Delay-constrained IP routing problems

MINLP meets computer networks

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Outline

Sometimes Things Just Don't Go as Planned (but Better)

- The Internet was built around a set of assumptions:
  - Integrity of information is crucial: lost packets are retransmitted
  - Timeliness does not matter: the sooner the better, but no deadline
  - Application adapt to the available rate
    (higher rate $\equiv$ higher user satisfaction, but no QoS agreements)

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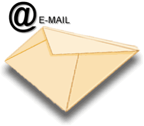
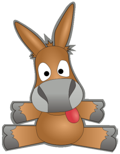
- Internet is built upon the "Best Effort" Service Model:
  routers do their best to relay packets to destination, but
  no guarantee that a given packet will arrive at all
- Traditional Internet applications play by these rules





It Is Possible to Succumb to One's Success

- Despite this, Internet has became a huge splash hit (doh!)

- This has made some technologies (TCP-IP, Ethernet) dominant, economy of scale dictates convergence of everything:
  - traditional internet applications (+ social stuff)
  - IP Telephony
  - live Internet Protocol Television
  - online gaming/MMORPGs
  - industrial control systems
  - remote sensing and surveillance systems
  - M2M communication, IoT/IoE (pick your favorite buzzword)

  irrespectively of the access medium (fixed, cellular, WiFi, BLE, . . .)

- Issue: many of these completely unsuitable for best effort

- Now what? Introduce QoS guarantees
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- What is QoS? "The ability of a network to offer different levels of service, in order to support different types of applications"

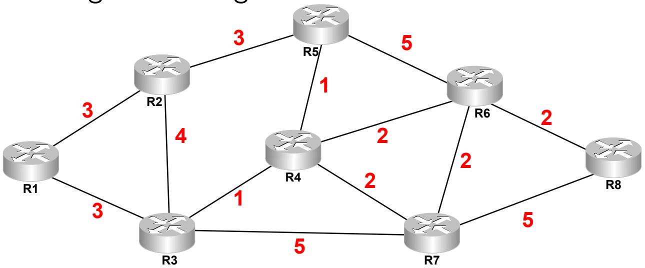
- Prime example: controlled end-to-end delay
- Critical in embedded systems (automative, avionics, . . .)
- Much easier said than done, the provisions simply weren't there
- Introducing QoS is a complex, multi-faceted effort

Introducing QoS

- Requires adding ad hoc algorithms, hw/sw components, protocols:
  - simple, scalable and cost-effective (10^6 routers, 10^9 devices)
  - effective $\equiv$ guarantee that QoS objectives are met (money involved)
  - distributed and cooperating (no central control & management)
- Some building blocks have been designed, a few standardized
- Big issue: cooperation at the various timescales (vertical)
  - years/months: network design/expansion
  - weeks/days: resource provisioning (traffic engineering, routing)
  - hours/seconds: flow lifetime (resource reservation, admission control)
  - sub-millisecond: transmission (packet scheduling)
- Horizontal cooperation is also needed
- All this within a distributed decision model

QoS Requires Optimization (doh!)

- Example: setting OSPF weights in a domain



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- Select the "best" path for a flow (can be many, horrible in practice)

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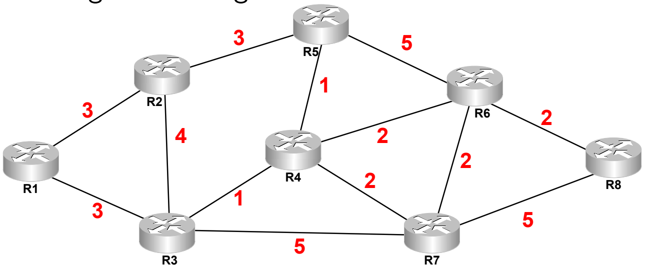
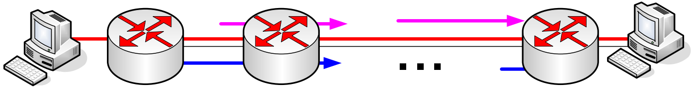


...a heinously complex problem for wanting too simple a system

- Select the "best" path for a flow (can be many, horrible in practice)
- Packets, not circuits: how will the packets behave?
- Can't say unless you reserve capacity for the flow (≈ circuits)

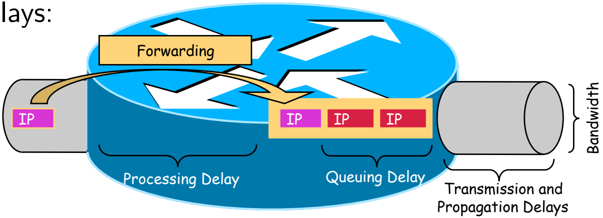


- How to do that optimally? It depends on many things

Outline

Flows, routers, links

- Flow: "distinguishable directed stream of packets with the same QoS requirements traveling from a source to one or more destinations"
- Lots of delays:



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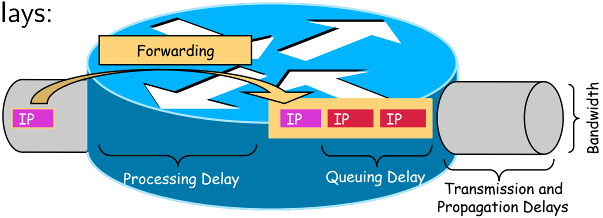
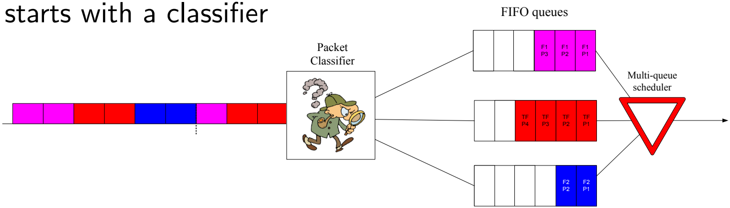


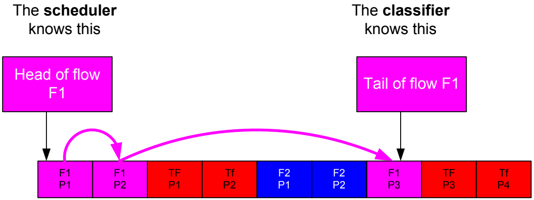
- Slowly creeping closer to our mathspeak:
  - IP Network $\equiv$ directed graph $G = (N, A)$ $(n = |N|,\ m = |A|)$
  - set of flows K: origin/destination (s^k, d^k), arrival curve $\mathcal{A}^k$ (???)
  - packet transmission cannot be preempted $\Longrightarrow$ packets size matters: maximum transfer unit L (MTU, max. packet length)
  - $(i, j) \in A$: link speed (bandwidth) $w_{ij} \Longrightarrow$ link delay l_{ij} $(\geq L / w_{ij})$
  - $i \in N$: node processing delay n_i, assumed constant (!)
- Queuing delay a relevant factor, depends on packet schedulers

A (very brief) Intro To Packet Schedulers

- It all starts with a classifier



- Multiple logical lists in a single memory buffer space



- The crucial part is the scheduler

The Ideal Packet Scheduler

- What we would want from a packet scheduler:
  - simplicity (low complexity)
  - isolation of flows
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  - guarantees
- Not at all easy
- Example: FIFO scheduler
  - simple: $O(1)$ ✓
  - no isolation of flows: a burst of a new flow can starve yours forever X
  - not controllable: can't change how it behaves X
  - no fairness: the first flow arriving takes it all X
  - no guarantees: can't prove anything on anything (e.g. max delay) X
- Strict priority list not much better

The Ideal Packet Scheduler: Generalized Processor Sharing

- What is the perfect formula of a scheduler?

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- Schedule packets so that flow k achieves effective rate

$$r_{ij}^{eff,k} = (\, w_{ij} / \bar{r}_{ij} \,) r_{ij}^k \geq r_{ij}^k \qquad \equiv \qquad \text{delay} \; = L / r_{ij}^{eff,k} \tag{1}$$

$\equiv r_{ij}^k$ if the arc loaded, more if spare bandwidth available

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- Provable perfect fairness (with appropriate definition)
- Can this be achieved? Almost, but not quite
- For once, GPS defined for idealized fluid model but we have packets
- Furthermore, it cannot be done in less than $O(\log |K|)$ (no $O(1)$)
- Yet, $O(\log |K|)$ good approximations exist
(e.g. Worst-case Fair Weighted Fair Queuing — WF^2Q)

A Good Approximation To The Ideal Packet Scheduler

- Notation: $r_{ij}^k > 0 \Longrightarrow$ flow k passes through $(i, j) \Longrightarrow k \in P(i, j)$
  $r_{ij}^{min} = \min\{\, r_{ij}^k \,:\, k \in P(i, j)\,\}$
- Strictly Rate-Proportional scheduler:

$$\theta_{ij}^k = \frac{L}{w_{ij}} + \begin{cases} L / r_{ij}^{eff,k} & \text{if } P(i,j) \setminus \{k\} \neq \emptyset \\ 0 & \text{otherwise} \end{cases} \qquad (2)$$

