Dynamic Elias-Fano Representation

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The 28-th Annual Symposium on
Combinatorial Pattern Matching (CPM 2017)

Warsaw, Poland

06/07/2017

Introduction

A **dynamic ordered set** S is a data structure representing n objects and supporting the following operations:

- Insert(x) inserts x in S
- Delete(x) deletes x from S
- Search(x) checks whether x belongs to S
- Minimum() returns the minimum element of S
- Maximum() returns the maximum element of S
- Predecessor(x) returns $\max\{y \in S : y < x\}$
- Successor(x) returns $\min\{y \in S : y \geq x\}$

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Challenge

How to **optimally** solve the **integer** dynamic ordered set problem in **compressed space**?

Motivation

Integer Data Structures

- van Emde Boas Trees
- X/Y-Fast Tries
- Fusion Trees
- Exponential Search Trees
- …

Elias-Fano Encoding

- EF($S(n,u)$) = $n \log(u/n) + 2n$ bits to encode an ordered integer sequence S
- O(1) **Access**
- O($1 + \log(u/n)$) **Predecessor**

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- space
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\+ time
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Can we grab the best from both?

Objectives

Extend the *static* Elias-Fano representation of S as to support

1. **Predecessor**

2. **Access/Insert/Delete**

in **optimal time** and using $n \log(u/n) + 2n$ bits

Objectives

Extend the *static* Elias-Fano representation of S as to support

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sublinear redundancy

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1. [Patrascu and Thorup, STC 2007]

- Optimal space/time trade-off
- m bits, where $a = \log(m/n) - \log w$

$$\Theta\left(\min\left\{\log_w n, \log\frac{w - \log n}{a}, \frac{\log\frac{w}{a}}{\log(\frac{a}{\log n}\log\frac{w}{a})}, \frac{\log\frac{w}{a}}{\log(\log\frac{w}{a} / \log\frac{\log n}{a})}\right\}\right)$$

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Dynamic List Representation Problem:

- Access/Insert/Delete in $\Omega(\log n / \log\log n)$ amortized time
- $w \leq \log^{\gamma} n$ for some γ

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For Elias-Fano, $a = \log(\log(u/n) + 2)$ bits:

the second branch becomes $O(\log\log n)$

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Results - Static Elias-Fano Optimal Predecessor Queries

For $u = n^\gamma$, $\gamma = \Theta(1)$:

- EF($\mathcal{S}(n,u)$) + o(n) bits
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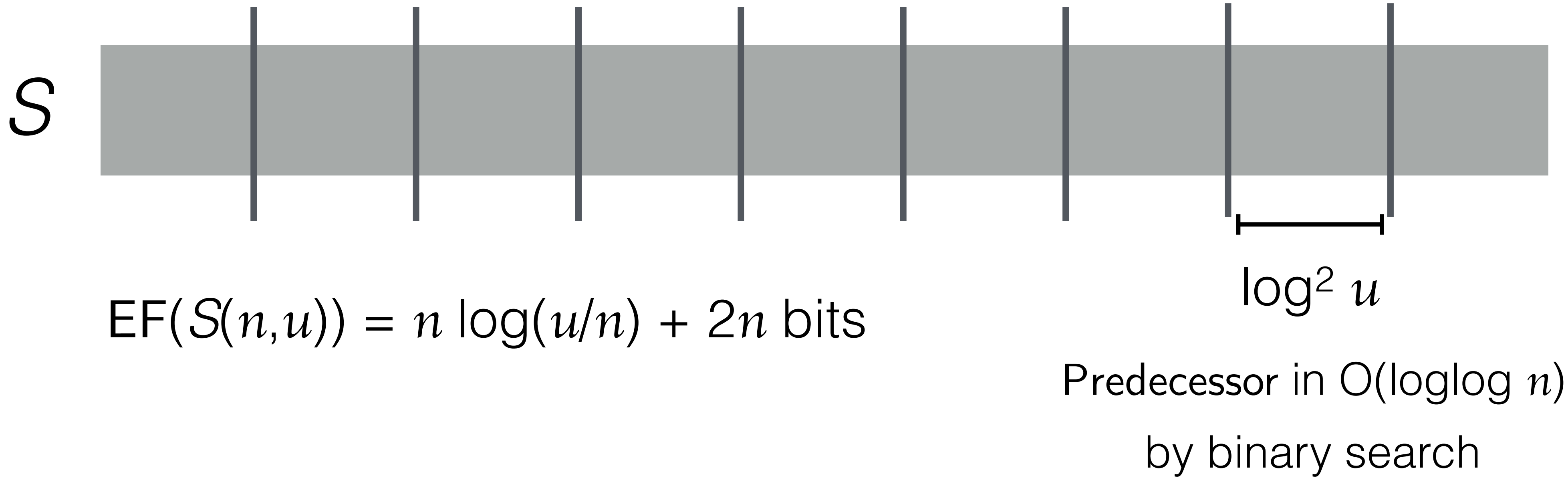

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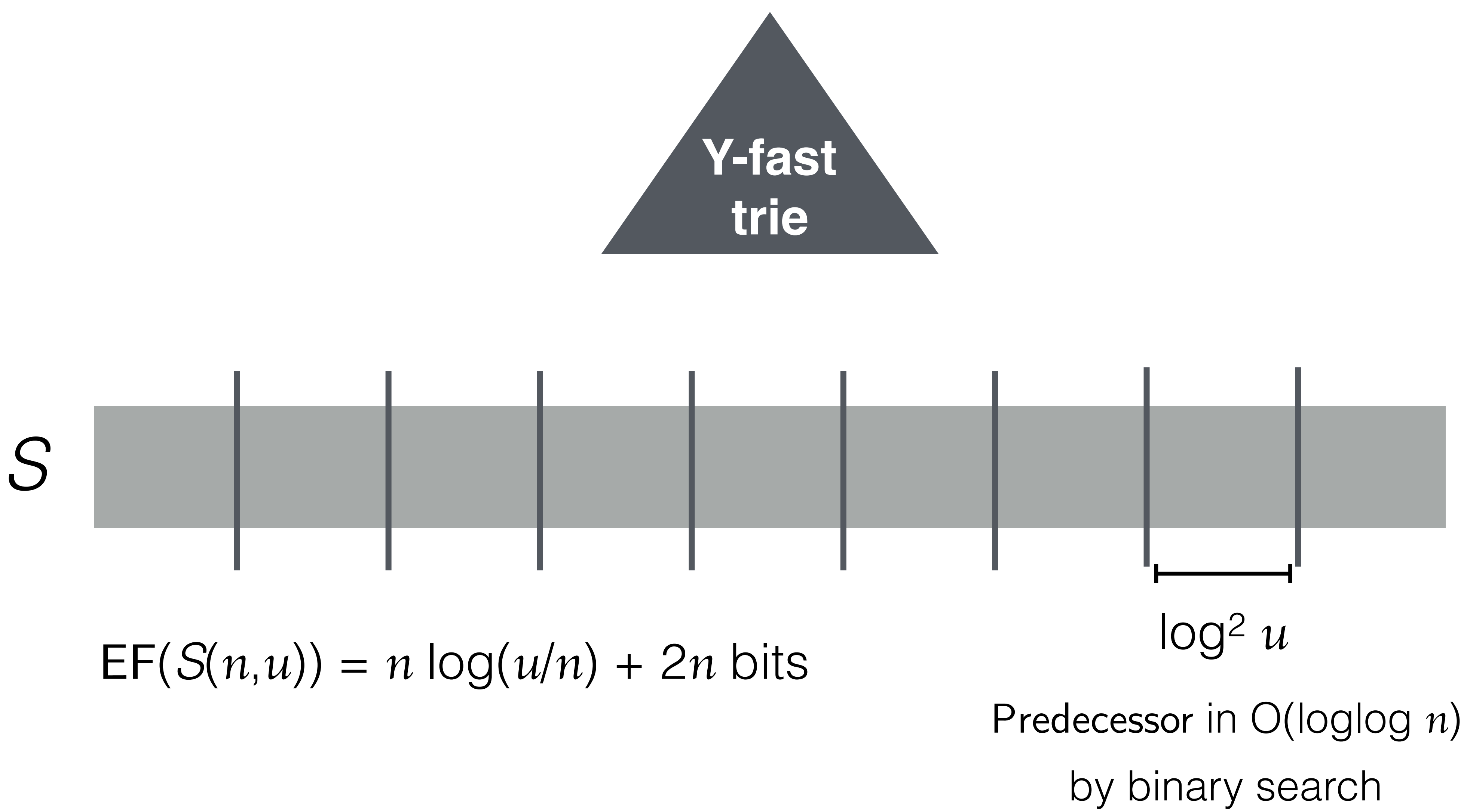

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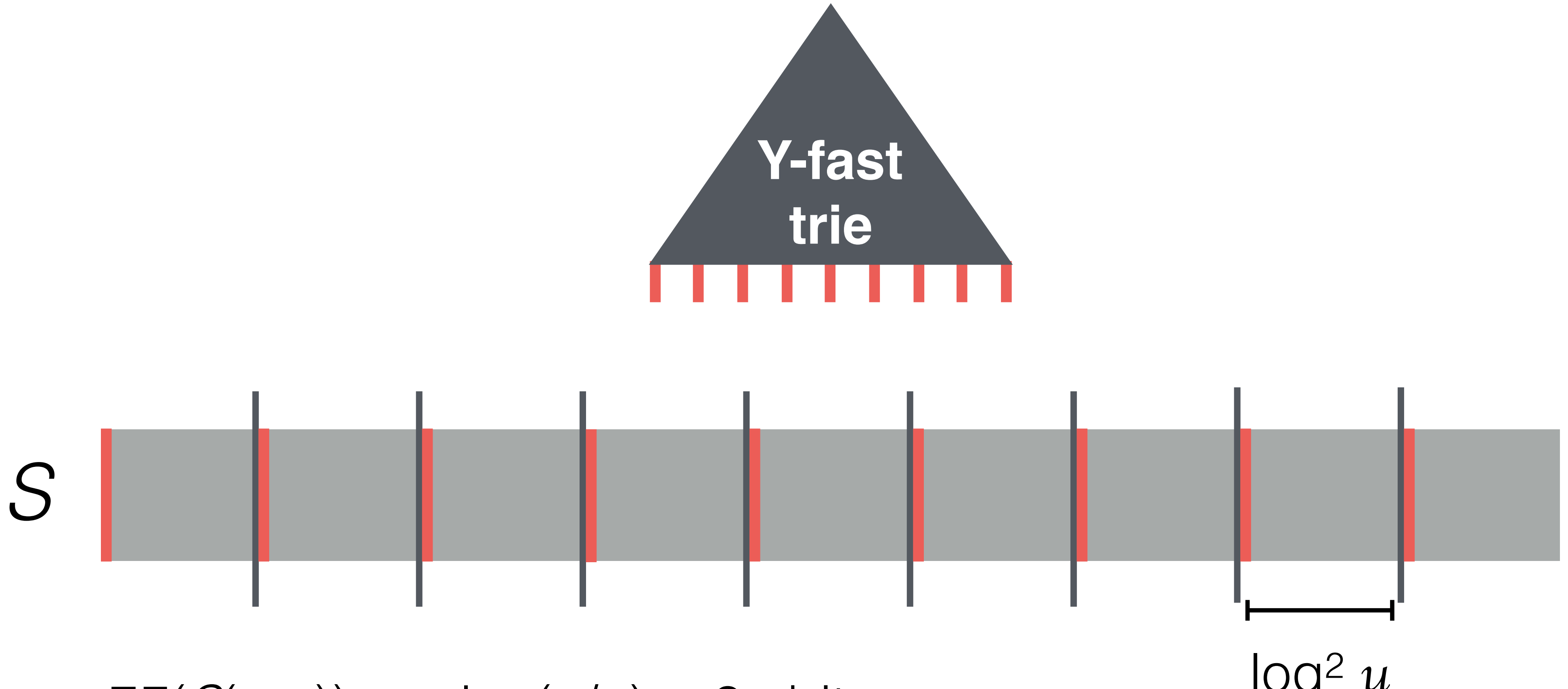

