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Dual-Ascent Methods and Multicommodity Flow Problems

Antonio Frangioni

Abstract. Lagrangean Duality provides an attractive way for exploiting the "easy part" of structured problems with "complicating" constraints: however, it calls for the (approximate) solution of large-scale, difficult NonDifferentiable Optimization problems. In the first part of the thesis, we study the theoretical and practical issues arising in the application of Bundle methods to the solution of Lagrangean Duals, showing how these methods are related with other approaches and giving some new contributions, such as: a new convergence proof where, rather than proving the correctness of any single implementation, we give general convergence conditions; a new algorithm for the solution of the Quadratic Programming (tentative ascent direction finding) subproblem that is solved at each step, whose effectiveness is demonstrated by a computational study; a new class of Bundle algorithms using (LP) subproblems; the extension to this new class of the convergence proof and the treatment of constrained and/or "separable" maximization; two new "Long Term" rules for on-line management of the trust region parameter, that seem to avoid the "stalling" on difficult problems. In the second part of the thesis, these methods are used for the solution of Multicommodity Flow problems: a (QP)-based Bundle code is tested against several other codes and on many classes of problems, proving to be competitive especially on instances with "many" commodities; the code is then parallelized, obtaining good efficiencies on the classes of instances where it is more effective in sequential. However, three other new Dual-Ascent approaches are also proposed for the problem, and preliminary computational reults are reported for one of them; on the other side, codes based on the Bundle technology are shown to be competitive w.r.t. standard Subgradient algorithms for rapidly obtaining lower bounds for two network-structured, difficult integer problems.

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ADDR: Dipartimento di Informatica, Corso Italia 40, 56125 Pisa, Italy • FAX: +39 50 887226
TEL: +39 50 887208 • E-MAIL: frangio@di.unipi.it • WEB: http://www.di.unipi.it/~frangio

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Foreword

Five years ago, before my doctorate began, Paolo Carraresi asked me to join Maddalena Nonato and himself in the development of an efficient code for the solution of the Lagrangean Dual of a large-scale, real-world scheduling problem: although the aim of the project was essentially practical, it was Paolo's style always to try keep practical effectiveness and theoretical significance together, and following the advice of Manlio Gaudioso we began investigating the properties of Bundle methods for NonDifferentiable Optimization. This line of work, where the computational results from practical applications lead to new theoretical investigations and vice-versa, was initially considered a "side" research, but nevertheless produced an amount of practical experiences and codes that later formed the basis of most of the applicative results of this work.
Meantime, my supervisor Giorgio Gallo draw my attention to continuous Multicommodity Flow problems: in this case also, the interest was for the practical applications of the model (especially in transportation and scheduling) as well as for its theoretical properties and possible advances in the algorithmic approaches. The focus of the research was from the beginning on Dual Ascent methods, for at least two reasons: the first was the expertise already present in our research group about dual methods for network problems, and the second the clear possibility of applying tools and methodologies from the developing NDO research to this area.
It wasn't without a certain surprise that we discovered how an almost "direct" application of the Bundle technology to Multicommodity Flow problems could result in efficient sequential and parallel algorithms: on the other hand, our understanding of Bundle methods (and our codes) greatly benefited of the research about Dual Ascent approaches to Multicommodity problems, that served as motivations and tests for most of the ideas that finally found their way in the practical Bundle codes and their accompanying theorems.

Hence, the structure of this thesis will try to reflect the historical and conceptual development of the material: to a more "methodological" Part I, dedicated to Bundle algorithms for NonDifferentiable Optimization with a particular stress on polyhedral functions, will follow a more "applicative" Part II, dedicated to (especially, but not only, Bundle-type) Dual Ascent approaches to Multicommodity Flow problems. However, neither the first part is only theoretical, nor the second one is exclusively applications-oriented: both will try to cover both the perspectives, in an attempt to keep the balance between the theoretical significance of a research and its "validation" through computational results that is one of the main characteristic of Operation Research as a discipline. Furthermore, the two parts are by no means to be considered separately from one another: rather, this work is perhaps another proof of the benefits that can be obtained by integrating approaches and methodologies from seemingly far away research areas, in this case NDO and Network Flows, that is typical of the variegated OR world.

I would like to end this introduction with two further remarks: the first is that these lines of research are not exhausted, on the contrary we now have (for both of them) more ideas and directions for future research than at the beginning. The second is that this thesis is not the only - and maybe not even the most important - outcome of these years of work: a large effort has been devoted to the development of efficient and flexible codes for the various problems under study. State-of-the-art codes are necessary if the practical significance of an approach is to be evaluated, as careless implementations may easily lead to biased results: even more important, they serve as a basis for future developments, since real-world difficult problems almost always call for the efficient solution of subproblems of some kind. For all of these reasons, some care will be taken in the following to describe the issues related to the actual implementation of the proposed approaches: the codes and the test problems described in this work will be made available to the academic community, in the hope of contributing to the development of the research in these areas.

Throughout the thesis, the following standard notations are adopted: symbols are written in *italic*, vectors are intended as columns and denoted by **lowercase bold** letters, matrices are denoted by **UPPERCASE BOLD** letters. The scalar product between $\boldsymbol{v}$ and $\boldsymbol{w}$ is denoted by $\boldsymbol{v}\boldsymbol{w}$ or $\boldsymbol{v}^T\boldsymbol{w}$, where $\boldsymbol{A}^T$ is the notation for the transpose of $\boldsymbol{A}$, according to the convenience; without any further specification, $\| \boldsymbol{v} \|$ stands for $(\boldsymbol{v}\boldsymbol{v})^{1/2}$, the Euclidean norm of the vector $\boldsymbol{v}$, but $\|\cdot\|_2$ is used when confusion may arise with

$$\| \boldsymbol{v} \|_1 = \Sigma_i \, | \, v_i \, | \qquad \text{and} \qquad \| \boldsymbol{v} \|_\infty = \max_i \{ \, | \, v_i \, | \, \}.$$

Given a problem

$$\text{(P)} \qquad \min \, [\max] \, \{ \, f(\boldsymbol{x}) : \boldsymbol{x} \in X \, \}$$

where $f()$ is a real-valued function, $z($ P $)$ denotes the optimal value of $f()$: following the customary assumption that the min [max] over an empty set is $+\infty$ $[-\infty]$, $X = \emptyset \Rightarrow z($ P $) = +\infty$ $[-\infty]$. If (some of) the variables $\boldsymbol{x}$ of (P) are restricted to a belong to $\boldsymbol{Z}^n$, the continuous relaxation of (P) where such integrality consraints are removed will be denoted by ($\underline{\text{P}}$).

Contents

Part I: Bundle Methods

Part II: Multicommodity Flows

Such also appeared the problem of human life. [...] A period [...] would then be a succession of random tentative solutions, all insufficient and, if taken individually, possibly wrong, from which, if Humanity would be able to resume, could finally follow the complete and exact solution.

R. Musil - *Der Mann ohne Eigenshaften*