Introduction

Modelling parallel systems

- Transition systems ⟵
- Modeling hard- and software systems
- Parallelism and communication

Linear Time Properties

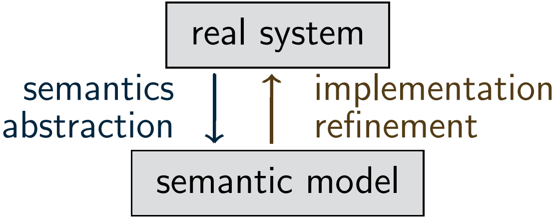
Regular Properties

Linear Temporal Logic

Computation-Tree Logic

Equivalences and Abstraction

Transition systems $\widehat{=}$ extended digraphs



The semantic model yields a formal representation of:

- the states of the system $\longleftarrow$ **nodes**
- the stepwise behaviour $\longleftarrow$ **transitions**
- the initial states
- additional information on
  communication $\longleftarrow$ **actions**
  state properties $\longleftarrow$ **atomic proposition**

Transition system (TS)

A transition system is a tuple

$$\mathcal{T} = (S, Act, \longrightarrow, S_0, AP, L)$$

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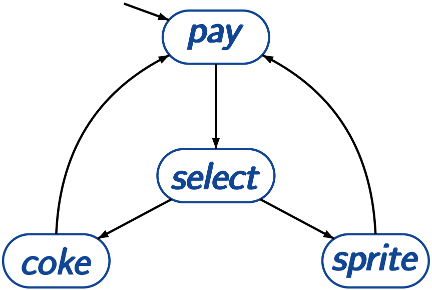
- $S_0 \subseteq S$ the set of initial states,
- AP a set of atomic propositions,
- $L : S \to 2^{AP}$ the labeling function

Transition system for beverage machine



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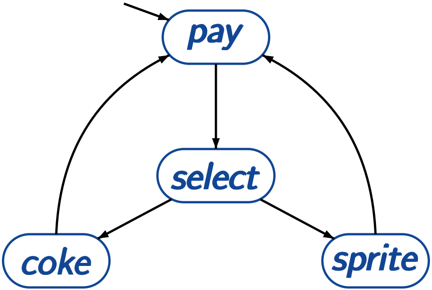

state space $S = \{pay, select, coke, sprite\}$

set of initial states: $S_0 = \{pay\}$

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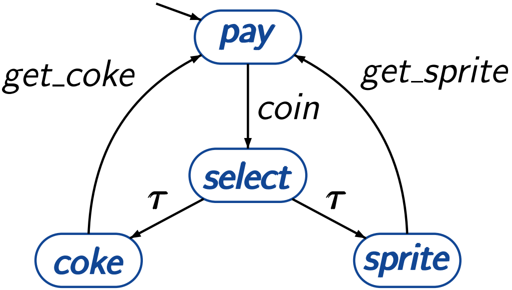

actions:
coin
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get_sprite
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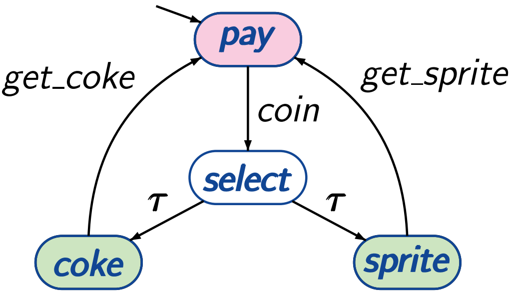


actions:
- *coin*
- τ
- *get_sprite*
- *get_coke*

state space $S = \{pay, select, coke, sprite\}$

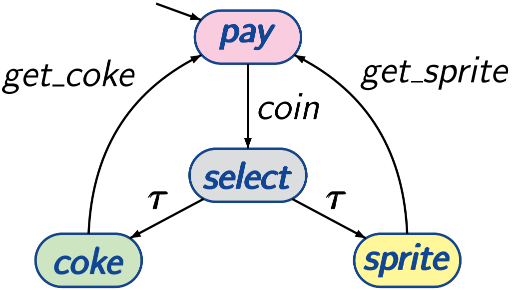
set of initial states: $S_0 = \{pay\}$

set of atomic propositions: $AP = \{pay, drink\}$

labeling function: $L(coke) = L(sprite) = \{drink\}$

$L(pay) = \{pay\}$, $L(select) = \emptyset$

Transition system for beverage machine



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set of atomic propositions: $AP = S$