EF($S(n,u)$) = $n \log(u/n) + 2n$ bits

Predecessor in O(loglog n) by binary search

Results - Static Elias-Fano Optimal Predecessor Queries

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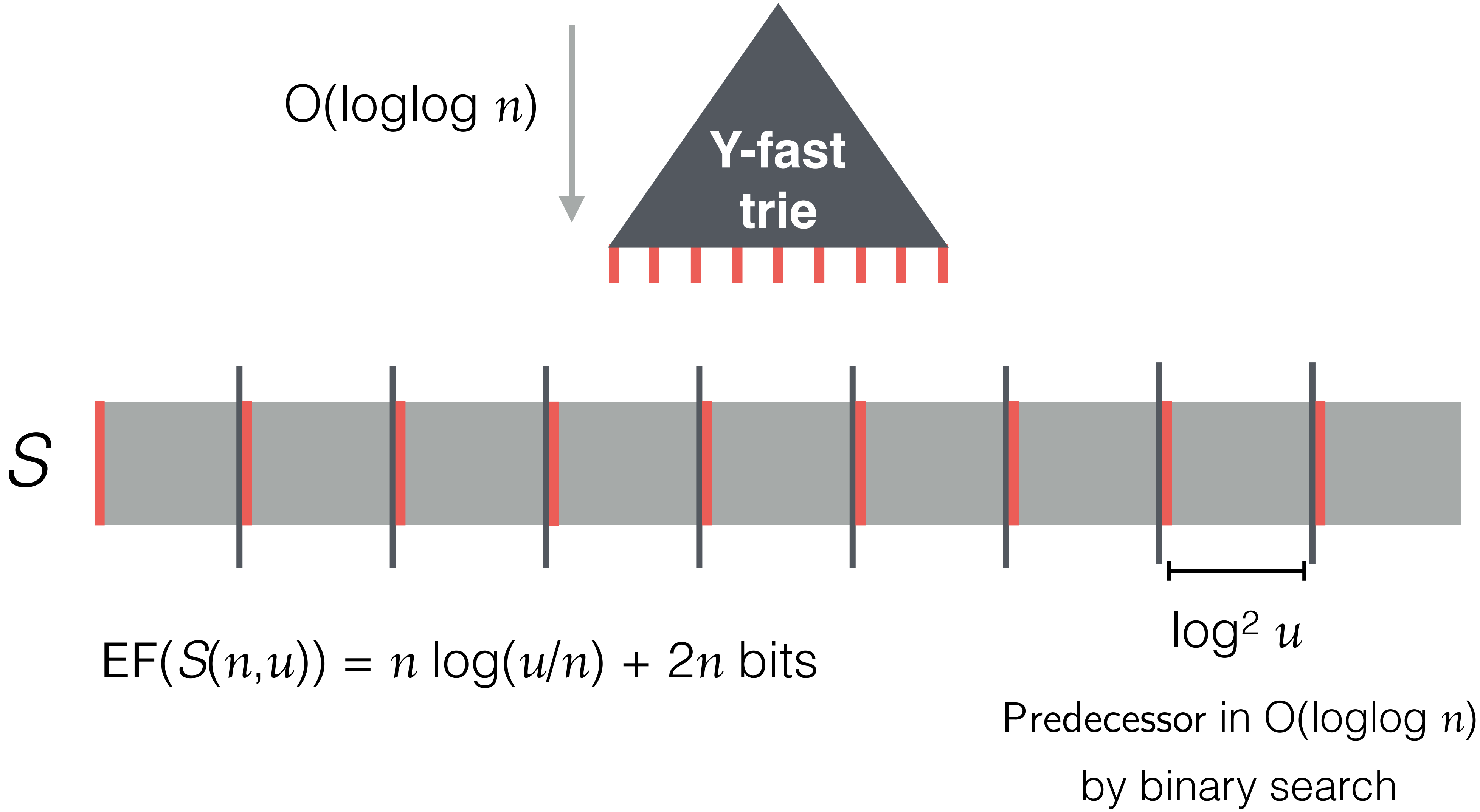

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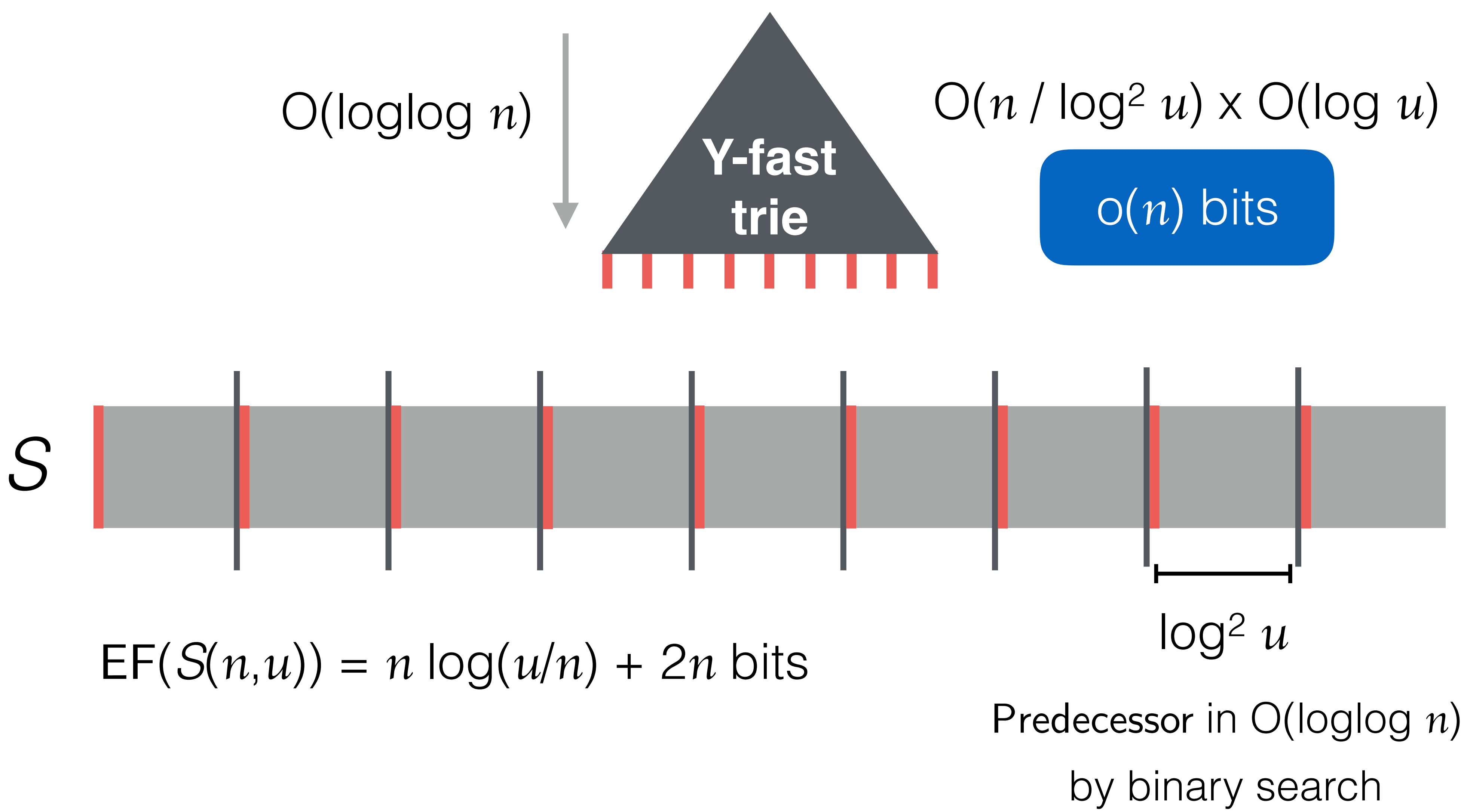

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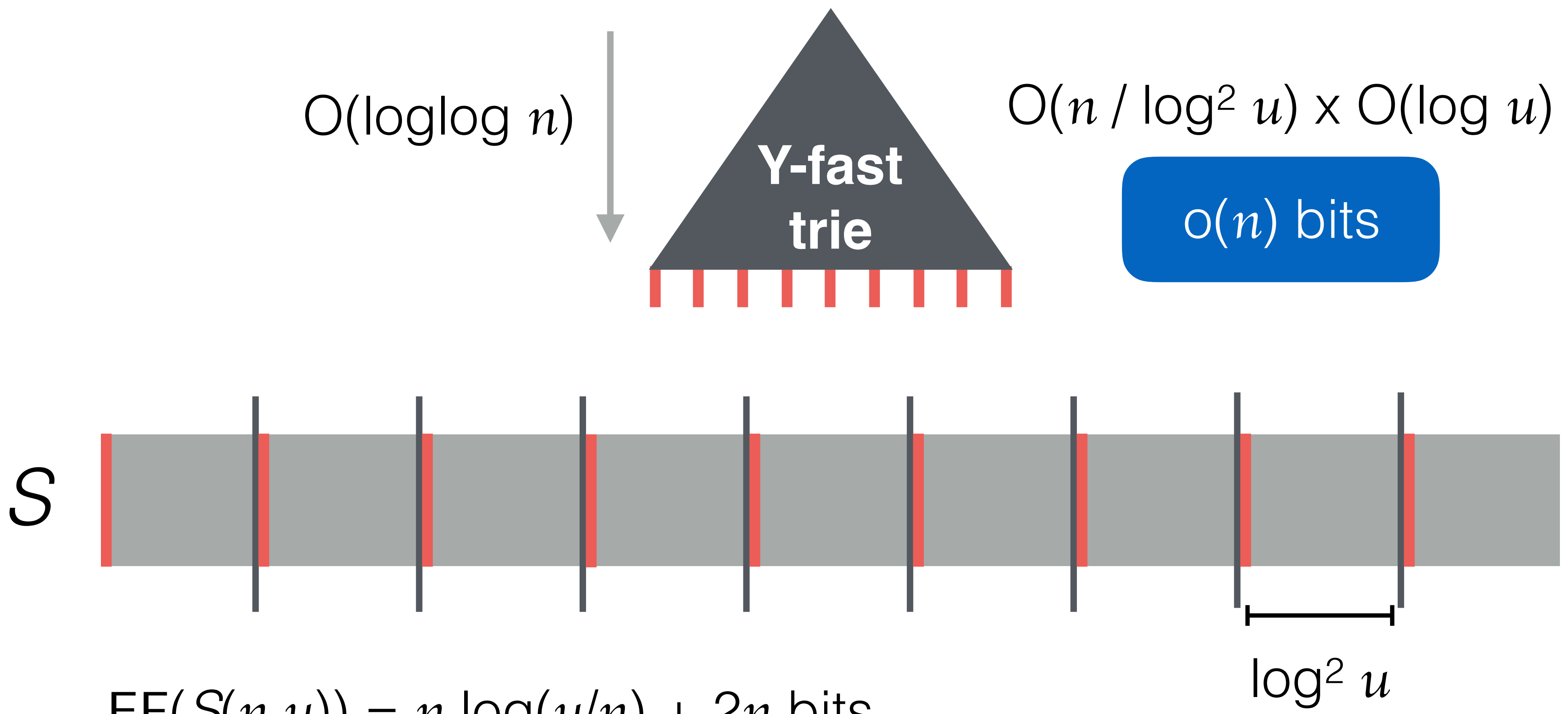
  for the tiny range

  $1 \leq \gamma \leq 1 + \text{loglog } n / \log n$



EF($S(n,u)$) = n log(u/n) + 2n bits

Predecessor in O(loglog n)

by binary search

Results - Extensible Elias-Fano

For $u = n^{\gamma}$, $\gamma = \Theta(1)$:

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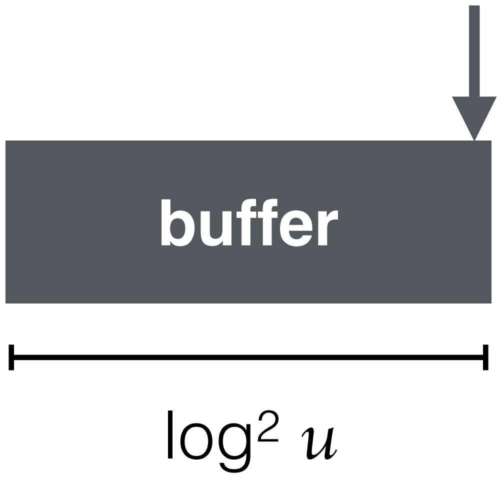

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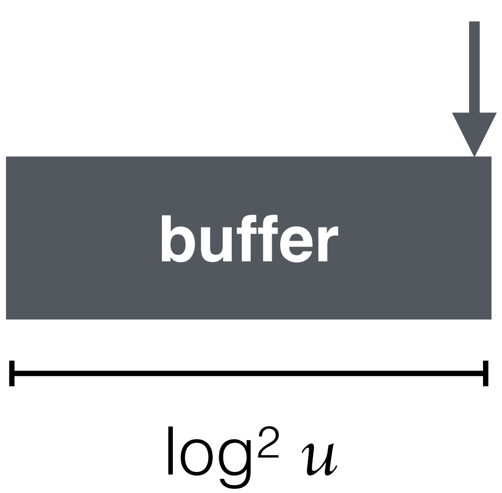

