Software Validation and Verification
Fifth Exercise Sheet – Computation Tree Logic

Exercise 1

Prove or disprove the following implications:

(a) Let $\Phi_1 = \forall\Diamond a \lor \forall\Diamond b$ and $\Phi_2 = \forall\Diamond(a \lor b)$.
Prove or disprove the following implications: $\Phi_1 \Longrightarrow \Phi_2$ and $\Phi_2 \Longrightarrow \Phi_1$.

(b) Now consider $\Psi_1 = \exists(a \mathsf{U} \exists(b \mathsf{U} c))$ and $\Psi_2 = \exists(\exists(a \mathsf{U} b) \mathsf{U} c)$.
Again, prove or disprove $\Psi_1 \Longrightarrow \Psi_2$ and $\Psi_2 \Longrightarrow \Psi_1$.

Exercise 2

Transform the CTL-formula $\Phi = \neg\forall\Diamond\big(\forall\,(\forall\Box b)\,\mathsf{U}\,(\forall\bigcirc a)\big)$ into an equivalent CTL-formula in

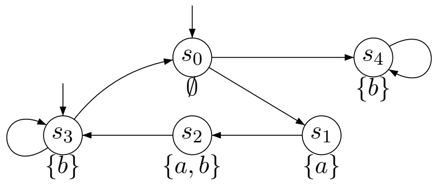
(a) existential normal form and

(b) positive normal form.

Exercise 3

Consider the following CTL formulas and the transition system TS outlined on the right:

$\Phi_1 = \forall(a \mathsf{U} b) \vee \exists \bigcirc (\forall \Box b)$
$\Phi_2 = \forall \Box \forall (a \mathsf{U} b)$
$\Phi_3 = (a \wedge b) \rightarrow \exists \Box \exists \bigcirc \forall (b \mathsf{W} a)$
$\Phi_4 = (\forall \Box \exists \Diamond \Phi_3)$



Give the satisfaction sets $Sat(\Phi_i)$ and decide whether $TS \models \Phi_i$ holds ($1 \leq i \leq 4$).