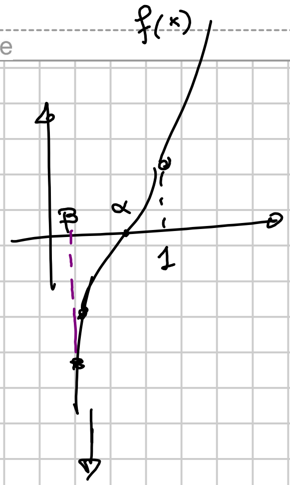


crescente crescente

1) $f(x) = \ln(x) + 2x^2 + 1$

$\alpha < 1 \qquad f(1) = 0 + 2 + 1 = 3$

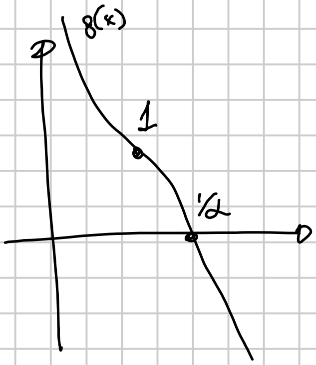
$f'(x) = \frac{1}{x} + 4x > 0 \quad \forall x \in \mathbb{R}^+ \Rightarrow f$ crescente

$\lim_{x \to 0} f(x) = \lim_{x \to 0} -\infty + 0 + 1 = -\infty$

Quindi esiste $\beta > 0$ t.c. $f(\beta) < 0$

$\Rightarrow (\beta, 1)$ è un intervallo di separazione per f

$\Rightarrow \exists\, \alpha$ t.c. $f(\alpha) = 0$, ed è unico perché $f'(x) > 0$



2) $g(x) = f(1/x) \qquad g(1) = f(1) > 0$

$g'(x) = \frac{d}{dx} f\left(\frac{1}{x}\right) = \frac{d}{dx} \ln\frac{1}{x} + \frac{2}{x^2} + 1 = \frac{d}{dx}(-\ln x) + \frac{2}{x^2} + 1$

$= -\frac{1}{x} - \frac{4}{x^3} < 0 \qquad \forall x \in \mathbb{R}^+$

$\lim_{x \to +\infty} g(x) = \lim_{x \to \infty} \ln\frac{1}{x} + \frac{2}{x^2} + 1 = -\infty + 0 + 1 = -\infty$

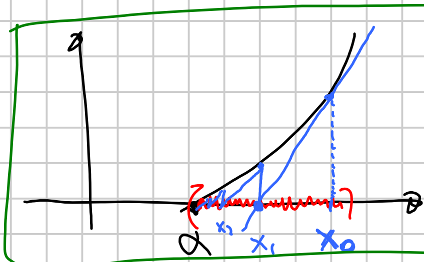
$\lim_{x \to 0^+} g(x) = \lim_{x \to 0^+} \ln\frac{1}{x} + \frac{2}{x^2} + 1 = +\infty + \infty + 1 = +\infty$

$f(\alpha) = 0 \Leftrightarrow g\left(\frac{1}{\alpha}\right) = 0$

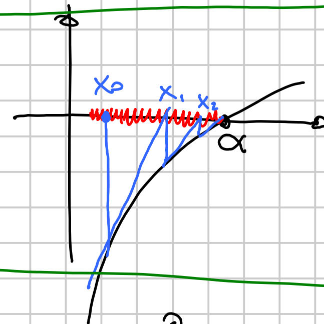
Teoremi di convergenza di Newton:

1) Teo. convergenza locale: $f(\alpha) = 0$, $f'(\alpha) \neq 0$, $f \in C^2 \Rightarrow \exists\, I_\alpha = [\alpha - \rho, \alpha + \rho]$ t.c. il metodo di Newton converge localmente per $x_0 \in I_\alpha$

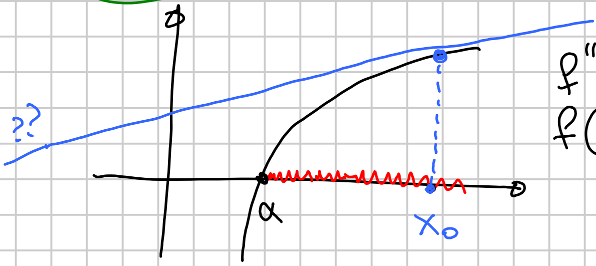
2) Teo. conv. "in grande": se ho un intervallo $I = [a, \alpha)$ oppure $(\alpha, b]$ di cui α è l'estremo destro oppure sinistro, tale che $f(x) f''(x) \geq 0$, $f'(x) \neq 0 \quad \forall x \in I$, allora il metodo di Newton converge per $x_0 \in I$



