Old written exams

Exam of Optimization Methods – June 6, 2016

1. Consider the following optimization problem:

$$\begin{cases} \min\ -x_1^2 + 2x_1 + x_2^2 - 2x_2 \\ x_1^2 - 4 \le 0 \\ x_2^2 - 4 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(0,0)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following optimization problem:

$$\begin{cases} \min\ 2x_1^2 + x_1x_3 + x_2^2 + 2x_2x_3 + 3x_3^2 + x_3x_4 + 2x_4^2 - 5x_1 - 4x_3 + 3x_4 \\ x \in \mathbb{R}^4 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Find the global minimum by using the gradient method with exact line search starting from the point $(0,0,0,0)$ [Use $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion]. How many iterations are needed?
   d) Find the global minimum by using the conjugate gradient method starting from the point $(0,0,0,0)$. How many iterations are needed? Write the point found by the method at any iteration.
   e) Find the global minimum by using the Newton method starting from the point $(0,0,0,0)$. How many iterations are needed?

3. Consider the following multi-objective problem:

$$\begin{cases} \min\ (x_2\ ,\ -x_1 - 2x_2) \\ x_1 \geq 0 \\ x_2 \geq 0 \\ x_1 + x_2 \leq 5 \end{cases}$$

a) Is it a convex problem? Why?
b) Do Pareto minima exist? Why?
c) Is the point $(0,2)$ a weak Pareto minimum? Why?
d) Find all the Pareto minima by using the scalarization method.
e) Find all the weak Pareto minima by using the scalarization method.
f) Find the ideal point.
g) Apply the goal method with $\|\cdot\|_1$.
h) Apply the goal method with $\|\cdot\|_2$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 3 & 2 & 2 \\ 4 & 12 & 4 \\ 5 & 10 & 1 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 6 & 5 & 6 \\ 4 & 2 & 3 \\ 9 & 8 & 1 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.
b) Do pure strategies Nash equilibria exist? Why?
c) Evaluate the gap function at the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/2, 1/2)$, in order to check if it is a Nash equilibrium.
d) Plot the polyhedra P and Q related to the game and find all the mixed strategies Nash equilibria.
e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – June 28, 2016

1. Consider the following optimization problem:

$$\begin{cases} \min\ -x_1^2 - x_2^2 + 4x_2 \\ x_1^2 + x_2^2 - 1 \leq 0 \\ -x_2 \leq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(0, 1)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following optimization problem:

$$\begin{cases} \min\ (x_1 - 7)^2 + (x_2 - 6)^2 + (x_3 - 3)^2 \\ x_1 + x_2 + x_3 \leq 10 \\ x_1 \geq 0 \\ x_2 \geq 0 \\ x_3 \geq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Solve the problem by using the active-set method starting from the point $(7, 0, 0)$. Write the point and the possible multipliers found by the method at any iteration.
   d) Solve the problem by using the Frank-Wolfe method with exact line search starting from the point $(5, 5, 0)$. How many iterations are needed?

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big(2x_1 + 3x_2 - x_3 - 2x_4 \ , \ -5x_1 - 2x_2 + 3x_3 + x_4\big) \\ x_1 + x_2 \leq 10 \\ x_3 + x_4 \leq 5 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(10, 0, 0, 5)$ a Pareto minimum? Why?

d) Is the point $(7, 3, 4, 1)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_2$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 6 & 3 & 1 & 2 \\ 1 & 4 & 5 & 7 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 9 & 1 & 3 & 5 \\ 1 & 2 & 4 & 3 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/4, 3/4)$, evaluate the gap function ψ at (x, y), the regularized gap function ψ_α, with $\alpha = 1$, at (x, y) and the D-gap function $\psi_{\alpha,\beta}$, with $\alpha = 1$ and $\beta = 3$ at (x, y).

d) Find the best response mapping of each player and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – July 18, 2016