Results - Extensible Elias-Fano 2

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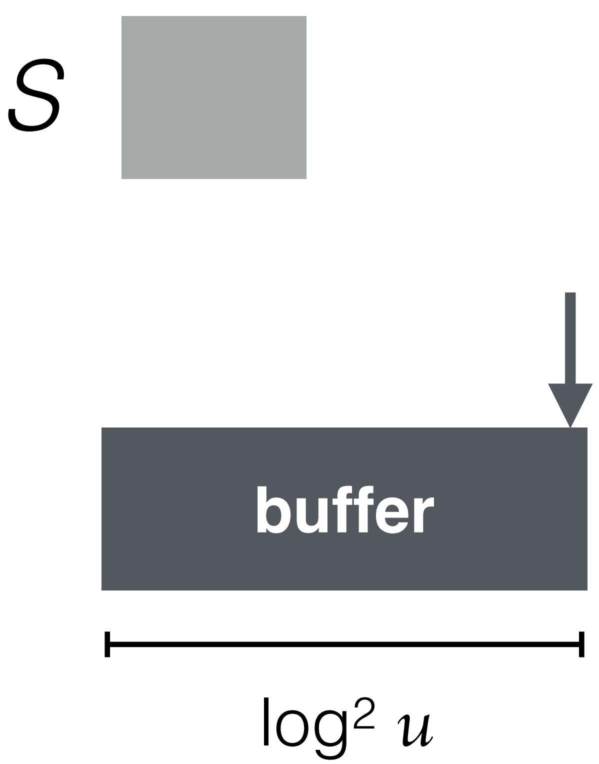

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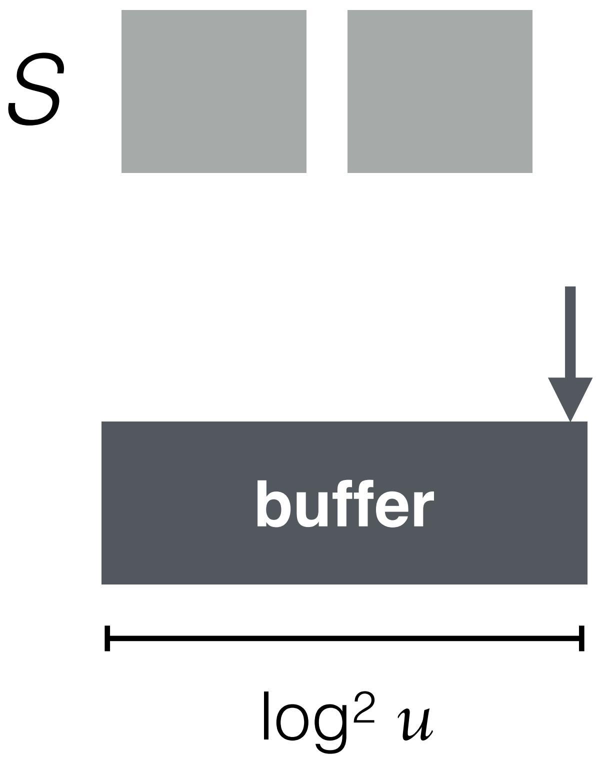

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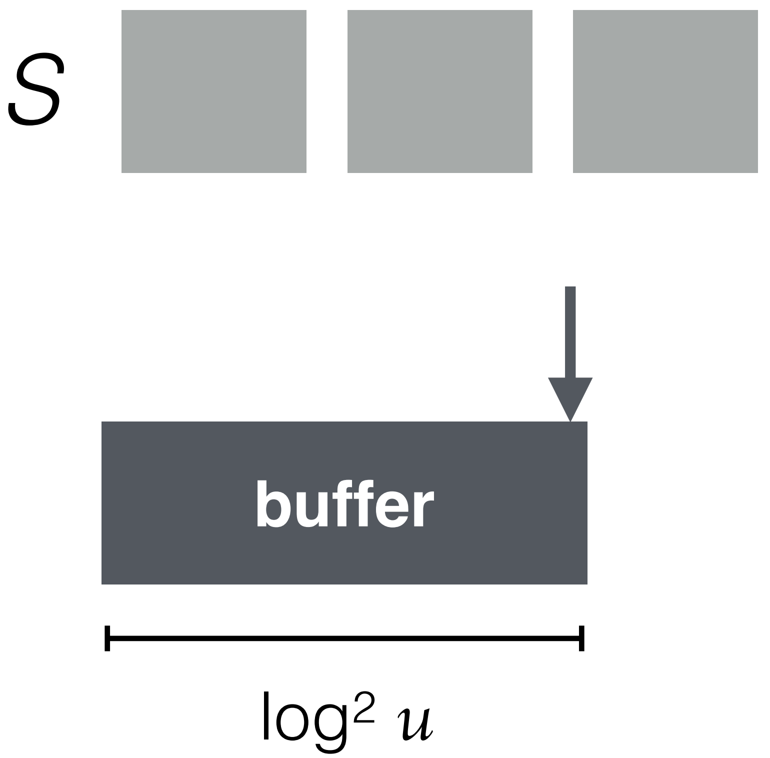

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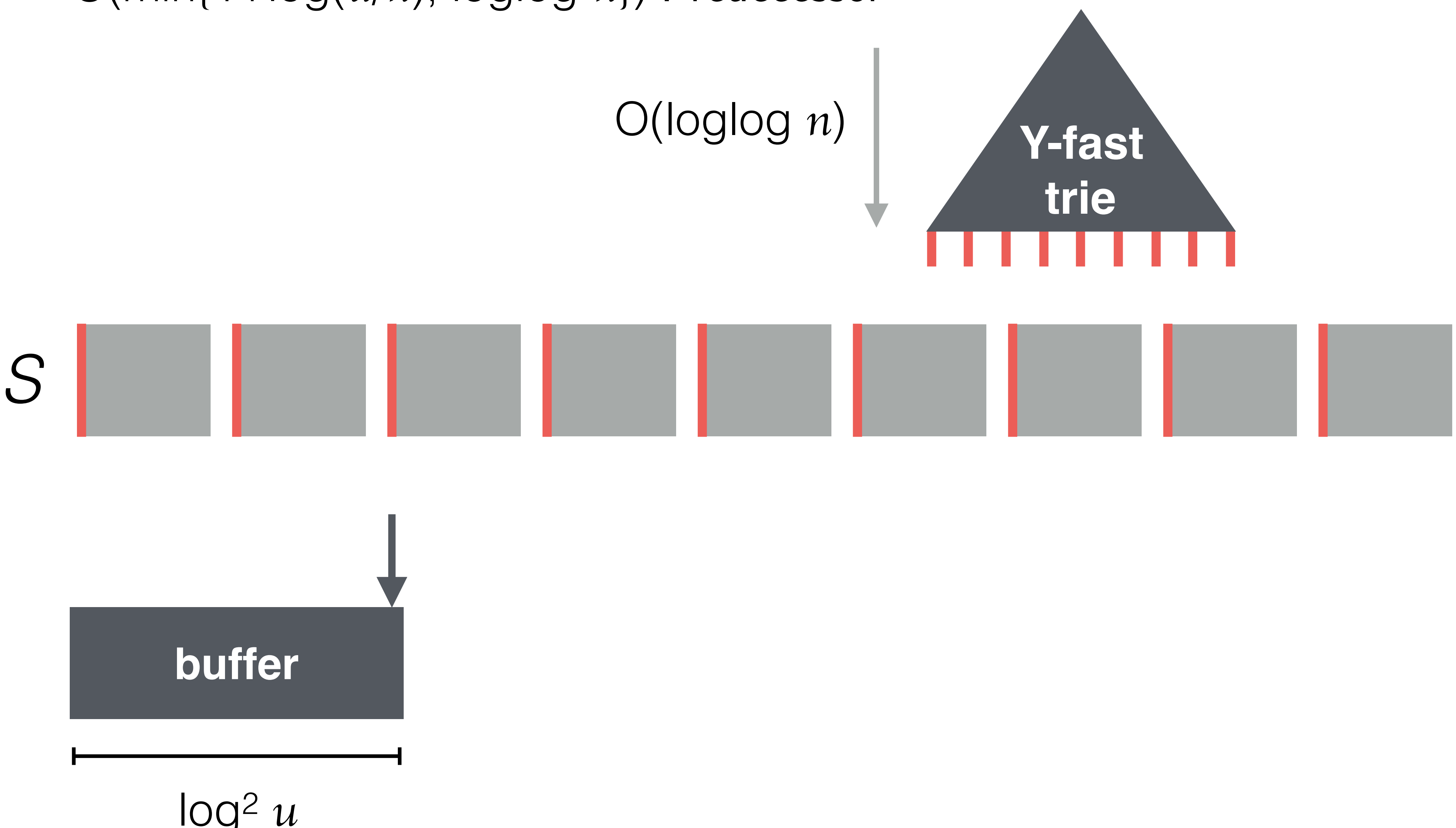

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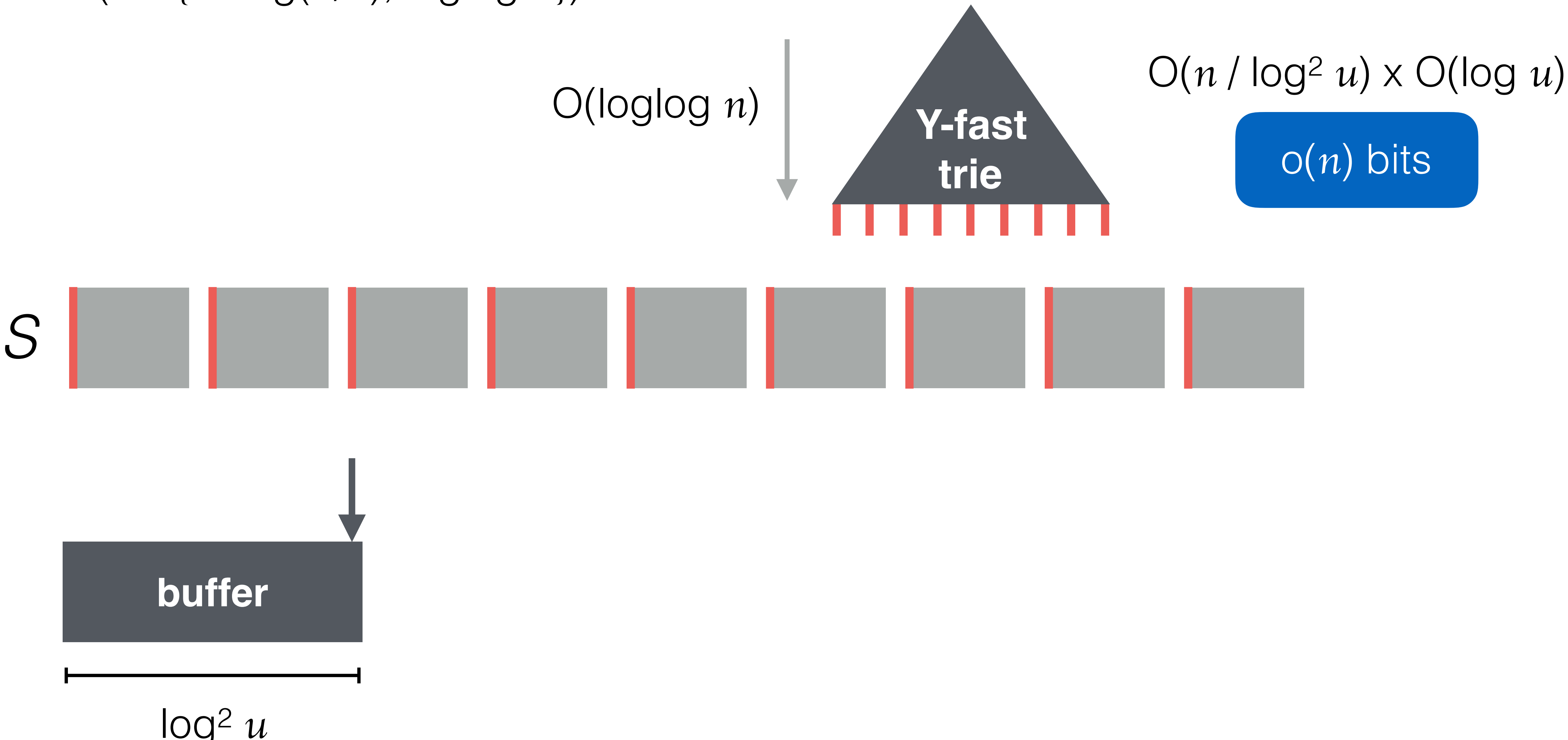

