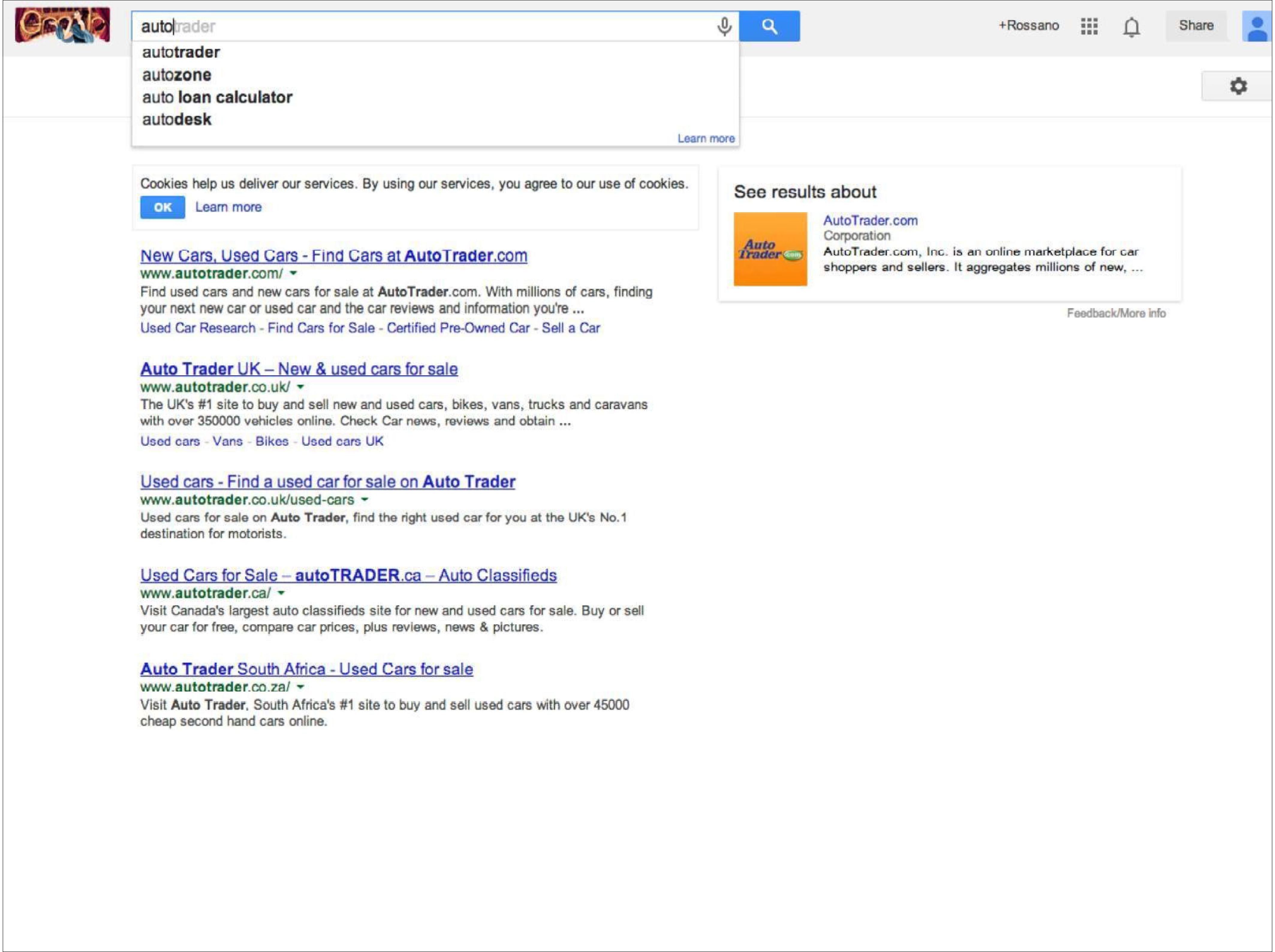
Succinct Data Structures

Auto-completion as our target application

With some of my changes