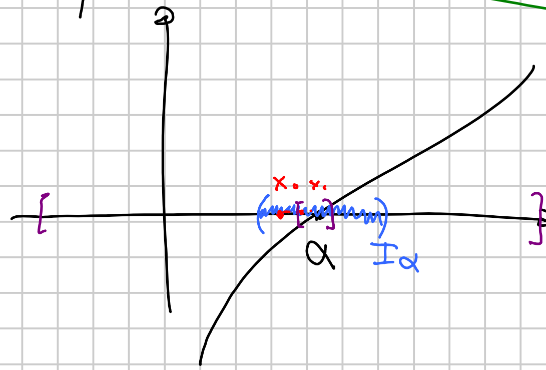
$f''(x)>0$
$f(x)>0$



$f''(x)<0$
$f(x)<0$

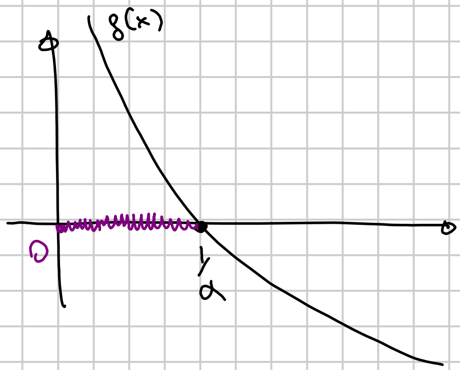


$f''(x)<0$
$f(x)>0$



$$g'(x) = -\frac{1}{x} - \frac{4}{x^3} = -x^{-1} - 4x^{-3}$$

$$g''(x) = -(-1)x^{-2} - 4\cdot(-3)x^{-4} = x^{-2} + 12x^{-4} > 0 \quad \forall x \in \mathbb{R}^+$$



Per ogni intervallo $I = [a, \frac{1}{\alpha})$, vale il teo. di convergenza in grande, e il metodo di Newton converge per $x_0 \in I$

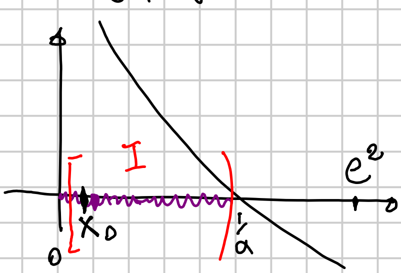
Ma $(0, \frac{1}{\alpha}]$ non è di questa forma!

Se $x_0 = \frac{1}{\alpha}$, allora $x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = \frac{1}{\alpha} - \frac{\overbrace{g(\frac{1}{\alpha})}^{=0}}{g'(\frac{1}{\alpha})} = \frac{1}{\alpha}$, $x_2 = x_3 = x_4 = \dots = \frac{1}{\alpha}$

Dobbiamo dimostrare che il metodo converge per ogni $x_0 \in (0, \frac{1}{\alpha}]$:

Se $x_0 = \frac{1}{\alpha}$, già dimostrato. Se $0 < x_0 < \frac{1}{\alpha}$, scelgo $I = [x_0, \frac{1}{\alpha})$

(oppure $I = [\frac{x_0}{2}, \frac{1}{\alpha})$)



e applico il teorema ⇒ il metodo di Newton converge per quel valore dato di x_0 ✓

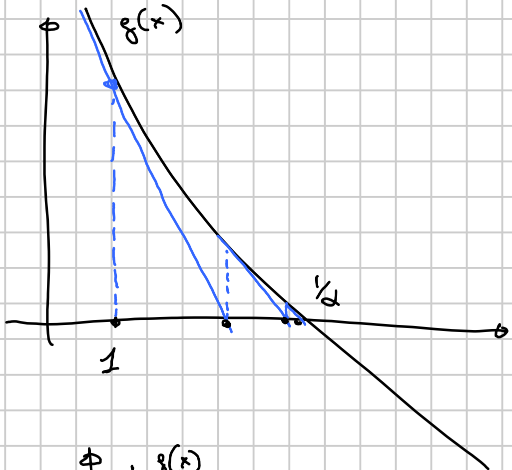
tol	num. it.	
10^{-4}	5	5
10^{-8}	6	10
10^{-12}	7	15

