# Space- and Time-Efficient Data Structures for Massive Datasets

#### Giulio Ermanno Pibiri

giulio.pibiri@di.unipi.it

Supervisor Rossano Venturini

Department of Computer Science University of Pisa

15/11/2018

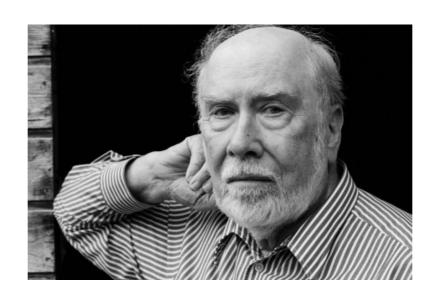


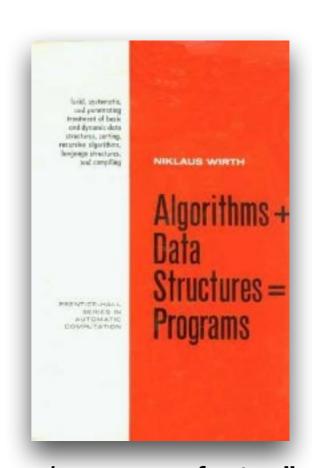
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The increase of information does **not** scale with technology.

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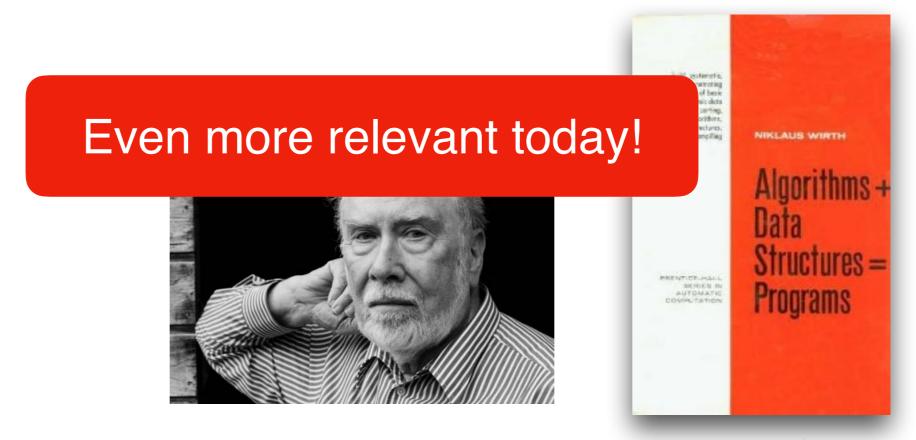


"Software is getting slower more rapidly than hardware becomes faster."

Niklaus Wirth, A Plea for Lean Software

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### Scenario

#### **Data structures**

PERFORMANCE

how quickly a program does its work - **faster** work





# **Algorithms**

**EFFICIENCY** 

how much work is required by a program - **less** work

#### **Scenario**

#### **Data structures**

PERFORMANCE

how quickly a program does its work - **faster** work time ✓ x space



# **Algorithms**

**EFFICIENCY** 

how much work is required by a program - **less** work



**Data compression** 



# The dichotomy problem

Small vs. fast?

# The dichotomy problem

Small vs. fast?

Choose one.

# The dichotomy problem

Small vs. fast?

Choose one.

NO

# **High level thesis**

Data Structures + Data Compression → Fast Algorithms

Design **space-efficient** *ad-hoc* data structures, both from a theoretical *and* practical perspective, that support **fast data extraction**.

Data Compression & Fast Retrieval together.

#### **Achieved results**

#### Clustered Elias-Fano Indexes

Journal paper

Giulio Ermanno Pibiri and Rossano Venturini ACM Transactions on Information Systems (TOIS) Full paper, 34 pages, 2017.

#### **Dynamic Elias-Fano Representation**

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short strings

integer sequences

# **Problem 1**

Consider a sorted integer sequence.

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How to represent it as a bit-vector where each original integer is uniquely-decodable, using **as few as possible** bits?

How to maintain **fast decompression speed**?

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This is a difficult problem that has been studied since the '60.

# **Applications**

#### **Inverted indexes**





#### **Databases**





#### **RDF** indexing





#### **E-Commerce**





# **Geo-spatial data**





### **Graph-compression**







# **Applications**

#### **Inverted indexes**





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### **Graph-compression**

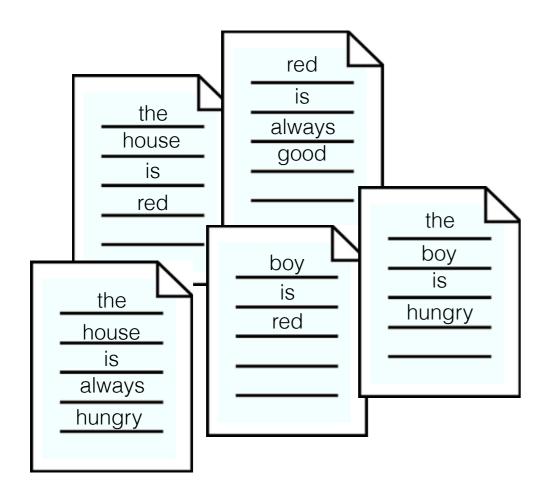




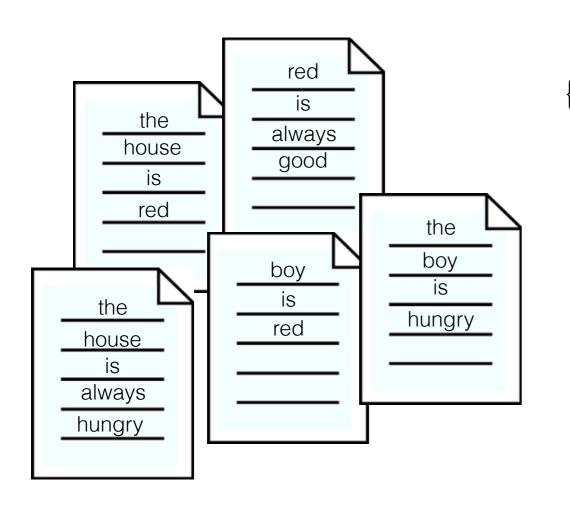


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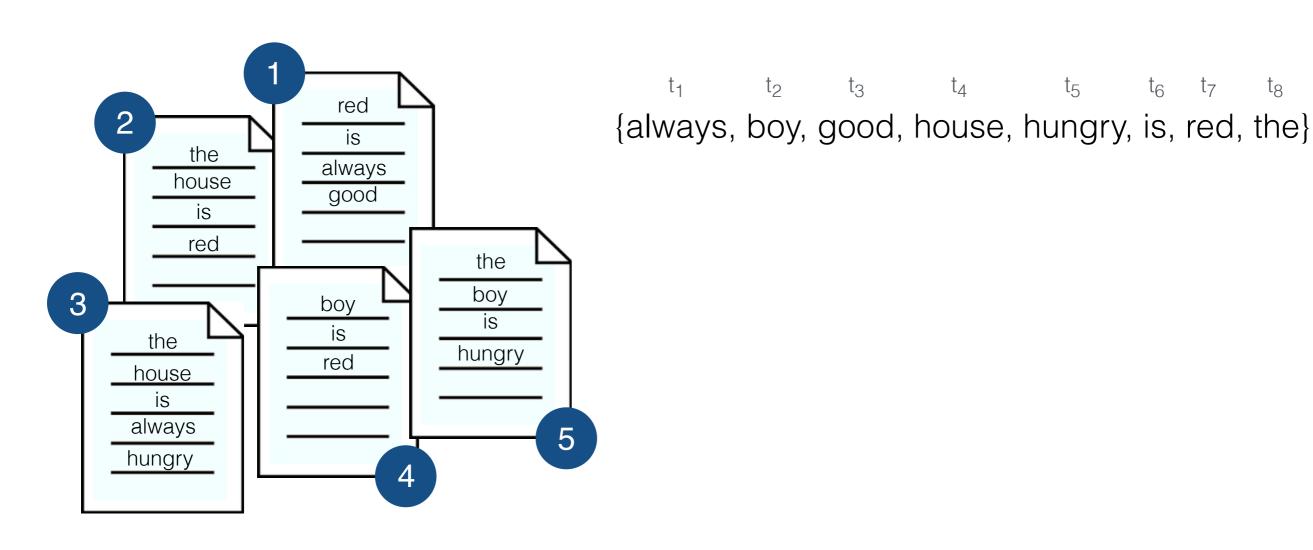
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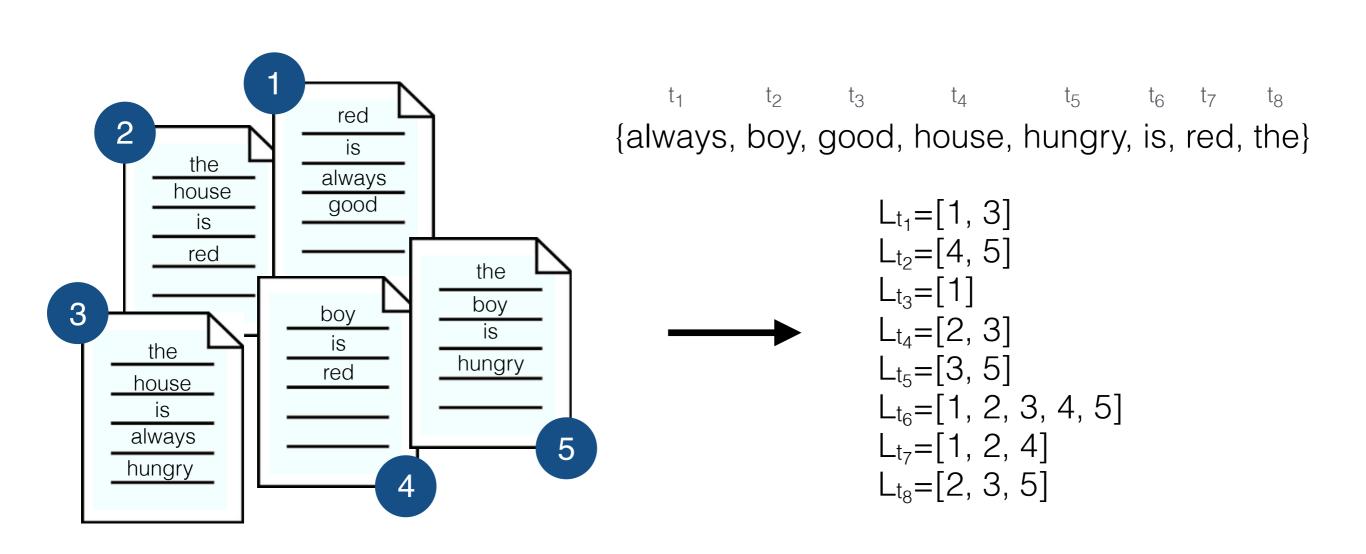
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 $t_7$ 