1. Consider the following optimization problem:

$$\begin{cases} \min\ x_1^2 + x_2^2 + x_3^2 - 2x_1 - 2x_2 - 2x_3 \\ x_1^2 + x_2^2 + x_3^2 - 1 \le 0 \\ -x_3 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(1, 0, 0)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following unconstrained optimization problem:

$$\begin{cases} \min\ e^{-x_1-x_2-x_3} + x_1^2 + 3x_2^2 + x_3^2 + x_1x_2 - x_2x_3 + x_1 - 3x_3 \\ x \in \mathbb{R}^3 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Solve the problem by means of the gradient method with inexact line search setting $\alpha = 0.1$, $\gamma = 0.9$, $\bar{t} = 1$, starting from the point $(0, 0, 0)$ and using $\|\nabla f(x)\| < 10^{-3}$ as stopping criterion. Which is the global optimal solution? How many iterations are needed?
   d) Solve the problem by means of the Newton method with inexact line search setting $\alpha = 0.1$, $\gamma = 0.9$, $\bar{t} = 1$, starting from the point $(0, 0, 0)$ and using $\|\nabla f(x)\| < 10^{-3}$ as stopping criterion. Which is the global optimal solution? How many iterations are needed?

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \left(x_1^2 + x_2^2 - 16x_1 - 12x_2 \ , \ -x_1 + 2x_2\right) \\ x_1 + x_2 \le 10 \\ x_1 \ge 0 \\ x_2 \ge 0 \end{cases}$$

a) Is it a convex problem? Why?
b) Do Pareto minima exist? Why?
c) Is the point $(5, 5)$ a Pareto minimum? Why?
d) Is the point $(10, 0)$ a weak Pareto minimum? Why?
e) Find a subset of Pareto minima by using the scalarization method.
f) Find the set of all weak Pareto minima by using the scalarization method.
g) Find the set of all Pareto minima.
h) Find the ideal point.

4. Consider the following matrix game:

$$C = \begin{pmatrix} 12 & 5 & 9 & 4 \\ 5 & 8 & 10 & 9 \\ 7 & 9 & 12 & 10 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.
b) Do pure strategies Nash equilibria exist? Why?
c) Given the point (x, y), where $x = (1/2, 1/2)$ and $y = (1/2, 1/2)$, evaluate the gap function ψ at (x, y), the regularized gap function ψ_α, with $\alpha = 1$, at (x, y) and the D-gap function $\psi_{\alpha,\beta}$, with $\alpha = 1$ and $\beta = 2$ at (x, y).
d) Find all the mixed strategies Nash equilibria using linear programming.
e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – September 19, 2016

1. Consider the following optimization problem:

$$\begin{cases} \min\ -x_1^2 + x_2 \\ x_1^2 + x_2^2 - 4 \leq 0 \\ -x_1^2 - x_2^2 + 1 \leq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(0, \sqrt{2})$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following optimization problem:

$$\begin{cases} \min\ 2x_1^2 + x_1x_2 + x_1x_3 + x_1x_4 + x_2^2 + 2x_3^2 + 2x_3x_4 + 3x_4^2 + 3x_1 + 2x_2 + x_3 - x_4 \\ x \in \mathbb{R}^4 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Find the global minimum by using the gradient method with exact line search starting from the point $(0, 0, 0, 0)$ [Use $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion]. How many iterations are needed? Which is the optimal value?
   d) Find the global minimum by using the conjugate gradient method starting from the point $(0, 0, 0, 0)$. How many iterations are needed? Write the point found by the method at any iteration.
   e) Find the global minimum by using the Newton method starting from the point $(0, 0, 0, 0)$. How many iterations are needed? Why?