labeling function: $L(s) = \{s\}$ for each state s

“Behaviour” of transition systems

possible behaviours of a TS result from:

```
select nondeterministically an initial state s ∈ S_0
WHILE  s is non-terminal DO
      select nondeterministically a transition s --α--> s'
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reachable fragment:

$Reach(\mathcal{T})$ = set of all states that are reachable from an initial state through some execution

Transition system for parallel actions

parallel execution of independent actions

parallel execution of dependent actions

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parallel execution of independent actions

e.g. $\underbrace{x := x+1}_{\text{action } \alpha} \;|||\; \underbrace{y := y-3}_{\text{action } \beta}$ $\quad \alpha, \beta$ independent

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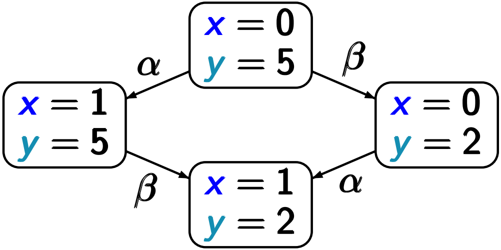

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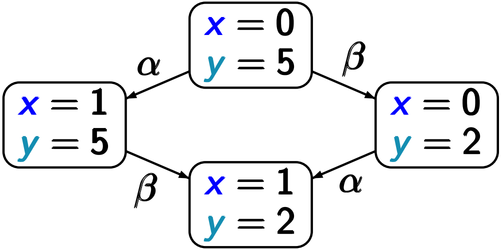
parallel execution of independent actions $\longleftarrow$ interleaving



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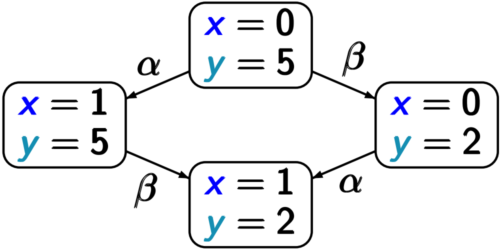

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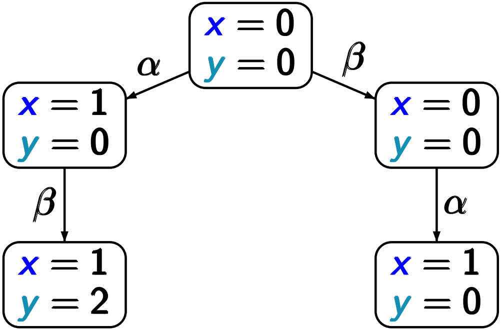

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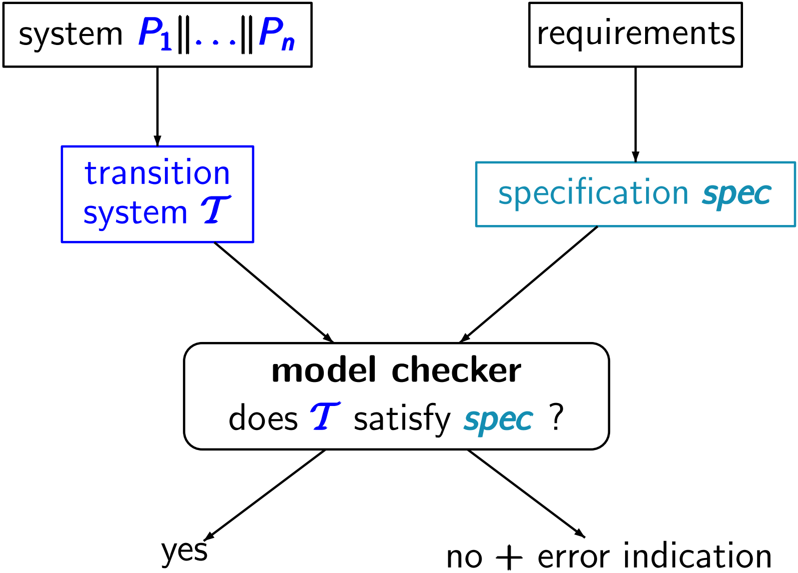
Overview

Model checking



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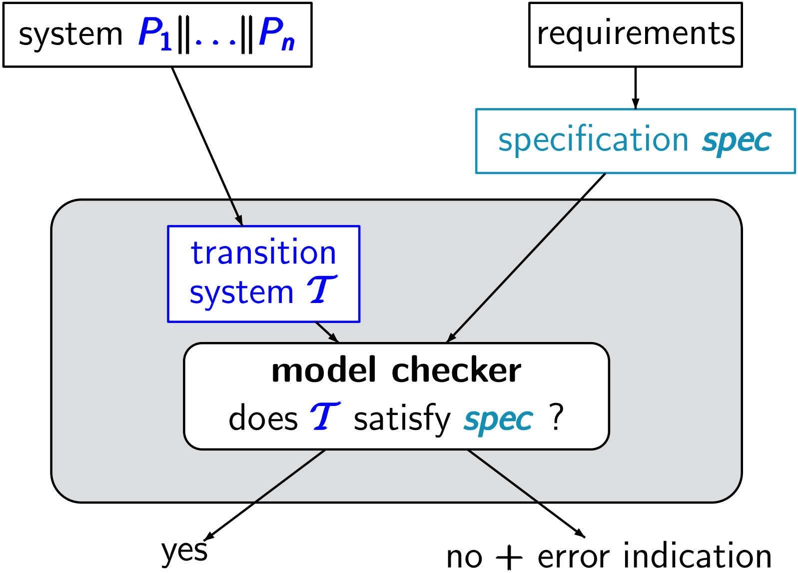

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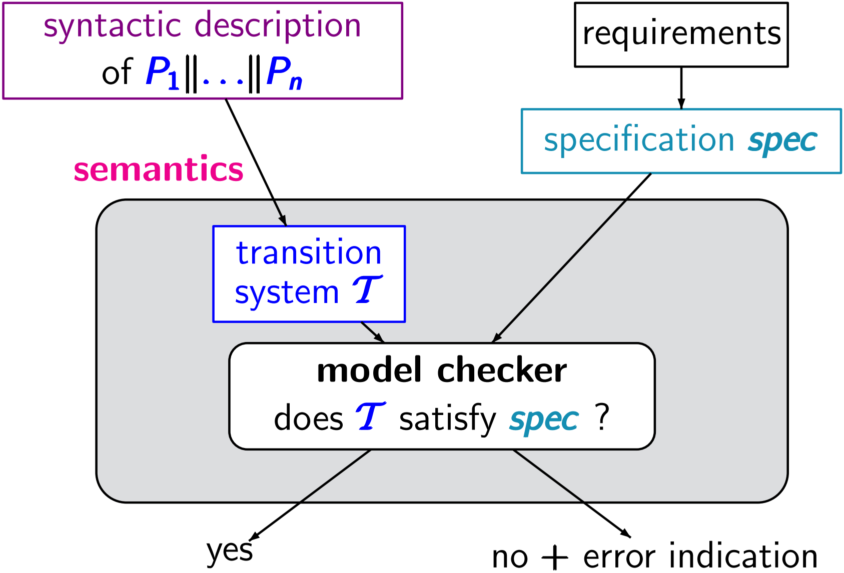
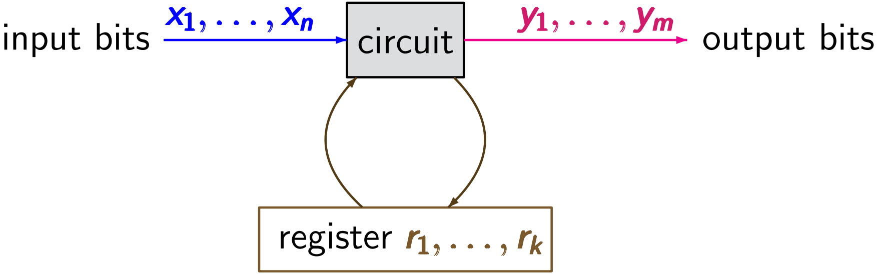


Modelling of sequential circuits by TS



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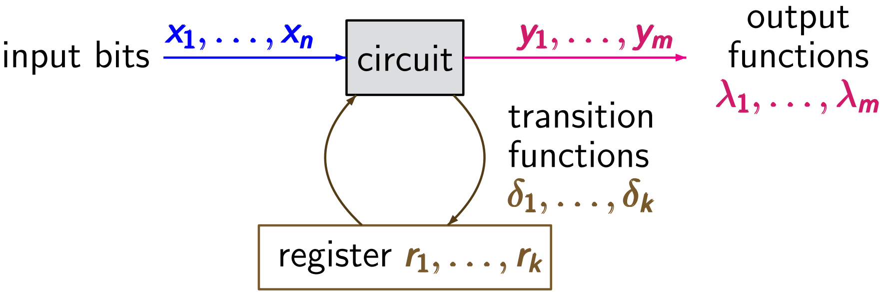

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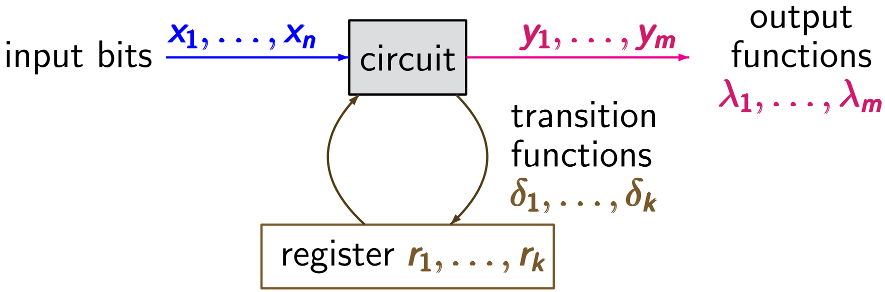

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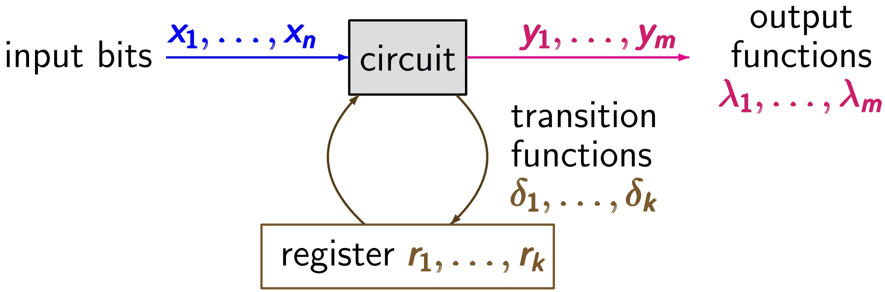