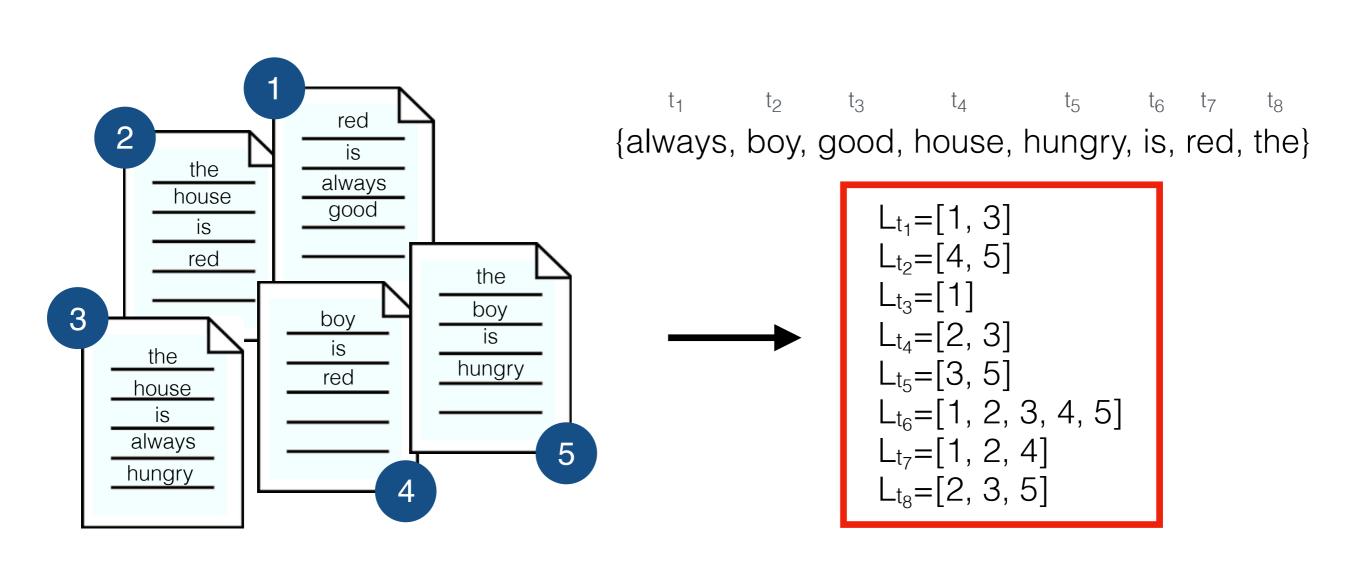
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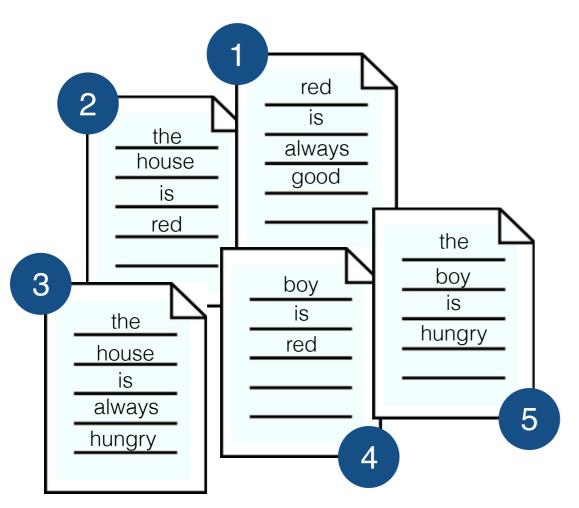


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$$L_{t_2}=[4, 5]$$

$$L_{t_3}=[1]$$

$$L_{t_4}=[2, 3]$$

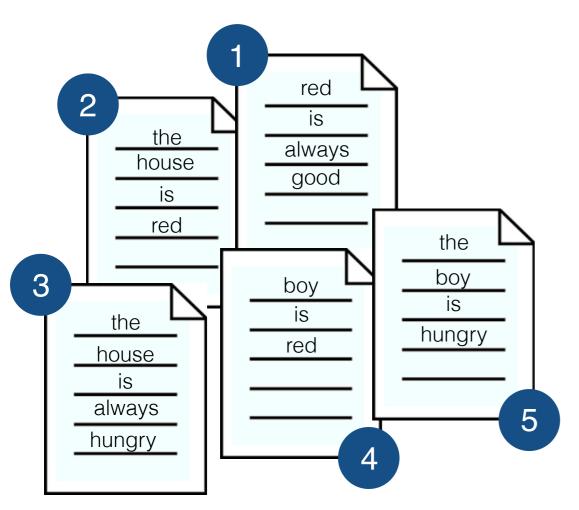
$$L_{t_5}=[3, 5]$$

$$L_{t_6}=[1, 2, 3, 4, 5]$$

$$L_{t_7}=[1, 2, 4]$$

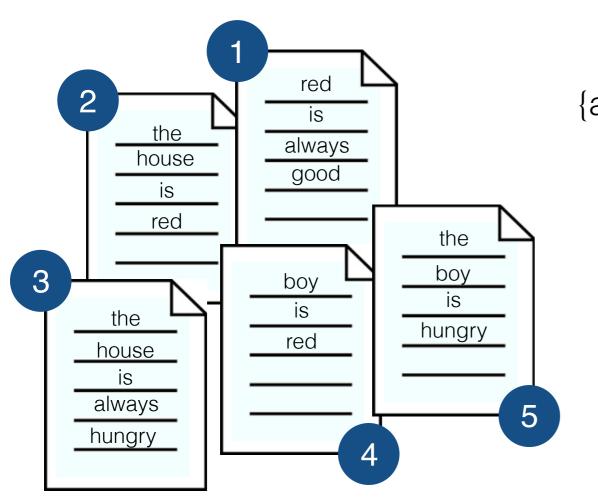
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# Many solutions

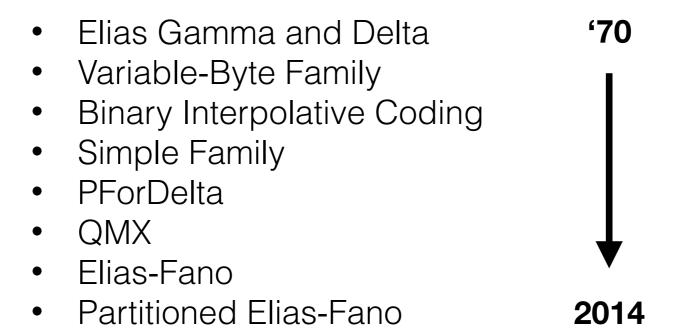
Huge research corpora describing different space/time trade-offs.

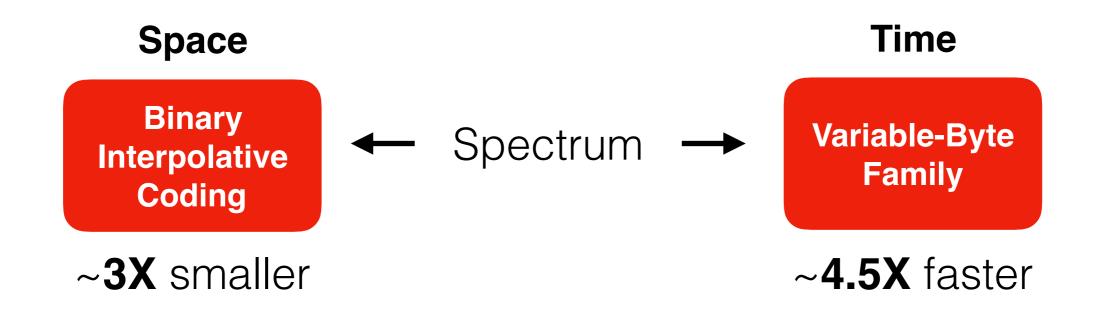
- Elias Gamma and Delta
- Variable-Byte Family
- Binary Interpolative Coding
- Simple Family
- PForDelta
- QMX
- Elias-Fano
- Partitioned Elias-Fano



# Many solutions

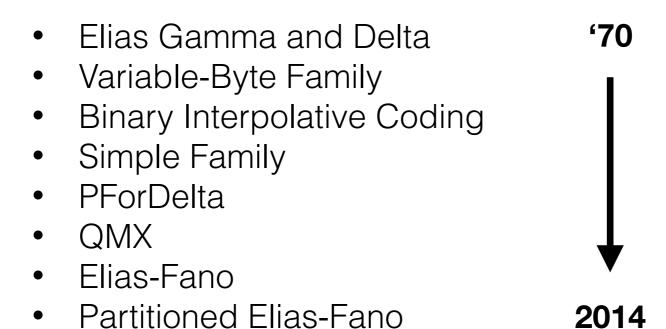
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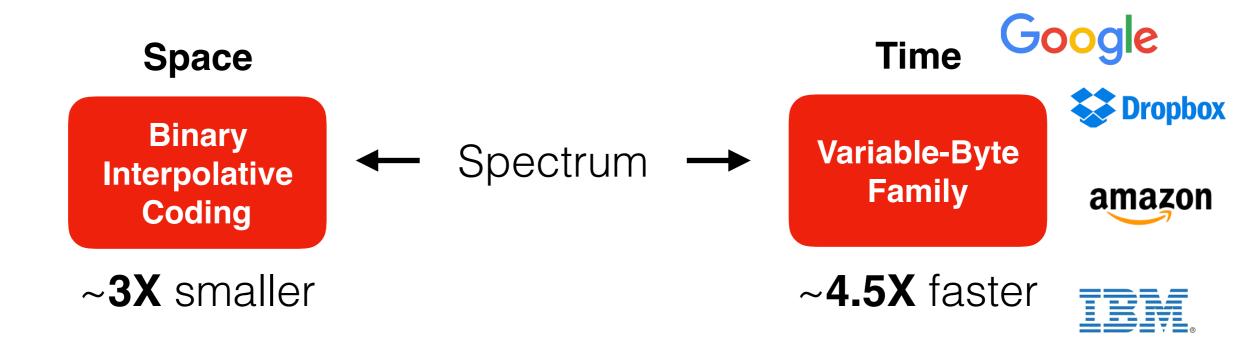