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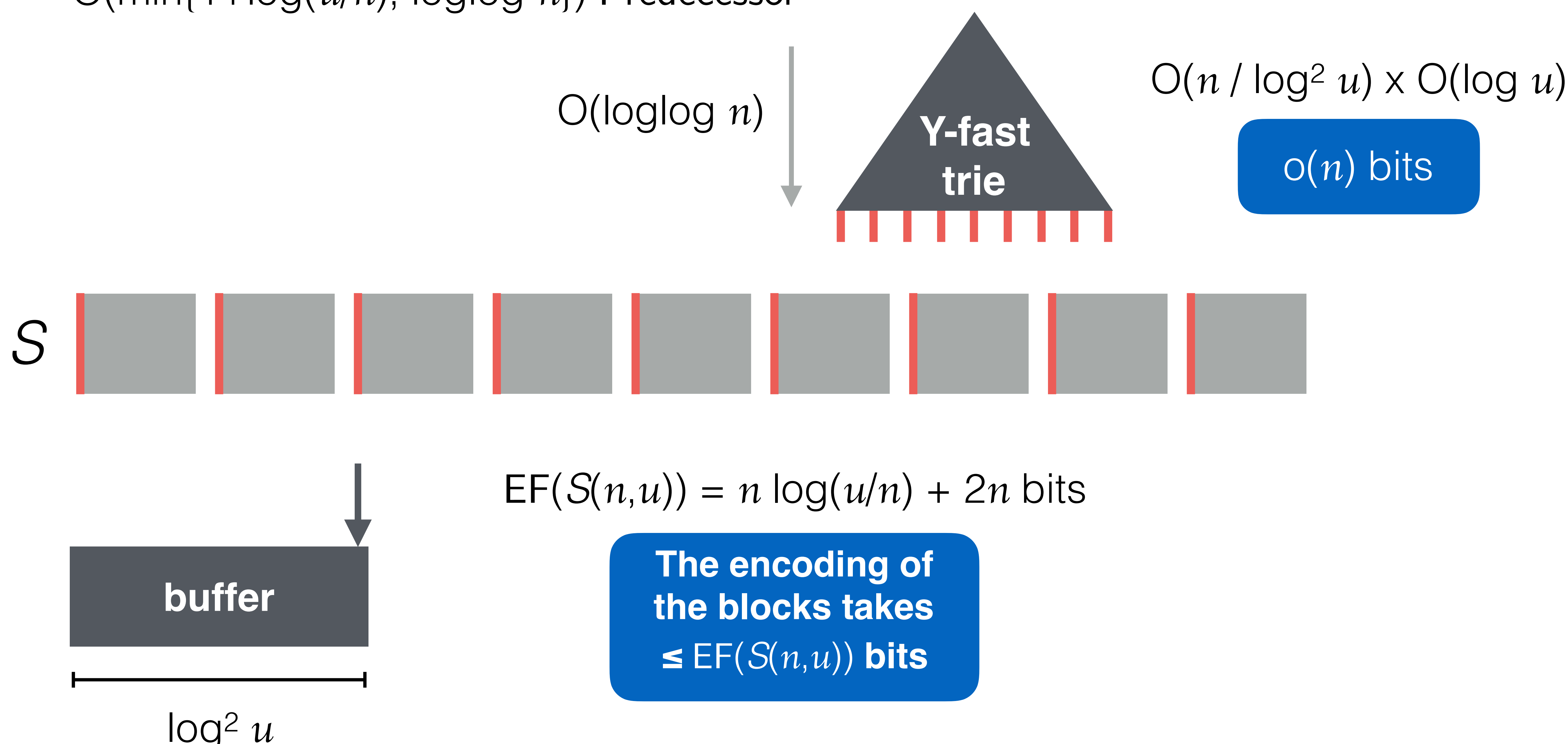

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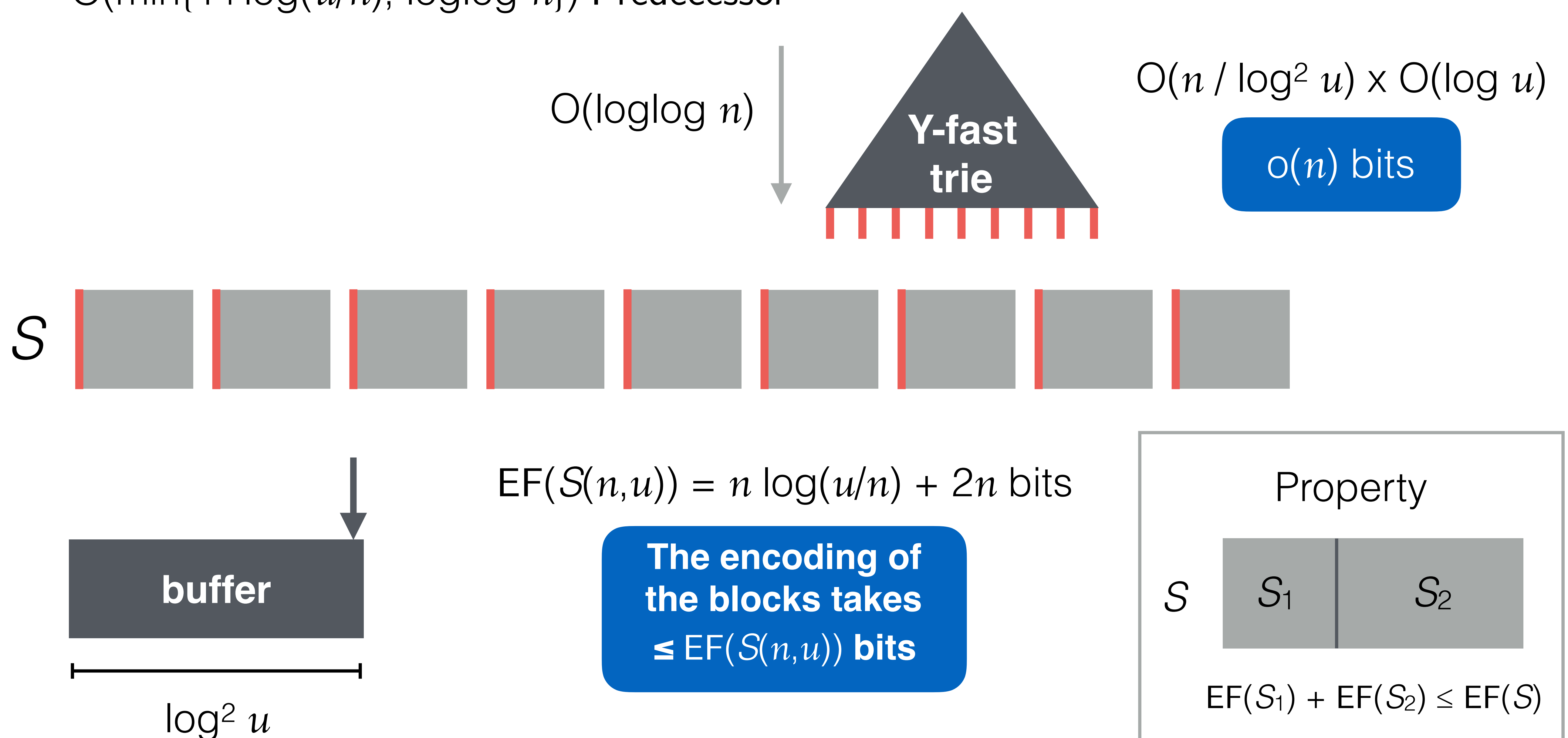

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EF($S(n,u)$) = n log(u/n) + 2n bits

mini block of size
b = log n / loglog n

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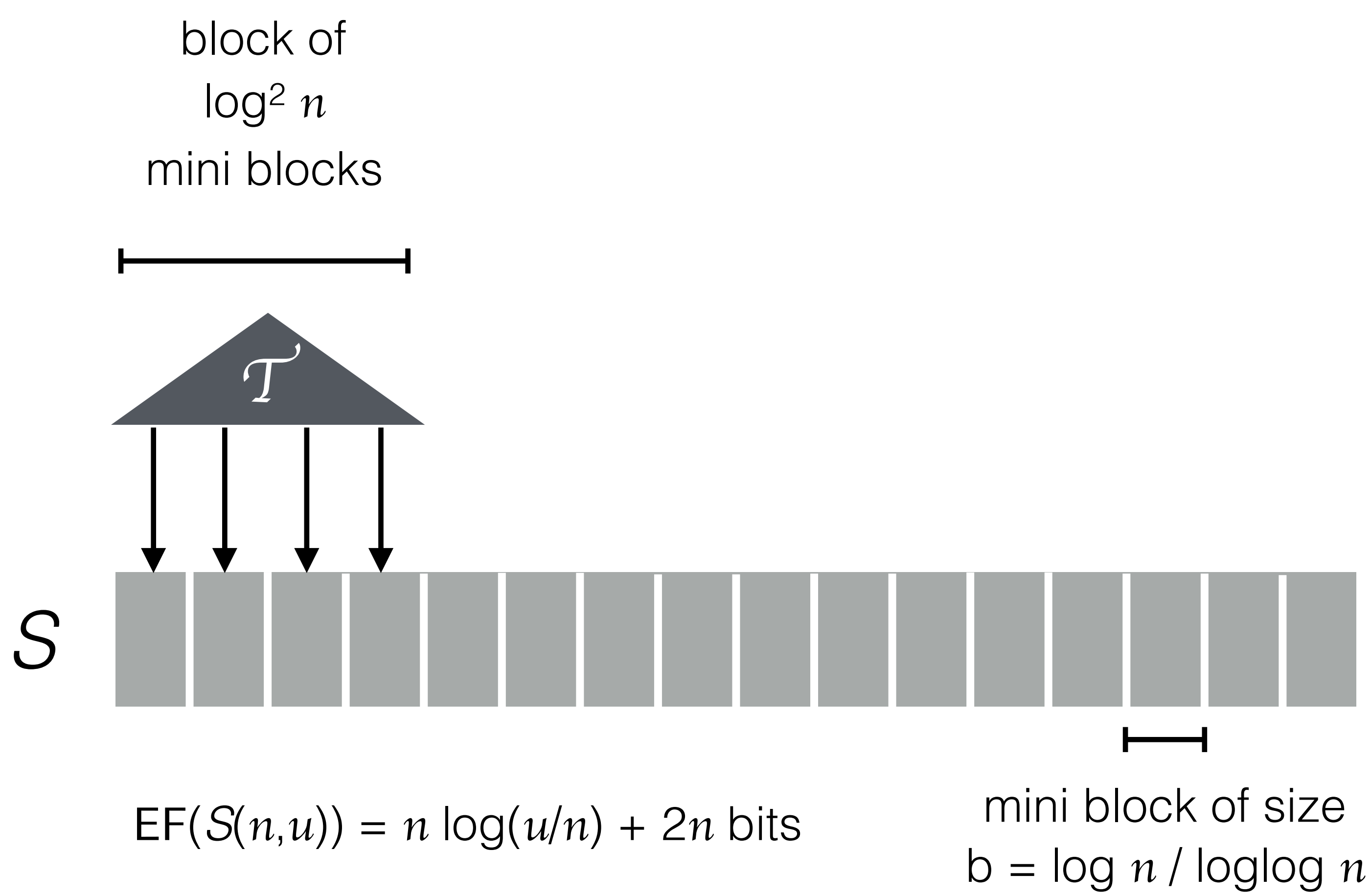

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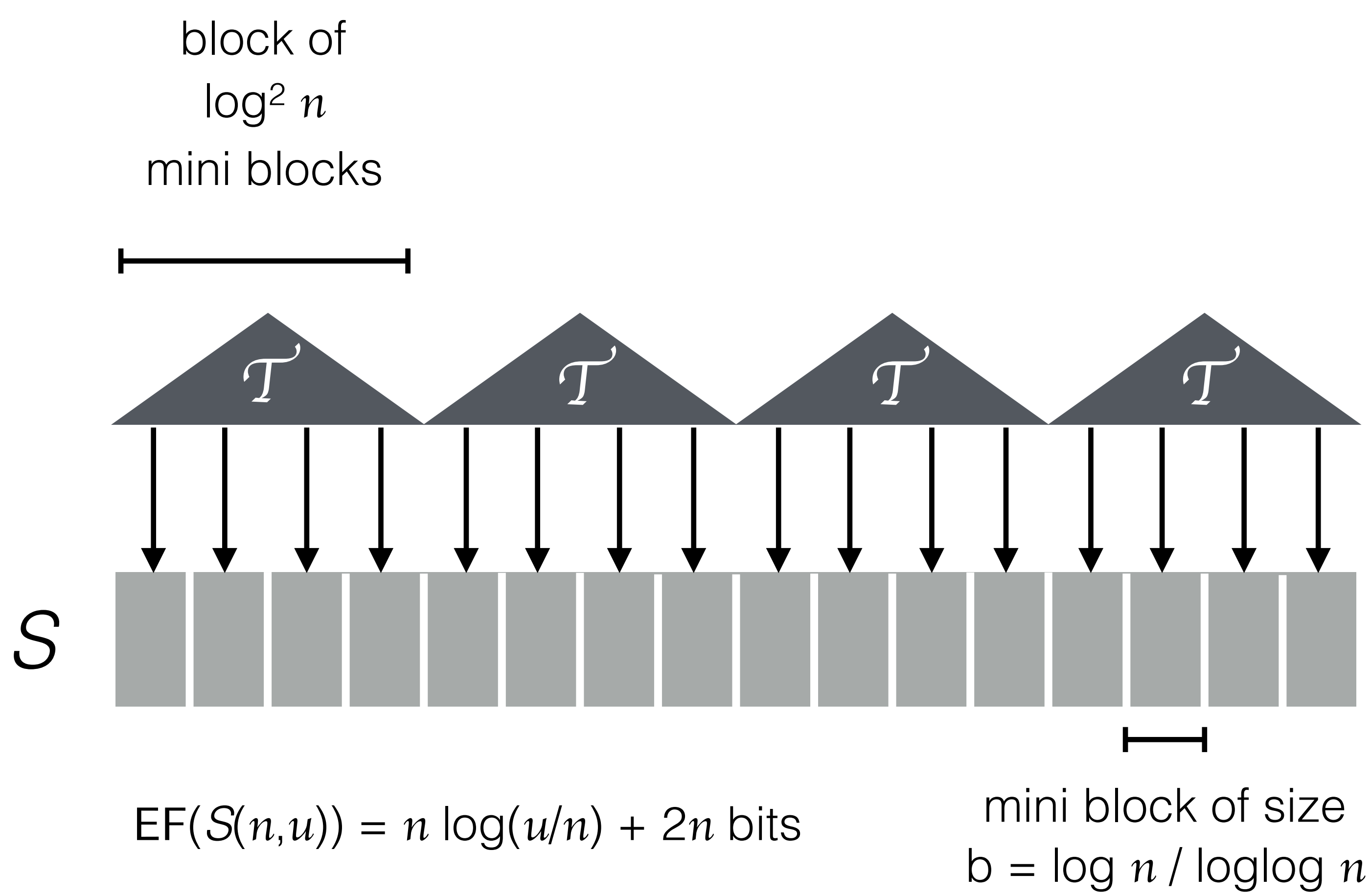