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input values $a_1, \ldots, a_n$ for the input variables		output value $\lambda_i(\ldots)$ for output variable y_i
+ current values $c_1, \ldots, c_k$ of the registers	$\mapsto$	next value $\delta_j(\ldots)$ for register r_j

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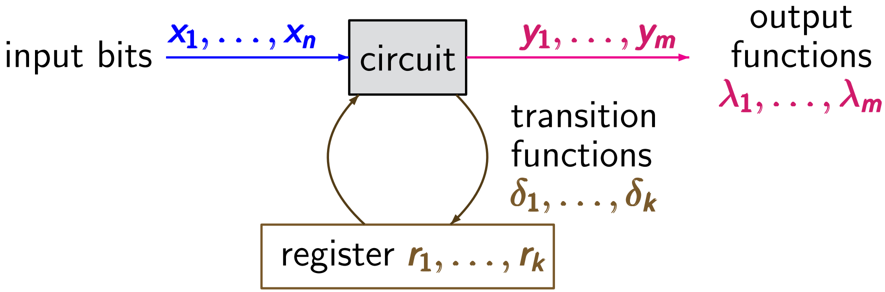

initial register evaluation $[r_1 = c_{01}, \ldots, r_k = c_{0k}]$

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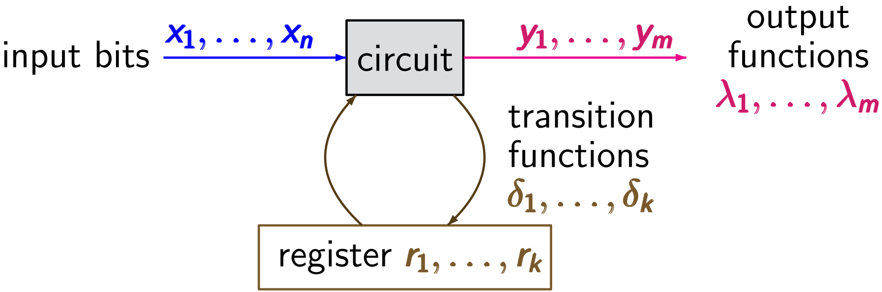

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- states: evaluations of $x_1, \ldots, x_n, r_1, \ldots, r_k$

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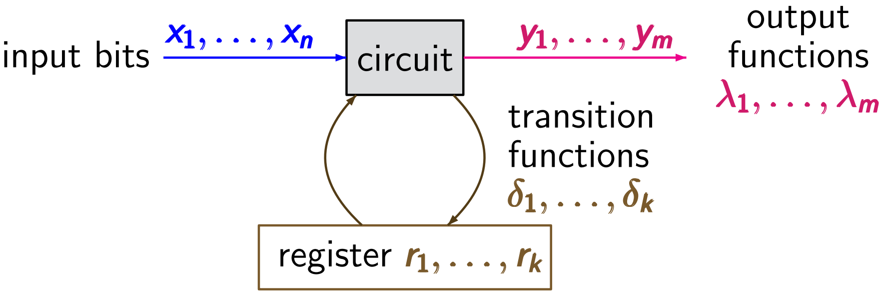

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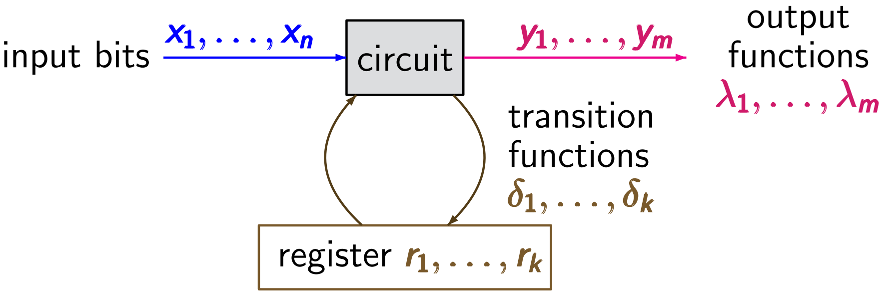

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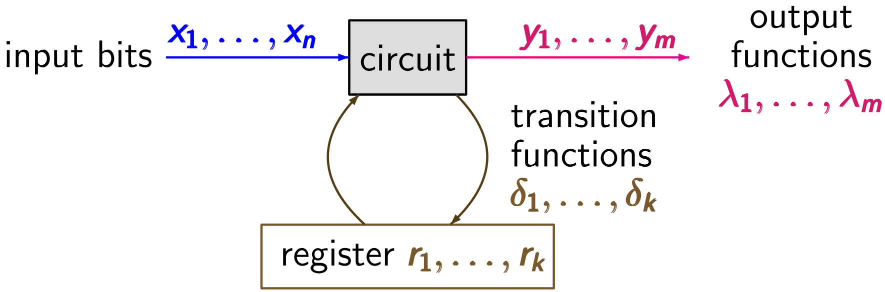

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- transitions represent the stepwise behavior
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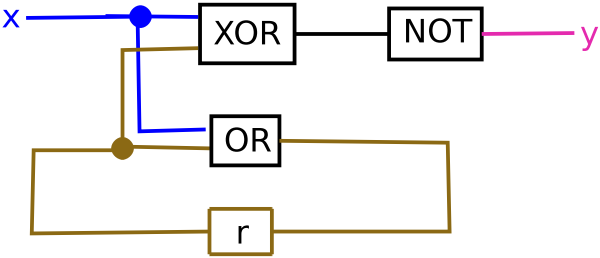
- atomic propositions: $x_1, \ldots, x_n, y_1, \ldots, y_m, r_1, \ldots, r_k$

Example: sequential circuit



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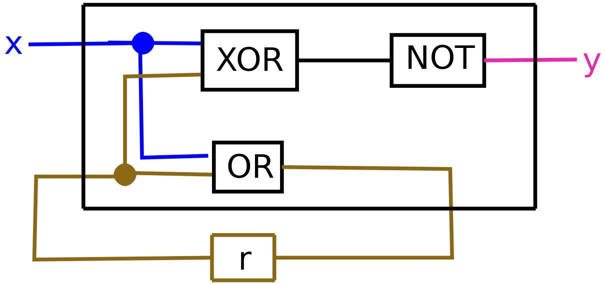
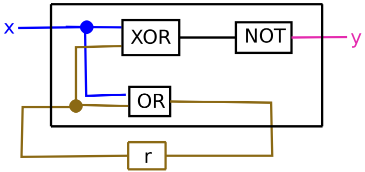


output function: $\lambda_y = \neg(x \oplus r)$

transition function: $\delta_r = x \vee r$

Example: TS for sequential circuit



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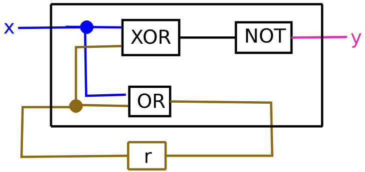

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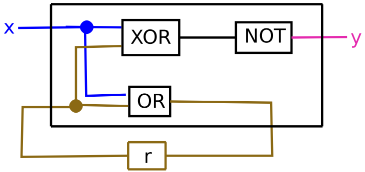

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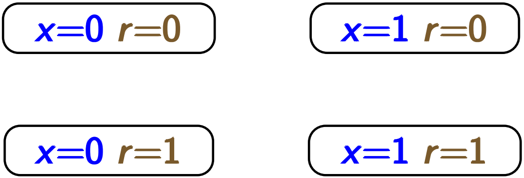

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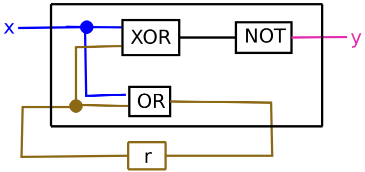

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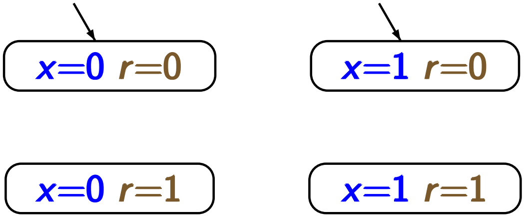

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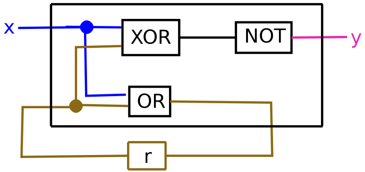