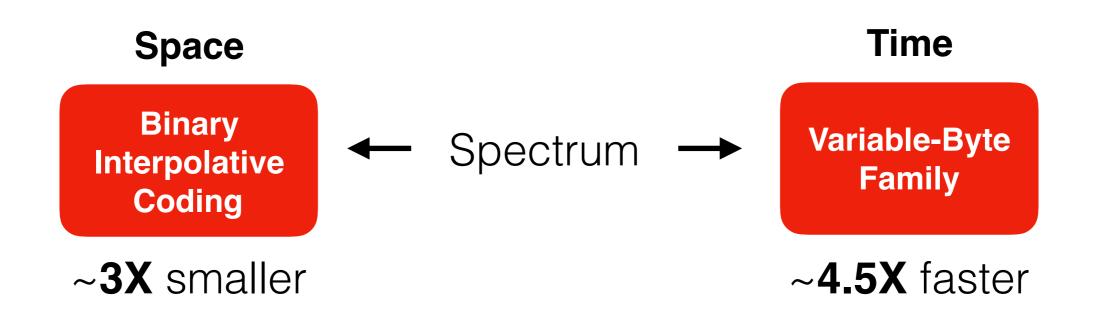


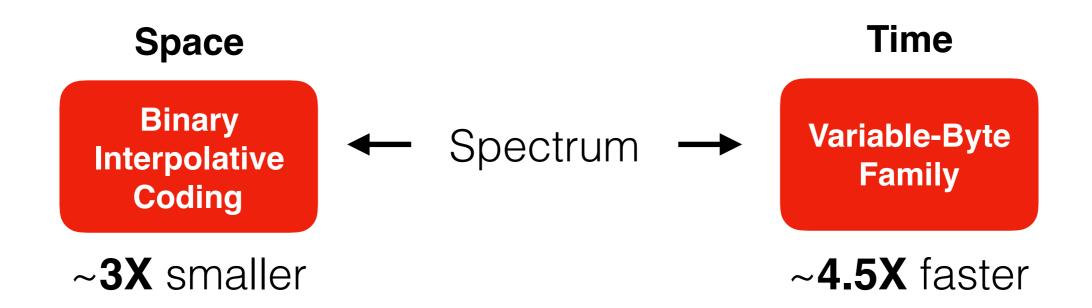
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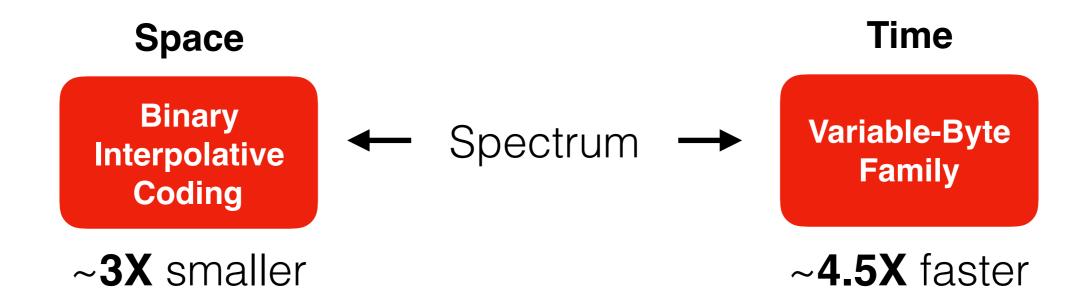






Is it possible to design an encoding that is as small as BIC and much faster?

1

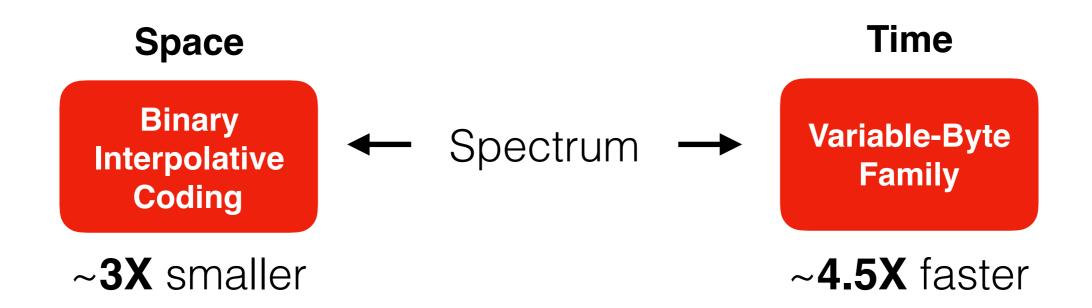


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Is it possible to design an encoding that is **as fast as**VByte and much smaller?

4

2



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2

1

What about **both** objectives at the same time?!

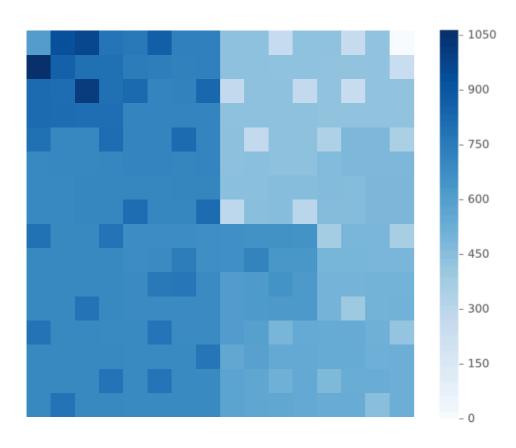
3

Every encoder represents each sequence individually.

No exploitation of redundancy.

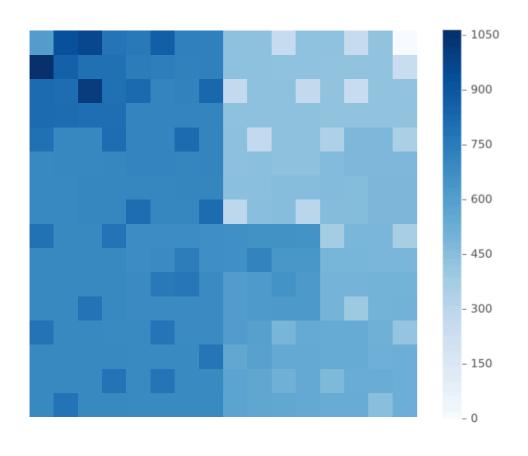
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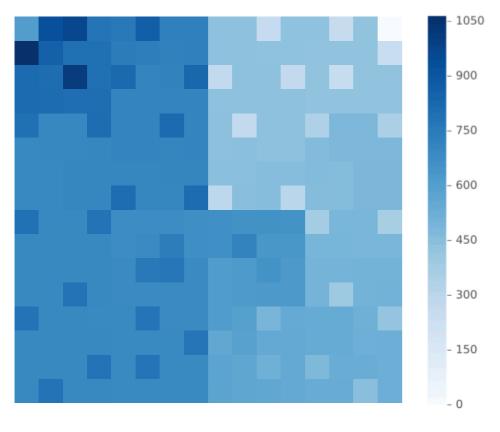
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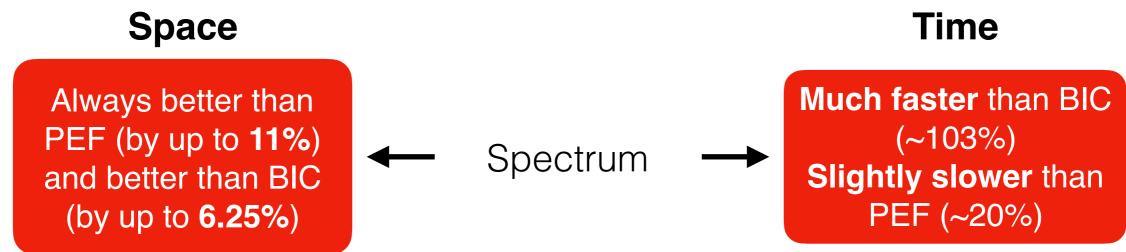
Encode **clusters** of inverted lists.

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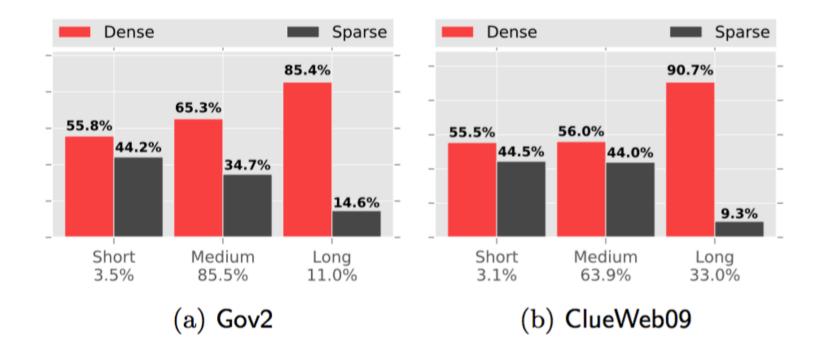


The majority of values are **small** (very small indeed).

VByte needs at least 8 bits per integer, that is sensibly far away from bit-level effectiveness (BIC: 3.54, PEF: 4.1 on Gov2).

