The Infinity Computer and numerical computations with infinities and infinitesimals

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Keywords. Infinity Computer, infinities and infinitesimals, numerical computations, grossone.

The lecture presents a recent methodology allowing one to execute numerical computations with finite, infinite, and infinitesimal numbers on a new type of a computer – the Infinity Computer – patented in EU, USA, and Russia (see [23]). The new approach is based on the principle 'The whole is greater than the part' (Euclid's Common Notion 5) that is applied to all numbers (finite, infinite, and infinitesimal) and to all sets and processes (finite and infinite). It is shown that it becomes possible to write down finite, infinite, and infinitesimal numbers by a finite number of symbols as particular cases of a unique framework different from that of the non-standard analysis. The new methodology evolves ideas of Cantor and Levi-Civita in a more applied way and, among other things, introduces new infinite integers that possess both cardinal and ordinal properties as usual finite numbers (its relations with traditional approaches are discussed in [8–11, 15, 21, 24, 34]).

It is emphasized that the philosophical triad – researcher, object of investigation, and tools used to observe the object – existing in such natural sciences as Physics and Chemistry, exists in Mathematics, too. In natural sciences, the instrument used to observe the object influences the results of observations. The same happens in Mathematics where numeral systems used to express numbers are among the instruments of observations used by mathematicians. The usage of powerful numeral systems gives the possibility to obtain more precise results in Mathematics, in the same way as the usage of a good microscope gives the possibility to obtain more precise results in Physics. A numeral system using a new numeral called *grossone* is described (see [16, 17, 20, 25]). It allows one to express easily infinities and infinitesimals offering rich capabilities for describing mathematical objects, mathematical modeling, and practical computations. The concept of the accuracy of numeral systems is introduced. The accuracy of the new numeral system is compared with traditional numeral systems used to work with infinity.

In order to see the place of the new approach in the historical panorama of ideas dealing with infinite and infinitesimal, see [8–11, 15, 21, 24, 34]. In particular, connections of the new approach with bijections are studied in [11] and metamathematical investigations on the theory can be found in [10]. The new methodology has been successfully used in a number of applications: Turing machines and lexicographic ordering [31, 34–36]), cellular automata (see [2–4]), percolation and biological processes (see [5, 6, 28, 37]), numerical differentiation, optimization, and ODEs (see [1, 26, 29, 39]), fractals (see [19, 22, 28, 33, 37]), infinite series (see [7, 21, 27, 38]), set theory (see [24, 32]), hyperbolic geometry (see [12, 13]), etc.

The Infinity Calculator using the Infinity Computer technology is presented during the talk.

Articles [1]– [39] and a lot of an additional information can be downloaded from the internet page `http://www.theinfinitycomputer.com`

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