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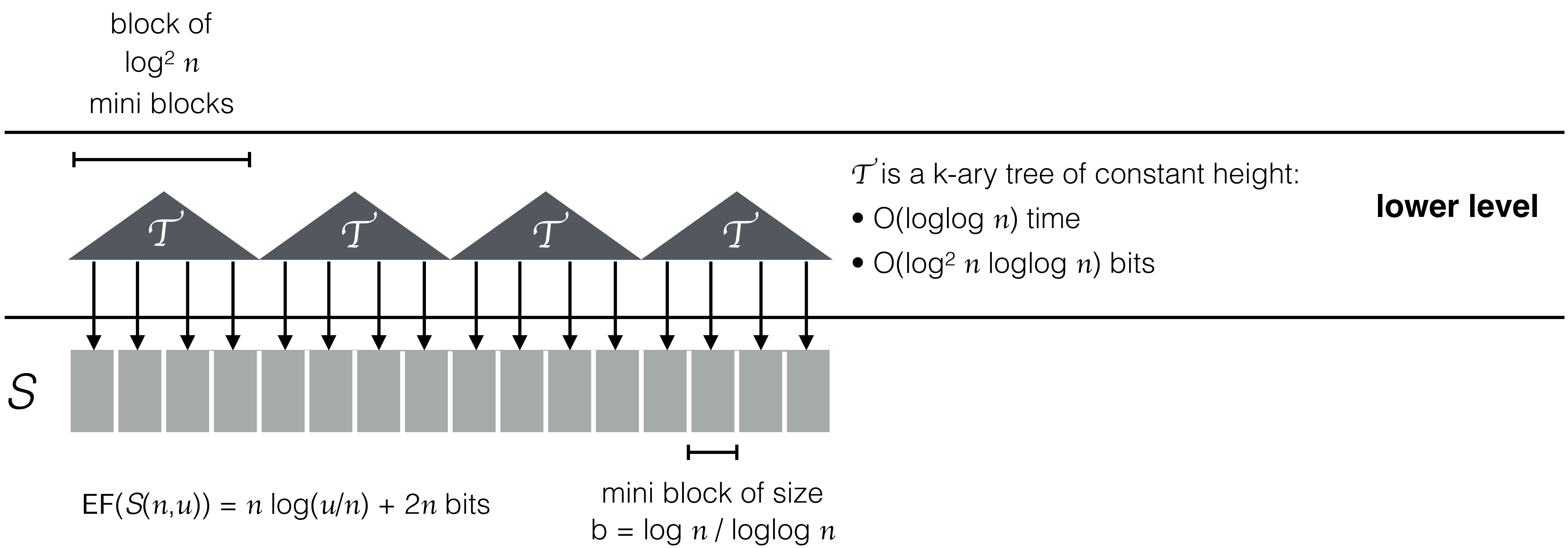

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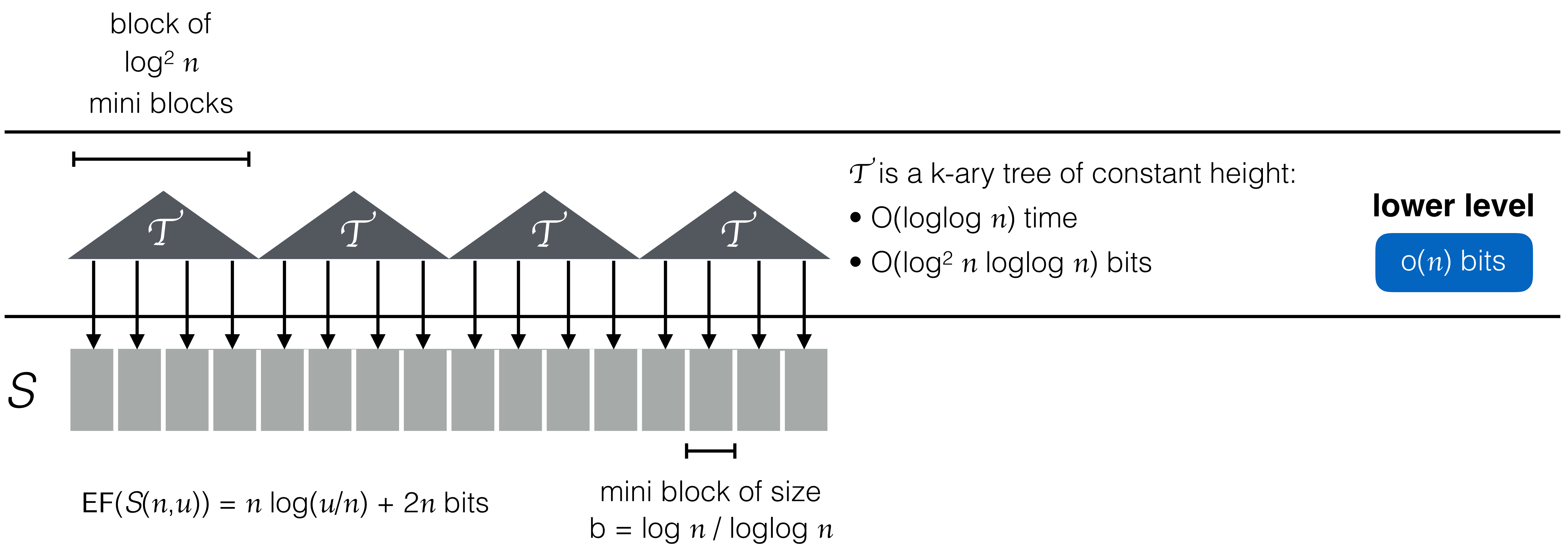

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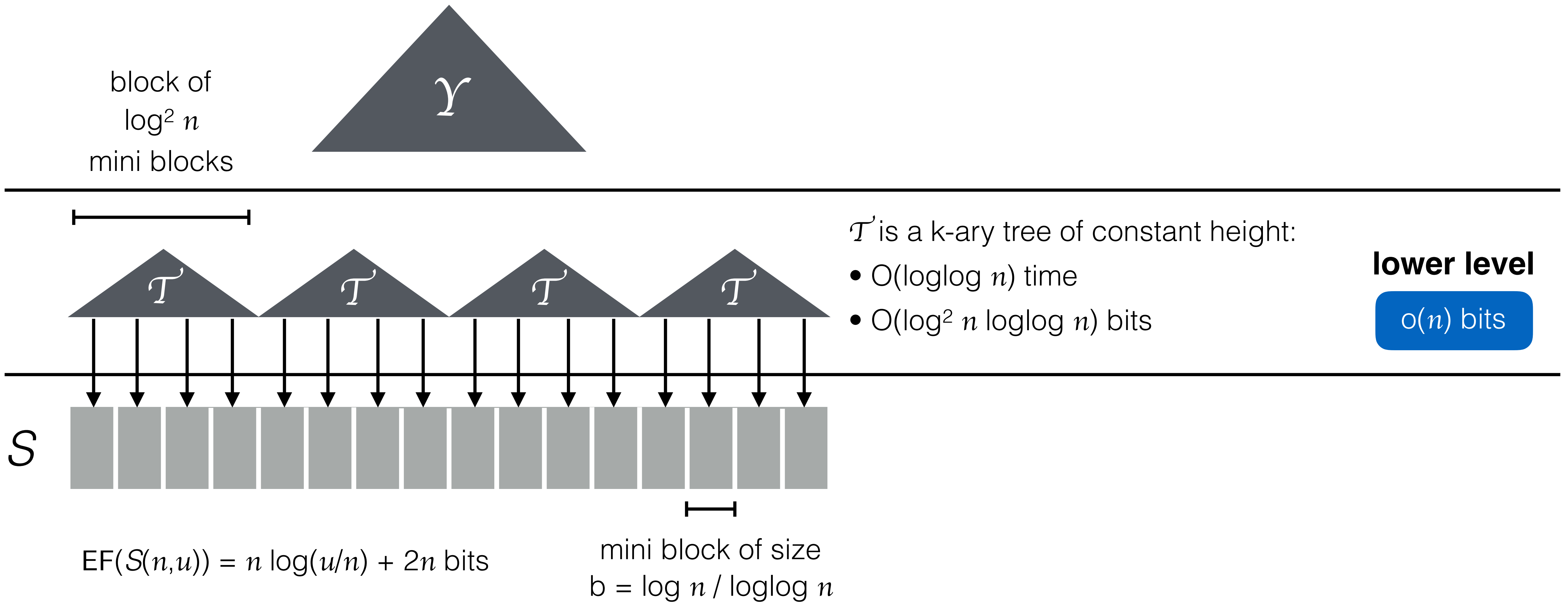

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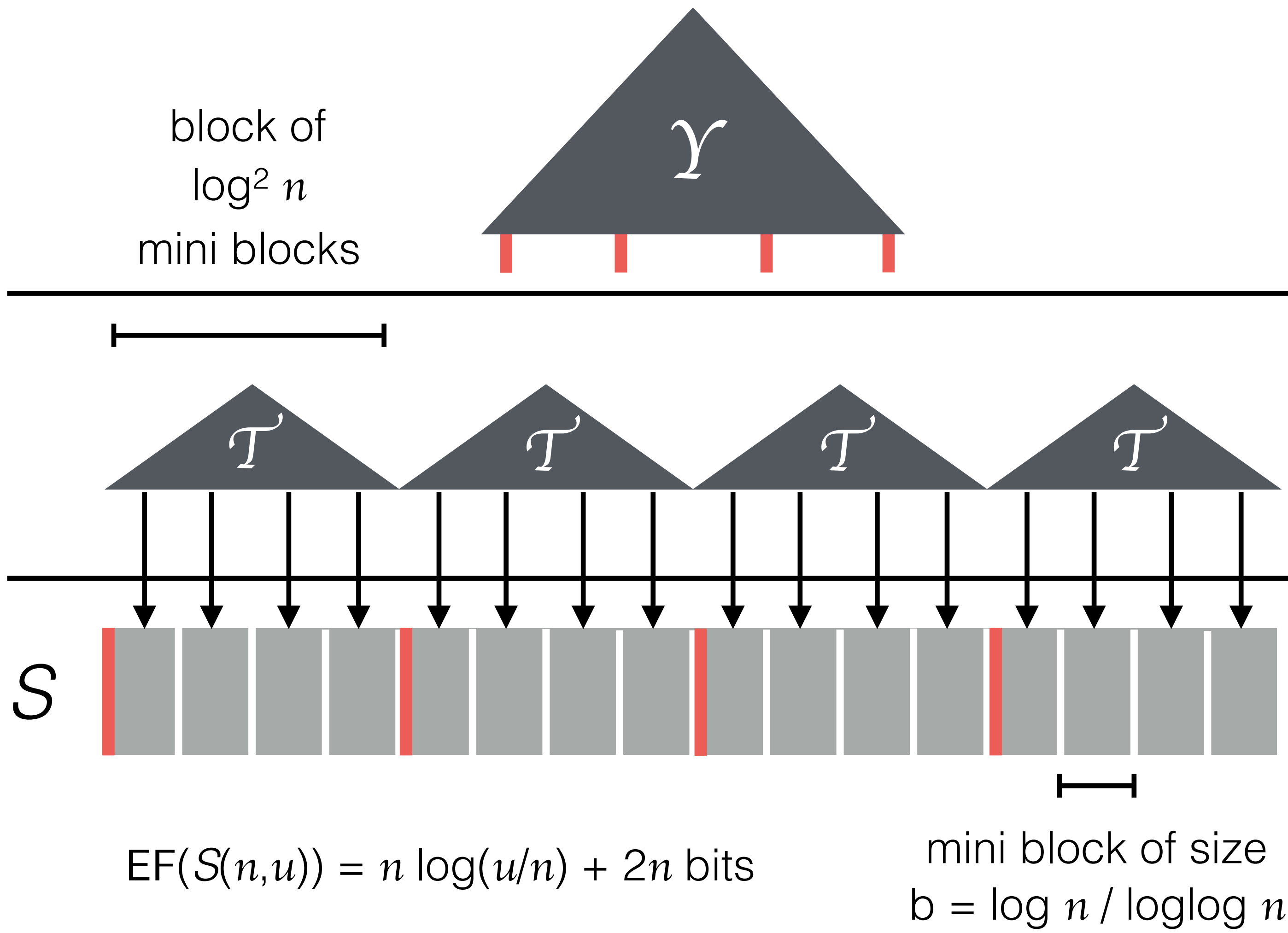