  L / w_{ij}: a packet has to be entirely received before anything happens

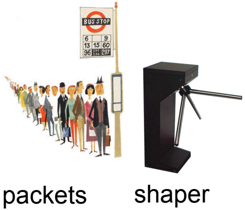
- Worst-worst case: $r_{ij}^{eff} = r_{ij}$, $P(i, j) \neq \emptyset \Longrightarrow$ coarser (but valid) estimate of the delay, somewhat simplified formula:

$$\theta_{ij}^k = \frac{L}{r_{ij}^k} + \frac{L}{w_{ij}} \qquad (3)$$

- Best possible, but is $O(\log |K|)$ (cheaper versions exist)

Putting It All Together

- Given all individual pieces, compute the end-to-end delay (e2ed)
- Could use queuing theory, but it would be very complex; plus: do your really know the arrival distribution?
- Alternative: worst case analysis, using network calculus
- Last crucial ingredient: the arrival function $\mathcal{A}^k$
- Not trivial to determine, but a nice trick: traffic shaper



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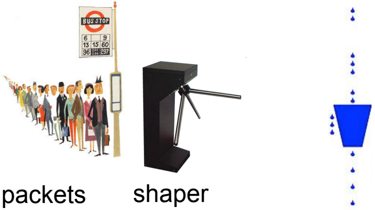

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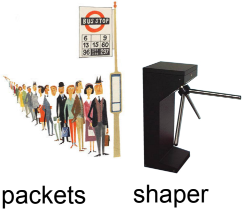

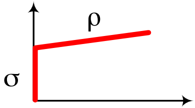

- In particular, leaky-bucket traffic shaper with burst σ^k and rate ρ^k makes for a very simple arrival function

The Worst-case End-To-End Delay Formula (at last!)

- Worst-case e2ed (WCD) of flow k with σ^k, ρ^k depends on:
  1. the selected s^k–d^k path P^k in G;
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- Necessary assumption for finite WCD:
  $r^k_{ij} \geq \rho^k$ for each $(i, j) \in P^k \equiv r^k_{min} = \min\{\, r^k_{ij} \;:\; (i, j) \in P^k \,\} \geq \rho^k$
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  (rate $\rho^k \equiv$ "steady-state" flow demand in usual flow models)
- General WCD formula (nonlinear!):

$$\frac{\sigma^k}{r^k_{min}} + \sum_{(i,j) \in P^k} \left(\theta^k_{ij} + l_{ij} + n_i \right) \tag{4}$$

  where θ^k_{ij} is the protocol-specific arc delay (also nonlinear!)
- σ^k / r^k_{min}: the burst can happen just before the worst-case packet, all of it has to go through the bottleneck arc
- Good news: (4) convex and SOCP-representable if θ^k_{ij} is

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Compute paths and reserve resources on arcs at minimum cost such that the maximum delay of each flow is $\leq$ deadline

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- Single-Flow Single-Path (SFSP) DCR: one new unsplittable flow (just about to enter the network, has to be routed now)
  - drop superscripts, r^k_{ij} = existing flows, fixed
  - $P(i, j)$ = set of paths passing through (i, j) excluding the new one
  - $\bar{r}_{ij} = \sum_{k \in P(i,j)} r^k_{ij}$, $r^{min}_{ij} = \min\{ r^k_{ij} : k \in P(i, j) \}$ exclude new flow
- Fixed deadline δ on the new flow
- Reservable capacity $w_{ij} \geq w_{ij} - \bar{r}_{ij} \geq c_{ij} \geq r_{ij}$
- Linear capacity reservation cost f_{ij} (often $= 1 \equiv$ Equal Cost (EC))
- All the other flows must remain feasible (access control): trivial for "bound" SRP (3), does not depend on other flows

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Combinatorial properties

- Only (bound-)SRP-SFSP-DCR studied in the literature before us
- $\sigma = L = 0 \Longrightarrow r_{ij} = \rho x_{ij} \Longrightarrow$ Constrained Shortest Path (CSP) (this gives more than one idea, and proves $\mathcal{NP}$-hardness)

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- But using (i,j) with "low" $c_{ij} \searrow r_{min} \Longrightarrow \nearrow$ the delay: $G^r = (N, A^r)$ with $A^r = \{\, (i,j) \in A \;:\; c_{ij} \geq r \,\} \Longrightarrow r_{min} \geq r$
- For each $r \in C = \{\, c_{ij} \;:\; (i,j) \in A \,\}$:
  - solve s–d shortest path P on G^r w.r.t. $\bar{l}$
  - if $\bar{l}(P) \leq \delta - \sigma/r$, then P feasible: stop
  - if no feasible P found, then problem unfeasible (for fixed P, both LHS and RHS of (4) increase with r)
- Keep f-best solution found: ERA-I heuristic

Equal-Cost, Equal Rate Allocation

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  - run Bellman-Ford on G^r with costs $l^r_{ij} = L/r + l'_{ij}$
  - at each round of BF, check path P entering d (if any)
  - if $l^r(P) \leq \delta - \sigma/r$ then stop: P optimal

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- Works because BF solves hop-constrained shortest path: find least-cost($=$ delay) path with that number of hops, but r fixed $\Longrightarrow$ true cost proportional to $|P|$
- Each round, cost($=$ delay) $\searrow$ but hop count ($=$ cost) $\nearrow$: first feasible path is optimal
- Repeating the above for all $r \in C$ does not solve $(\ldots)$DCR counterexample: returned path P with delay constraint not tight

$$\Delta(r, P) = \frac{\sigma + L|P|}{r} + \sum_{(i,j) \in P} l'_{ij} < \delta$$

Equal-Cost, Equal Rate Allocation

- Obvious solution: for each feasible P reduce r until constraint tight

$$\tilde{r}(P) = (\,\sigma + L|P|\,)/(\,\delta - l'(P)\,)$$

$$\Longrightarrow \tilde{r}(P) \leq r,\ \Delta(\tilde{r}(P), P) = \delta \text{ (keep feasibility, improve objective)}$$

Theorem

For all $r \in C$ run BF on G^r, for all P decrease r, keep best P (don't stop at first feasible): solves EC-ERA-SRP-SFSP-DCR in $O(|C|nm) \leq O(nm^2)$

- Standard Bellman-Ford is $\Omega(mn)$, slow in practice
- Alternative implementation: SPT.L.Queue with label pairs $(l, d) \equiv$ shortest path on acyclic graph $n \times G^r$, not much better
- Heuristic alternative: SPT.L.Queue on original G^r, each time d extracted from Q check delay of that P
- Can miss the optimal path but rarely, much faster

Extending ERA: Non-equal f_{ij}

- Non-equal but integer f_{ij}: use dynamic programming instead of BF

Theorem

If $f_{ij} \in \mathbb{N}$, then the algorithm solves ERA-SRP-SFSP-DCR in $O(|C|\bar{f}m) \leq O(nm^2 f_{max})$ (pseudo-polynomial)

- NE continuous f_{ij}: standard cost rounding approximation algorithm
- Standard idea: scaling factor $f \in F = \{ f_{ij} \ : \ (i,j) \in A \}$ ($|F| \leq m$), scaled costs $\tilde{f}_{ij} = \lceil f_{ij}/K \rceil$ where $K = (\varepsilon f)/(n-1)$
- Algorithm: cycle over all scaling factors f, apply pseudo-poly algorithm to G_f, keep best f-solution of all these found

Theorem

The algorithm finds a ε-optimal solution for ERA-SFSP-SRP-DCR (with unscaled f_{ij}) in $O(|F|n^2m^2/\varepsilon)$ (Fully Poly-time Approximation Scheme)

- Yet, we don't really want to solve the ERA version

Instances

- Real-world IP network topologies (GARR, SNDlib, TopoZOO): 10 – 65 nodes, 12 – 170 arcs, few 10s – several 100s flows
- Realistic random topologies (Waxman model): $\leq$ 200 nodes, 1500 arcs
- Equal (reservation) Costs $f_{ij} = 1$
- FNSS tool for realistic traffic matrices ($\mu(T) = 0.8$ Gbps and $\sigma^2(T) = 0.05$) and link-capacity assignment ($1, 10, 40$ Gbps)
- DCR-generator for the remaining network parameters ($L = 1500$, $n_i = l_{ij} = L/w_{ij}$, $\sigma = 3L$)
- Distributed at `http://www.di.unipi.it/optimize/Data/MMCF.html#UMMCF`
- Experiments with "unloaded networks", but "loaded" case analogous