Rossano Venturini

rossano@di.unipi.it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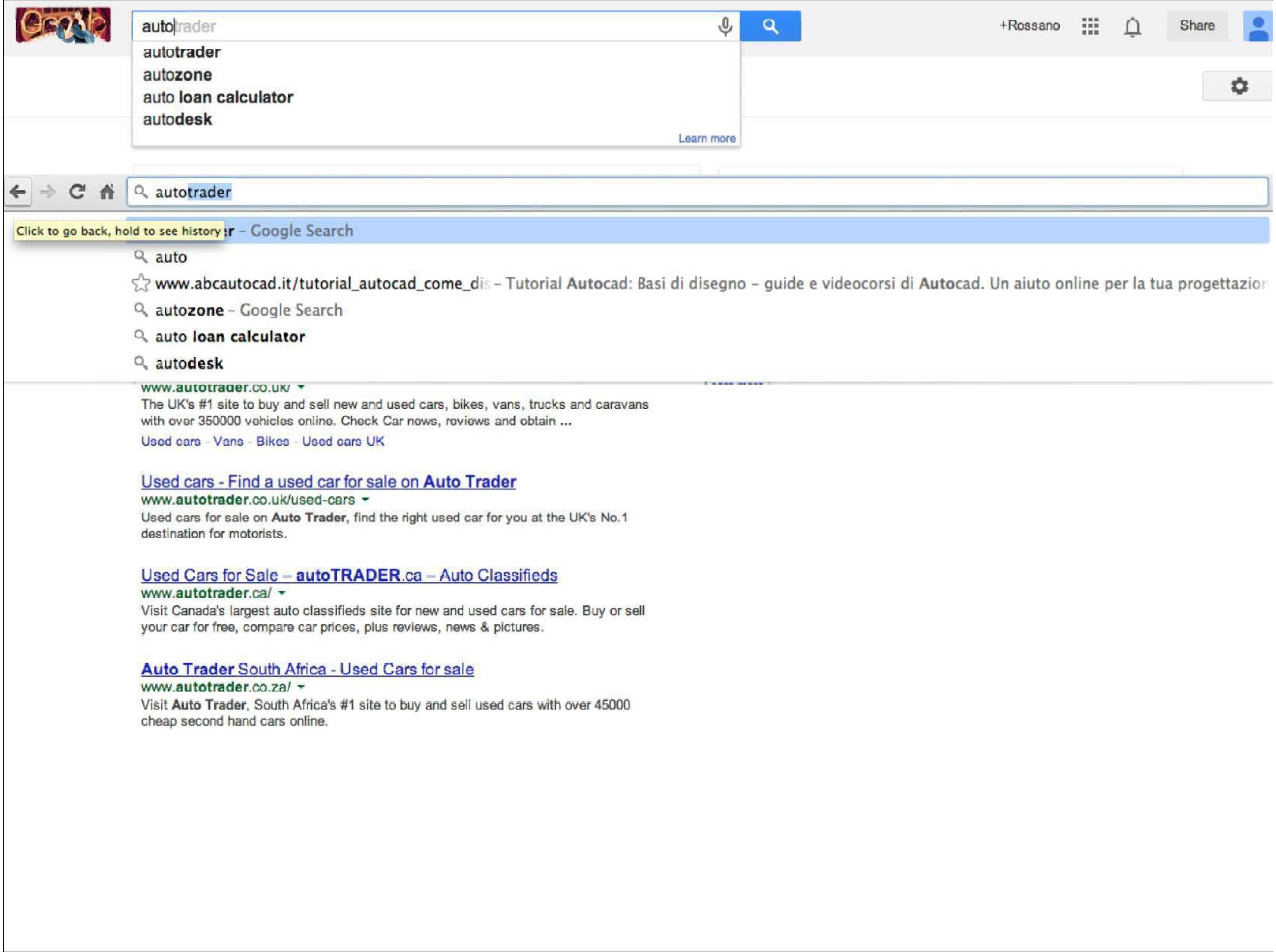