$\mathcal{T}$ is a k-ary tree of constant height:

- O(loglog n) time
- O($\log^2 n$ loglog n) bits

lower level

o(n) bits

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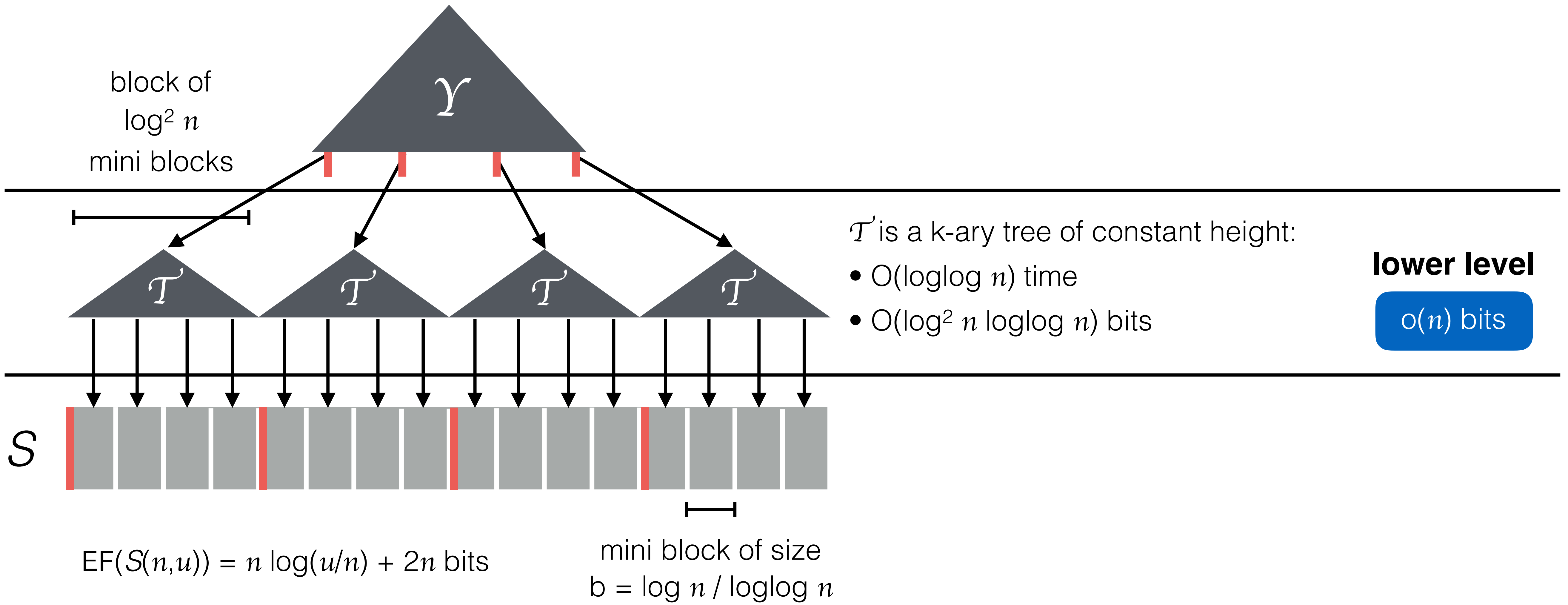
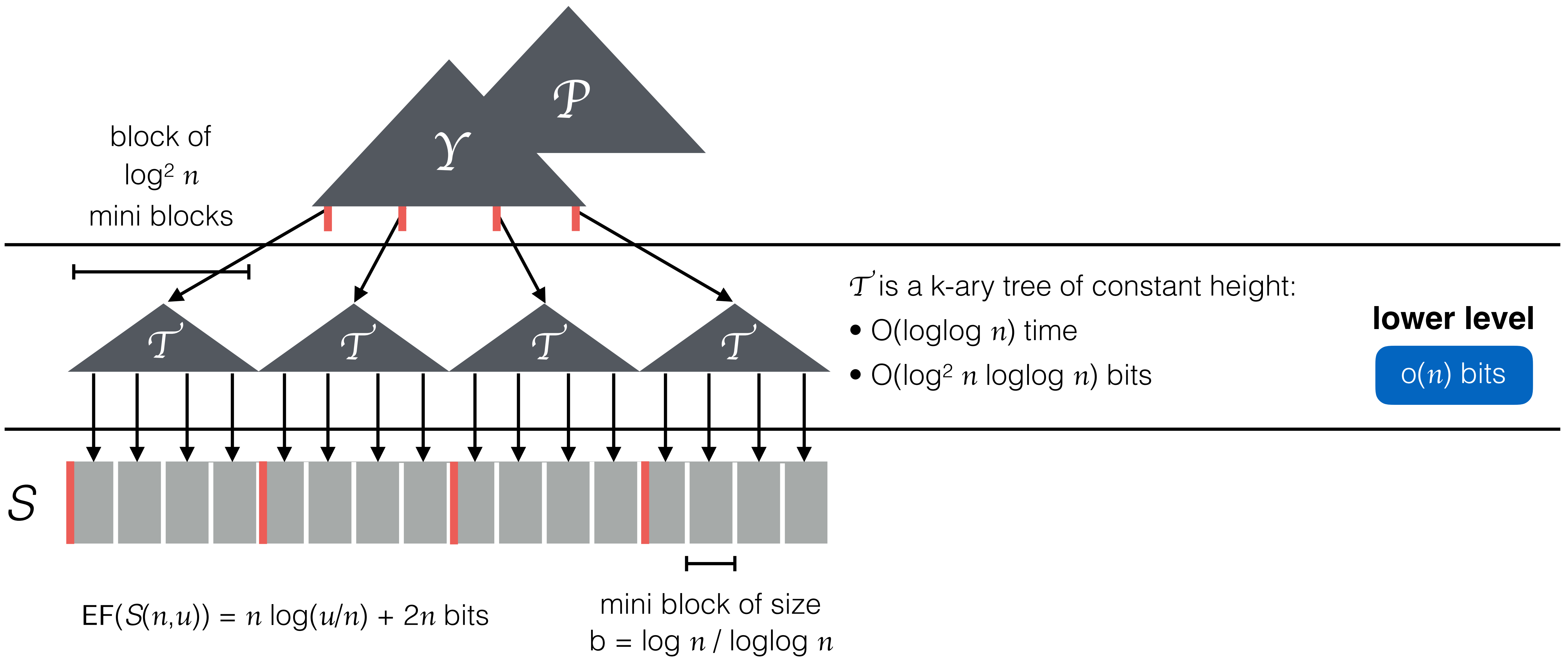


For $u = n^{\gamma}$, $\gamma = \Theta(1)$:

- EF($S(n,u)$) + $o(n)$ bits
- $O(\log n / \log\log n)$ Access
- $O(\log n / \log\log n)$ Insert/Delete (amortized)
- $O(\min\{1+\log(u/n), \log\log n\})$ Predecessor



Results - Dynamic Elias-Fano

For $u = n^{\gamma}$, $\gamma = \Theta(1)$:

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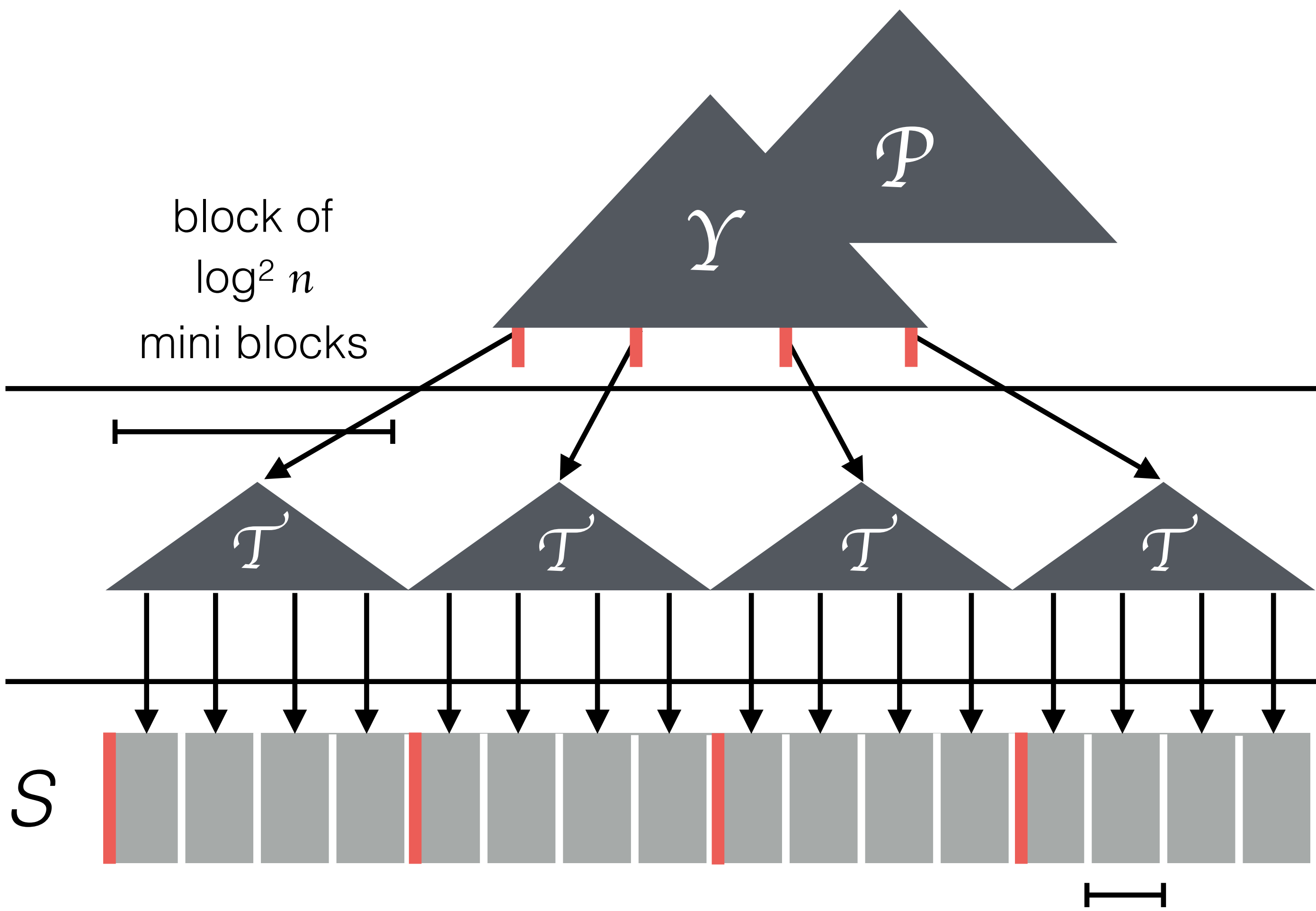

Υ is an Y-fast trie

- O(loglog n) time

$\mathcal{P}$ is a dynamic prefix-sums DS [1]

- O(b) time

O(n / (b x log² n)) x O(log u) = o(n) bits each

upper level

$\mathcal{T}$ is a k-ary tree of constant height:

- O(loglog n) time
- O(log² n loglog n) bits

lower level

o(n) bits

EF($S(n,u)$) = n log(u/n) + 2n bits

mini block of size
b = log n / loglog n

[1] Philip Bille, Patrick Hagge Cording, Inge Li Gørtz, Frederik Rye Skjoldjensen, Hjalte Wedel Vildhøj, and Søren Vind. *Dynamic relative compression*. *CoRR*, abs/1504.07851, 2015.