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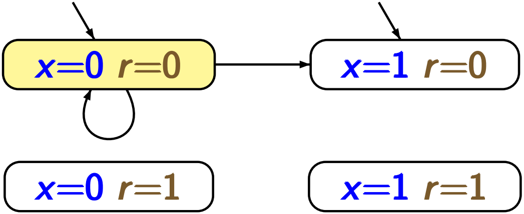

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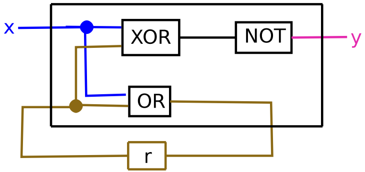

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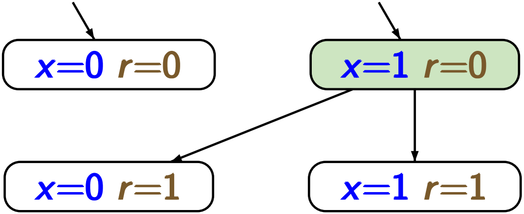

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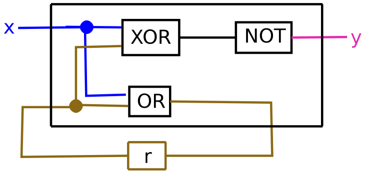

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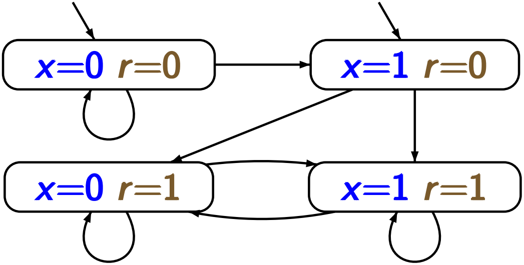

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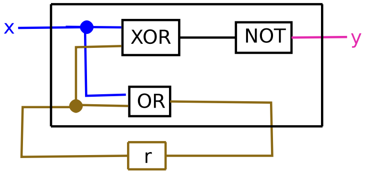

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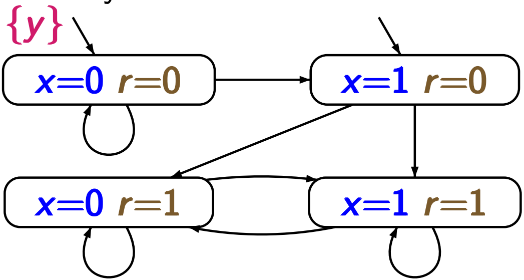

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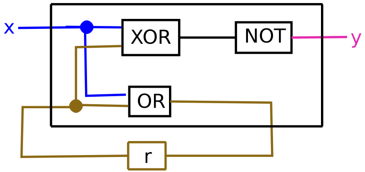

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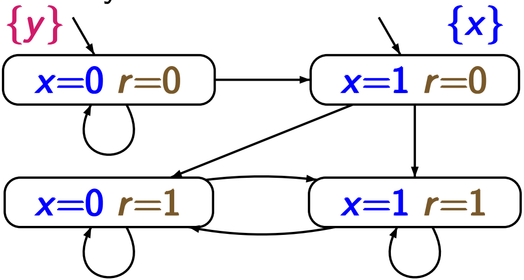

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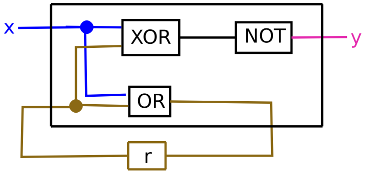

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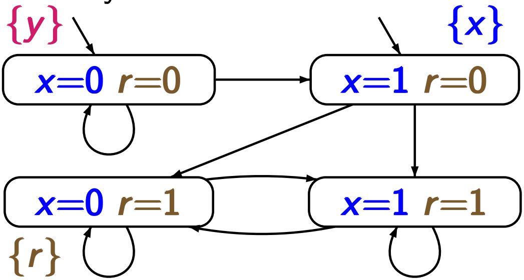

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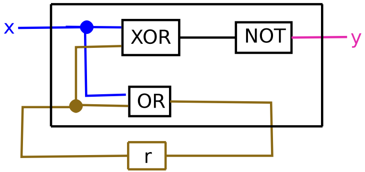

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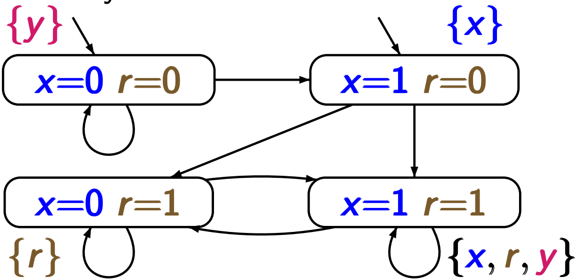

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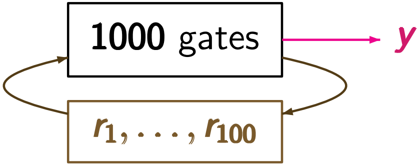

1 output bit
no input
100 registers

How many states . . .

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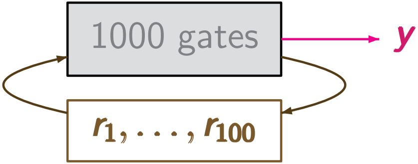

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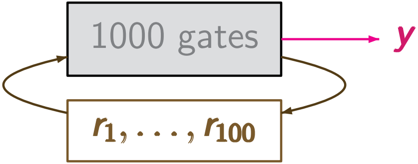

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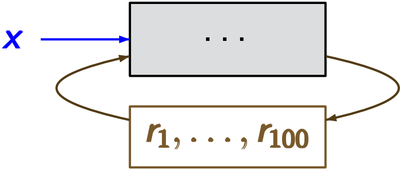

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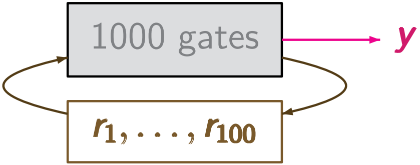

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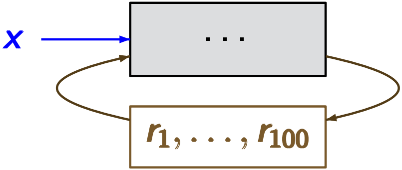

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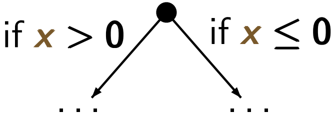
