Exercises on Noncooperative game theory

Answer the following questions for any of the matrix games reported below:

a) Find strictly dominated strategies (if any) and reduce the cost matrix accordingly.

b) Do pure strategies Nash equilibria exist?

c) Find all the mixed strategies Nash equilibria of the game by solving the related linear programming problems.

1. $C = \begin{pmatrix} 6 & 9 & 1 & 4 \\ 3 & 4 & 12 & 7 \end{pmatrix}$

2. $C = \begin{pmatrix} 1 & 5 & 11 & 9 \\ 10 & 9 & 8 & 7 \end{pmatrix}$

3. $C = \begin{pmatrix} 7 & 15 & 2 & 3 \\ 4 & 2 & 3 & 10 \\ 5 & 3 & 4 & 12 \end{pmatrix}$

4. $C = \begin{pmatrix} 5 & 2 & 11 & 15 \\ 1 & 13 & 5 & 1 \end{pmatrix}$

5. $C = \begin{pmatrix} 3 & 10 & 3 & 8 \\ 13 & 6 & 7 & 2 \end{pmatrix}$

6. $C = \begin{pmatrix} 10 & 7 & 12 & 10 \\ 7 & 10 & 6 & 7 \end{pmatrix}$

Answer the following questions for any of the bimatrix games reported below:

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist?

c) Find all the mixed strategies Nash equilibria of the game by solving the related KKT system.

d) Plot the polyhedra P and Q related to the game and find all the mixed strategies Nash equilibria.

e) Find the best response mapping of each player and find all the mixed strategies Nash equilibria.

1. $C_1 = \begin{pmatrix} 8 & 1 & 3 \\ 6 & 3 & 1 \\ 5 & 2 & 0 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 9 & 5 & 6 \\ 3 & 7 & 8 \\ 1 & 2 & 3 \end{pmatrix}$

2. $C_1 = \begin{pmatrix} 3 & 1 \\ 1 & 9 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 7 & 10 \\ 4 & 1 \end{pmatrix}$

3. $C_1 = \begin{pmatrix} 2 & 2 & 6 \\ 1 & 4 & 2 \\ 3 & 5 & 3 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 3 & 4 & 5 \\ 9 & 1 & 2 \\ 2 & 3 & 1 \end{pmatrix}$

4. $C_1 = \begin{pmatrix} 1 & 6 \\ 8 & 3 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 1 & 4 \\ 4 & 1 \end{pmatrix}$

5. $C_1 = \begin{pmatrix} 4 & 9 & 3 \\ 2 & 7 & 1 \\ 5 & 1 & 8 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 3 & 6 & 2 \\ 2 & 9 & 1 \\ 9 & 5 & 8 \end{pmatrix}$

6. $C_1 = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ $\quad C_2 = \begin{pmatrix} 1 & 3 \\ 4 & 3 \end{pmatrix}$

Answer the following questions for any of the convex games reported below:

a) Find all the Nash equilibria by solving the related KKT system.

b) Find the best response mapping of each player and find all the Nash equilibria.

1. Player 1: $\begin{cases} \min\limits_{x} \; y_1 (x_1 - 2)^2 + y_2 (x_2 - 2)^2 \\ 0 \le x_1 \le 3 \\ 1 \le x_2 \le 4 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \; x_1 \left(y - \dfrac{1}{2} \right)^2 + y_2 \\ y_1 \ge 0 \\ 0 \le y_2 \le 1/2 \end{cases}$

2. Player 1: $\begin{cases} \min\limits_{x} \; y_1 (x + 2)^2 + y_2 \, x \\ x \ge 0 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \; (y_1 + 3)^2 + x (y_2 + 4)^2 \\ y_1 \ge 0 \\ y_2 \ge 0 \end{cases}$

3. Player 1: $\begin{cases} \min\limits_{x} \; y (x - 2)^2 \\ 0 \le x \le 1 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \; \dfrac{1}{2} y^2 + 2 \, x \, y - 2 \, y \\ 0 \le y \le 1 \end{cases}$

4. Player 1: $\begin{cases} \min\limits_{x} \; 2 \, x^2 - x \, y + 3 \, x \\ x \ge 0 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \; y^2 + y - 3 \, x \, y \\ y \ge 0 \end{cases}$

5. Player 1: $\begin{cases} \min\limits_{x} \; 2 \, x_1^2 + \dfrac{5}{2} x_2^2 + 2 \, x_1 \, x_2 + 2 \, x_1 \, y + 4 \, x_2 \, y - 14 \, x_2 \\ x_1 \ge 0 \\ x_2 \ge 0 \end{cases}$

   Player 2: $\begin{cases} \min\limits_{y} \; 5 \, y^2 + 6 \, x_1 \, y + 8 \, x_2 \, y - 26 \, y \\ y \ge 0 \end{cases}$

6. Player 1: $\begin{cases} \min\limits_{x} \; 2 \, x^2 + 3 \, x \, y_1 + 2 \, x \, y_2 + 2 \, x \\ x \ge 0 \end{cases}$

   Player 2: $\begin{cases} \min\limits_{y} \; \dfrac{5}{2} y_1^2 + 5 \, y_2^2 + 6 \, y_1 \, y_2 + x \, y_1 + 6 \, x \, y_2 - 9 \, y_1 + 8 \, y_2 \\ y_2 \ge 0 \end{cases}$

7. Player 1: $\begin{cases} \min\limits_{x} \; y_1 (x + 2)^2 + x \, y_2 \\ x \ge 0 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \; (y_1 + 3)^2 + x (y_2 + 4)^2 \\ y_1 \ge 0 \\ y_2 \ge 0 \end{cases}$

8. Player 1: $\begin{cases} \min\limits_{x} (x+7)(2-y) \\ -2 \leq x \leq 5 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} \dfrac{1}{2} y^2 - y\,(x_1^2 - 4\,x_1) \\ -3 \leq y \leq 5 \end{cases}$

9. Player 1: $\begin{cases} \min\limits_{x} x^2 - x\,(4\,y+6) \\ -4 \leq x \leq 6 \end{cases}$ Player 2: $\begin{cases} \min\limits_{y} (x+3)(4-y) \\ -5 \leq y \leq 5 \end{cases}$