Exam of Optimization Methods – July 1, 2019

1. Consider the following constrained optimization problem:

$$\begin{cases} \min \ x_1^2 + x_2^2 - 4x_1 + 2x_2 \\ -x_2 \leq 0 \\ x_1 + x_2 - 1 \leq 0 \end{cases}$$

   a) Do global optimal solutions exist? Why?
   b) Is it a convex problem? Why?
   c) Does the Abadie constraints qualification hold in any feasible point? Why?
   d) Find all the solutions of the KKT system.
   e) Find all local minima and global minima.
   f) Write and solve the Lagrangian dual problem.

2. Consider the following constrained optimization problem:

$$\begin{cases} \min \ 4x_1^2 + 2x_2^2 + x_3^2 + x_1x_2 + x_1x_3 + 2x_2x_3 + 3x_1 - 3x_2 + 3x_3 \\ x_1 + 2x_2 + x_3 \leq 10 \\ x \geq 0 \end{cases}$$

   a) Is it a convex problem? Why?
   b) Do global minima exist? Why?
   c) Is the global minimum unique? Why?
   d) Solve the problem by means of the Frank-Wolfe method with exact line search, tolerance 10^{-6} and starting from the point $(0, 0, 10)$. What is the global minimum? How many iterations are needed?
   e) Solve the problem by means of the logarithmic barrier method with $\tau = 0.1$, $\varepsilon_0 = 1$, tolerance 10^{-4} and starting from the point $(1, 1, 1)$. What is the global minimum? How many iterations are needed?

      Hint: at each iteration use the `fminunc` *function with the following options:*

```
options = optimoptions('fminunc','GradObj','on',...
'Algorithm','quasi-newton','Display','off').
```

3. Consider the following multiobjective problem:

$$\begin{cases} \min \ \left(x_1 + 2x_2 \ , \ -x_1 - 3x_2\right) \\ -x_1 - 2x_2 \leq -1 \\ x_1 \leq 9 \\ x_1 + 4x_2 \leq -3 \end{cases}$$

a) Is it a convex problem? Why?

b) Do Pareto minima exist? Why?

c) Find all Pareto minima by using the scalarization method.

d) Find all weak Pareto minima by using the scalarization method.

e) Find the ideal point.

f) Apply the goal method with $\|\cdot\|_1$.

g) Apply the goal method with $\|\cdot\|_\infty$. Is the found point a Pareto minimum? Why?

4. Consider the following bimatrix game:

$$C_1 = \begin{pmatrix} 2 & 6 & 1 \\ 10 & 5 & 4 \end{pmatrix} \qquad C_2 = \begin{pmatrix} 4 & 1 & 3 \\ 6 & 7 & 5 \end{pmatrix}$$

a) Find strictly dominated strategies (if any) and reduce the cost matrices accordingly.

b) Do pure strategies Nash equilibria exist? Why?

c) Evaluate the D-gap function $\psi_{\alpha,\beta}$ at the point (x, y), where $\alpha = 1$, $\beta = 5$, $x = (3/4, 1/4)$ $y = (1/3, 2/3)$, in order to check if it is a Nash equilibrium.

d) Find the best response mapping of each player and find all the mixed strategies Nash equilibria.

e) Find the KKT multipliers related to the mixed strategies Nash equilibrium.