answer: $2^{100} * 2^1 = 2^{101}$

problem: TS-representation of conditional branchings **?**



Data-dependent systems

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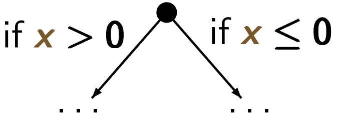

example: sequential program

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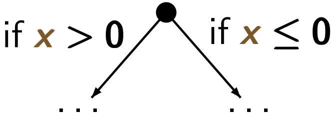

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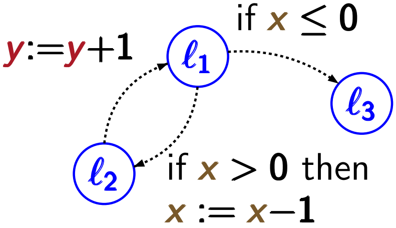

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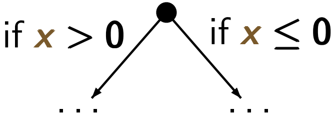

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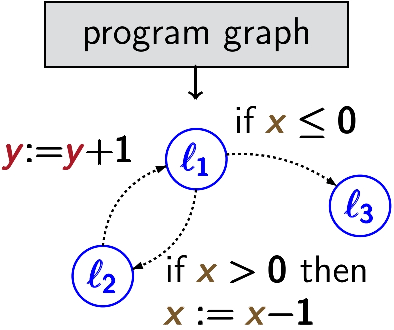

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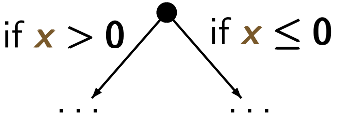

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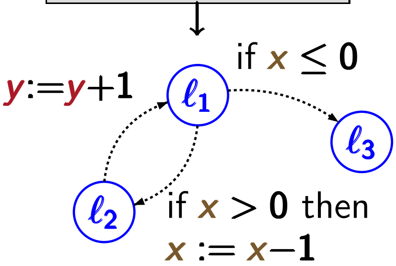

ℓ_1, ℓ_2, ℓ_3 are locations,
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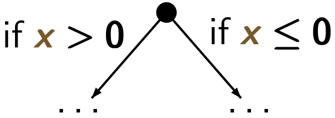

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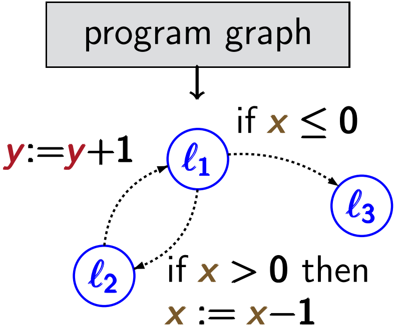

states of the transition system:

locations + relevant data (*here:* values for x and y)

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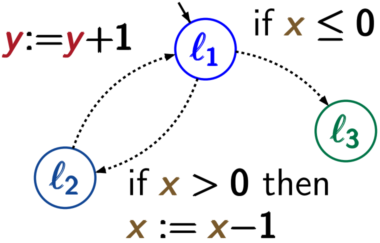

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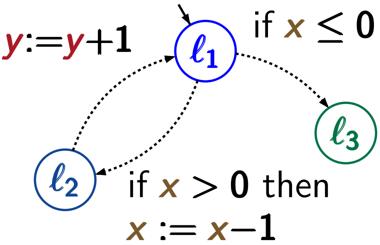

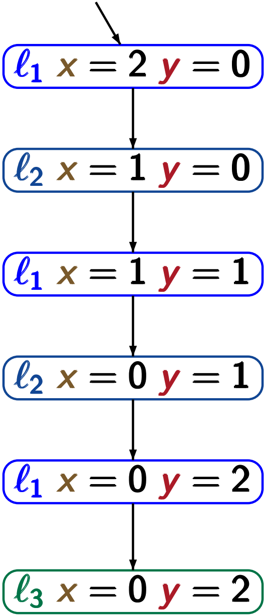

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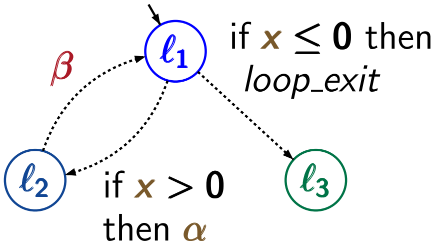

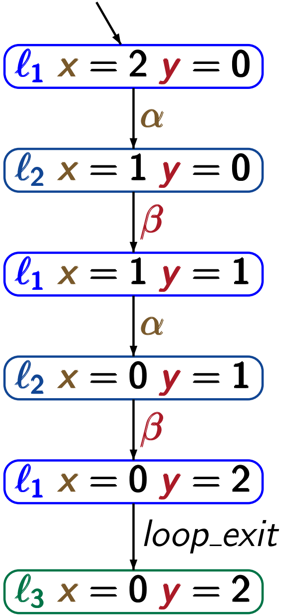

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Notation: $Eval(Var)$ = set of evaluations for Var

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Example:

$$[x=0, y=3, z=6] \models \neg x \wedge y<z$$

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if γ is "$(x,y):=(2x+y, 1-x)$" then:

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example: if α is the assignment $x{:=}x{+}y$ then

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$$\Downarrow$$

transition system $\mathcal{T}_{\mathcal{P}}$