tol	num. it
10^{-4}	7
10^{-8}	8
10^{-12}	8

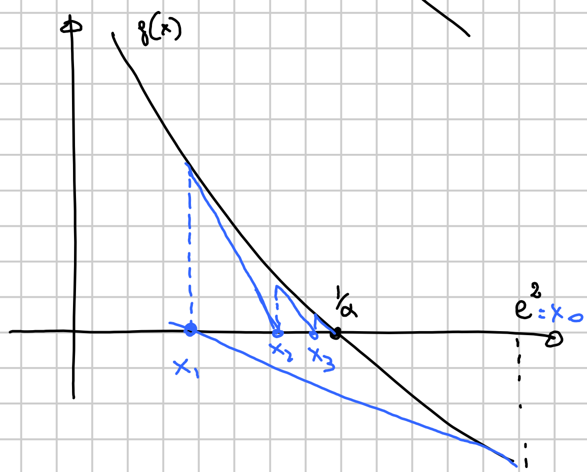
$g(x_7) = 10^{-6}$ $\quad$ $g(x_8) = 10^{-13}$

---

per $x_0 = 1$



per $x_0 = e^2$



$g(x) = -\ln x + \frac{2}{x^2} + 1$

$g'(x) = -\frac{1}{x} - \frac{4}{x^3}$

Quanto vale x_1?

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)} = e^2 - \frac{-2 + \frac{2}{e^4} + 1}{-\frac{1}{e^2} - \frac{4}{e^6}} = e^2 + \frac{-e^6 + 2e^2}{e^4+4} = \frac{\cancel{e^6} + 4e^2 - \cancel{e^6} + 2e^2}{e^4+4}$$

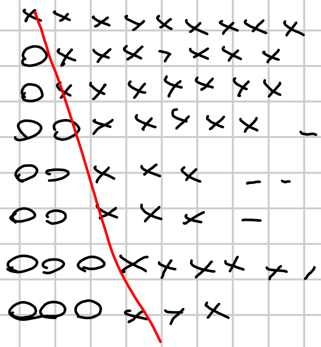
$$= \frac{6e^2}{e^4+4}$$

$2.7 < e < 2.8$

R: Si calcola che $x_1 = 0.75.. < \frac{1}{\alpha}$, quindi la successione $x_1, x_2, x_3 \dots$ converge a $\frac{1}{\alpha}$ per il teorema di conv. in grande (come già dimostrato sopra).

