**Premise.** This homework is optional but warmly advised to check the understanding of the student lecture-by-lecture. It should be used to improve the student preparation during the course. All the answers must be properly and clearly explained.

**Exercise 1.** Given a farm parallelism form with emitter, workers and collector modules (at any level), provide a formal proof that the cost model of the ideal service time of the parallelization is the following: \( T_{id-farm} = \max\{T_{id-E}, T_{id-C}, T_{id-W}/n\} \) where \( n \geq 1 \) is the number of worker modules.

**Exercise 2.** Given a map parallelism form with a sequential distributor module (scatter or multicast), workers and gather modules (at any level), provide a formal proof that the cost model of the ideal service time of the parallelization is the following: \( T_{id-map} = \max\{T_{id-D}, T_{id-G}, T_{id-W}\} \).

**Exercise 3.** Given the acyclic computation graph with a single source in the figure:

![Acyclic Computation Graph]

Provide the performance analysis of the graph by finding the values of:
- the effective service time, utilization factor and relative efficiency of each module;
- the ideal service time, effective service time and efficiency of the system;
- the processing latency per stream element and the completion time of the whole system by assuming a stream length of \( M = 10^4 \) input items.

**Exercise 4.** A process \( P_2 \) receives a stream of pairs of floating point numbers \((x_i, y_i)\) and for each pair executes the following computation, where \( s \) is an initialized floating point variable: \( \forall \) input \((x, y)\) : \( z = (r = F_1(x, y); s = F_2(y, s); t = F_3(y, s); z = F_4(r, t);) \). The process receives the stream from a process \( P_1 \) with ideal service time \( 4.5 \cdot 10^2 \tau \). Assume that:
- the calculation times of the four functions are: \( T_{F_1} = 200 \tau, T_{F_2} = 300 \tau, T_{F_3} = 150 \tau, T_{F_4} = 400 \tau \);
- we can neglect the communication latency, i.e. \( L_{com} \approx 0 \).

Determine whether \( P_2 \) is a bottleneck and in case study a data-flow parallelization by discussing the achieved effective service time, relative efficiency and processing latency. Although the communication latency can be neglected in this problem, discuss how the data-flow graph is interfaced with the pre-existing process \( P_1 \), i.e. which kinds of collective communications \( P_1 \) executes to forward the necessary operands to the processes of the data-flow graph.