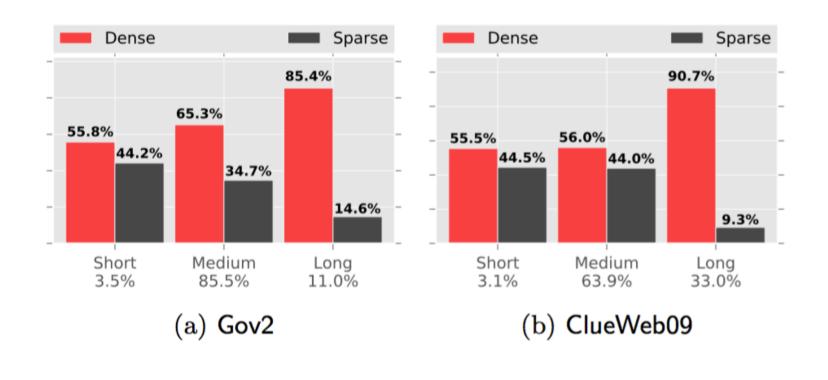
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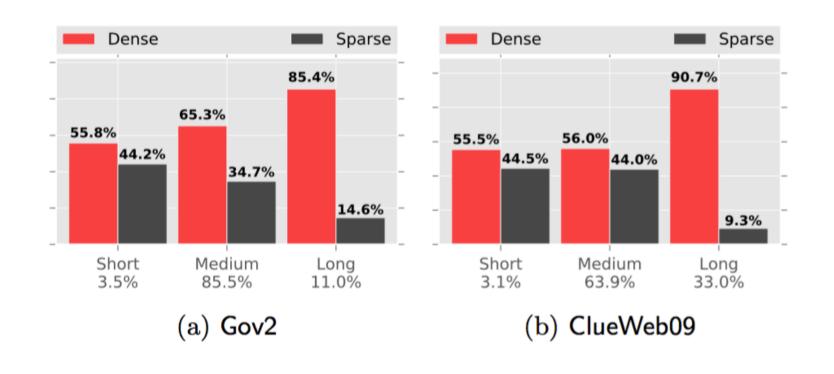
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Encode **dense** regions with unary codes, **sparse** regions with VByte.

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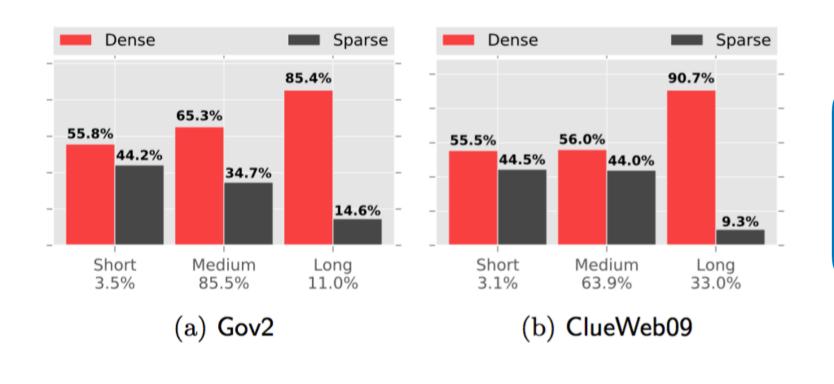


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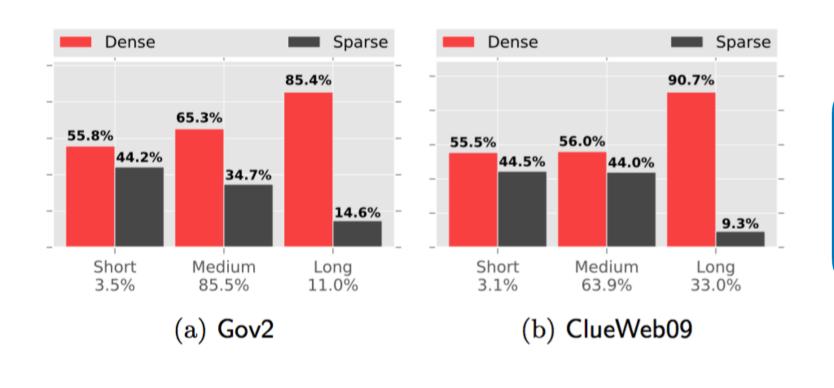
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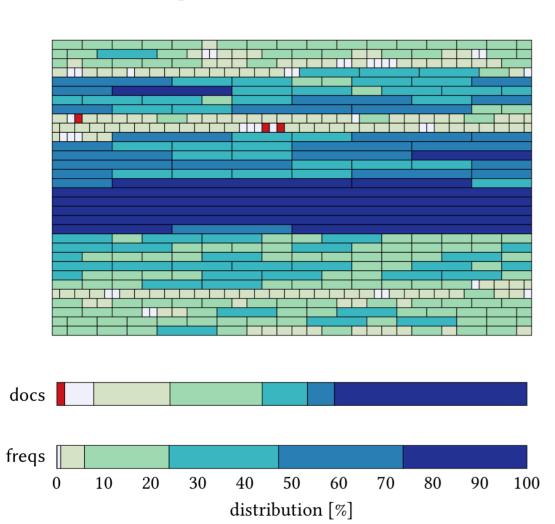
Query processing speed and sequential decoding **not affected**.

with M. Petri and A. Moffat (University of Melbourne)

If we consider subsequences of *d*-gaps in inverted lists, these are **repetitive** across the whole inverted index.

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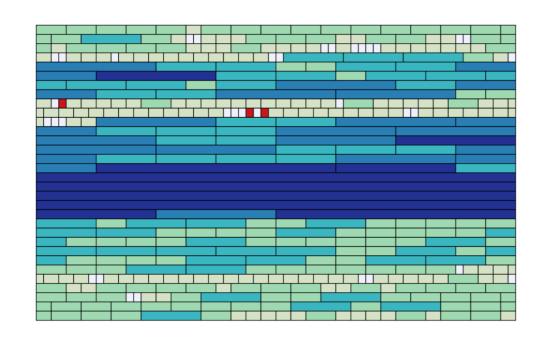
 $\blacksquare$  exceptions  $\square 1 \square 2 \square 4 \square 8 \square 16 \square$  runs

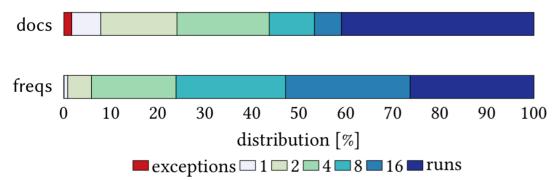
Put the **top-***k* **frequent patters** in a dictionary of size *k*.

Then encode inverted lists as sequences of log *k*-bit codewords.

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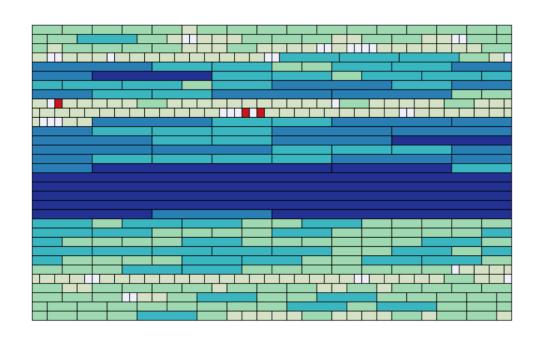
Close to the most space-efficient representation (~7% away from BIC).

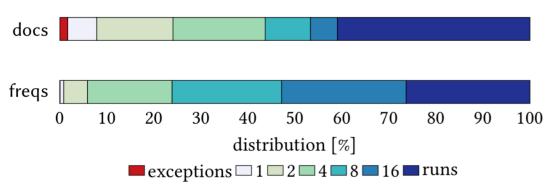
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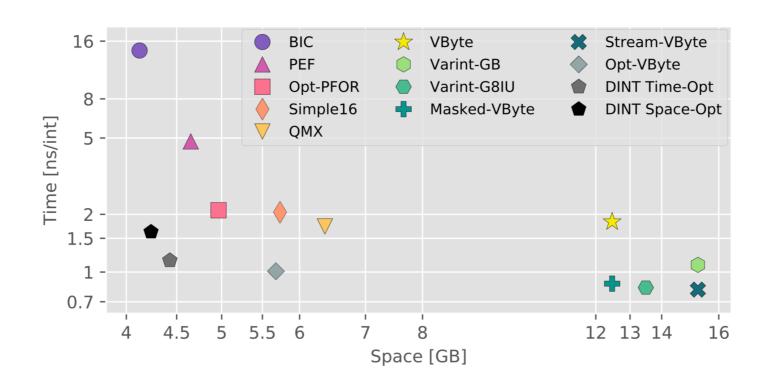
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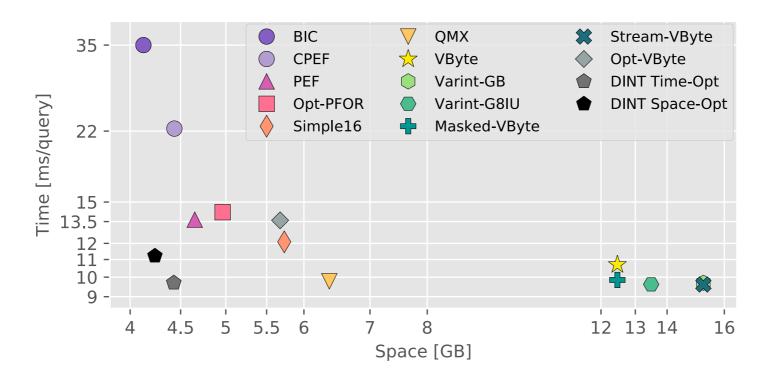
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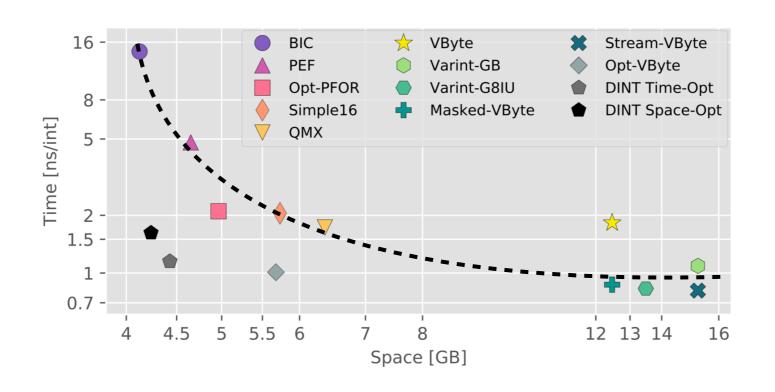
Almost **as fast as** the fastest SIMD-ized decoders.