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transition system $\mathcal{T}_{\mathcal{P}}$

states in $\mathcal{T}_{\mathcal{P}}$ have the form

$$\langle \ell, \eta \rangle$$

ℓ: location — η: variable evaluation

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$$\frac{\textit{premise}}{\textit{conclusion}}$$

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It means that $\longrightarrow$ is the smallest relation such that:

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transition system $\mathcal{T}_{\mathcal{P}} = (S, Act, \longrightarrow, S_0, AP, L)$

- state space: $S = Loc \times Eval(Var)$
- initial states: $S_0 = \{\langle \ell, \eta \rangle : \ell \in Loc_0, \eta \models g_0\}$
- $\longrightarrow$ is given by the following rule:

$$\frac{\ell \overset{g:\alpha}{\hookrightarrow} \ell' \;\wedge\; \eta \models g}{\langle \ell, \eta \rangle \overset{\alpha}{\longrightarrow} \langle \ell', Effect(\eta, \alpha) \rangle}$$

- atomic propositions: $AP = Loc \cup Cond(Var)$
- labeling function:

$$L(\langle \ell, \eta \rangle) = \{\ell\} \cup \{g \in Cond(Var) : \eta \models g\}$$

Guarded Command Language (GCL)

by Dijkstra

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- high-level modeling language that contains features of imperative languages and nondeterministic choice

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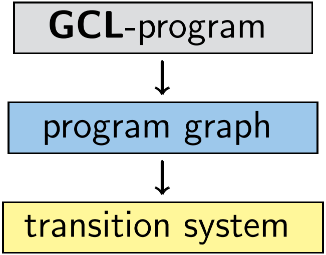
- high-level modeling language that contains features of imperative languages and nondeterministic choice
- semantics:



Guarded Command Language (GCL)

guarded command $g \Rightarrow stmt$

g	: guard, i.e., Boolean condition on the program variables
$stmt$	: statement

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DO $:: g \Rightarrow stmt$ OD

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conditional command:

IF :: $g \Rightarrow stmt_1$
:: $\neg g \Rightarrow stmt_2$
FI

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conditional command:

```
IF :: g  ⇒ stmt1
   :: ¬g ⇒ stmt2
FI
```

⟵

```
IF g THEN stmt1
     ELSE stmt2
FI
```

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conditional command:

```
IF :: g ⇒ stmt1
   :: ¬g ⇒ stmt2
FI
```

←

```
IF g THEN stmt1
     ELSE stmt2
FI
```

symbol :: stands for the nondeterministic choice between enabled guarded commands modeling language with nondeterministic choice

$$
\begin{aligned}
stmt \stackrel{\text{def}}{=} \; & x := expr \quad \mid \quad stmt_1 ; stmt_2 \quad \mid \\
& \texttt{DO} :: g_1 \Rightarrow stmt_1 \quad \ldots \quad :: g_n \Rightarrow stmt_n \; \texttt{OD} \\
& \texttt{IF} :: g_1 \Rightarrow stmt_1 \quad \ldots \quad :: g_n \Rightarrow stmt_n \; \texttt{FI} \\
& \vdots
\end{aligned}
$$

where x is a typed variable and $expr$ an expression of the same type

Guarded Command Language (GCL)

modeling language with nondeterministic choice

$$
\begin{array}{rcl}
\mathit{stmt} & \stackrel{\text{def}}{=} & x := \mathit{expr} \quad \big| \quad \mathit{stmt}_1 ; \mathit{stmt}_2 \quad \big| \\
& & \mathtt{DO} :: g_1 \Rightarrow \mathit{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathit{stmt}_n \ \mathtt{OD} \\
& & \mathtt{IF} :: g_1 \Rightarrow \mathit{stmt}_1 \quad \ldots \quad :: g_n \Rightarrow \mathit{stmt}_n \ \mathtt{FI} \\
& & \vdots
\end{array}
$$

where x is a typed variable and *expr* an expression of the same type

semantics of a **GCL**-program: program graph

GCL-program for beverage machine

GCL-program for beverage machine

uses two variables $\#\textit{sprite}, \#\textit{coke} \in \{0, 1, \ldots, \textit{max}\}$
for the number of available drinks (sprite or coke)

GCL-program for beverage machine

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uses the following actions:

	enabled	effect
get_coke	if $\#coke > 0$	$\#coke := \#coke - 1$
get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$

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refill	any time	$\#sprite := max$ $\#coke := max$

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insert_coin	any time	no effect on variables

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get_sprite	if $\#sprite > 0$	$\#sprite := \#sprite - 1$
refill	any time	$\#sprite := max$ $\#coke := max$
insert_coin	any time	no effect on variables
return_coin	if machine is empty and user has entered a coin (no effect on variables)	

GCL-program for beverage machine

$$
\begin{array}{l}
\texttt{DO} :: \text{true} \Rightarrow \text{insert\_coin}; \\
\quad \texttt{IF} :: \#sprite = \#coke = 0 \Rightarrow \text{return\_coin} \\
\quad\quad :: \#coke > 0 \Rightarrow \#coke := \#coke - 1 \\
\quad\quad :: \#sprite > 0 \Rightarrow \#sprite := \#sprite - 1 \\
\quad \texttt{FI} \\
\quad :: \text{true} \Rightarrow \#sprite := max; \#coke := max \\
\texttt{OD}
\end{array}
$$

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin

         :: #coke  > 0 ⇒ #coke := #coke − 1

         :: #sprite > 0 ⇒ #sprite := #sprite−1

      FI
   :: true ⇒ #sprite := max; #coke := max

OD
```

