data”})}{\mathbf{f}(\text{“space and time-efficient”})}$$

Next word prediction



$$\mathbf{P}(\text{“data”} \mid \text{“space and time-efficient”}) \approx \frac{\mathbf{f}(\text{“space and time-efficient data”})}{\mathbf{f}(\text{“space and time-efficient”})}$$

amazon alexa

Google Translate

Problem 3

Integer data structures

- van Emde Boas Trees
- X/Y-Fast Tries
- Fusion Trees
- Exponential Search Trees
- …

+ time
- space
+ dynamic

Elias-Fano encoding

- EF($S(n,u)$) = $n \log(u/n) + 2n$ bits to encode a sorted integer sequence S
- O(1) **Access**
- O(1 + $\log(u/n)$) **Predecessor**

+ time
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+ time
- space
+ dynamic

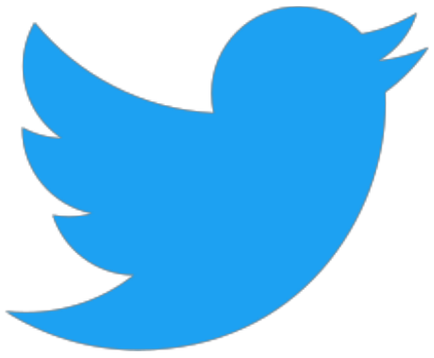
+ time
+ space
- static

Can we grab the best from both?

Dynamic inverted indexes

Classic solution: use two indexes.
One is big and **static**; the other is small and **dynamic**.
Merge them periodically.

Append-only inverted indexes.



Integer dictionaries in succinct space (CPM 2017)

Result 1

For $u = n^{\gamma}$, $\gamma = \Theta(1)$:

- **EF**($S(n,u)$) + o(n) bits
- O(1) **Access**
- O(min{1+log(u/n), loglog n}) **Predecessor**

Result 2

- **EF**($S(n,u)$) + o(n) bits
- O(1) **Access**
- O(1) **Append** (amortized)
- O(min{1+log(u/n), loglog n}) **Predecessor**

Result 3

- **EF**($S(n,u)$) + o(n) bits
- O(log n / loglog n) **Access**
- O(log n / loglog n) **Insert/Delete** (amortized)
- O(min{1+log(u/n), loglog n}) **Predecessor**

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- O(log n / loglog n) **Insert/Delete** (amortized)
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Optimal time bounds for all operations using a sublinear redundancy.