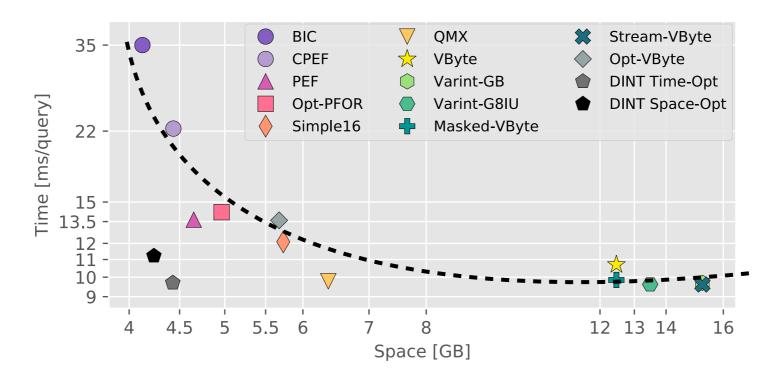
# The bigger picture



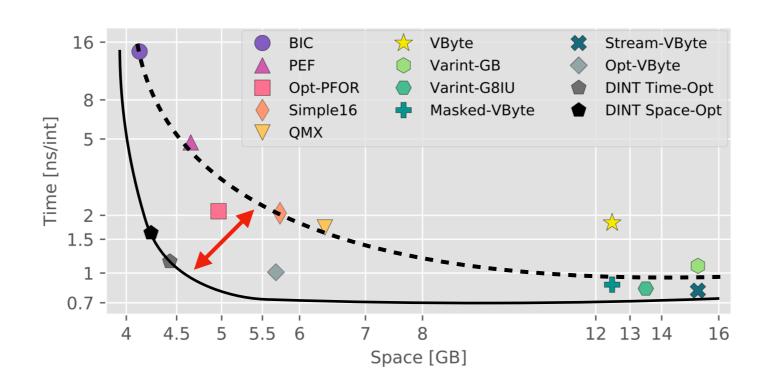


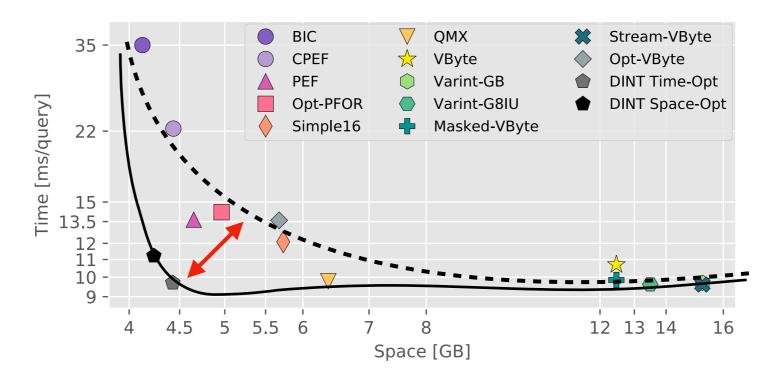
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# The bigger picture





#### **Integer data structures**

- van Emde Boas Trees
- X/Y-Fast Tries
- Fusion Trees
- Exponential Search Trees
- . . . .
- + time
- space
- + dynamic

#### **Elias-Fano encoding**

- EF(S(n,u)) =  $n \log(u/n) + 2n$  bits to encode a sorted integer sequence S
- O(1) **Access**
- O(1 +  $\log(u/n)$ ) Predecessor
  - + time
  - + space
  - static

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Can we grab the best from both?

# **Dynamic inverted indexes**

Classic solution: use two indexes.

One is big and cold; the other is small and hot.

Merge them periodically.

**Append-only** inverted indexes.









#### Integer dictionaries in succinct space (CPM '17)

For  $u = n^{\gamma}$ ,  $\gamma = \Theta(1)$ :

**Result 1** 

- $\mathsf{EF}(S(n,u)) + \mathsf{o}(n)$  bits
- O(1) Access
- O(min{1+log(u/n), loglog n}) Predecessor

•  $\mathsf{EF}(S(n,u)) + \mathsf{o}(n)$  bits

**Result 2** 

- O(1) Access
- O(1) Append (amortized)
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**Result 3** 

- O(log n / loglog n) Access
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Optimal time bounds for all operations using a sublunar redundancy.

Consider a large text.

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How to represent all its substrings of size  $1 \le k \le N$  words for fixed N (e.g., N = 5), using **as few as possible** bits?

Fast **Access** to individual N-grams?

How to **estimate** the probability of occurrence of the patterns under a given probability model?

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This is problem is central to applications in IR, ML, NLP, WSE.

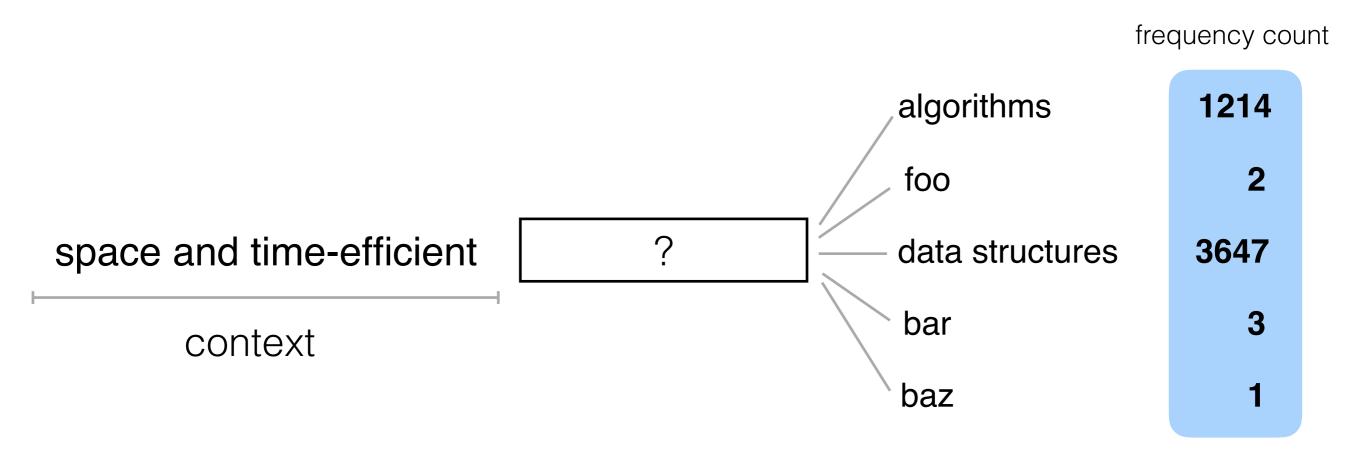
Next word prediction.

Next word prediction.

space and time-efficient ?

context

#### Next word prediction.



#### Next word prediction.

space and time-efficient ? data structures bar baz 1

context baz 1

$$\mathcal{P}(\text{"data structures" I "space and time-efficient"}) \approx \frac{\int (\text{"space and time-efficient data structures"})}{\int (\text{"space and time-efficient"})}$$

# What can I help you with?





The latest news from Research at Google

#### All Our N-gram are Belong to You

Thursday, August 03, 2006

Posted by Alex Franz and Thorsten Brants, Google Machine Translation Team

Here at Google Research we have been using word n-gram models for a variety of R&D projects, such as statistical machine translation, speech recognition, spelling correction, entity detection, information extraction, and others. While such models have usually been estimated from training corpora containing at most a few billion words, we have been harnessing the vast power of Google's datacenters and distributed processing infrastructure to process larger and larger training corpora. We found that there's no data like more data, and scaled up the size of our data by one order of magnitude, and then another, and then one more - resulting in a training corpus of one trillion words from public Web pages.



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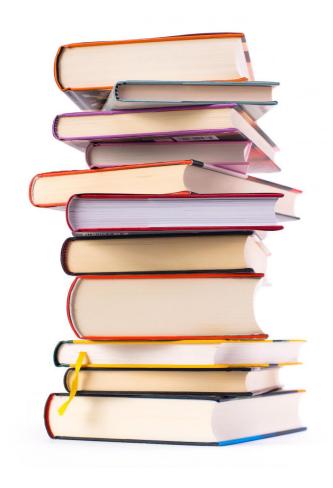
#### All Our N-gram are Belong to You

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# Indexing



# Google Books

~6% of the books ever published

n	number of $n$ -grams
1	24,359,473
2	667,284,771
3	7,397,041,901
4	1,644,807,896
5	1,415,355,596

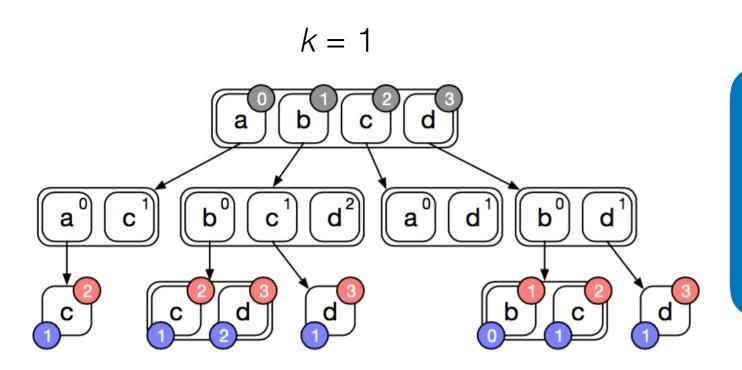
More than 11 billion n-grams.

#### Idea 1 - Context-based remapped tries (SIGIR '17)

The number of words following a given context is **small**.

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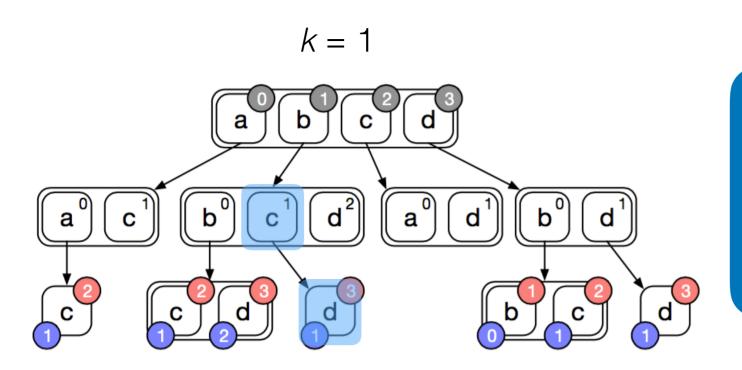
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Map a word ID to the **position** it takes within its *sibling* IDs (the IDs following a context of fixed length *k*).

