Spatial P Systems

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Motivation: modelling of ecosystems (1)

The adjective computational is becoming widely used in life sciences to qualify disciplines such as biology, ecology, epidemiology, and so on.

In the study of the dynamics of systems it often holds:

Computational = Mathematical models + simulation.

Notations and analisys techniques of theoretical computer science have found application in systems biology

We believe that they could be applied fruitfully also in population biology and ecology:

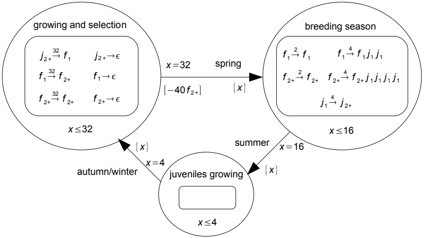
- reintroduction of species
- effects of climate changes
- spread of diseases
- evolution and genetics

Motivation: modelling of ecosystems (2)

Models of ecosystems based on P systems have been developed in Sevilla

- Bearded vultures at the Catalonian-Pyreneean area
- Zebra mussels at Ribarroja reservoir (Zaragoza)

In the past, we developed timed P automata to describe periodical changes in environmental conditions (seasons, periodical hunt/harvest)



Now, we want to study spatial aspects of population dynamics

Spatial P systems

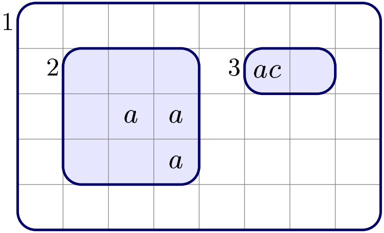
We have extended P systems with spatial features

- membranes are embedded in a two-dimensional discrete space
- objects are associated with positions in membranes
- rules are extended to specify the resulting position of objects
- two disjoint sets of objects:
  - V *ordinary objects*, representing object of negligible size
  - E *mutually-exclusive* objects, representing "bigger" objects
    - at most one is allowed in each position

We have investigated the universality of spatial P systems with only *non-cooperative* rules

We have modelled a simple example of ecosystem in which spatiality matters.

Spatial P system: example



Membrane 1 contains 2 and 3

- membrane 1: size is 8×5, all positions are empty
- membrane 2: size is 3×3, position is $(1, 1)$ w.r.t membrane 1
  - a copy of object *a* is contained in each position: $(1, 1)$, $(2, 1)$, $(2, 0)$
- membrane 3: size is 2×1, position is $(5, 3)$ w.r.t membrane 1
  - objects *ac* are contained in position $(0, 0)$

Note that objects *a* and *c* cannot be both mutually exclusive objects!

Evolution rules

As usual, evolution rules are associated with membranes

A rule $u \rightarrow v$ can be applied only to objects u that are all at the same position

In a rule $u \rightarrow v$, we have that v is a multiset of messages such as:

- $a_{\delta p}$, with $\delta p \in \mathbb{Z}^2$
  - object a has to be added to pos. $p + \delta p$ relative to position p of u
- a_{out}
  - object a has to be sent out of the membrane
- a_{in_l}
  - object a has to be sent into the child membrane l

Example: $ac \rightarrow a_{(0,0)}\ c_{(1,0)}\ e_{out}$

Let $N = (0,1)$, $S = (0,-1)$, $W = (-1,0)$ and $E = (0,1)$.

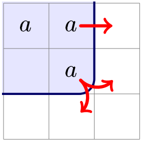
On the applicability of evolution rules

An evolution rule $u \rightarrow v$ can be applied to objects at position p only if:

- for each $a_{\delta p}$ in v
  - $p + \delta p$ does not exceed the bounds of the membrane
  - $p + \delta p$ is not a position occupied by a inner membrane
- in case a_{out} is in v
  - p must be adjacent (from inside) to the border of the membrane
- in case a_{in_l} is in v
  - p must be adjacent (from outside) to the border of child membrane l

When an evolution rule $u \rightarrow v$ is applied:

- objects to be sent out (or into a child membrane) are moved to the closest position of the parent (or child) membrane



Maximal parallelism and mutually exclusive objects

Similarly to standard P systems, rules are applied with maximal parallellelism.

Two mutually exclusive objects cannot be at the same position at the same time, hence:

- if there are several rules willing to place mutually exclusive objects at the same position, only one of them is applied
- a rule willing to place a mutually exclusive object at a position already containing one of such objects can be applied only if at the same time a rule is applied which removes the already present object

Spatial P systems without mutually-exclusive objects

Spatial P systems

- with only non-cooperative rules, and
- without mutually-exclusive objects

are not universal.

Theorem

$PsSP_*(ncoo, nme) \subseteq PsP_*(ncoo)$

Proof.