ERA-Based Heuristics: Experiments

				ERA-I		ERA-H		
instance	n	m	k	avg	max	avg	max	inf
abilene	12	15	31	0.52	0.92	0.000	0.000	0.06
atlanta	15	22	45	0.57	0.88	0.000	0.000	0.07
cost266	37	57	120	0.48	0.95	0.000	0.000	0.17
dfn-bwin	10	45	45	0.03	0.06	0.000	0.000	0.00
dfn-gwin	11	47	53	0.16	0.86	0.000	0.000	0.02
di-yuan	11	42	58	0.48	0.90	0.000	0.000	0.12
france	25	45	66	0.44	0.90	0.000	0.000	0.02
geant	22	36	63	0.46	0.89	0.000	0.001	0.06
germany50	50	88	276	0.50	0.90	0.000	0.001	0.21
giul39	39	172	1482	0.67	0.97	0.011	0.570	0.10
india35	35	80	195	0.53	0.93	0.000	0.000	0.11
janos-us	26	84	650	0.71	0.95	0.004	0.275	0.18
janos-us-ca	39	122	1482	0.68	0.95	0.010	0.289	0.23
newyork	16	49	89	0.50	0.90	0.000	0.000	0.03
nobel-eu	28	41	106	0.55	0.93	0.000	0.000	0.23
nobel-ger	17	26	51	0.49	0.93	0.000	0.000	0.10

- gap with optimum, inf = feasible wrongly declared unfeasible

ERA-Based Heuristics: Experiments (cont.)

nobel-us	14	21	24	0.35	0.90	0.000	0.001	0.00
norway	27	51	341	0.71	0.94	0.000	0.000	0.12
pdh	11	34	54	0.64	0.90	0.000	0.001	0.04
pioro40	40	89	204	0.40	0.89	0.000	0.000	0.25
polska	12	18	24	0.59	0.90	0.000	0.000	0.00
sun	27	102	702	0.76	0.95	0.008	0.431	0.06
ta2	65	108	388	0.45	0.92	0.000	0.000	0.31
garr 1999-01	16	36	240	0.65	0.88	0.000	0.001	0.02
garr 1999-04	23	50	506	0.57	0.94	0.000	0.001	0.75
garr 1999-05	23	50	506	0.55	0.94	0.000	0.000	0.75
garr 2001-09	22	48	462	0.60	0.94	0.000	0.000	0.74
garr 2001-12	24	52	552	0.59	0.94	0.000	0.000	0.75
garr 2004-04	22	48	462	0.56	0.94	0.000	0.000	0.75
garr 2009-08	54	136	2862	0.65	0.94	0.001	0.386	0.85
garr 2009-09	55	138	2970	0.67	0.94	0.000	0.000	0.85
garr 2009-12	54	136	2862	0.67	0.94	0.001	0.240	0.85
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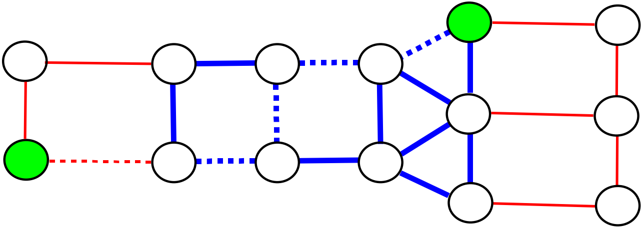
w1-100-04	100	414	664	0.77	0.95	0.015	0.739	0.07
w1-200-04	200	1550	1528	0.71	0.96	0.015	0.814	0.05

Why does ERA fail so often?



- Hub-and-spoke-like network with well-connected core (40/100 Gb) but weaker links to the periphery (1 Gb)
- Path from a core node to a peripheral one has to cross a weak link
- ERA has to allocate the same rate to all links $\Longrightarrow$ no more than the weak link's (residual) capacity $\Longrightarrow$ cannot meet the deadline
- The deadline can be met by reserving more capacity on core links: how do we know that?

Outline

A MI-SOCP Model for SFSP-DCR

- Path binary variables x_{ij}, reserve continuous variables r_{ij}

$$\min \textstyle\sum_{(i,j)\in A} f_{ij} r_{ij} \qquad (5)$$

$$\sum_{(j,i)\in BS(i)} x_{ji} - \sum_{(i,j)\in FS(i)} x_{ij} = \begin{cases} -1 & \text{if } i = s \\ 1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \qquad i \in N \qquad (6)$$

$$0 \leq r_{ij} \leq c_{ij} x_{ij} \qquad (i,j) \in A \qquad (7)$$

$$\rho \leq r_{min} \leq r_{ij} + c_{max}(1 - x_{ij}) \qquad (i,j) \in A \qquad (8)$$

$$t + \textstyle\sum_{(i,j)\in A} \big(\theta_{ij} + (l_{ij} + n_i) x_{ij} \big) \leq \delta \qquad (9)$$

$$t\, r_{min} \geq \sigma \quad , \quad t \geq 0 \qquad (10)$$

$$x_{ij} \in \{0,1\} \quad , \quad r_{ij} \in \mathbb{R} \qquad (i,j) \in A$$

- (10) rotated SOCP constraint $\equiv t \geq \sigma / r_{min}$ (since $t \geq 0$)
- $c_{max} = \max\{ c_{ij} \ : \ (i,j) \in A \} =$ big-M, but cannot use c_{ij} (otherwise $r_{min} \leq c_{ij}$ even if $(i,j) \notin P$)

A big-M Formulation for SRP-SFSP-DCR

- $\theta_{ij} = L/r_{ij}$, add L/w_{ij} to the coefficient of x_{ij} in (9)
- Issue: how to write "$x_{ij} = 1 \Longrightarrow \theta_{ij} = L/r_{ij}$, $x_{ij} = 0 \Longrightarrow \theta_{ij} = 0$"; can't use $r_{ij}\,\theta_{ij} \geq L$ for that $\Longrightarrow \theta_{ij} > 0$ always

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- Solution: two extra sets of variables s_{ij} and r'_{ij}

$$0 \leq \theta_{ij} \leq M x_{ij}$$

$$\theta_{ij} \geq s_{ij} - M(1 - x_{ij})$$

$$s_{ij}\, r'_{ij} \geq L \ , \quad s_{ij} \geq 0$$

$$0 \leq r'_{ij} \leq r_{ij} + M(1 - x_{ij})$$

  - $\theta_{ij} \geq s_{ij}$ if $x_{ij} = 1$, while θ_{ij} and s_{ij} are "free" if $x_{ij} = 0$
  - $r'_{ij} \leq r_{ij}$ if $x_{ij} = 1$, while r'_{ij} and r_{ij} are "free" if $x_{ij} = 0$
  - $s_{ij} \geq L / r'_{ij} \Longrightarrow \theta_{ij} \geq s_{ij} \geq L / r'_{ij} \geq L / r_{ij}$ if $x_{ij} = 1$
- $M = \max(\sqrt{L} \, , \, L / \rho)$ suffices, still it's big-M: can we do better?

Outline

A Step Up: Convex Indicator Constraints

- Vector of n variables x, convex function $f(x)$, one binary variable y
- Constraint $f(x) \leq 0$ "active" $\iff y = 1$, or more in general, constraint $\ldots + s + \ldots \leq d$ with $s = f(x)$ if $y = 1$, $s = 0$ otherwise

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- What can we do to improve on this? If f is linear, nothing ...

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- What can we do to improve on this? If f is linear, nothing ...
  ... but if f is nonlinear, we can indeed do something

Constructing a better formulation

- General result: $conv(\mathcal{P}_0 \cup \mathcal{P}_1) = pr_{(x,y)}(cl(\mathcal{P}^*))$, where

$$\mathcal{P}^* = \left\{ \begin{array}{l} (x, x', y) \in \mathbb{R}^{2n+1} \ : \ y\, f(x'/y) \leq 0 \ , \ y \in (0, 1] \\ y l_1 \leq x' \leq y u_1 \ , \ (1-y) l_0 \leq x - x' \leq (1-y) u_0 \end{array} \right.$$

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- Even simpler to see: nonlinear convex-cost semi-continuous variable

$$f(x,y) = \begin{cases} 0 & \text{if } y = 0 \text{ and } x = 0 \\ f(x) + c & \text{if } y = 1 \text{ and } l_1 \leq x \leq u_1 \\ +\infty & \text{otherwise} \end{cases}$$

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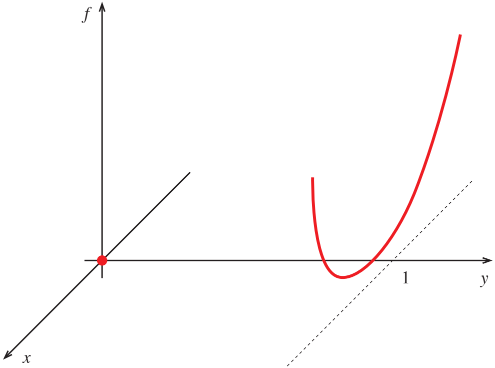
- $f(x,y) = y\, f(x/y)$ is the perspective function of f

The Perspective what?