Results - Dynamic Elias-Fano

For $u = n^{\Upsilon}$, $\gamma = \Theta(1)$:

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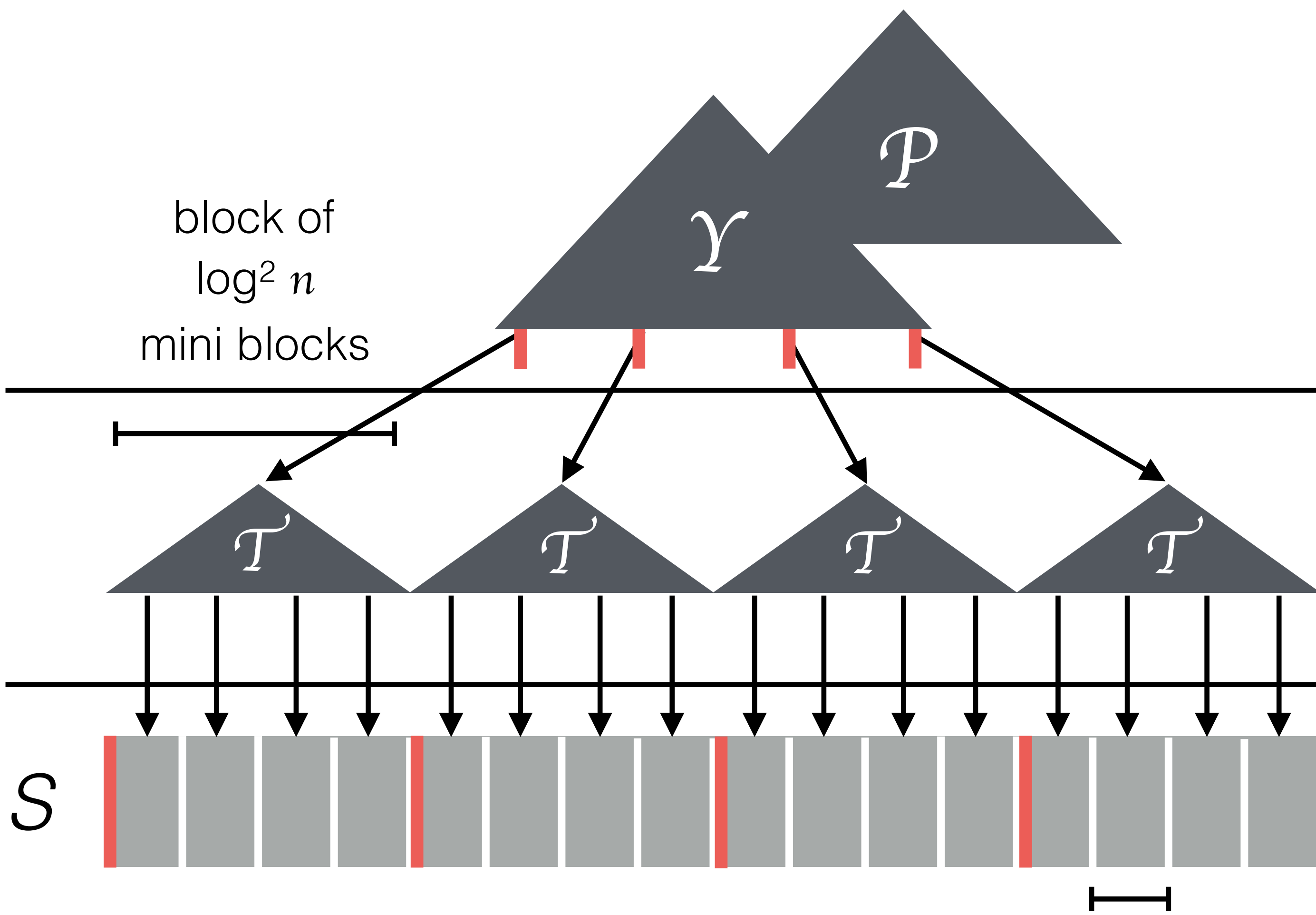
- $O(\log n / \log\log n)$ Access
- $O(\log n / \log\log n)$ Insert/Delete (amortized)
- $O(\min\{1+\log(u/n), \log\log n\})$ Predecessor



Υ is an Y-fast trie

- $O(\log\log n)$ time

$\mathcal{P}$ is a dynamic prefix-sums DS [1]

- O(b) time

$O(n / (b \times \log^2 n)) \times O(\log u) = o(n)$ bits each

upper level

$o(n)$ bits

$\mathcal{T}$ is a k-ary tree of constant height:

- $O(\log\log n)$ time
- $O(\log^2 n \log\log n)$ bits

lower level

$o(n)$ bits

EF($S(n,u)$) = $n \log(u/n) + 2n$ bits

mini block of size
b = $\log n / \log\log n$

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Results - Dynamic Elias-Fano

For $u = n^{\Upsilon}$, $\gamma = \Theta(1)$:

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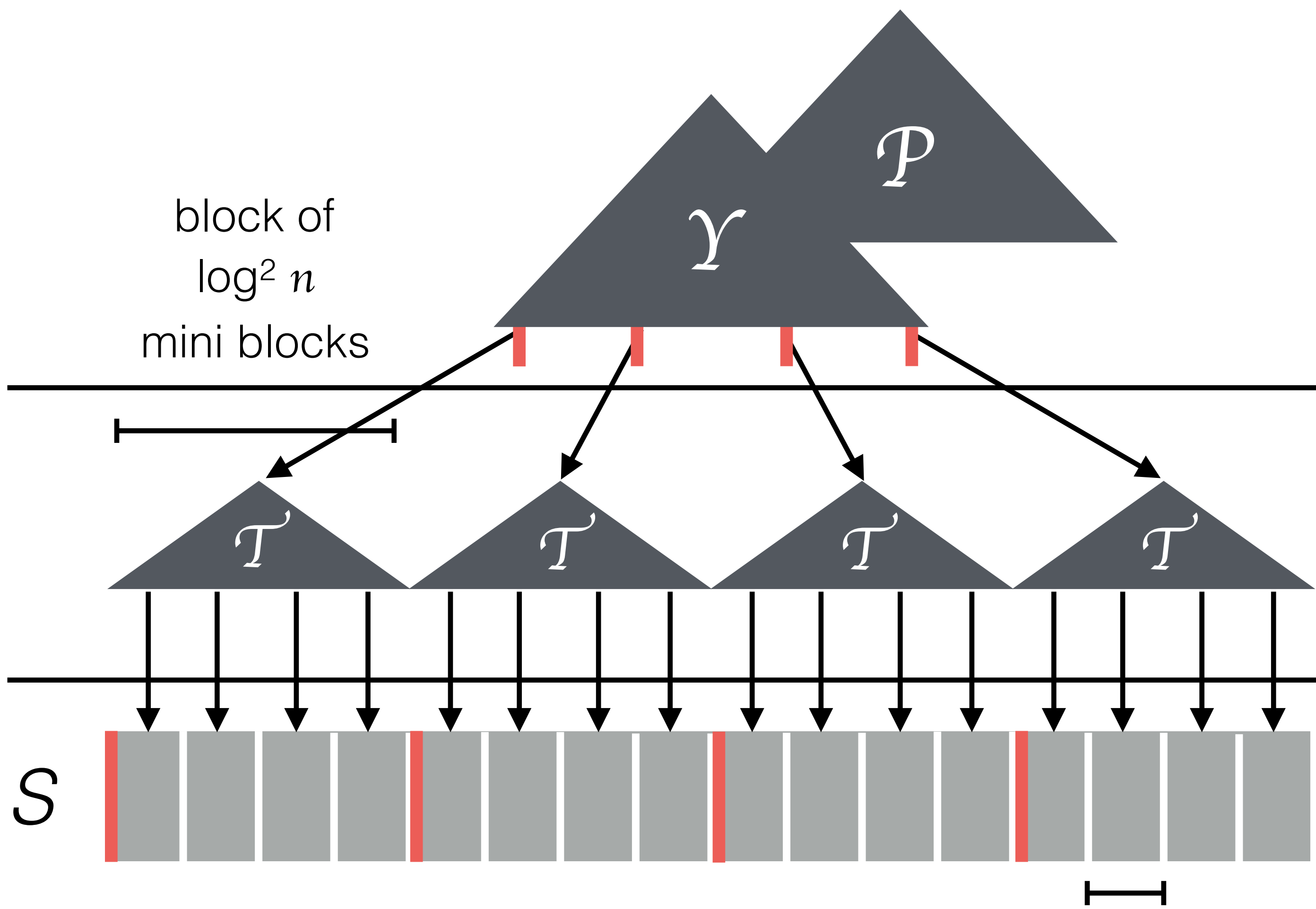

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$\mathcal{T}$ is a k-ary tree of constant height:
- O(loglog n) time
- O(log² n loglog n) bits

lower level

o(n) bits

mini block of size
b = log n / loglog n

EF($S(n,u)$) = n log(u/n) + 2n bits

The encoding of the mini blocks takes ≤ EF($S(n,u)$) + o(n) **bits**

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Results - Dynamic Elias-Fano

For $u = n^\gamma$, $\gamma = \Theta(1)$:

- EF($S(n,u)$) + o(n) bits
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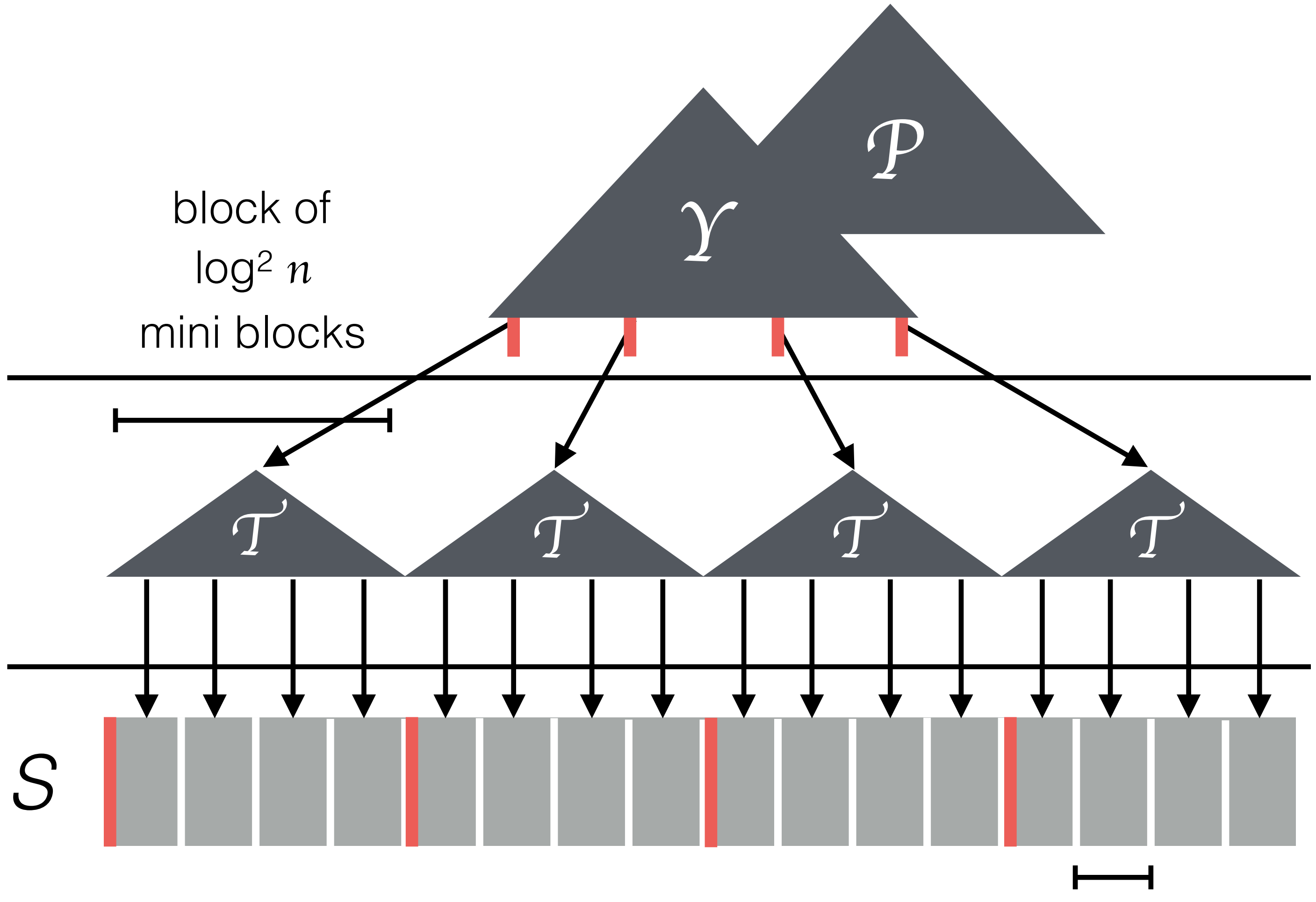

