Note Title 2021-03-02

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Es: rappresentiamo in macchina il numero $x = \frac{20}{3} = 6,6666\ldots$

$B = 10$ $t = 4$ cifre significative

$$x = \frac{20}{3} = 6.666\ldots = 10^{1} \times 0,6666\ldots$$

$$6 \cdot B^{-1} + 6 \cdot B^{-2} + 6 \cdot B^{-3} + \ldots$$

$$B^{p} \sum_{i=1}^{\infty} d_i B^{-i}$$

troncamento:

$$\tilde{x} = 10^{1} \times 0,6666 \qquad \tilde{y} = 10^{1} \times 0,6667$$

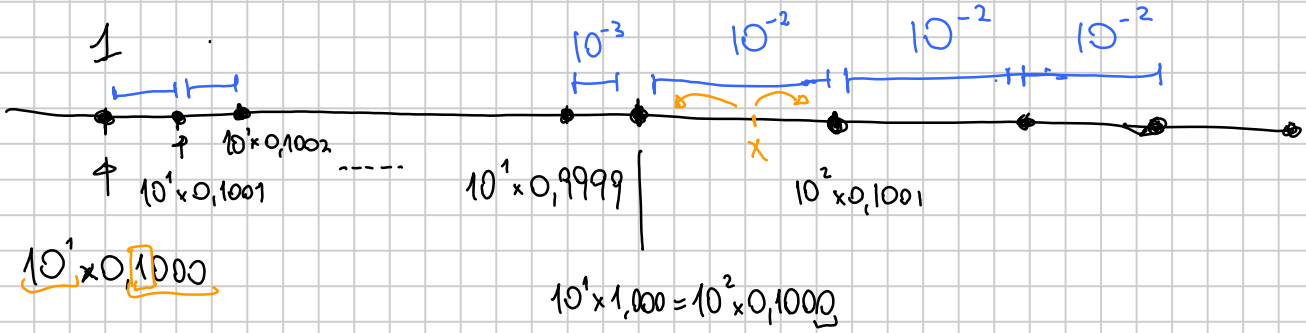
($t = 4$ cifre)

$$\tilde{x} = 10^{1} \times 0,6666 \leq x = \frac{20}{3} \leq \tilde{y} = 10^{1} \times 0,6667$$

$$\tilde{y} - \tilde{x} = B^{1} \cdot B^{-4}$$

$$B^{p} \cdot B^{-t}$$

numeri di macchina ≥ 1: $B = 10$ $t = 4$



$10^{1} \times 0,1000$

$10^{1} \times 1,000 = 10^{2} \times 0,1000$

$x = 10$ $y = 6,666$ $x + y = 16,666$

$x \oplus y = 10^{2} \cdot 0,1667$

$B = 10$
$t = 4$

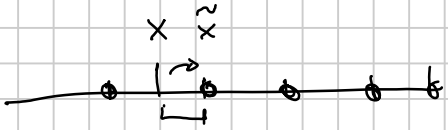
$a = 1/3$ | $a = 0,3333$

$b = 3*a - 1$ | $b = 0,9999 - 1 = 0,0001$

Matlab: $B = 2$ $t = 53$ ("double", 64-bit)

$x = \frac{20}{3} = 2^3 \cdot 0,1101010101\ldots$

$B^{-1} + B^{-2} + 0 \cdot B^{-3} + 1 \cdot B^{-4} + \ldots$



$x = \frac{1}{10}$

$x \in \mathbb{R}$ $\tilde{x}$ num. macchina

$\frac{|x - \tilde{x}|}{|x|} \leq \underbrace{B^{1-t}}_{u}$

$\frac{x - \tilde{x}}{x} = \epsilon_x$

$x - \tilde{x} = x \cdot \epsilon_x \Rightarrow \tilde{x} = x(1 + \epsilon_x)$

$|\epsilon_x| \leq u$

Esempio: $f(x) = x^2 - 1 = (x+1)(x-1)$

$x \approx 1$

$(x+1)(x-1)$

Due algoritmi:

① calcolo $x \cdot x$, poi sottraggo 1

```
a = x * x
b = a - 1
return b
```

② calcolo $x+1$, $x-1$ e li moltiplico

```
a = x+1
b = x-1
c = a * b
return c
```

① $\tilde{x} = x(1+\epsilon_x)$ $\tilde{x} \cdot \tilde{x}$

$\tilde{x} \otimes \tilde{x} = x(1+\epsilon_x) \times (1+\epsilon_x)(1+\epsilon_1)$

$\tilde{x} \otimes \tilde{x} \ominus 1 = \left(x^2 \underbrace{(1+\epsilon_x)\ (1+\epsilon_x)(1+\epsilon_1)}_{} - 1\right)(1+\epsilon_2)$

$\doteq \left(x^2(1 + 2\epsilon_x + \epsilon_1) - 1\right)(1+\epsilon_2) =$

$|\epsilon_1| \leq u$
$|\epsilon_2| \leq u$
$|\epsilon_x| \leq u$

$\doteq x^2(1+2\epsilon_x+\epsilon_1+\epsilon_2) - 1 - \epsilon_2$

errore totale: $\dfrac{g(x) - f(x)}{f(x)} = \dfrac{x^2(1+2\epsilon_x+\epsilon_1+\epsilon_2) - 1 - \epsilon_2 - (x^2-1)}{x^2-1}$

$\doteq \dfrac{2x^2}{x^2-1}\epsilon_x + \dfrac{x^2}{x^2-1}\epsilon_1 + \epsilon_2$

```
for k = a:t:b
   (corpo)
end
```

esegue il corpo del ciclo assegnando a k i valori $a,\ a+t,\ a+2t,\ a+3t,\ \ldots$ fino a quando supera b

`for a:b` fa stessa cosa, ma assume $t=1$

`end`

Esercizi)

1) scrivi una function f = fattoriale(n)

2) scrivi function y = pow(x,n) che calcola $y = x^n$ (per n naturale)