- $f(x, y) = y\, f(x/y)$ is convex for $y > 0$ if f is
- *epi* $f(x, y)$ is a cone emanating from $(0, 0)$ with the "shape of f"



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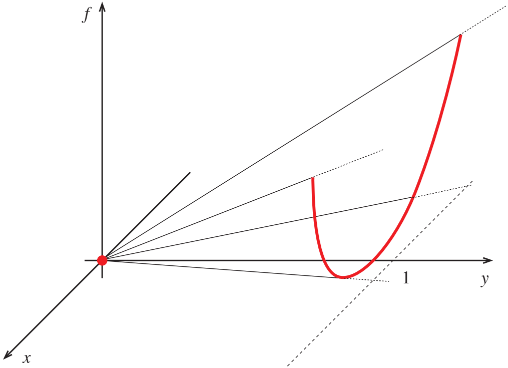

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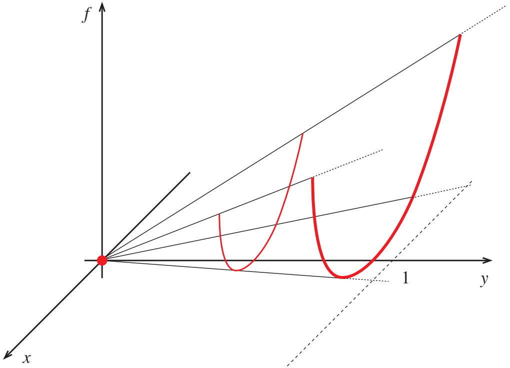

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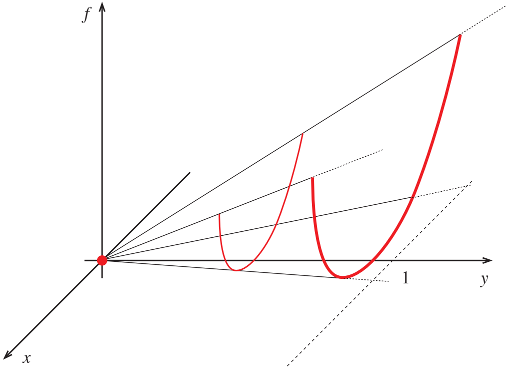

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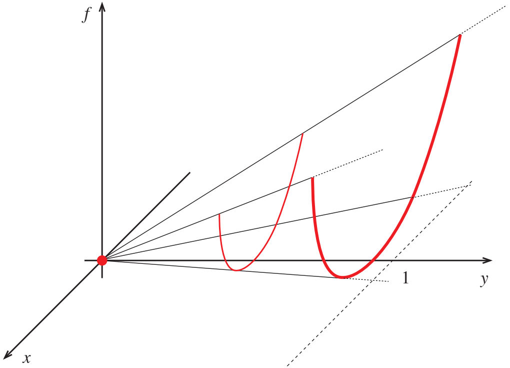

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notes: I) $a/y > a$ for $y < 1$; II) for $a = 0$ nothing happens

The Perspective Reformulation (Relaxation)

- Slightly more general: $Ax \leq b$ compact ($\equiv \{Ax \leq 0\} = \{0\}$), MINLP

$$\min \{ \ f(x) + cy \ : \ Ax \leq by \ , \ y \in \{0,1\} \ \} \tag{11}$$

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- Even better: (13) continuous relaxation of Perspective Reformulation

$$\min \{\ y\,f(x/y) + cy\ :\ Ax \leq by\ ,\ y \in \{0,1\}\ \} \tag{14}$$

$\equiv$ (11) (requires assuming $0f(0/0) = 0$, not really an issue)

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- Linearize the cones in (15)/(16) $\Longrightarrow$ a Semi-Infinite MILP;
  can be done automatically . . . but better done by hand:
  "Perspective Cuts" (don't know why)

Application to SRP-SFSP-DCR

- "Perspectivized" formulation (first two not even strictly necessary):

$$\rho x_{ij} \leq r_{ij} \leq c_{ij} x_{ij} \quad , \quad 0 \leq \theta_{ij} \leq (L/\rho) x_{ij} \quad , \quad \theta_{ij} r_{ij} \geq L x_{ij}^2$$

original variables + a(nother rotated) SOCP constraint

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  original variables + a(nother rotated) SOCP constraint

- Looks much better than

$$0 \leq \theta_{ij} \leq M x_{ij}$$
$$\theta_{ij} \geq s_{ij} - M(1 - x_{ij})$$
$$s_{ij}\, r'_{ij} \geq L \ , \ s_{ij} \geq 0$$
$$0 \leq r'_{ij} \leq r_{ij} + M(1 - x_{ij})$$

  not only better bound, but also fewer variables/constraints

- Is it? Time for computational tests

- Don't even bother with Perspective Cuts, just call a general-purpose MI-SOCP solver

Outline

MI-SOCP models – Cplex

	Cplex P				Cplex bM			
	avg		max		avg		max	
	t	n	t	n	t	n	t	n
abilene	0.011	0.000	0.03	0	0.02	0.03	0.09	1
atlanta	0.015	0.044	0.18	1	0.03	0.07	0.17	1
cost266	0.015	0.017	0.06	1	0.05	0.03	0.26	1
dfn-bwin	0.012	0.000	0.03	0	0.05	0.02	0.11	1
dfn-gwin	0.020	0.151	0.10	1	0.05	0.00	0.16	0
di-yuan	0.051	1.190	0.34	18	0.11	1.36	0.62	31
france	0.014	0.000	0.05	0	0.04	0.02	0.16	1
geant	0.011	0.016	0.06	1	0.03	0.03	0.19	1
germany50	0.024	0.025	0.10	1	0.09	0.06	0.70	1
giul39	0.245	0.547	0.99	13	1.27	15.33	6.68	610
india35	0.021	0.036	0.27	1	0.08	0.07	0.58	4
janos-us	0.093	0.108	0.63	7	0.43	2.65	1.55	30
janos-us-ca	0.141	0.138	0.83	8	0.80	5.76	2.76	243
newyork	0.018	0.034	0.14	1	0.07	0.05	0.28	1
nobel-eu	0.016	0.009	0.08	1	0.04	0.05	0.26	1
nobel-ger	0.011	0.020	0.04	1	0.04	0.08	0.24	3

MI-SOCP models – Cplex (cont.)

nobel-us	0.015	0.083	0.10	1	0.04	0.04	0.19	1
norway	0.035	0.079	0.32	8	0.11	0.36	0.96	8
pdh	0.042	0.444	0.38	8	0.11	0.74	0.38	13
pioro40	0.019	0.039	0.27	1	0.10	0.14	0.57	6
polska	0.020	0.042	0.11	1	0.03	0.08	0.09	1
sun	0.165	0.587	0.89	13	0.65	7.68	2.36	257
ta2	0.020	0.015	0.13	1	0.12	0.08	0.89	4
garr 1999-01	0.022	0.017	0.13	1	0.09	0.21	0.33	1
garr 1999-04	0.029	0.000	0.07	0	0.10	0.07	0.45	3
garr 1999-05	0.029	0.004	0.09	1	0.10	0.08	0.40	3
garr 2001-09	0.030	0.000	0.10	0	0.11	0.10	0.44	3
garr 2001-12	0.029	0.000	0.08	0	0.09	0.16	0.32	3
garr 2004-04	0.028	0.000	0.18	0	0.09	0.05	0.31	3
garr 2009-08	0.087	0.005	0.46	2	0.57	0.47	1.99	27
garr 2009-09	0.089	0.011	0.62	4	0.60	0.61	2.19	36
garr 2009-12	0.090	0.013	0.78	4	0.60	0.59	2.47	44
garr 2010-01	0.093	0.013	0.50	4	0.61	0.57	2.32	32
w1-100-04	1.854	3.176	43.14	85	8.88	164.49	43.87	2585
w1-200-04	24.231	25.366	413.95	4075	231.09	2714.68	9088.54	127429

MI-SOCP models – GUROBI

	GUROBI P				GUROBI bM			
	avg		max		avg		max	
	t	n	t	n	t	n	t	n
abilene	0.011	0.0	0.03	0	0.032	0.1	0.06	3
atlanta	0.012	0.5	0.03	8	0.044	1.6	0.08	15
cost266	0.012	0.4	0.05	11	0.099	0.8	0.30	27
dfn-bwin	0.007	0.0	0.01	0	0.068	0.0	0.08	0
dfn-gwin	0.017	0.0	0.04	0	0.104	0.1	0.31	4
di-yuan	0.028	2.0	0.21	46	0.116	4.9	0.46	74
france	0.011	0.3	0.03	6	0.079	1.2	0.18	17
geant	0.011	0.7	0.04	11	0.062	1.2	0.17	22
germany50	0.016	1.1	0.26	34	0.166	2.5	0.93	52
giul39	0.424	67.6	6.69	1308	1.795	138.5	30.02	2212
india35	0.014	0.4	0.12	14	0.132	1.8	0.34	29
janos-us	0.150	21.2	2.14	767	0.717	85.4	16.54	1168
janos-us-ca	0.285	47.1	7.87	916	1.741	158.4	25.93	1595
newyork	0.013	0.8	0.04	14	0.091	2.2	0.22	22
nobel-eu	0.013	0.2	0.09	9	0.080	0.4	0.25	31
nobel-ger	0.012	0.4	0.04	11	0.056	1.4	0.33	38

MI-SOCP models – GUROBI (cont.)