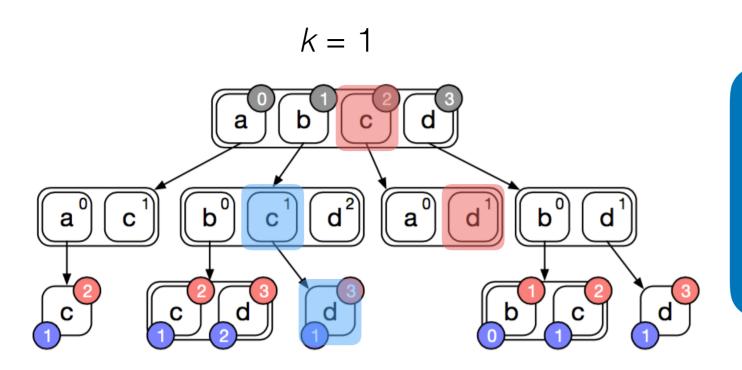
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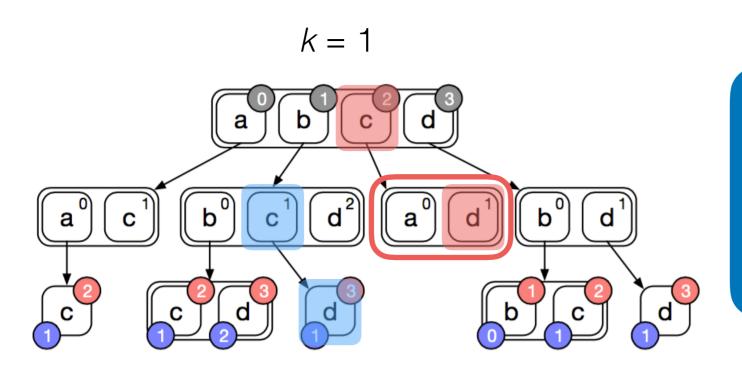
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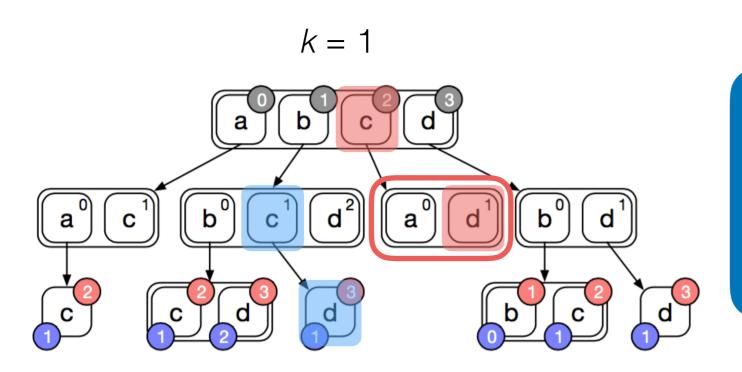
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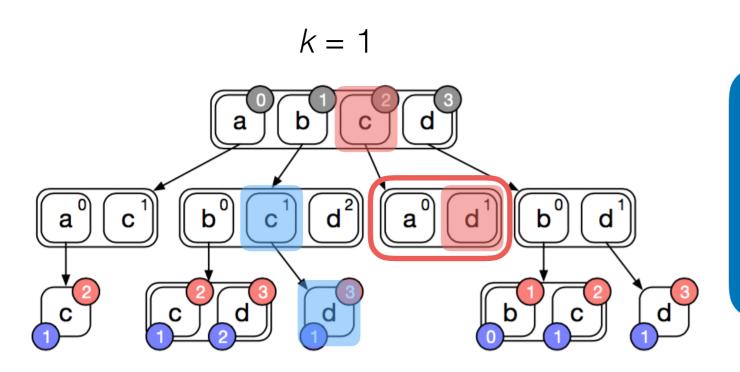
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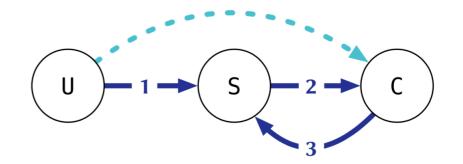


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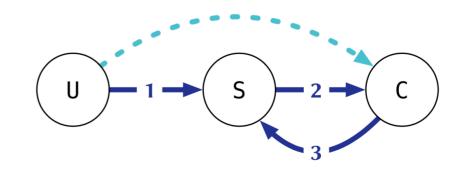
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The (Elias-Fano) context-based remapped trie is **even smaller** than the most space-efficient competitors, that are lossy and with false-positives allowed, and up to **5X faster**.

To compute the modified Kneser-Ney probabilities of the *n*-grams, the fastest algorithm in the literature uses **3 sorting steps** in external memory.

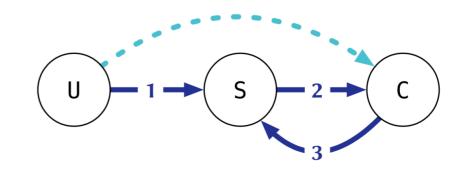


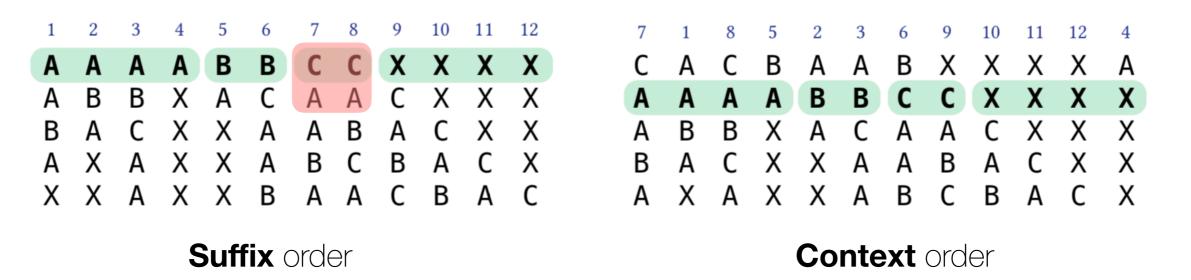
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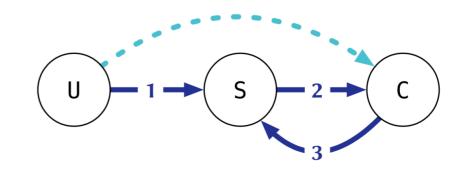
1	2	3	4	5	6	7	8	9	10	11	12	7	1	8	5	2	3	6	9	10	11	12	4
Α	Α	Α	Α	В	В	C	C	X	X	X	X	C	Α	C	В	Α	Α	В	Χ	Χ	Χ	Χ	Α
Α	В	В	Χ	Α	C	Α	Α	C	Χ	Χ	Χ	Α	Α	Α	A	В	В	C	C	X	X	X	X
В	Α	C	Χ	Χ	Α	Α	В	Α	C	Χ	Χ	Α	В	В	Χ	Α	C	Α	Α	C	Χ	Χ	Χ
Α	Χ	Α	Χ	Χ	Α	В	C	В	Α	C	Χ	В	Α	C	Χ	Χ	Α	Α	В	Α	C	Χ	Χ
Χ	Χ	Α	Χ	Χ	В	Α	Α	C	В	Α	C	Α	Χ	Α	Χ	Χ	Α	В	C	В	Α	C	Χ
	Suffix order											Co	nte	yt	$\cap$ r $\circ$	ler							

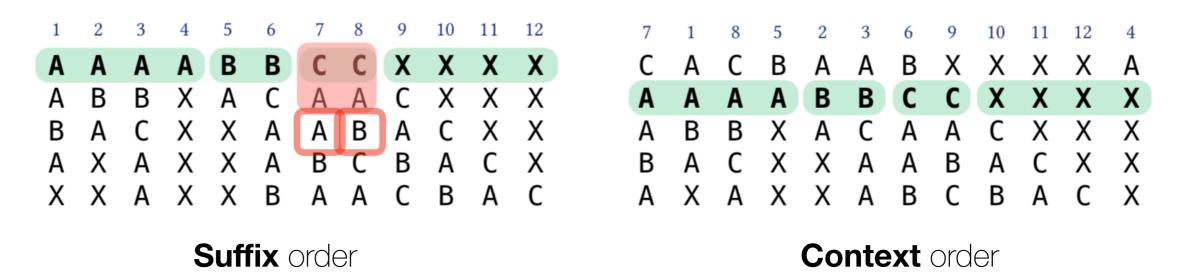
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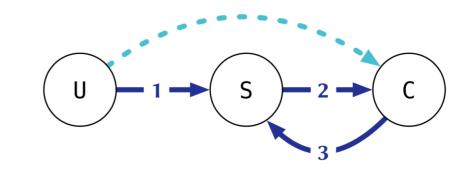


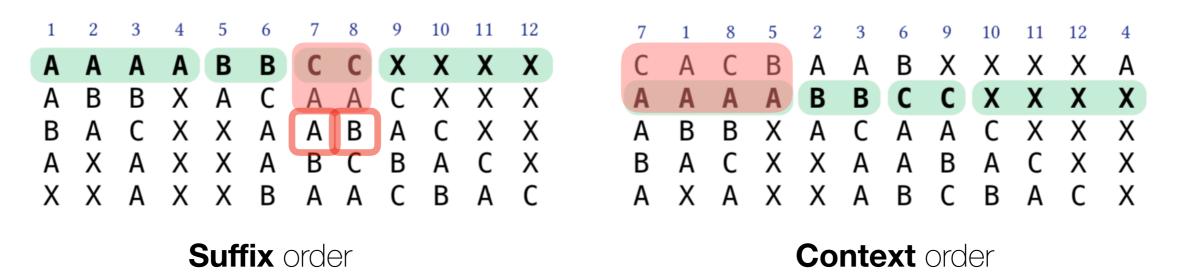
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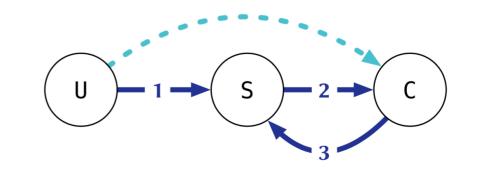


