Exam of Optimization Methods – July 22, 2019

1. Consider the following constrained optimization problem:

$$\begin{cases} \min\ -x_1^2 - x_2^2 \\ -4x_1^2 - x_2^2 + 4 \leq 0 \\ 4x_1^2 + x_2^2 - 16 \leq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?

   b) Is it a convex problem? Why?

   c) Does the Abadie constraints qualification hold in any feasible point? Why?

   d) Find all the solutions of the KKT system.

   e) Find all local minima and global minima.

   f) Write and solve the Lagrangian dual problem.

2. Consider the following constrained optimization problem:

$$\begin{cases} \min\ 2x_1^2 + 3x_2^2 + 4x_3^2 + x_1x_2 + x_1x_3 + x_2x_3 - 16x_1 - 18x_2 - 20x_3 \\ x_1 + 2x_2 + 3x_3 \leq 10 \\ x \geq 0 \end{cases}$$

   a) Is it a convex problem? Why?

   b) Do global minima exist? Why?

   c) Is the global minimum unique? Why?

   d) Solve the problem by using the active-set method starting from the point (2, 4, 0). What is the global minimum? How many iterations are needed?

   e) Solve the problem by means of the logarithmic barrier method with $\tau = 0.1$, $\varepsilon_0 = 1$, tolerance 10^{-4} and starting from the point $(1, 1, 1)$. What is the global minimum? How many iterations are needed?

   Hint: at each iteration use the `fminunc` *function with the following options:*

```
options = optimoptions('fminunc','GradObj','on',...
'Algorithm','quasi-newton','Display','off').
```

3. Consider the following multiobjective problem:

$$\begin{cases} \min \ \left(x_1^2 + 2x_2^2 - 2x_1 + 2x_2 \ , \ 2x_1^2 + x_2^2 + 2x_1 - 2x_2\right) \\ 0 \leq x_1 \leq 2 \\ 0 \leq x_2 \leq 2 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Find all weak Pareto minima by using the scalarization method.

d) Find all Pareto minima by using the scalarization method.

e) Find the KKT multipliers related to the Pareto minimum $(0, 0)$.

f) Find the ideal point.

4. Consider the following two-person noncooperative game:

$$\begin{cases} \min\limits_{x} \ \dfrac{1}{4}x^2 + (2y-1)x + 4 - 3y \\ 0 \leq x \leq 1 \end{cases} \qquad \begin{cases} \min\limits_{y} \ \dfrac{1}{4}y^2 + (3x-1)y + 2 - x \\ 0 \leq y \leq 1 \end{cases}$$

a) Do Nash equilibria exist? Why?

b) Is it a convex game? Why?

c) Find the best response mapping of each player.

d) Find all Nash equilibria of the game.

e) Find the KKT multipliers related to the Nash equilibria.