nobel-us	0.012	0.8	0.05	11	0.047	0.9	0.15	11
norway	0.033	2.8	0.44	30	0.141	7.7	0.63	55
pdh	0.023	4.6	0.09	47	0.081	7.1	0.23	45
pioro40	0.015	0.6	0.09	13	0.160	2.6	0.57	44
polska	0.010	0.5	0.03	7	0.038	1.2	0.06	9
sun	0.189	39.6	0.76	282	0.961	126.9	5.68	583
ta2	0.018	0.6	0.12	27	0.214	1.9	1.52	33
garr 1999-01	0.034	0.5	0.09	9	0.096	6.6	0.38	17
garr 1999-04	0.016	1.9	0.11	26	0.115	2.7	0.55	35
garr 1999-05	0.018	2.0	0.08	25	0.139	3.5	0.79	36
garr 2001-09	0.020	2.0	0.09	19	0.156	4.0	0.97	29
garr 2001-12	0.015	0.0	0.04	0	0.116	0.1	0.31	17
garr 2004-04	0.021	3.0	0.06	14	0.128	3.5	0.57	27
garr 2009-08	0.070	7.6	0.72	124	0.776	18.8	5.39	164
garr 2009-09	0.071	7.6	0.59	202	0.918	21.8	4.85	212
garr 2009-12	0.071	7.6	0.55	123	0.920	22.7	6.21	352
garr 2010-01	0.073	7.6	0.68	114	0.916	22.8	5.76	339
w1-100-04	2.372	159.3	7.09	703	14.064	407.2	110.36	5339
w1-200-04	9.575	241.4	63.37	1395	134.145	637.0	2384.84	10943

3-Pronged Approach

- MI-SOCP approach accurate but slow
  ERA-* approaches fast but inaccurate
- Best of both worlds: 3-pronged approach
  1. run ERA-I, if instance unfeasible terminate
  2. otherwise run ERA-H: if a solution found, report it and terminate
  3. if all else fails, then run model P and report its solution
- So crude, does it really work?

3-Pronged Approach: Experiments

Cplex				GUROBI								
SOCP		3P		SOCP		3P		Gaps		ERA-H		
avg	max	avg	max	avg	max	avg	max	avg	max	avg	max	inf
0.009	0.02	0.001	0.01	0.009	0.02	0.001	0.01	0.00	0.00		0.00	0.06
0.016	0.16	0.001	0.02	0.010	0.03	0.001	0.02	0.00	0.00		0.00	0.07
0.013	0.05	0.002	0.03	0.012	0.04	0.003	0.04	0.00	0.00		0.00	0.17
0.011	0.02	0.000	0.00	0.007	0.01	0.000	0.01	0.00	0.00		0.00	0.00
0.019	0.09	0.000	0.01	0.015	0.04	0.000	0.01	0.00	0.00		0.00	0.02
0.050	0.35	0.017	0.35	0.028	0.22	0.012	0.23	0.00	0.00		0.00	0.12
0.015	0.04	0.000	0.01	0.010	0.03	0.000	0.01	0.00	0.00		0.00	0.02
0.013	0.05	0.001	0.01	0.010	0.04	0.001	0.03	0.00	0.00		0.00	0.06
0.021	0.09	0.005	0.08	0.017	0.24	0.007	0.27	0.00	0.00	7e-5	0.01	0.21
0.254	1.01	0.019	0.66	0.449	7.57	0.087	6.52	0.01	0.57	3e-4	0.01	0.10
0.019	0.25	0.002	0.04	0.016	0.11	0.002	0.07	0.00	0.00		0.00	0.11
0.091	0.62	0.013	0.33	0.153	2.25	0.051	2.19	0.00	0.28	1e-4	0.01	0.18
0.144	0.84	0.026	0.49	0.298	9.59	0.118	7.70	0.01	0.29	2e-4	0.01	0.23
0.017	0.13	0.000	0.02	0.015	0.04	0.001	0.02	0.00	0.00		0.00	0.03
0.014	0.05	0.004	0.05	0.016	0.09	0.005	0.09	0.00	0.00		0.00	0.23
0.010	0.03	0.002	0.03	0.015	0.04	0.002	0.04	0.00	0.00		0.00	0.10

3-Pronged Approach: Experiments (cont.)

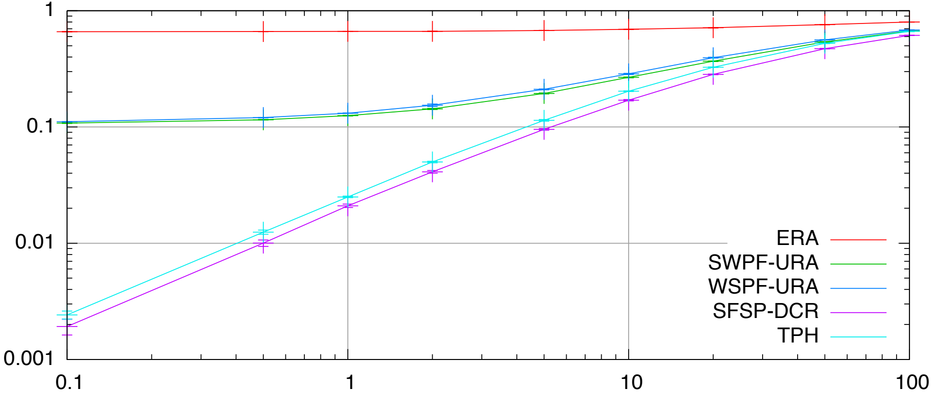
0.013	0.09	0.000	0.00	0.014	0.05	0.000	0.00	0.00	0.00		0.00	0.00
0.032	0.30	0.005	0.25	0.035	0.32	0.005	0.13	0.00	0.00	6e-5	0.01	0.12
0.034	0.30	0.001	0.02	0.026	0.10	0.002	0.10	0.00	0.00		0.00	0.04
0.019	0.27	0.007	0.25	0.018	0.09	0.007	0.09	0.00	0.00	5e-5	0.01	0.25
0.016	0.09	0.000	0.00	0.014	0.03	0.000	0.00	0.00	0.00		0.00	0.00
0.154	0.89	0.006	0.36	0.188	0.87	0.009	0.40	0.01	0.43	2e-4	0.01	0.06
0.019	0.12	0.008	0.05	0.020	0.13	0.009	0.13	0.00	0.00	8e-5	0.01	0.31
0.025	0.12	0.001	0.03	0.035	0.10	0.001	0.03	0.00	0.00	4e-5	0.01	0.02
0.030	0.08	0.022	0.06	0.017	0.12	0.016	0.10	0.00	0.00	4e-5	0.01	0.75
0.028	0.08	0.021	0.06	0.018	0.08	0.016	0.08	0.00	0.00	6e-5	0.01	0.75
0.026	0.09	0.021	0.08	0.022	0.09	0.018	0.09	0.00	0.00	4e-5	0.01	0.74
0.027	0.07	0.022	0.07	0.016	0.04	0.012	0.04	0.00	0.00	4e-5	0.01	0.75
0.026	0.17	0.020	0.05	0.022	0.06	0.019	0.06	0.00	0.00	4e-5	0.01	0.75
0.084	0.44	0.075	0.44	0.069	0.70	0.065	0.71	0.00	0.39	2e-4	0.01	0.85
0.086	0.62	0.078	0.62	0.069	0.56	0.063	0.57	0.00	0.00	2e-4	0.01	0.85
0.088	0.75	0.078	0.73	0.071	0.52	0.061	0.50	0.00	0.24	2e-4	0.01	0.85
0.087	0.46	0.076	0.45	0.074	0.61	0.066	0.59	0.00	0.24	2e-4	0.01	0.85
1.906	46.7	0.034	1.84	2.354	8.35	0.150	3.54	0.01	0.74	2e-3	0.01	0.07
23.660	357.7	0.247	54.29	9.033	63.19	0.399	12.36	0.01	0.81	1e-2	0.02	0.05

Outline

Does it really matter in practice?

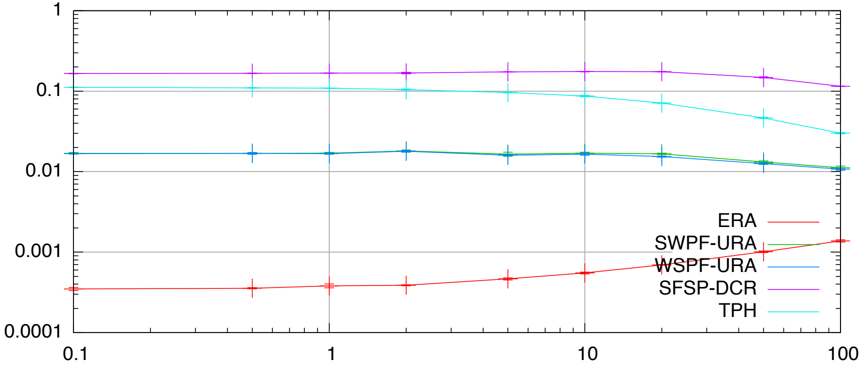
- Simulating the network behavior, large number of path computations
- Exponential interarrival (avg $= \lambda$), exponential duration (avg $=$ 1s)
- $\sigma = 3$ MTU and δ random in $[\, d_{min}\,,\, d_{min} + \beta(d_{max} - d_{min})\,]$
  d_{min} = minimum feasible deadline, d_{max} = delay constraint inactive
- Average of five independent replicas, and 95% confidence intervals
- Comparing all practical approaches known so far (2 new):
  1. ERA (equal rate allocation)
  2. SWPF-URA: shortest-widest-path + optimal (unequal) rate allocation
  3. WSPF-URA: widest-shortest-path + optimal (unequal) rate allocation
  4. SFSP-DCR: MI-SOCP model (perspective version)
  5. TPH: 3-pronged heuristic
- Same real-world topologies, realistic capacities