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big(3x_1 + x_2 - 2x_3 \ , \ -4x_1 - 3x_2 + 2x_3\big) \\ x_1 + x_2 + x_3 \leq 7 \\ x_1, x_2, x_3 \geq 0 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(0, 7, 0)$ a Pareto minimum? Why?

d) Is the point $(3, 0, 4)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_1$.

i) Apply the goal method with $\|\cdot\|_2$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 9 & 7 & 3 \\ 8 & 4 & 1 \\ 5 & 3 & 2 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 8 & 3 & 1 \\ 9 & 6 & 5 \\ 1 & 4 & 2 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Evaluate the gap function at the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/4, 3/4)$, in order to check if it is a Nash equilibrium.

d) Plot the polyhedra P and Q related to the game and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – November 8, 2016

1. Consider the following optimization problem:

$$\begin{cases} \min\ (x_1-2)^2+(x_2-2)^2+(x_3+1)^2 \\ x_1^2+x_2^2+x_3^2-1 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(0,1,0)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following optimization problem:

$$\begin{cases} \min\ \dfrac{1}{2}x_1^2+\dfrac{3}{2}x_2^2-x_1x_2+2x_1-x_2 \\ 2x_1+x_2 \le 6 \\ -x_1-x_2 \le -1 \\ x_1, x_2 \ge 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Solve the problem by using the active-set method starting from the point $(2,0)$. Write the point and the possible multipliers found by the method at any iteration.
   d) Solve the problem by using the Frank-Wolfe method with exact line search starting from the point $(2,0)$. How many iterations are needed?

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big(x_1 + x_2 \ , \ 2x_1 - 3x_2\big) \\ 2x_1 + x_2 \le 6 \\ -x_1 - x_2 \le -1 \\ x_1, x_2 \ge 0 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(0, 3)$ a Pareto minimum? Why?

d) Is the point $(2, 0)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_2$.

i) Apply the goal method with $\|\cdot\|_1$.

4. Consider the following matrix game:

$$C = \begin{pmatrix} 10 & 6 & 4 & 5 \\ 2 & 3 & 6 & 4 \\ 4 & 6 & 7 & 5 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x, y), where $x = (1/2, 1/2)$ and $y = (1/2, 1/2, 0, 0)$, evaluate at (x, y) the gap function ψ, the regularized gap function ψ_α with $\alpha = 1$, and the D-gap function $\psi_{\alpha,\beta}$ with $\alpha = 1$ and $\beta = 2$.

d) Find all the mixed strategies Nash equilibria using linear programming.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – February 21, 2017

1. Consider the following optimization problem:

$$\begin{cases} \min \ -x_1^2 - x_2^2 + 10x_1 + 6x_2 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \\ x_1 + x_2 \leq 12 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(3, 0)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.

2. Consider the following optimization problem:

$$\begin{cases} \min \ 2x_1^2 + x_2^2 - x_1x_2 - 2x_1 + x_2 \\ -x_1 \leq 0 \\ -x_2 \leq 0 \\ x_1 + x_2 \leq 12 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Solve the problem by using the active-set method starting from the point $(0, 10)$.
   d) Solve the problem by using the Frank-Wolfe method with exact line search, tolerance 10^{-3} and starting point $(0, 10)$.

3. Consider the following multi-objective optimization problem:

$$\begin{cases} \min \ \big(2x_1 - x_2 \ , \ x_1 + 3x_2\big) \\ -x_1 - 3x_2 \leq 0 \\ 3x_1 + 2x_2 \leq 10 \\ -21x_1 + 7x_2 \leq -10 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(2, 0)$ a weak Pareto minimum? Why?

d) Find all the Pareto minima by using the scalarization method.

e) Find all the weak Pareto minima by using the scalarization method.

f) Find the ideal point.

g) Apply the goal method with $\|\cdot\|_2$.

h) Apply the goal method with $\|\cdot\|_1$.

4. Consider the following matrix game:

$$C = \begin{pmatrix} 9 & 3 & 15 & 4 \\ 5 & 10 & 3 & 6 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/3, 1/3, 1/3, 0)$, evaluate at (x, y) the gap function ψ, the regularized gap function ψ_α with $\alpha = 1$, and the D-gap function $\psi_{\alpha,\beta}$ with $\alpha = 1$ and $\beta = 5$.

d) Find all the mixed strategies Nash equilibria using linear programming.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – June 13, 2017

1. Consider the following constrained optimization problem:

$$\begin{cases} \min \ x_1 + x_2 \\ x_1^2 + x_2^2 - 5 \le 0 \\ x_2^2 - 1 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $(2, 1)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.
   h) Is (1, 0) an optimal solution of the Lagrangian dual problem? Why?