GCL-program for beverage machine

$$
\begin{array}{lll}
\texttt{DO} & :: & true \Rightarrow insert\_coin; \text{ (* user inserts a coin *)} \\
 & & \texttt{IF} \; :: \; \#sprite = \#coke = 0 \Rightarrow return\_coin \\
 & & \qquad\qquad\qquad\qquad \text{(* no beverage available *)} \\
 & & \quad\;\; :: \; \#coke > 0 \Rightarrow \#coke := \#coke - 1 \\
 & & \quad\;\; :: \; \#sprite > 0 \Rightarrow \#sprite := \#sprite - 1 \\
 & & \texttt{FI} \\
 & :: & true \Rightarrow \#sprite := max; \#coke := max \\
\texttt{OD} & &
\end{array}
$$

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                         (* no beverage available *)
         :: #coke > 0 ⇒ #coke := #coke − 1
                         (* user selects coke *)
         :: #sprite > 0 ⇒ #sprite := #sprite−1
                         (* user selects sprite *)
      FI
   :: true ⇒ #sprite := max; #coke := max
                         (* refilling of the machine *)
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin; (* user inserts a coin *)
      IF :: #sprite = #coke = 0 ⇒ return_coin
                                   (* no beverage available *)
         :: #coke > 0 ⇒ get_coke
                                   (* user selects coke *)
         :: #sprite > 0 ⇒ get_sprite
                                   (* user selects sprite *)
      FI
   :: true ⇒ refill
                                   (* refilling of the machine *)
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin;
      IF :: #sprite = #coke = 0 ⇒ return_coin
         :: #coke > 0 ⇒ get_coke
         :: #sprite > 0 ⇒ get_sprite
      FI
   :: true ⇒ refill
OD
```

GCL-program for beverage machine


```
DO :: true ⇒ insert_coin;
      IF :: #sprite = #coke = 0
                      ⇒ return_coin
         :: #coke   > 0 ⇒ get_coke
         :: #sprite > 0 ⇒ get_sprite
      FI
   :: true ⇒ refill
OD
```

... yields a program graph with

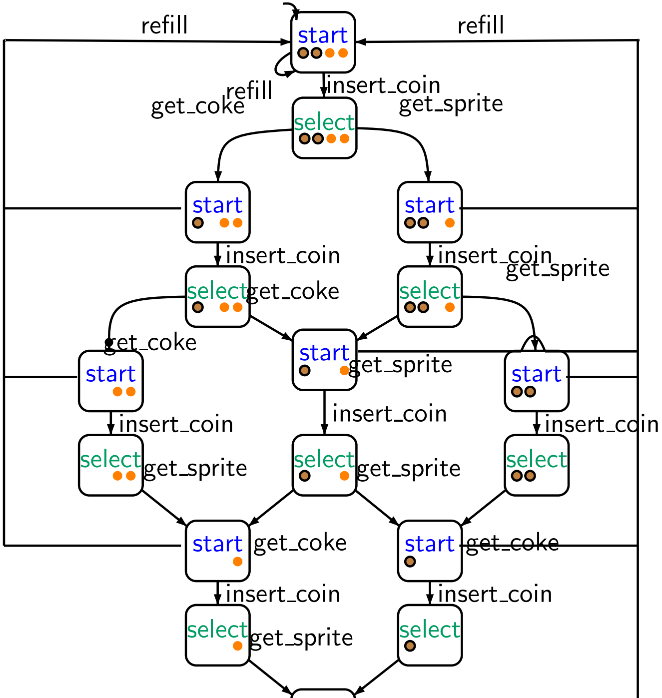
- two variables $\#sprite, \#coke \in \{0, 1, \ldots, max\}$

GCL-program for beverage machine

$$
\begin{array}{lllll}
\textit{start} \longrightarrow \texttt{DO} & :: & \text{true} \Rightarrow \text{insert\_coin}; & & \\
\textit{select} \longrightarrow & & \texttt{IF} & :: & \#\textit{sprite} = \#\textit{coke} = 0 \\
 & & & & \quad \Rightarrow \text{return\_coin} \\
 & & & :: & \#\textit{coke} > 0 \Rightarrow \text{get\_coke} \\
 & & & :: & \#\textit{sprite} > 0 \Rightarrow \text{get\_sprite} \\
 & & \texttt{FI} & & \\
 & :: & \text{true} \Rightarrow \text{refill} & & \\
\texttt{OD} & & & &
\end{array}
$$

... yields a program graph with

- two variables $\#\textit{sprite}, \#\textit{coke} \in \{0, 1, \ldots, \textit{max}\}$
- two locations ***start*** and ***select***


refill
refill
start
insert_coin
refill
get_coke
select
get_sprite
start
start
insert_coin
insert_coin
get_sprite
select
get_coke
select
get_coke
start
get_sprite
start
start
insert_coin
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