Simulation results: blocking probability



- ERA fails far too much (allocating the same rate a bad idea)
- both ERA and $*$-URA perform considerably worse than SFSP-DCR
- TPH performs quite close to the optimum
- Similar on all topologies, $\sigma \in \{ 1, 3, 10 \}$MTU, $\beta \in \{ 0.2, 0.5, 1.0 \}$

Simulation results: time



- SFSP-DCR slower but still affordable
- TPH much faster and almost as good
- "large" networks: $|N| = 70+$, $|A| = 230+$
- Path Computation Element makes this technically feasible

Outline

Other Schedulers

- Other practical scheduling protocols:

$$\theta_{ij} = \Big(|P(i,j)| + 1\Big)\frac{L}{w_{ij}} + \frac{L}{r_{ij}^{eff}} \qquad \text{Weakly Rate-Proportional} \qquad (17)$$

$$\theta_{ij} = \left(|P(i,j)| + \frac{\bar{r}_{ij}}{r_{ij}^{min}}\right)\frac{L}{w_{ij}} + \frac{L}{r_{ij}^{eff}} \qquad \text{Frame-Based} \qquad (18)$$

$$\theta_{ij} = 3\frac{2^{\lceil \log_2 w_{ij} L / r_{ij} \rceil}}{w_{ij}} + 2\frac{L}{w_{ij}} \qquad \text{Group-Based} \qquad (19)$$

$$\text{for which} \qquad 3\frac{L}{r_{ij}} + 2\frac{L}{w_{ij}} \leq \theta_{ij} \leq 6\frac{L}{r_{ij}} + 2\frac{L}{w_{ij}} \qquad (20)$$

- SRP $\leq$ WRP $\leq$ FB $\lesssim$ GB (depends on "quantum" $\geq L$ in FB)
- SRP and WRP $O(\log|K|)$, FB and GB $O(1)$ ($\neq$ in practice)

"Bound" Versions

- "Bound" Versions of the Delay Formulæ:

$$\theta_{ij} = \frac{L}{r_{ij}} + |P(i,j)| \frac{L}{w_{ij}} \qquad \text{WRP (21)}$$

$$\theta_{ij} = \frac{L}{r_{ij}} + \left(|P(i,j)| + \frac{w_{ij} - r_{ij}}{\min\{ r_{ij} , r_{ij}^{min} \}} \right) \frac{L}{w_{ij}} \qquad \text{FB (22)}$$

- (3) independent of other flows, convex, SOCP-representable
- (21) $\approx$ (3) but not flow-independent
- (22) (surprisingly) also convex but only for SFSP, less trivial
- No "worst-case" version of (19)/(20)
- All but (3) and (19) not flow-independent $\Longrightarrow$ need admission control

Outline

MI-SOCP Model for WRP-B

- $\theta_{ij} = L/r_{ij} + |P(i,j)|L/w_{ij} \approx (3) \Longrightarrow$ (basically) same model
- But requires access control: not to make existing flows unfeasible

MI-SOCP Model for WRP-B

- $\theta_{ij} = L/r_{ij} + |P(i,j)|L/w_{ij} \approx (3) \Longrightarrow$ (basically) same model
- But requires access control: not to make existing flows unfeasible
- Delay slack:
$$\bar{\delta}^k = \delta^k - \frac{\sigma^k}{r^k_{min}} - \sum_{(i,j)\in P^k} \left(\frac{L}{r^k_{ij}} + |P(i,j)|\frac{L}{w_{ij}} + l_{ij} + n_i \right)$$
- Access control constraint, one for each $k \in K$
$$\sum_{(i,j)\in P^k} \frac{L}{w_{ij}} x_{ij} \leq \bar{\delta}^k$$
$|P(i,j)|$ increases by one in all (i,j) that the new path traverses
- Can be used to "preprocess away" some arcs
- The coefficients are the same for each flow, can use path (+ RHS) dominance to detect redundant ones
- Still, possibly many constraints ($|K| \approx n^2$)

Modeling FB-B

- $\theta_{ij} = L/r_{ij} + (\,|P(i,j)| + \phi(r_{ij})\,)L/w_{ij}$ ($\approx$ WRP), where

$$\phi(r) = (w_{ij} - r)/\min\{\, r\,,\; r_{ij}^{min}\,\}$$

Modeling FB-B

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$$\phi(r) = (w_{ij} - r)/\min\{\, r\,,\, r_{ij}^{min}\,\}$$

- Since r_{ij}^{min} is fixed, can be rewritten as

$$\phi(r) = \begin{cases} \phi_1(r) = w_{ij}/r - 1 & \text{if } 0 < r \leq r_{ij}^{min} \\ \phi_2(r) = (w_{ij} - r)/r_{ij}^{min} & \text{if } r_{ij}^{min} \leq r \leq c_{ij}\,(\leq w_{ij}) \end{cases}$$

- Convex!: both ϕ_1 and ϕ_2 are, and $\phi_1'(r_{ij}^{min}) \leq \phi_2'(r_{ij}^{min})$

Modeling FB-B

- $\theta_{ij} = L/r_{ij} + (\,|P(i,j)| + \phi(r_{ij})\,)L/w_{ij}$ ($\approx$ WRP), where

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- Convex!: both ϕ_1 and ϕ_2 are, and $\phi'_1(r_{ij}^{min}) \leq \phi'_2(r_{ij}^{min})$
- Could use the classical variable splitting reformulation

$$\phi(r) = \phi_1(r') + \phi_2(r'' + r_{ij}^{min}) - \phi(r_{ij}^{min}) \qquad \text{s.t.}$$

$$0 \leq r'_{ij} \leq r_{ij}^{min},\;\; 0 \leq r''_{ij} \leq (c_{ij} - r_{ij}^{min}),\;\; r = r' + r''$$

- But we can do better, because $\phi(r) = \max\{\,\phi_1(r)\,,\;\phi_2(r)\,\}$

MI-SOCP Model for FB-B and GB

- Can use the "cutting planes" representation of ϕ

$$v \geq \phi_1(r) = w_{ij}/r - 1 \quad , \quad v \geq \phi_2(r) = (w_{ij} - r)/r_{ij}^{min}$$

- Formulation (recall the L/w_{ij} factor):

$$\theta_{ij} = v_{ij} + v'_{ij} + \frac{L}{w_{ij}}(|P(i,j)| + 1)x_{ij}$$

$$v_{ij} r_{ij} \geq L x_{ij}^2 \ , \quad v_{ij} \geq 0$$

$$v'_{ij} \geq v_{ij} - L/w_{ij}$$

$$v'_{ij} \geq (L/r_{ij}^{min})x_{ij} - L r_{ij}/(w_{ij} r_{ij}^{min})$$

only one extra conic constraint, two extra variables

- Note the $x_{ij} \cdot w_{ij}/r_{ij}^{min}$ in ϕ_2: otherwise, $v'_{ij} \geq L/r_{ij}^{min}$ even if $x_{ij} = 0$
- GB: obvious from SRP using (20) (we use "optimistic" factor 3).

Admission Control for FB-B

- "Abstract" admission control constraint for FB: same $\bar{\delta}^k$ as WRP,

$$\sum_{(i,j)\in P^k} \frac{L}{w_{ij}} \left(x_{ij} + \frac{w_{ij} - r_{ij}^k}{\min\{\, r_{ij}\,,\, r_{ij}^{min}\,\}} \right) \leq \bar{\delta}^k$$

$|P(i,j)| \mathrel{+}= 1$, plus r_{ij}^{min} decreases $\iff r_{ij} \leq r_{ij}^{min}$

Admission Control for FB-B

- "Abstract" admission control constraint for FB: same $\bar{\delta}^k$ as WRP,

$$\sum_{(i,j)\in P^k} \frac{L}{w_{ij}} \left(x_{ij} + \frac{w_{ij} - r^k_{ij}}{\min\{\, r_{ij} \,,\, r^{min}_{ij} \,\}} \right) \leq \bar{\delta}^k$$

$|P(i,j)| \mathrel{+}= 1$, plus r^{min}_{ij} decreases $\iff r_{ij} \leq r^{min}_{ij}$

- Extra term $(w_{ij} - r^k_{ij})/r_{ij}$, but only if $r_{ij} \leq r^{min}_{ij}$; otherwise, constant term $(w_{ij} - r^k_{ij})/r^{min}_{ij} \quad \Longrightarrow$

$$\sum_{(i,j)\in P^k} \frac{L}{w_{ij}} \Big(x_{ij} + (w_{ij} - r^k_{ij}) z_{ij} \Big) \leq \bar{\delta}^k$$

$$s_{ij} \leq r_{ij} \;\;,\;\; s_{ij} \leq r^{min}_{ij} \;\;,\;\; s_{ij} z_{ij} \geq x^2_{ij} \;\;,\;\; z_{ij} \geq 0$$