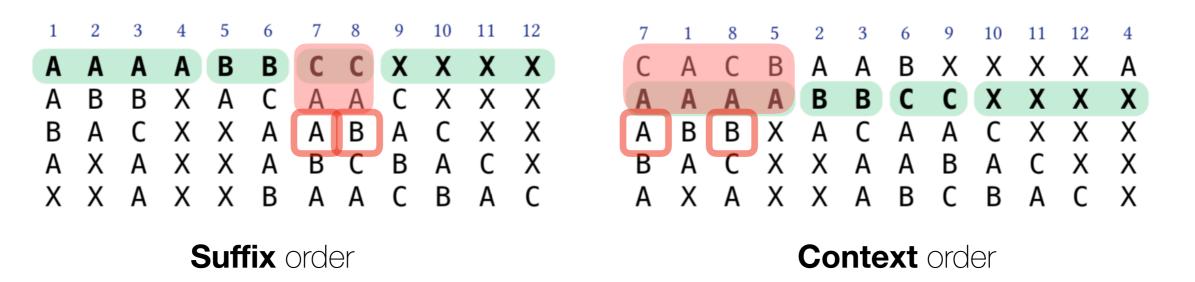
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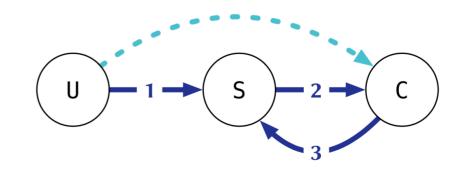


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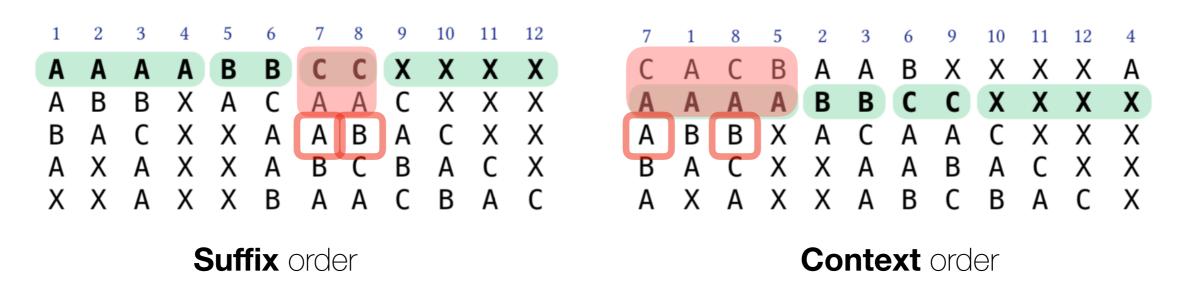




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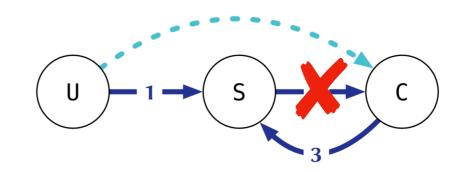


#### Computing the distinct left extensions.

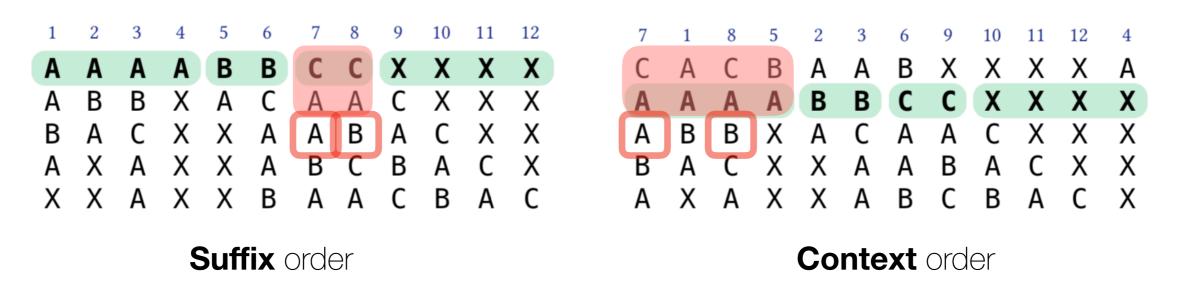


Using a scan of the block and O(|V|) space.

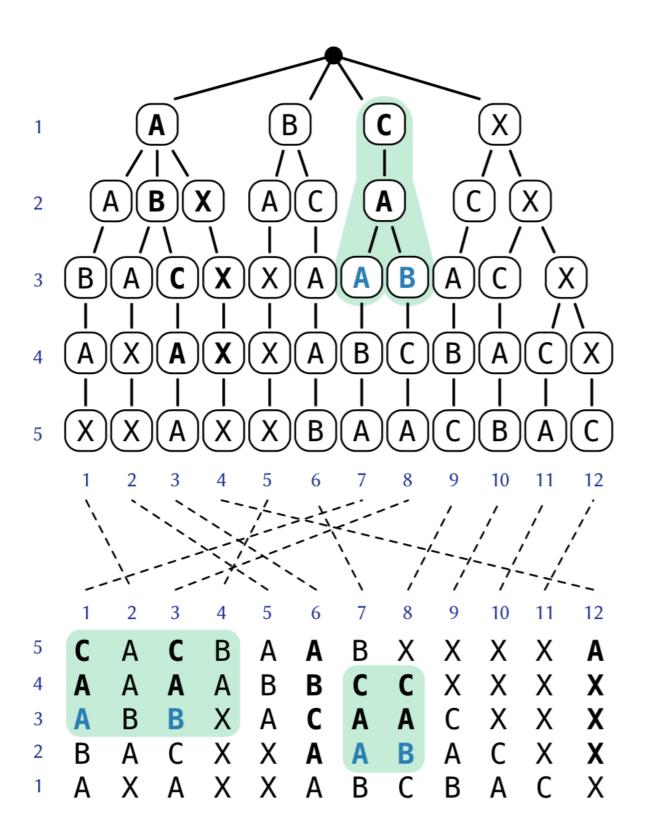
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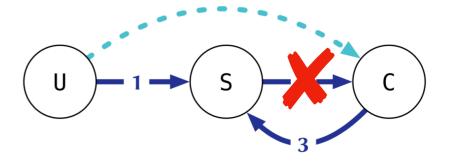


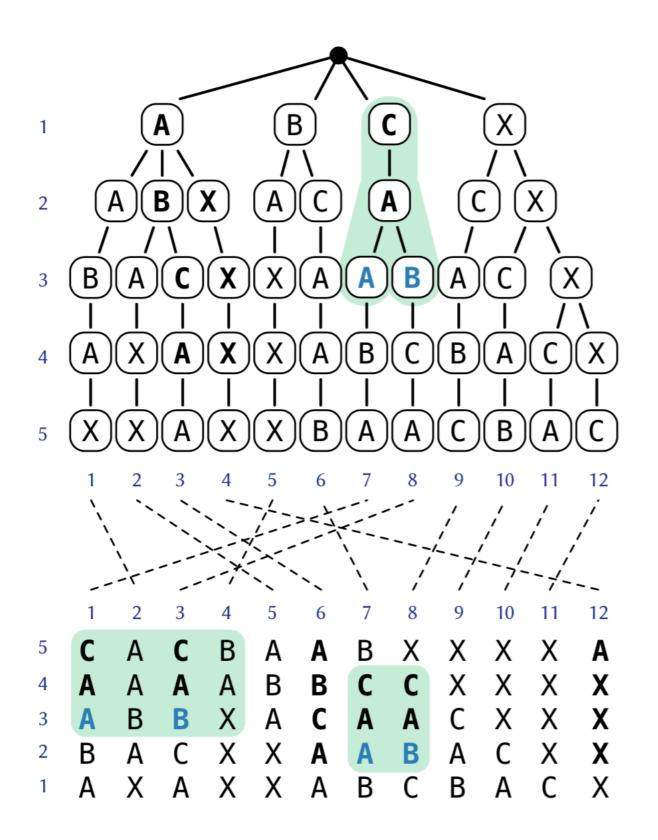
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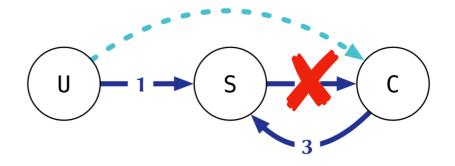


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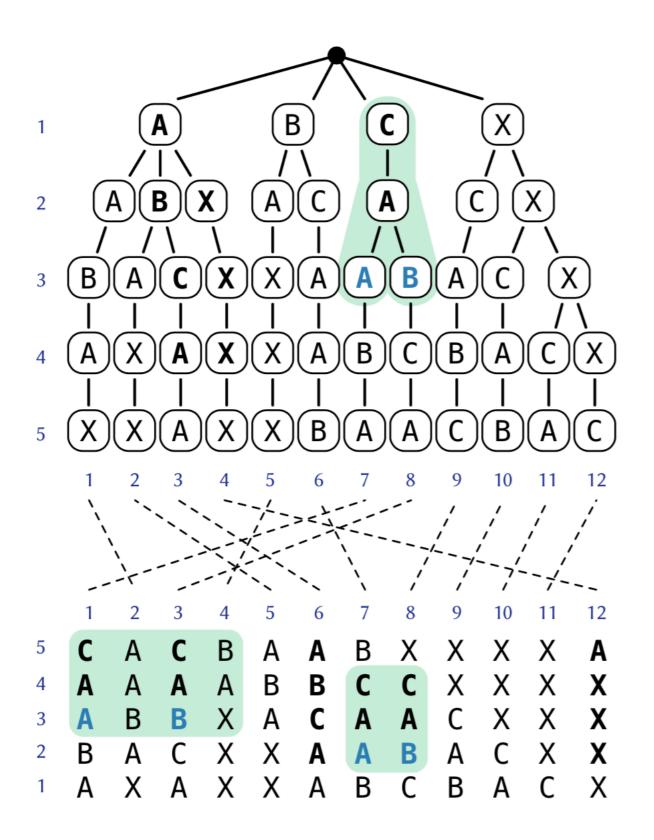


