Exercises on multiobjective optimization problems

Answer the following questions for any of the multiobjective optimization problems reported below:

a) Is it a convex problem? Is it a linear problem?

b) Do Pareto minima exist? Why?

c) Does the Abadie constraint qualification hold in any feasible point?

d) Find a subset of Pareto minima by using the scalarization method. Are there other Pareto minima?

e) Find a subset of Pareto weak minima by using the scalarization method. Are there other Pareto weak minima?

f) Find all the solutions of the KKT system. Are they Pareto weak minima? Are they Pareto minima?

g) Plot the set $f(\Omega)$ (image of the feasible region). Find all the efficient points and weakly efficient points of $f(\Omega)$. Find all the Pareto minima and Pareto weak minima.

1. $$\begin{cases} \min\ (x_2, x_1) \\ x_2 - 4 \leq 0 \\ x_1^2 - x_2 \leq 0 \end{cases}$$

2. $$\begin{cases} \min\ (2\,x_1 - x_2, -x_1) \\ x_2 - 2 \leq 0 \\ x_1^2 - x_2 \leq 0 \end{cases}$$

3. $$\begin{cases} \min\ (2\,x_2 + 1, x_1 - 2) \\ x_2^2 - 9 \leq 0 \\ x_1^2 - x_2^2 - 1 \leq 0 \end{cases}$$

4. $$\begin{cases} \min\ (x_2 - 1, x_1 + 3) \\ x_1\,x_2 \geq 1 \\ 0 \leq x_1 \leq 5 \\ 0 \leq x_2 \leq 4 \end{cases}$$

5. $$\begin{cases} \min\ (x_2^2 - 1, x_1) \\ x_1\,x_2 \geq 4 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$$

6. $\begin{cases} \min\ (x_1 + x_2, -x_1 + x_2) \\ -x_1 + x_2 \leq 0 \\ x_1 + x_2 \leq 0 \\ -x_1 - x_2 \leq 2 \\ -2\,x_2 \leq 3 \\ x_1 - x_2 \leq 2 \end{cases}$

7. $\begin{cases} \min\ (1 - 2\,x_2, x_2 - x_1) \\ x_1^2 - 25 \leq 0 \\ x_1 + x_2 - 7 \leq 0 \end{cases}$

8. $\begin{cases} \min\ (x_2 + 1, 5 - x_1) \\ (x_1 + x_2)^2 - 25 \leq 0 \\ 1 - x_1\,x_2 \leq 0 \\ x_1 \geq 0 \\ x_2 \geq 0 \end{cases}$

9. $\begin{cases} \min\ (x_1, x_2) \\ x_2 - \sqrt{-x_1} \geq 0 \\ x_1 \leq 0 \end{cases}$

10. $\begin{cases} \min\ (x_1 + 1, 2\,x_2 - 3) \\ -x_2 - x_1^3 \leq 0 \\ x_1 \leq 0 \\ x_2 - 3 \leq 0 \end{cases}$

11. $\begin{cases} \min\ (-x_1 + 2\,x_2, x_1 - 1) \\ x_2 - 2\,x_1 + 2 \geq 0 \\ x_1^2 + x_2 \leq 0 \end{cases}$

12. $\begin{cases} \min\ (2 - x_1, 3 - x_2) \\ (x_1 - 2)^2 + (x_2 - 2)^2 - 1 \geq 0 \\ x_1^2 + x_2^2 - 36 \leq 0 \end{cases}$

13. $\begin{cases} \min\ (x_1 - x_2, x_1 + x_2) \\ -3\,x_1 + x_2 \leq 2 \\ -x_1 - x_2 \leq 0 \\ x_1 + x_2 \leq 2 \\ x_1 - 3\,x_2 \leq 2 \end{cases}$