2. Consider the following unconstrained optimization problem:

$$\begin{cases} \min \ 2x_1^2 + x_2^2 + x_3^2 + 2x_4^2 + x_1x_2 + x_1x_4 + 2x_3x_4 - x_1 + 4x_2 - 2x_3 + 4x_4 \\ x \in \mathbb{R}^4 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the gradient method with exact line search starting from the point $(0, 0, 0, 0)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? What is the optimal value? How many iterations are needed?
   e) Solve the problem by means of the gradient method with inexact line search setting $\alpha = 0.5$, $\gamma = 0.9$, $\bar{t} = 1$, starting from the point $(0, 0, 0, 0)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? How many iterations are needed?
   f) Solve the problem by means of the conjugate gradient method starting from the point $(0, 0, 0, 0)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? Write the vector x found at each iteration.

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big(x_1 - 3x_2 + x_3 - 2x_4 \ , \ -2x_1 + 2x_2 - 3x_3 + x_4\big) \\ x_1 + x_3 \leq 5 \\ x_2 + x_4 \leq 6 \\ x_1, x_2, x_3, x_4 \geq 0 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(0, 6, 2, 0)$ a Pareto minimum? Why?

d) Is the point $(0, 3, 5, 3)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\| \cdot \|_2$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 8 & 1 \\ 5 & 2 \\ 9 & 3 \\ 6 & 4 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 9 & 5 \\ 1 & 2 \\ 4 & 1 \\ 3 & 3 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x, y), where $x = (1/2, 1/2)$ and $y = (2/3, 1/3)$, evaluate the gap function ψ at (x, y), the regularized gap function ψ_α, with $\alpha = 1$, at (x, y) and the D-gap function $\psi_{\alpha,\beta}$, with $\alpha = 1$ and $\beta = 5$ at (x, y).

d) Find the best response mapping of each player and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – July 4, 2017

1. Consider the following constrained optimization problem:

$$\begin{cases} \min \ -x_1^2 - 4x_2^2 - 2x_1 + 8x_2 \\ x_1^2 - 2x_1 \leq 0 \\ x_2^2 - 2x_2 \leq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $x = (0, 1)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Find the objective function and constraints of the Lagrangian dual problem.
   h) Is $\lambda = (2, 4)$ an optimal solution of the Lagrangian dual problem? Why?

2. Consider the following constrained optimization problem:

$$\begin{cases} \min \ x_1^2 + 2x_2^2 + 2x_3^2 + x_4^2 - x_1x_2 - x_1x_4 + 2x_2x_3 + 5x_1 + 5x_2 + 5x_3 + 5x_4 \\ x_1 + 2x_2 + 2x_3 + x_4 \leq 10 \\ x \geq 0 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the Frank-Wolfe method with exact line search, tolerance 10^{-6} and starting from the point $(10, 0, 0, 0)$. What is the global minimum? What is the optimal value? How many iterations are needed?
   e) Solve the problem by means of the penalty method with $\tau = 0.5$, $\varepsilon_0 = 1$ and $\min(b - Ax) > -10^{-6}$ as stopping criterion. What is the global minimum? How many iterations are needed?

      Hint: at each iteration use the `fminunc` *function with the following options:*
      `options = optimoptions('fminunc','GradObj','on',...`
      `'Algorithm','quasi-newton','Display','off').`
   f) Solve the problem by means of the logarithmic barrier method with $\tau = 0.5$, $\varepsilon_0 = 1$, tolerance 10^{-6} and starting from the point $(1, 1, 1, 1)$. What is the global minimum? How many iterations are needed?