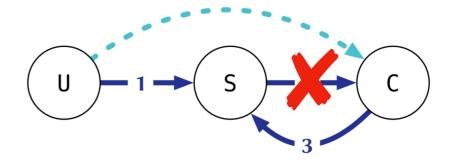


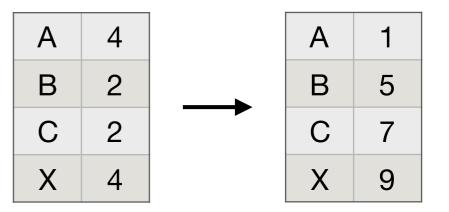


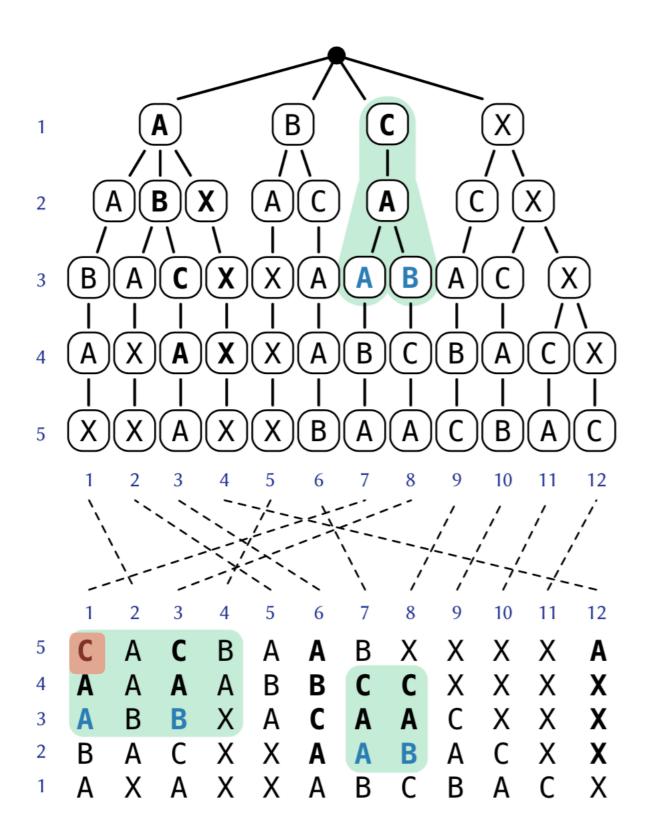


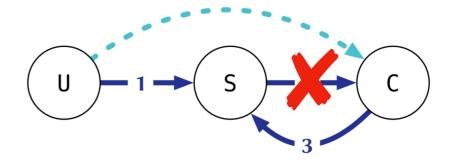
Α	4
В	2
С	2
Χ	4

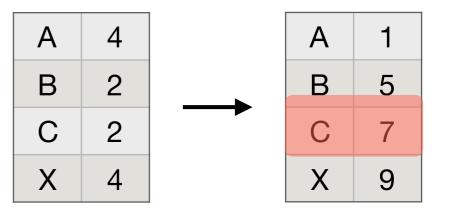


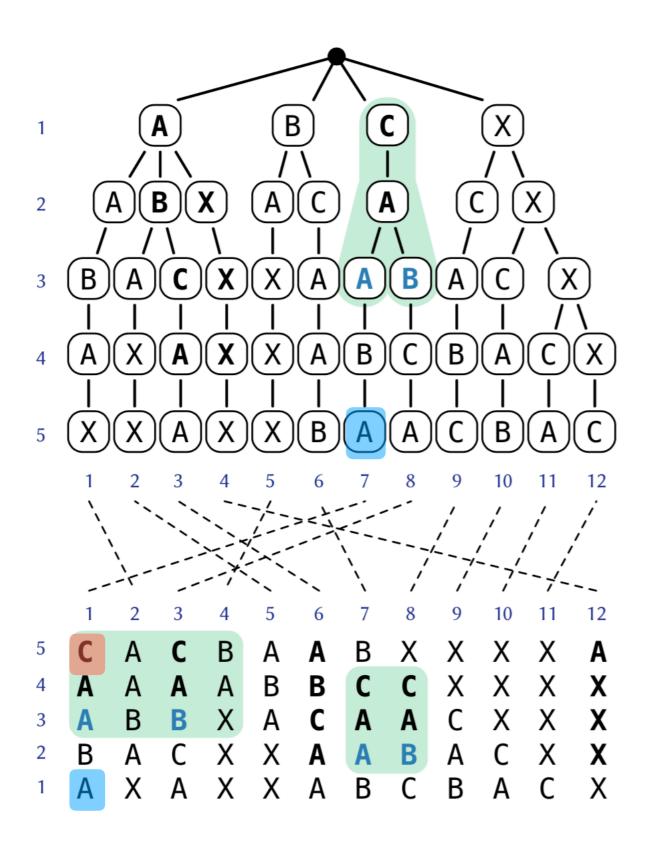


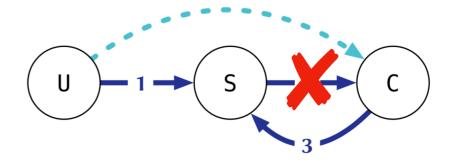


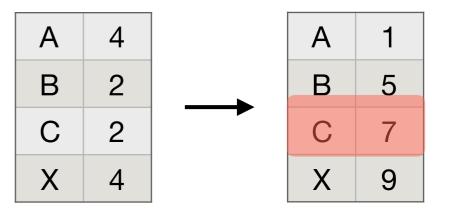


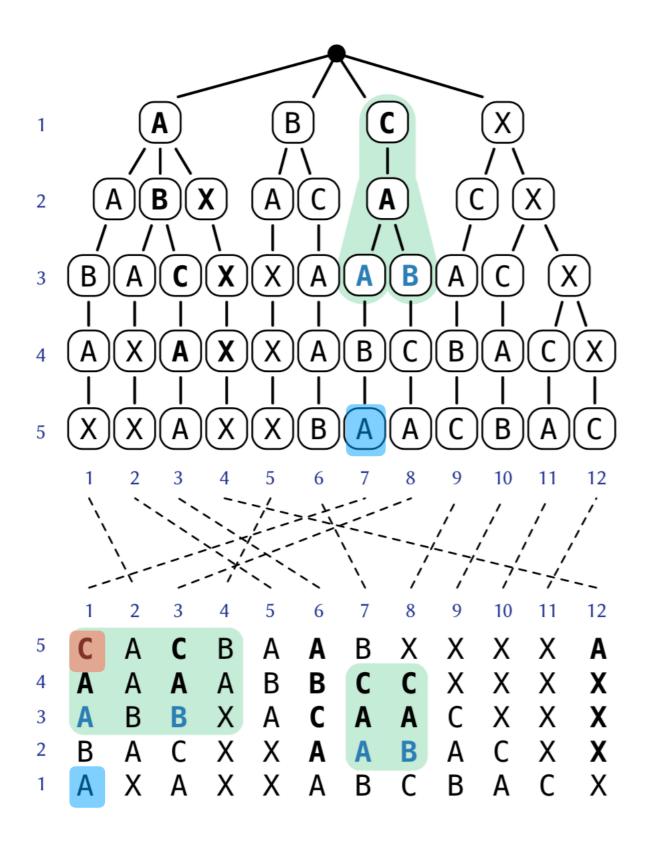


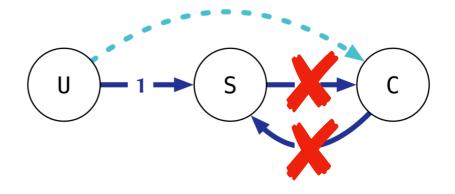


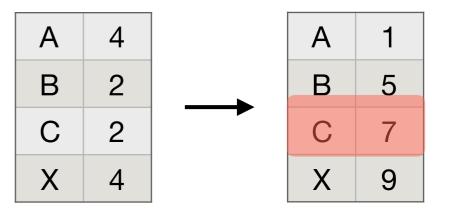


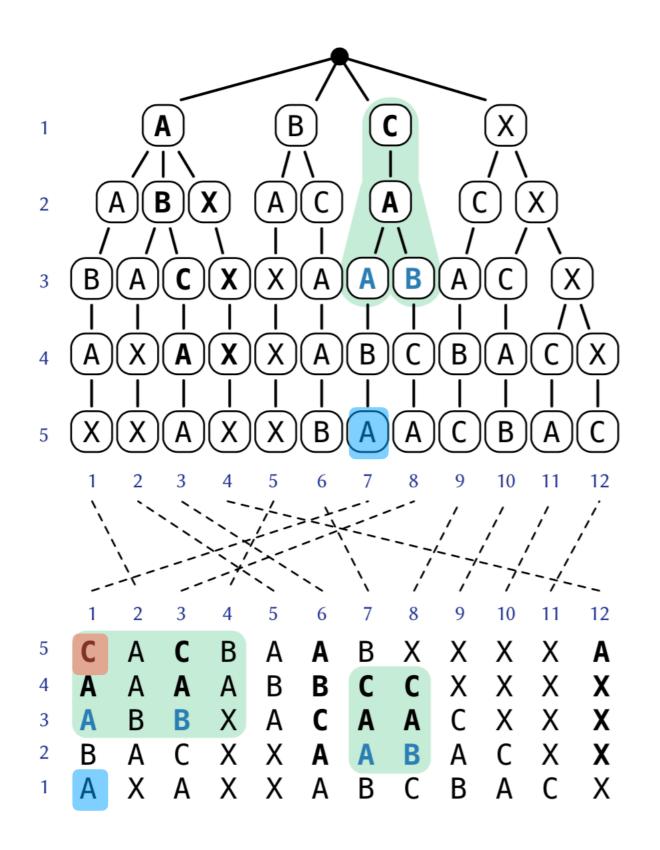


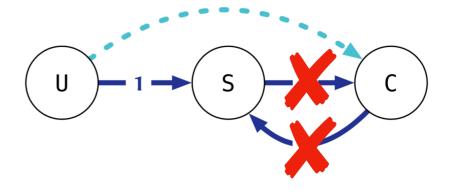












### Rebuilding the last level of the trie.

Α	4	Α	1
В	2	 В	5
С	2	С	7
Χ	4	Χ	9

Estimation runs 4.5X faster with billions of strings.

# Thanks for your attention, time, patience!

Any questions?