$Ax=b \quad A=M-N \quad Mx_1 = Nx_0 + b$

$x_1 = M^{-1}Nx_0 + M^{-1}b$

$x_{k+1} = \underbrace{M^{-1}N}_{P} x_k + M^{-1}b \qquad k=1,2,\dots$

Jacobi: $M = \text{diag}(A)$ Gauß-Seidel: $\text{tril}(A)$

$\to M = \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \quad N = \begin{bmatrix} 0 & a_{12} & a_{13} \\ a_{21} & 0 & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix}$

... creo M, N

```
>> x = M \ (N * x + b);
```

$O(n^3)$

$O(n^2)$, anche meno se A ha tanti zeri

(1) $\to \begin{bmatrix} a_{11} & 0 & 0 \\ 0 & a_{22} & 0 \\ 0 & 0 & a_{33} \end{bmatrix} \begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ x_3^{k+1} \end{bmatrix} = \begin{bmatrix} 0 & -a_{12} & -a_{13} \\ -a_{21} & 0 & -a_{23} \\ -a_{31} & -a_{32} & 0 \end{bmatrix} \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

(1) $a_{11} x_1^{k+1} = -a_{12} x_2^k - a_{13} x_3^k + b_1$

$x_1^{k+1} = \dfrac{-a_{12} x_2^k - a_{13} x_3^k + b_1}{a_{11}}$

(i) $\underbrace{x_i^{k+1}}_{\text{x\_new}} = \dfrac{-\sum_{j=1}^{i-1} a_{ij} x_j^k - \sum_{j=i+1}^{n} a_{ij} \underbrace{x_j^k}_{\text{x\_old}} + b_i}{a_{ii}} \qquad i=1,\dots,n$

function x= jacobi(A, b, x0, m)

\# di passi

(1) (2) (3) $\begin{bmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & a_{33} \end{bmatrix}_M \begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ x_3^{k+1} \end{bmatrix} = \begin{bmatrix} 0 & -a_{12} & -a_{13} \\ 0 & 0 & -a_{23} \\ 0 & 0 & 0 \end{bmatrix}_N \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \end{bmatrix} + \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$

(1) $a_{11} \underline{x_1^{k+1}} = b_1 - a_{12} x_2^k - a_{13} x_3^k$

(2) $a_{21} x_1^{k+1} + a_{22} \underline{x_2^{k+1}} = -a_{23} x_3^k + b_2$

(3) $a_{31} x_1^{k+1} + a_{32} x_2^{k+1} + a_{33} \underline{x_3^{k+1}} = b_3$

$$\begin{cases} x_1^{k+1} = \dfrac{b_1 - a_{12} x_2^k - a_{13} x_3^k}{a_{11}} & \leftarrow x = \begin{bmatrix} x_1^k \\ x_2^k \\ x_3^k \end{bmatrix} \\ x_2^{k+1} = \dfrac{b_2 - a_{23} x_3^k - a_{21} x_1^{k+1}}{a_{22}} & \leftarrow x = \begin{bmatrix} x_1^{k+1} \\ x_2^k \\ x_3^k \end{bmatrix} \\ x_3^{k+1} = \dfrac{b_3 - a_{31} x_1^{k+1} - a_{32} x_2^{k+1}}{a_{33}} & \leftarrow x = \begin{bmatrix} x_1^{k+1} \\ x_2^{k+1} \\ x_3^k \end{bmatrix} \end{cases}$$