Spatial P systems without mutually-exclusive objects can be translated into standard P systems without adding any cooperative rule. □

Universality of spatial P systems

If mutually-exclusive objects are allowed, then spatial P systems are universal even when using only non-cooperative rules

Theorem

$PsSP_1(ncoo, me) = PsRE$

Proof.

Simulation of matrix grammars with appearence checking. □

An idea of the proof of universality (1)

Matrix Grammar:

$m_0 : S \rightarrow X_{init} A_{init}$

$\ldots$

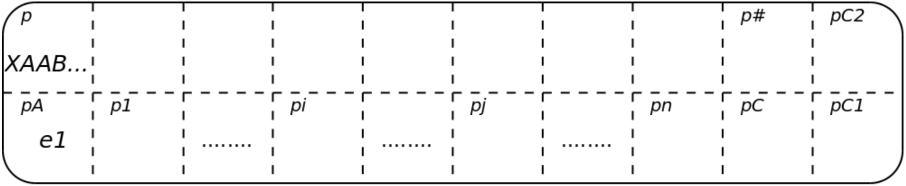
$m_i : (X \rightarrow Y \; ; \; A \rightarrow x)$

$\ldots$

$mj : (Y \rightarrow Z \; ; \; C \rightarrow \#)$

$\ldots$

$m_n : \ldots$



An idea of the proof of universality (1)

Matrix Grammar:

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$m_i : (X \rightarrow Y \; ; \; A \rightarrow x)$

$\ldots$

$mj : (Y \rightarrow Z \; ; \; C \rightarrow \#)$

$\ldots$

$m_n : \ldots$

Spatial P system rules:

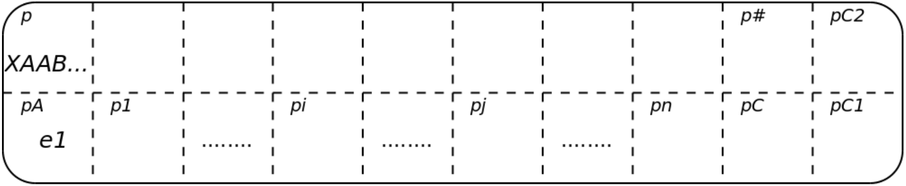
$e3 \rightarrow e2$

$e2 \rightarrow e1$

$e1 \rightarrow e0$

$e0 \rightarrow \lambda$

$\# \rightarrow \#$



An idea of the proof of universality (2)

Matrix Grammar:

$m_0 : S \rightarrow X_{init} A_{init}$

$\ldots$

$m_i : (X \rightarrow Y \ ; \ A \rightarrow x)$

$\ldots$

$mj : (Y \rightarrow Z \ ; \ C \rightarrow \#)$

$\ldots$

$m_n : \ldots$

Spatial P system rules:

$X \rightarrow Y \ \{e3_{pk} \mid k \neq i\} \ c1_{pC} \ e2_{pC1}$

$A \rightarrow x \ e3_{pA} \ e2_{pi} \ c2_{pC} \ e1_{pC2}$

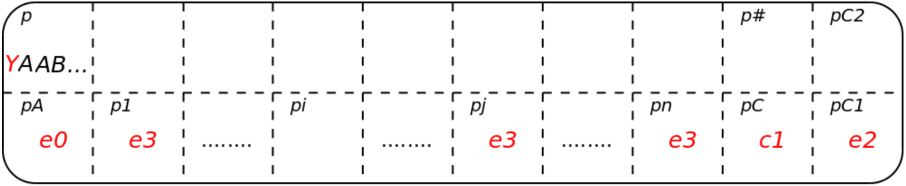
$c1 \rightarrow c1'$

$\{c1' \rightarrow \#e0_{pk} \mid 1 \leq k \leq n\}$

$\{c2 \rightarrow \#e0_{pk} \mid 1 \leq k \leq n\}$

$c1' \rightarrow e0_{pC1}$

$c2 \rightarrow e0_{pC2}$



An idea of the proof of universality (3)

Matrix Grammar:

$m_0 : S \to X_{init} A_{init}$

$\ldots$

$m_i : (X \to Y \; ; \; A \to x)$

$\ldots$

$mj : (Y \to Z \; ; \; C \to \#)$

$\ldots$

$m_n : \ldots$

Spatial P system rules:

$X \to Y \, \{e3_{pk} \mid k \neq i\} \, c1_{pC} \, e2_{pC1}$

$A \to x \, e3_{pA} \, e2_{pi} \, c2_{pC} \, e1_{pC2}$

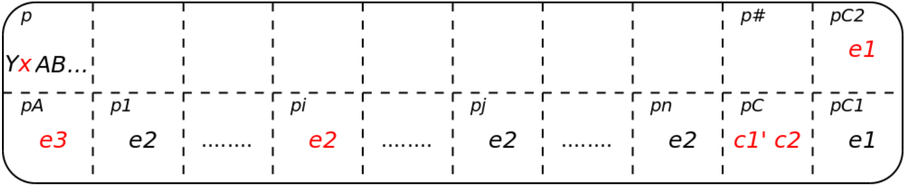
$c1 \to c1'$

$\{c1' \to \#e0_{pk} \mid 1 \leq k \leq n\}$

$\{c2 \to \#e0_{pk} \mid 1 \leq k \leq n\}$

$c1' \to e0_{pC1}$

$c2 \to e0_{pC2}$



An idea of the proof of universality (4)

Matrix Grammar:

$m_0 : S \rightarrow X_{init} A_{init}$

$\ldots$

$m_i : (X \rightarrow Y \; ; \; A \rightarrow x)$

$\ldots$

$mj : (Y \rightarrow Z \; ; \; C \rightarrow \#)$

$\ldots$

$m_n : \ldots$

Spatial P system rules:

$X \rightarrow Y \, \{e3_{pk} \mid k \neq i\} \, c1_{pC} \, e2_{pC1}$

$A \rightarrow x \, e3_{pA} \, e2_{pi} \, c2_{pC} \, e1_{pC2}$

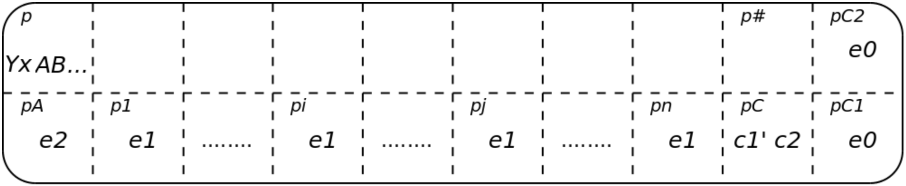
$c1 \rightarrow c1'$

$\{c1' \rightarrow \# e0_{pk} \mid 1 \leq k \leq n\}$

$\{c2 \rightarrow \# e0_{pk} \mid 1 \leq k \leq n\}$

$c1' \rightarrow e0_{pC1}$

$c2 \rightarrow e0_{pC2}$



An idea of the proof of universality (5)

Matrix Grammar:

$m_0 : S \rightarrow X_{init} A_{init}$

$\ldots$

$m_i : (X \rightarrow Y \ ; \ A \rightarrow x)$

$\ldots$

$mj : (Y \rightarrow Z \ ; \ C \rightarrow \#)$

$\ldots$

$m_n : \ldots$

Spatial P system rules:

$X \rightarrow Y \ \{e3_{pk} \mid k \neq i\} \ c1_{pC} \ e2_{pC1}$

$A \rightarrow x \ e3_{pA} \ e2_{pi} \ c2_{pC} \ e1_{pC2}$

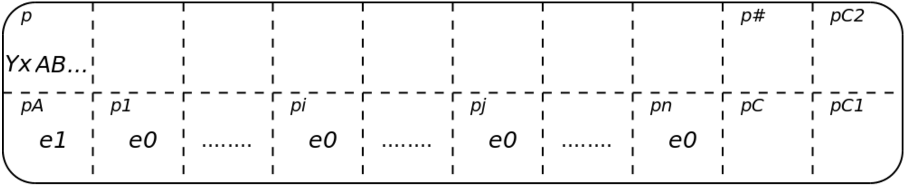
$c1 \rightarrow c1'$

$\{c1' \rightarrow \#e0_{pk} \mid 1 \leq k \leq n\}$

$\{c2 \rightarrow \#e0_{pk} \mid 1 \leq k \leq n\}$

$c1' \rightarrow e0_{pC1}$

$c2 \rightarrow e0_{pC2}$



An idea of the proof of universality (5)

Matrix Grammar:

$m_0 : S \rightarrow X_{init} A_{init}$

$\dots$

$m_i : (X \rightarrow Y \; ; \; A \rightarrow x)$

$\dots$

$mj : (Y \rightarrow Z \; ; \; C \rightarrow \#)$

$\dots$

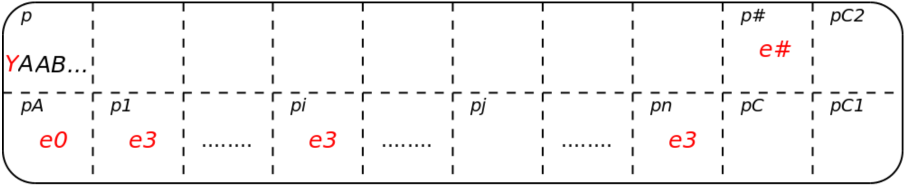
$m_n : \dots$

Spatial P system rules:

$Y \rightarrow Z \, \{e3_{pk} \mid k \neq j\} \, e\#_{p\#}$

$C \rightarrow \# \, e0_{pA} \, e0_{pj}$

$e\# \rightarrow e3_{pA}$



Open problem

At the beginning, we wanted to prove universality with only rules of the form

$$u \longrightarrow v$$

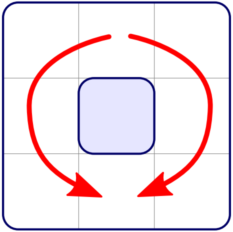
where $a_{\delta p} \in v$ implies $\delta p \in \{(0,0), N, S, W, E\}$.

This problem is still open.

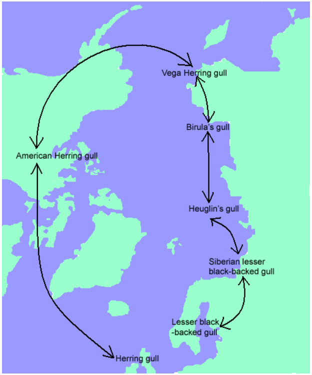
Example: ring species

Ring species is a species whose population expanded along two pathways around a geographycal barrier

- in each pathway, the genotype of populations gradually changed
- intermediate contiguous forms are similar enough to interbreed
- however final forms, along the two pathways, are too much different to interbreed



A ring species: Larus gulls



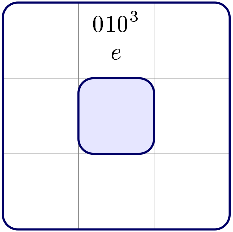
(pictures from Wikipedia)

Example: ring species

Each population is represented by its genotype as a string $xyz \in \{0,1\}^3$

- the colonization of a new space results in a small change in population genotype
- a colonized position is represented with a mutually-exclusive object e
- two populations can interbreed only if their genotypes differ in at most one position

Initial configuration



Evolution rules (1)

For the sake of simplicity we consider an additional form for evolution rules:

$$u_1 - u_2 \longrightarrow v_1 - v_2$$

where u_1 and u_2 are multisets of objects, and v_1, v_2 are multisets of messages (objects + target indications).

A rule $u_1 - u_2 \longrightarrow v_1 - v_2$ denotes a simultaneous application of rules

$$u_1 \longrightarrow v_1$$

$$u_2 \longrightarrow v_2$$

to two adjacent positions inside the membrane

Evolution rules (2)

(1) $xyz \rightarrow xyz$ (2) $xyz \rightarrow \lambda$ (3) $xyz^2 \rightarrow xyz^3$

(4) $xyz - xyz \rightarrow xyz^2 - xyz$ (5) $xyz - \overline{x}yz \rightarrow xyz^2 - \overline{x}yz$

(6) $xyz - \overline{x}yz \rightarrow xyz - \overline{x}yz^2$ (7) $xyz - x\overline{y}z \rightarrow xyz^2 - x\overline{y}z$

(8) $xyz - x\overline{y}z \rightarrow xyz - x\overline{y}z^2$ (9) $xyz - xy\overline{z} \rightarrow xyz^2 - xy\overline{z}$

(10) $xyz - xy\overline{z} \rightarrow xyz - xy\overline{z}^2$

(11) $xyz \rightarrow xyz_d\ e_d$

Rules 11-14 model colonization

(12) $xyz \rightarrow \overline{x}yz_d\ e_d$

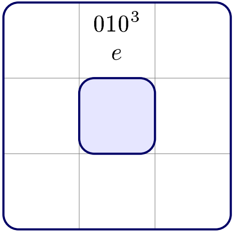
(13) $xyz \rightarrow x\overline{y}z_d\ e_d$ (14) $xyz \rightarrow xy\overline{z}_d\ e_d$

where

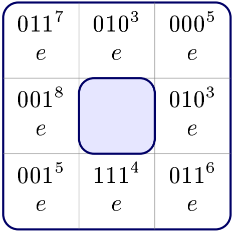
- symbol $\overline{x}$ represents the complement of x
- d denotes a direction $d \in \{N, S, E, W\}$

Example

Initial configuration



A possible final configuration



Populations with genotype 001 and 111 cannot interbreed anymore

Reference

R. Barbuti, A. Maggiolo-Schettini, P. Milazzo, G. Pardini, L. Tesei. *Spatial P systems*. Natural Computing, to appear.