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Results - Dynamic Elias-Fano 3

For $u = n^{\gamma}$, $\gamma = \Theta(1)$:

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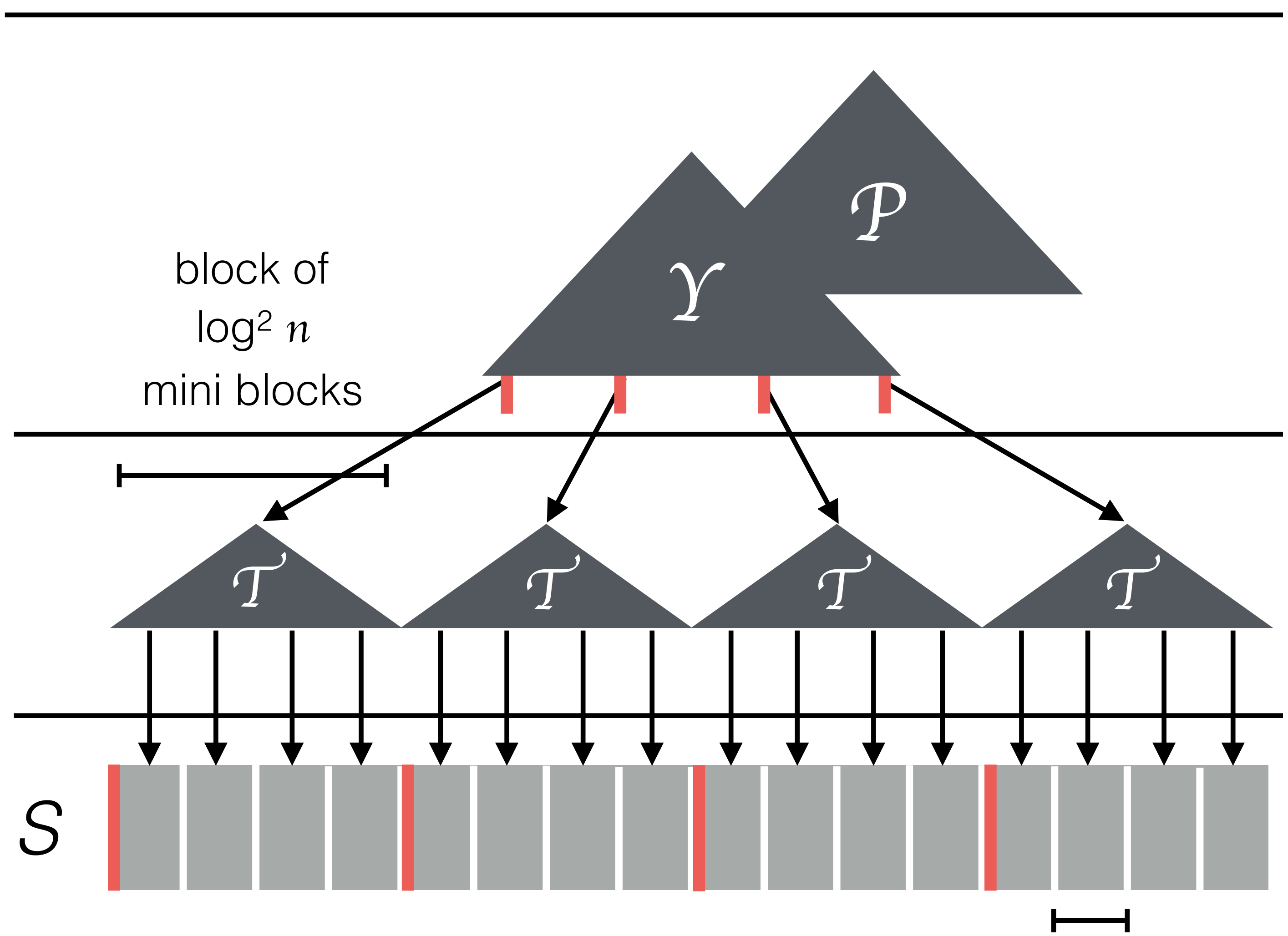

EF($S(n,u)$) = n log(u/n) + 2n bits

mini block of size
b = log n / loglog n

The encoding of the mini blocks takes ≤ EF($S(n,u)$) + o(n) bits

Memory management for the mini blocks:

LOW HIGH

b log(u/b) + 2b bits

Corollary 3 from [3]: random Access in O(1).

Theorem 6 from [2]: address and allocate the high part of a mini block in O(1).

The overall redundancy is o(n) bits.

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Results - Dynamic Elias-Fano

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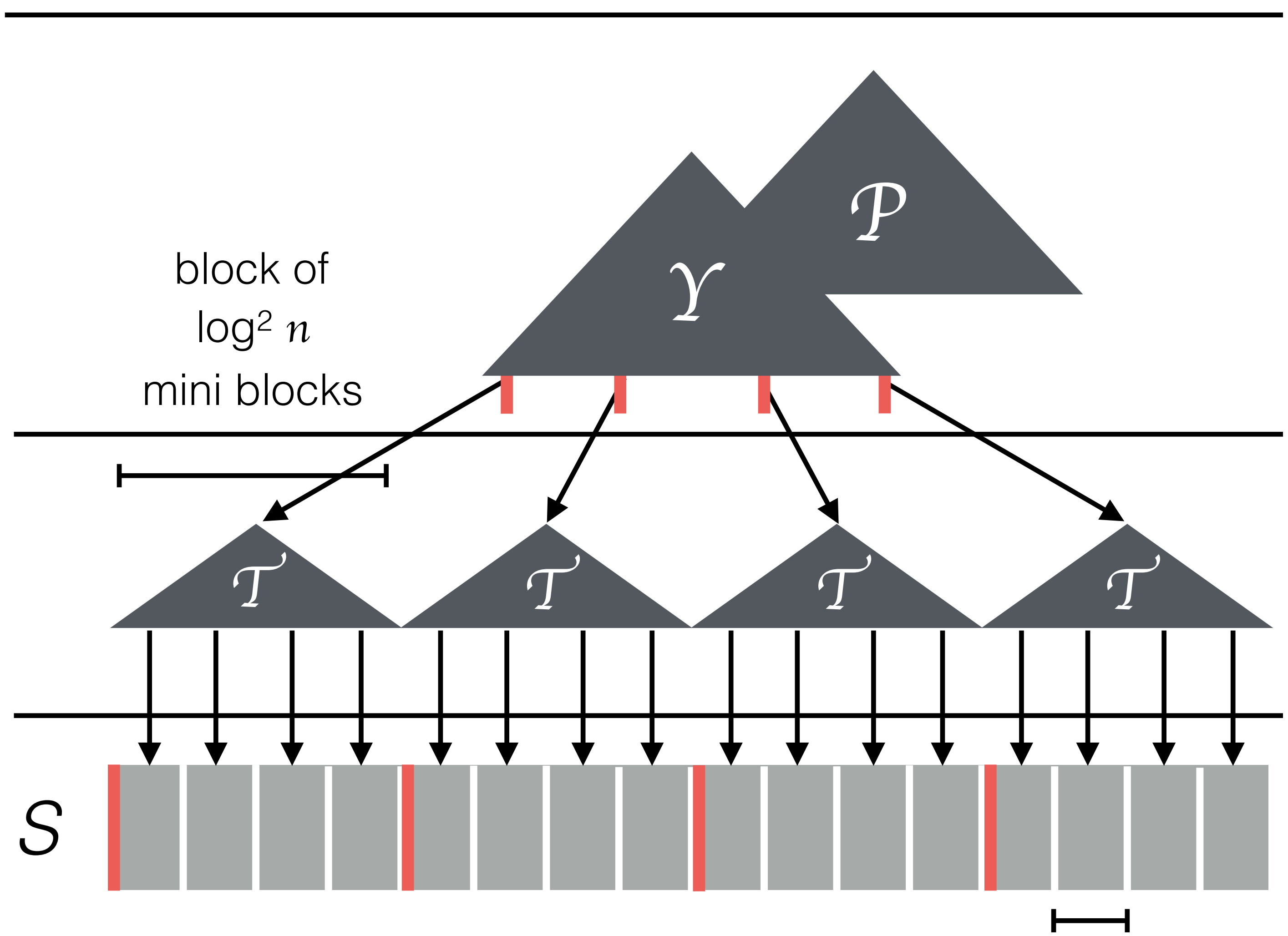

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[2] J. Jansson, K. Sadakane, and Wing-Kin Sung. *CRAM: Compressed random access memory*. ICALP 2012.
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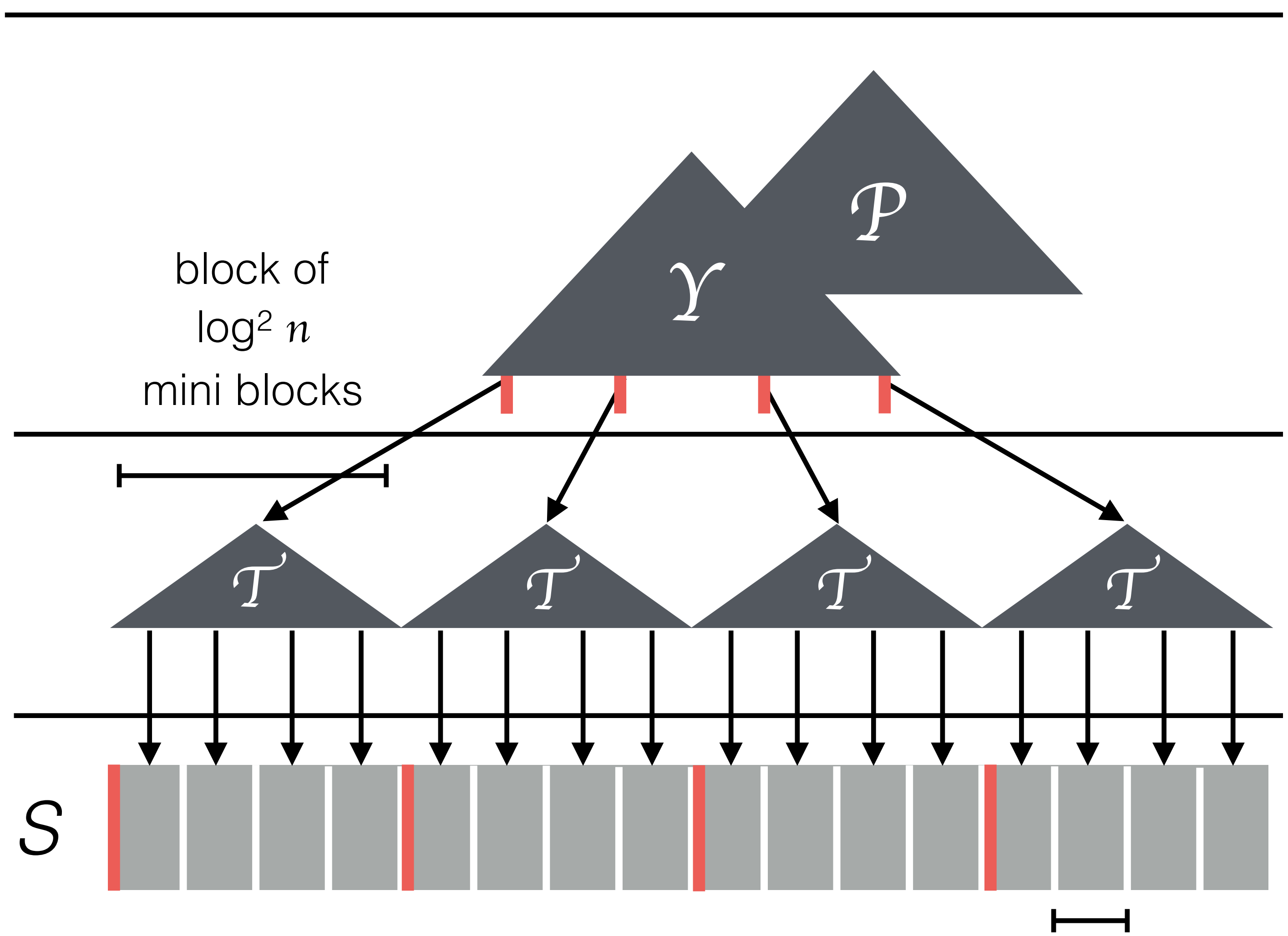

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The encoding of the mini blocks takes ≤ EF($S(n,u)$) + o(n) bits

Memory management for the mini blocks:

LOW: b log(u/b) + HIGH: 2b bits

Corollary 3 from [3]: random Access in O(1).

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The overall redundancy is o(n) bits.

Property

EF(S_1) + EF(S_2) ≤ EF(S)

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Thanks for your attention, time, patience!

Any questions?