      Hint: at each iteration use the `fminunc` *function with the same options as in e).*

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \left(x_1^2 + x_2^2 - 6x_1 \ , \ x_1^2 + x_2^2 - 12x_1 - 6x_2\right) \\ 2 \leq x_1 \leq 4 \\ 2 \leq x_2 \leq 4 \end{cases}$$

a) Is it a convex problem? Why?
b) Do Pareto minima exist? Why?
c) Is the point $(3, 2)$ a Pareto minimum? Why?
d) Is the point $(4, 4)$ a weak Pareto minimum? Why?
e) Find a subset of Pareto minima by using the scalarization method.
f) Find the set of all weak Pareto minima by using the scalarization method.
g) Find the set of all Pareto minima.
h) Find the ideal point.

4. Consider the following matrix game:

$$C = \begin{pmatrix} 13 & 15 & 5 & 3 \\ 9 & 6 & 12 & 15 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.
b) Do pure strategies Nash equilibria exist? Why?
c) Given the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/2, 1/2, 0, 0)$, evaluate at (x, y) the gap function ψ, the regularized gap function ψ_α with $\alpha = 1$, and the D-gap function $\psi_{\alpha,\beta}$ with $\alpha = 1$ and $\beta = 2$.
d) Find all the mixed strategies Nash equilibria using linear programming.
e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – July 25, 2017

1. Consider the following constrained optimization problem in $\mathbb{R}^n$:

$$\begin{cases} \min \ \dfrac{1}{2}\sum\limits_{i=1}^{n}(x_i-1)^2 \\ \sum\limits_{i=1}^{n} x_i \le 1 \\ x_i \ge 0 \qquad \forall\ i=1,\dots,n \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Is the point $x=(0,\dots,0)$ a local minimum? Why?
   e) Find all the solutions of the KKT system.
   f) Find local minima and global minima.
   g) Write the Lagrangian dual problem as follows

$$\begin{cases} \max \ \frac{1}{2}\lambda^T Q\lambda + c^T\lambda + d \\ \lambda \ge 0 \end{cases}$$

   What are the matrix Q, the vector c and the scalar d?
   h) Is $\lambda=(0,1,\dots,1)$ an optimal solution of the Lagrangian dual problem? Why?

2. Consider the following unconstrained optimization problem:

$$\begin{cases} \min \ \dfrac{5}{2}x_1^2 + 2x_2^2 + 2x_3^2 + \dfrac{5}{2}x_4^2 + 2x_5^2 - x_1x_4 - 2x_2x_5 + 6x_1 + 6x_2 - 6x_4 - 6x_5 \\ x \in \mathbb{R}^5 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the gradient method with exact line search starting from the point $(1,1,1,1,1)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? What is the optimal value? How many iterations are needed?
   e) Solve the problem by means of the gradient method with inexact line search setting $\alpha=0.5$, $\gamma=0.9$, $\bar{t}=1$, starting from the point $(1,1,1,1,1)$ and using $\|\nabla f(x)\| <$ 10^{-6} as stopping criterion. What is the global minimum? How many iterations are needed?

f) Solve the problem by means of the conjugate gradient method starting from the point $(1,1,1,1,1)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? Write the vector x found at each iteration.

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big(3x_1 + x_2 \ , \ -x_1 - 4x_2\big) \\ 0 \leq x_1 \leq 10 \\ 0 \leq x_2 \leq 10 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(5,10)$ a Pareto minimum? Why?

d) Is the point $(5,0)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_2$.

i) Apply the goal method with $\|\cdot\|_1$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 7 & 1 & 5 \\ 1 & 8 & 2 \\ 9 & 2 & 1 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 9 & 1 & 4 \\ 5 & 8 & 9 \\ 4 & 2 & 3 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x,y), where $x = (1/3, 2/3)$ and $y = (1/2, 1/2)$, evaluate the gap function ψ and the regularized gap function ψ_α, with $\alpha = 1$, at (x,y) in order to check if it is a Nash equilibrium.

d) Plot the polyhedra P and Q related to the game and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – September 12, 2017

1. Consider the following constrained optimization problem:

$$\begin{cases} \min\ -x_1^2 - x_2^2 + 4x_1 + 6x_2 \\ -x_1 \le 0 \\ x_1^2 + x_2^2 - 6x_2 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Find all the solutions of the KKT system.
   e) Find local minima and global minima.
   f) Find the objective function and constraints of the Lagrangian dual problem.
   g) Find the optimal solution of the Lagrangian dual problem.