$+2|A|$ variables, $+|A|$ conic constraints but shared among flows

- Different coefficients (to share the z_{ij}), dominance more difficult
- Arc-based preprocessing still possible (using $r_{ij} = c_{ij}$)

MI-SOCP Model for SRP-W

- Distinguish the "empty" arcs: $A'' = \{ (i,j) \, : \, P(i,j) \neq \emptyset \}$
- To model (2), just use (1) to get

$$\theta_{ij} = \frac{L}{w_{ij}} + \begin{cases} \dfrac{L}{w_{ij}} \left(\dfrac{\bar{r}_{ij}}{r_{ij}} + 1 \right) & \text{if } (i,j) \in A'' \\ 0 & \text{otherwise} \end{cases} \tag{23}$$

- The "empty" arcs have constant latency
- For the others: yet a different constant + same "$1/r_{ij}$" form

$$t + \sum_{(i,j) \in A} \bar{l}_{ij} x_{ij} + \sum_{(i,j) \in A''} \left(\frac{\bar{r}_{ij}}{w_{ij}} v_{ij} + \frac{L}{w_{ij}} x_{ij} \right) \leq \delta \tag{24}$$

Admission Control for SRP-W

- The arrival of the new flow does impact the delay formula
- $\bar{r}_{ij}^{-q} = \sum_{q \in P(i,j) \setminus \{p\}} r_{ij}^q$ = rates of existing flows except p: (23) reads

$$\theta_{ij}^p = \frac{L}{w_{ij}} \left(\frac{\bar{r}_{ij}^{-q} + r_{ij}}{r_{ij}^p} + 2 \right) \quad ,$$

- New flow pass from $(i,j) \Longrightarrow$ increase in latency

$$\Delta\theta_{ij}^p = \begin{cases} \dfrac{L\, r_{ij}}{w_{ij}\, r_{ij}^p} & \text{if } |P(i,j) \setminus \{p\}| > 0 \\ \dfrac{L}{w_{ij}} \left(\dfrac{r_{ij}}{r_{ij}^p} + 1 \right) & \text{otherwise} \end{cases} \quad .$$

- Admission control constraint (linear)

$$\sum_{(i,j) \in p} \frac{L}{w_{ij}\ r_{ij}^p} r_{ij} + \sum_{(i,j) \in p'} \frac{L}{w_{ij}} x_{ij} \leq \bar{\delta}^p \tag{25}$$

for usual "delay slack" $\bar{\delta}^p$ of flow p

MI-SOCP Model for WRP-W

- Delay formula for WRP-W

$$\theta_{ij} = |P(i,j)|\frac{L}{w_{ij}} + \frac{L}{w_{ij}}\left(\frac{\bar{r}_{ij}}{r_{ij}} + 1\right) \tag{26}$$

which easily gives

$$t + \sum_{(i,j)\in A}\left[\frac{\bar{r}_{ij}}{w_{ij}}v_{ij} + \left(|P(i,j)|\frac{L}{w_{ij}} + \bar{l}_{ij}\right)x_{ij}\right] \leq \delta \tag{27}$$

- Increase in latency for an existing flow p

$$\Delta\theta^p_{ij} = \frac{L}{w_{ij}}\left(\frac{r_{ij}}{r^p_{ij}} + 1\right) \tag{28}$$

- Properly defined $\bar{\delta}^p$, admission control constraint

$$\sum_{(i,j)\in p}\frac{L}{w_{ij}}\left(\frac{r_{ij}}{r^p_{ij}} + x_{ij}\right) \leq \bar{\delta}^p \tag{29}$$

once again linear

MI-SOCP Model for FB-W

- Worst-case FB formula is

$$\theta_{ij} = \frac{L}{w_{ij}} \left[\frac{\bar{r}_{ij}}{\min\{\, r_{ij} \,,\, r_{ij}^{min} \,\}} + \frac{\bar{r}_{ij}}{r_{ij}} + |P(i,j)| + 1 \right] \quad , \qquad (30)$$

i.e., the same as (26) plus the extra term

$$\frac{L}{w_{ij}} \frac{\bar{r}_{ij}}{\min\{\, r_{ij} \,,\, r_{ij}^{min} \,\}} \quad . \qquad (31)$$

- Same tricks as above give

$$t + \sum_{(i,j) \in A} \left(|P(i,j)| \frac{L}{w_{ij}} + \bar{l}_{ij} \right) x_{ij} + \sum_{(i,j) \in A''} \frac{\bar{r}_{ij}}{w_{ij}} (s_{ij} + v_{ij}) \leq \delta$$

$$s_{ij} \geq v_{ij} \quad , \quad s_{ij} \geq L / r_{ij}^{min} \quad , \quad (i,j) \in A''$$

Admission Control for FB-W

- Latency increase = "WRP part" of the latency formula + (31)
- First leads to delay increase (28)
- Second is

$$\sum_{(i,j)\in p} \frac{L}{w_{ij}} \left(\frac{\bar{r}_{ij}^{-p} + r_{ij}}{\min\{\, r_{ij} \,,\, r_{ij}^{min} \,\}} + \frac{r_{ij}}{r_{ij}^{p}} + x_{ij} \right) \leq \bar{\delta}^{p} \qquad (32)$$

- All in all

$$\sum_{(i,j)\in p} \frac{L}{w_{ij}} \left(z_{ij} + \frac{r_{ij}}{r_{ij}^{p}} + x_{ij} \right) \leq \bar{\delta}^{p} \qquad (33)$$

$$z_{ij} \geq (\bar{r}_{ij} + r_{ij})/r_{ij}^{min} \;\;,\;\; z_{ij} \geq \bar{r}_{ij} v_{ij}/L + 1 \qquad (i,j) \in p \qquad (34)$$

Outline

Computational results for all the models

	Mean Time							Max Time						
	S-B	GRB	W-B	F-B	S-W	W-W	F-W	S-B	GRB	W-B	F-B	S-W	W-W	F-W
abilene	0.002	0.002	0.002	0.001	0.002	0.002	0.002	0.01	0.01	0.01	0.01	0.01	0.01	0.01
atlanta	0.004	0.003	0.004	0.003	0.002	0.002	0.002	0.03	0.01	0.02	0.02	0.01	0.01	0.01
cost266	0.004	0.004	0.004	0.004	0.001	0.001	0.002	0.02	0.01	0.02	0.02	0.01	0.01	0.01
Dfn-bwin	0.003	0.003	0.007	0.003	0.002	0.002	0.002	0.01	0.01	0.03	0.02	0.01	0.01	0.01
Dfn-gwin	0.007	0.005	0.010	0.005	0.002	0.002	0.003	0.03	0.03	0.05	0.03	0.01	0.01	0.01
Di-yuan	0.014	0.007	0.014	0.009	0.004	0.004	0.004	0.10	0.03	0.08	0.06	0.02	0.02	0.02
france	0.005	0.005	0.006	0.004	0.002	0.002	0.002	0.02	0.01	0.03	0.02	0.01	0.01	0.01
geant	0.004	0.004	0.004	0.003	0.002	0.002	0.002	0.02	0.01	0.02	0.02	0.01	0.01	0.01
ger50	0.007	0.007	0.007	0.006	0.002	0.002	0.002	0.03	0.05	0.05	0.03	0.02	0.02	0.02
giul39	0.097	0.075	0.132	0.051	0.077	0.078	0.068	0.49	0.35	1.51	0.35	0.72	0.74	0.50
india35	0.007	0.007	0.009	0.006	0.002	0.002	0.003	0.11	0.03	0.06	0.04	0.02	0.02	0.02
Janos-us	0.038	0.033	0.050	0.033	0.025	0.026	0.023	0.23	0.14	0.28	0.22	0.36	0.37	0.19
Janos-ca	0.059	0.055	0.085	0.058	0.036	0.036	0.033	0.40	0.25	0.45	0.49	0.49	0.48	0.32
newyork	0.006	0.005	0.007	0.004	0.002	0.002	0.003	0.04	0.02	0.03	0.02	0.02	0.02	0.01
Nobel-eu	0.005	0.004	0.005	0.004	0.002	0.002	0.002	0.02	0.01	0.03	0.03	0.01	0.01	0.01
Nobel-ge	0.003	0.003	0.003	0.002	0.002	0.002	0.002	0.01	0.01	0.01	0.01	0.01	0.01	0.01
Nobel-us	0.003	0.003	0.003	0.002	0.001	0.001	0.001	0.02	0.02	0.01	0.01	0.01	0.01	0.01
norway	0.011	0.008	0.015	0.008	0.005	0.005	0.005	0.11	0.05	0.15	0.08	0.06	0.06	0.06
pdh	0.009	0.006	0.012	0.008	0.004	0.004	0.005	0.06	0.06	0.06	0.05	0.02	0.02	0.03