Punto fisso

conv. lineare



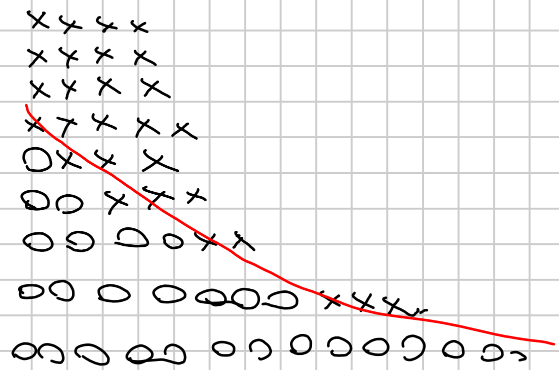
$\Updownarrow$

$|\beta - x_k| \le \lambda^k \rho$

$$\frac{|\beta - x_{k+1}|}{|\beta - x_k|} \le C$$

Newton

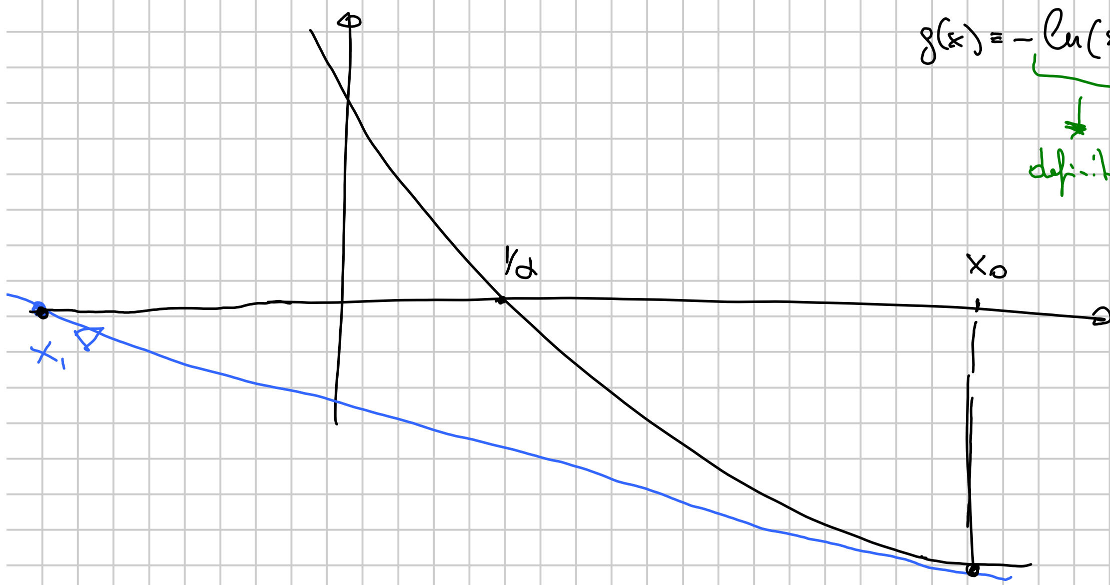
conv. quadratica



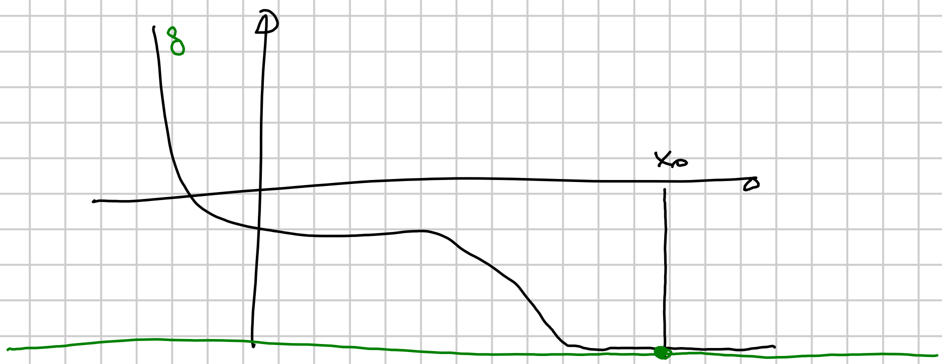
$\Updownarrow$

$$\lim_{k \to \infty} \frac{|\beta - x_{k+1}|}{|\beta - x_k|^2} = C$$

$$g(x) = -\underbrace{\ln(x)}_{\text{definito solo per } x>0} + \frac{2}{x^2} + 1$$

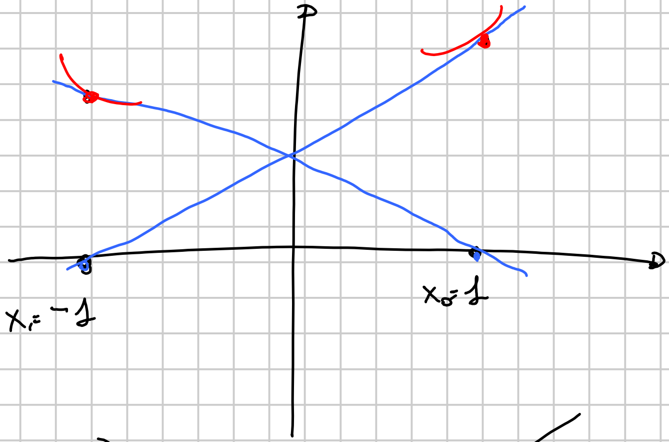


$e'' < 0$



se $g'(x_0) = 0$, tangente orizzontale

$$x_1 = x_0 - \frac{g(x_0)}{g'(x_0)}$$ non def.!



$x_0 = 1$
$x_1 = -1$
$x_2 = 1$
$x_3 = -1$
$x_4 = 1$
$\vdots$

Ad es.

$f(1) = 2$

$f'(1) = 1$

$f(-1) = 2$

$f'(-1) = -1$

e non è complicato trovare un polinomio della forma $f(x) = ax^2 + bx + c$