2. Consider the following constrained optimization problem:

$$\begin{cases} \min\ 3x_1^2 + 2x_2^2 + x_3^2 + 2x_4^2 - x_1x_4 - x_2x_3 - 3x_2x_4 - x_1 - 2x_2 + 3x_3 + 4x_4 \\ 4x_1 + 3x_2 + 2x_3 + x_4 \le 20 \\ 2x_1 + 3x_2 + 4x_3 + 5x_4 \le 40 \\ x \ge 0 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the Frank-Wolfe method with exact line search, tolerance 10^{-6} and starting from the point $(1, 2, 1, 2)$. What is the global minimum? What is the optimal value? How many iterations are needed?
   e) Solve the problem by means of the penalty method with $\tau = 0.5$, $\varepsilon_0 = 1$ and $\min(b - Ax) > -10^{-6}$ as stopping criterion. What is the global minimum? How many iterations are needed?
      Hint: at each iteration use the `fminunc` *function with the following options:*
      ```
      options = optimoptions('fminunc','GradObj','on',...
      'Algorithm','quasi-newton','Display','off').
      ```
   f) Solve the problem by means of the logarithmic barrier method with $\tau = 0.5$, $\varepsilon_0 = 1$, tolerance 10^{-6} and starting from the point $(1, 2, 1, 2)$. What is the global minimum? How many iterations are needed?
      Hint: at each iteration use the `fminunc` *function with the same options as in e).*

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \big((x_1-2)^2+(x_2-7)^2 \ , \ -x_1+x_2\big) \\ 0 \le x_1 \le 5 \\ 0 \le x_2 \le 5 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(4,5)$ a solution of the KKT system? Why?

d) Is the point $(5,5)$ a weak Pareto minimum? Why?

e) Find the set of all weak Pareto minima by using the scalarization method.

f) Find the set of all Pareto minima.

g) Find the ideal point.

4. Consider the following matrix game:

$$C = \begin{pmatrix} 9 & 13 & 14 \\ 12 & 3 & 2 \\ 4 & 10 & 7 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x,y), where $x=(1/2,1/2,0)$ and $y=(1/2,1/3,1/6)$, evaluate at (x,y) the gap function ψ, the regularized gap function ψ_α with $\alpha=1$, and the D-gap function $\psi_{\alpha,\beta}$ with $\alpha=1$ and $\beta=2$.

d) Find all the mixed strategies Nash equilibria using linear programming.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – February 20, 2018

1. Consider the following constrained optimization problem:

$$\begin{cases} \min\ x_1 + x_2 \\ x_1^2 + x_2^2 - 8x_1 - 8x_2 + 31 \le 0 \\ 4 - x_2 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Find all the solutions of the KKT system.
   e) Find local minima and global minima.
   f) Find the objective function and constraints of the Lagrangian dual problem.
   g) Find the optimal solution of the Lagrangian dual problem.

2. Consider the following unconstrained optimization problem:

$$\begin{cases} \min\ 2x_1^4 + x_2^4 + x_3^4 + x_1^2 + x_2^2 + x_3^2 + x_1x_2 + x_2x_3 - 5x_1 - 3x_2 - x_3 \\ x \in \mathbb{R}^3 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the gradient method with inexact line search setting $\alpha = 0.1$, $\gamma = 0.9$, $\bar{t} = 1$, starting from the point $(10, 10, 10)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? How many iterations are needed?
   e) Solve the problem by means of the Newton method with inexact line search setting $\alpha = 0.1$, $\gamma = 0.9$, $\bar{t} = 1$, starting from the point $(10, 10, 10)$ and using $\|\nabla f(x)\| < 10^{-6}$ as stopping criterion. What is the global minimum? How many iterations are needed?