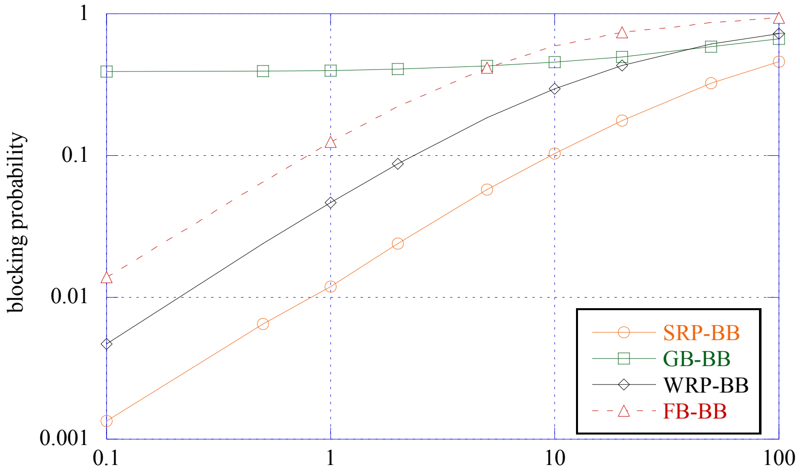
Computational results for all the models (cont.d)

	Mean Time							Max Time						
	S-B	GRB	W-B	F-B	S-W	W-W	F-W	S-B	GRB	W-B	F-B	S-W	W-W	F-W
garr9901	0.008	0.009	0.010	0.006	0.006	0.006	0.007	0.04	0.03	0.05	0.04	0.03	0.03	0.03
garr9904	0.008	0.008	0.011	0.010	0.003	0.003	0.003	0.03	0.04	0.06	0.07	0.03	0.03	0.02
garr9905	0.008	0.007	0.010	0.009	0.003	0.003	0.003	0.03	0.04	0.07	0.06	0.02	0.02	0.02
garr0109	0.008	0.009	0.009	0.009	0.003	0.003	0.003	0.03	0.04	0.05	0.04	0.02	0.02	0.02
garr0112	0.008	0.007	0.010	0.009	0.003	0.003	0.003	0.03	0.03	0.06	0.06	0.02	0.02	0.02
garr0404	0.007	0.007	0.010	0.010	0.003	0.003	0.003	0.05	0.03	0.04	0.06	0.06	0.06	0.06
garr0908	0.031	0.032	0.033	0.057	0.009	0.009	0.010	0.17	0.12	0.22	0.39	0.27	0.29	0.26
garr0909	0.032	0.036	0.034	0.067	0.009	0.009	0.009	0.18	0.14	0.23	0.45	0.26	0.25	0.18
garr0912	0.036	0.042	0.038	0.068	0.010	0.010	0.010	0.28	0.16	0.26	0.49	0.29	0.30	0.21
garr1001	0.033	0.039	0.036	0.067	0.010	0.009	0.009	0.19	0.15	0.25	0.45	0.26	0.26	0.19
w-100-L3	0.663	0.308	0.982	0.193	0.505	0.508	0.474	12.67	5.04	18.26	3.55	3.77	3.86	3.55
w-100-L2	0.619	0.311	0.585	0.001	0.488	0.483	0.490	12.21	4.19	12.34	0.01	3.68	3.55	3.92

- In a nutshell: not significantly different (as running time goes)
- Sometimes funny things happen: more complex models but "worse" schedulers $\Longrightarrow$ more failures $\equiv$ faster

Outline

Simulation results: bound versions



- GB fails far too much (that factor of 3 kills it)
- As expected, SRP (significantly) better than WRP better than FB
- One would expect the worst-case formulæ to further improve ...

The Mystery of Worst-Case Formulæ

- ... but does not happen: worst-case formulæ as bad as bound ones
- True also at low loads where $r^{eff} \gg r$: should not happen
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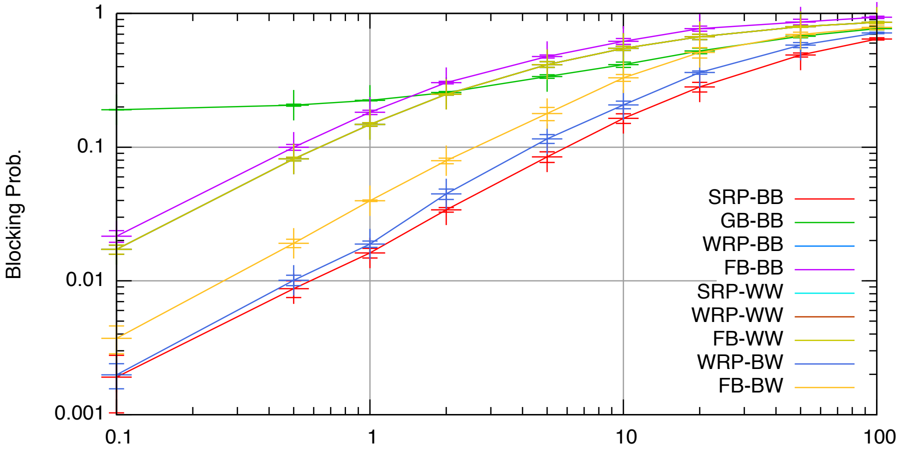
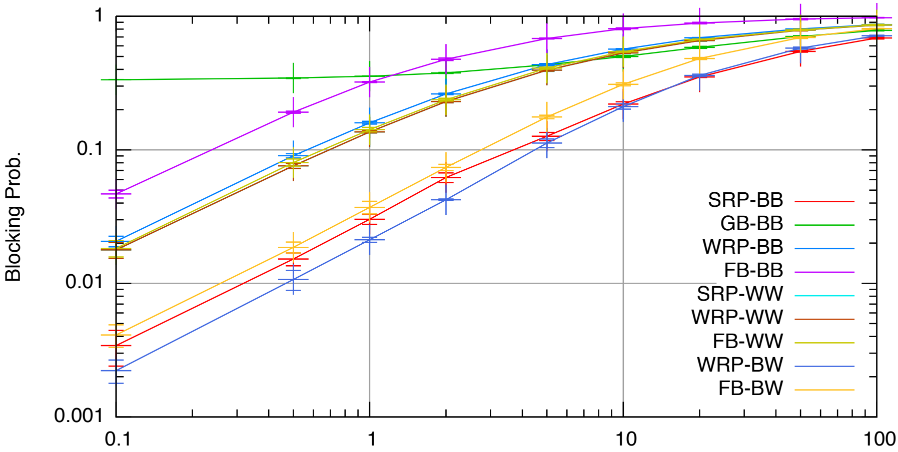
- Bound formulæ over-provision a little, so that new flows can survive

Simulation results: hybrid versions



- BW (much) better than WW, WW (slightly, if any) better than BB
- WRP-BW not much worse than SRP(-BB) (note: ∄ SRP-WB)

Simulation results: hybrid versions better than SRP



- Actually, WRP-BW can be better than SRP(-BB)
- My take: still ways to improve on DCR
- Need a better proxy of future network performances!

Outline

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  - the domain experts are happy, the results are good
  - it was easy: "just write the model and solve it"
  - "everything is convex" because "a lot of things are fixed"
  - SDN is all the rage, a "fat" Path Computation Element is OK
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    10 years ago it would have been a different story
  - we just happened to know the right modeling trick(s)
- Our only merit: we fearlessly took the nonlinear bull by the horns

Conclusions II

- The world is indeed nonlinear, but is can be nicely convex/SOCP
- DCR: interesting generalization of classical "steady state" flows
- MI-SOCP with substantial network structure =
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  - MINLP techniques useful (Perspective Reformulation, SOCP, . . .)
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- Lots of fun. Join in! :-)

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References

Scheduling Algorithms, Network Calculus

Cruz "A calculus for network delay" (part I/part II), *IEEE Trans. on Information Theory*, 1991

Zhang "Service Disciplines for Guaranteed Performance Service in Packet-Switching Networks", *Proceedings of the IEEE*, 1995

Shreedhar, Varghese "Efficient Fair Queueing Using Deficit Round Robin", *IEEE Trans. on Networking*, 1996.

Stiliadis, Varma "Latency-Rate Servers: A General Model for Analysis of Traffic Scheduling Algorithms", *IEEE Trans. on Networking*, 1998

Second-Order Cone Programs

Ben Tal "Lecture Notes of the Course on Conic and Robust Optimization", 2002

References

Perspective stuff

F., Gentile "Perspective Cuts for a Class of Convex 0-1 Mixed Integer Programs", *Math. Prog.*, 2006

Günlük, Linderoth "Perspective reformulations of MINLPs with indicator variables", *Math. Prog.*, 2010

Hijazi, Bonami, Cornuejols, Ouorou "Mixed integer nonlinear programs featuring "On/Off" constraints: Convex analysis and applications" *Electronic Notes in Discrete Mathematics*, 2010

DCR

Orda "Routing with end-to-end QoS guarantees in broadband networks", *IEEE Trans. on Networking*, 1999

F., Galli, Scutellà "Delay-Constrained Shortest Paths: Approximation Algorithms and Second-Order Cone Models", *JOTA*, 2014

F., Galli, Stea "Optimal Joint Path Computation and Rate Allocation for Real-time Traffic", *The Computer Journal*, 2014