3. Consider the following multi-objective problem:

$$\begin{cases} \min \ \left(x_1 + 3x_2 \ , \ -2x_1 - x_2\right) \\ x_1 - 2x_2 \leq -5 \\ x_2 \leq 2 \\ -x_1 - 3x_2 \leq 0 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(-3, 2)$ a Pareto minimum? Why?

d) Is the point $(-3, 1)$ a weak Pareto minimum? Why?

e) Find all the Pareto minima by using the scalarization method.

f) Find all the weak Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_1$.

i) Apply the goal method with $\|\cdot\|_2$.

4. Consider the following bimatrix game, where the cost matrices are

$$C_1 = \begin{pmatrix} 6 & 3 & 2 \\ 1 & 4 & 3 \\ 3 & 5 & 4 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 9 & 1 & 3 \\ 1 & 2 & 4 \\ 6 & 3 & 5 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x, y), where $x = (1/3, 2/3)$ and $y = (1/3, 2/3)$, evaluate the gap function ψ and the regularized gap function ψ_α, with $\alpha = 1$, at (x, y) in order to check if it is a Nash equilibrium.

d) Plot the polyhedra P and Q related to the game and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.

Exam of Optimization Methods – April 23, 2018

1. Consider the following constrained optimization problem:

$$\begin{cases} \min\ -x_1^2 + 4x_1 + x_2 \\ x_1^2 + x_2^2 - 4x_1 \le 0 \\ -x_2 \le 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Find all the solutions of the KKT system.
   e) Find local minima and global minima.
   f) Find the objective function and constraints of the Lagrangian dual problem.

2. Consider the following constrained optimization problem:

$$\begin{cases} \min\ 2x_1^2 + 3x_2^2 + 3x_3^2 + 2x_4^2 - x_1x_2 - x_1x_4 + 2x_2x_3 + x_3x_4 - 2x_1 - 3x_2 + 4x_3 + 2x_4 \\ 2x_1 + x_2 + 2x_3 + 3x_4 \le 20 \\ x \ge 0 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the Frank-Wolfe method with exact line search, tolerance 10^{-4} and starting from the point $(2, 2, 2, 2)$. What is the global minimum? What is the optimal value? How many iterations are needed?
   e) Solve the problem by means of the penalty method with $\tau = 0.5$, $\varepsilon_0 = 1$ and $\min(b - Ax) > -10^{-4}$ as stopping criterion. What is the global minimum? How many iterations are needed?
   Hint: at each iteration use the `fminunc` *function with the following options:*
   `options = optimoptions('fminunc','GradObj','on',...`
   `'Algorithm','quasi-newton','Display','off').`
   f) Solve the problem by means of the logarithmic barrier method with $\tau = 0.5$, $\varepsilon_0 = 1$, tolerance 10^{-4} and starting from the point $(1, 1, 1, 1)$. What is the global minimum? How many iterations are needed?
   Hint: at each iteration use the `fminunc` *function with the same options as in e).*

3. Consider the following multi-objective problem:

$$\begin{cases} \min\ (3\,x_1 + 2\,x_2\ ,\ -x_1 - 2\,x_2) \\ x_1 + 2\,x_2 \leq 0 \\ -x_1 \leq -1 \\ x_1 - 2\,x_2 \leq 4 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Is the point $(2,-1)$ a Pareto minimum? Why?

d) Is the point $(1,-1)$ a weak Pareto minimum? Why?

e) Find the set of all weak Pareto minima by using the scalarization method.

f) Find the set of all Pareto minima by using the scalarization method.

g) Find the ideal point.

h) Apply the goal method with $\|\cdot\|_2$.

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 1 & 5 \\ 5 & 1 \\ 7 & 3 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 4 & 1 \\ 5 & 6 \\ 2 & 3 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Given the point (x,y), where $x = (1/2, 1/2)$ and $y = (1/3, 2/3)$, evaluate the gap function ψ at (x,y), the regularized gap function ψ_α, with $\alpha = 1$, at (x,y) and the D-gap function $\psi_{\alpha,\beta}$, with $\alpha = 1$ and $\beta = 5$ at (x,y).

d) Find the best response mapping of each player and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.