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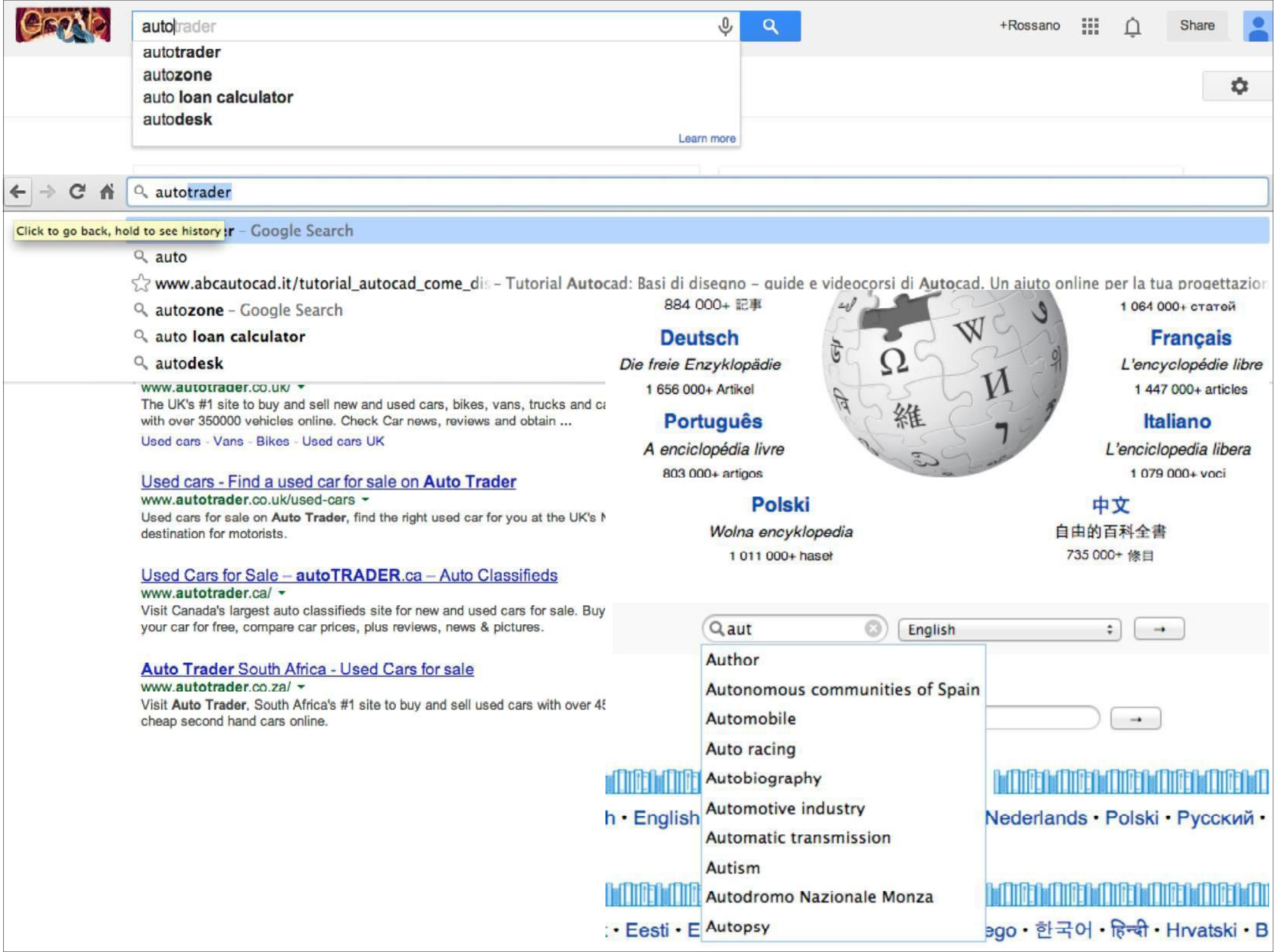
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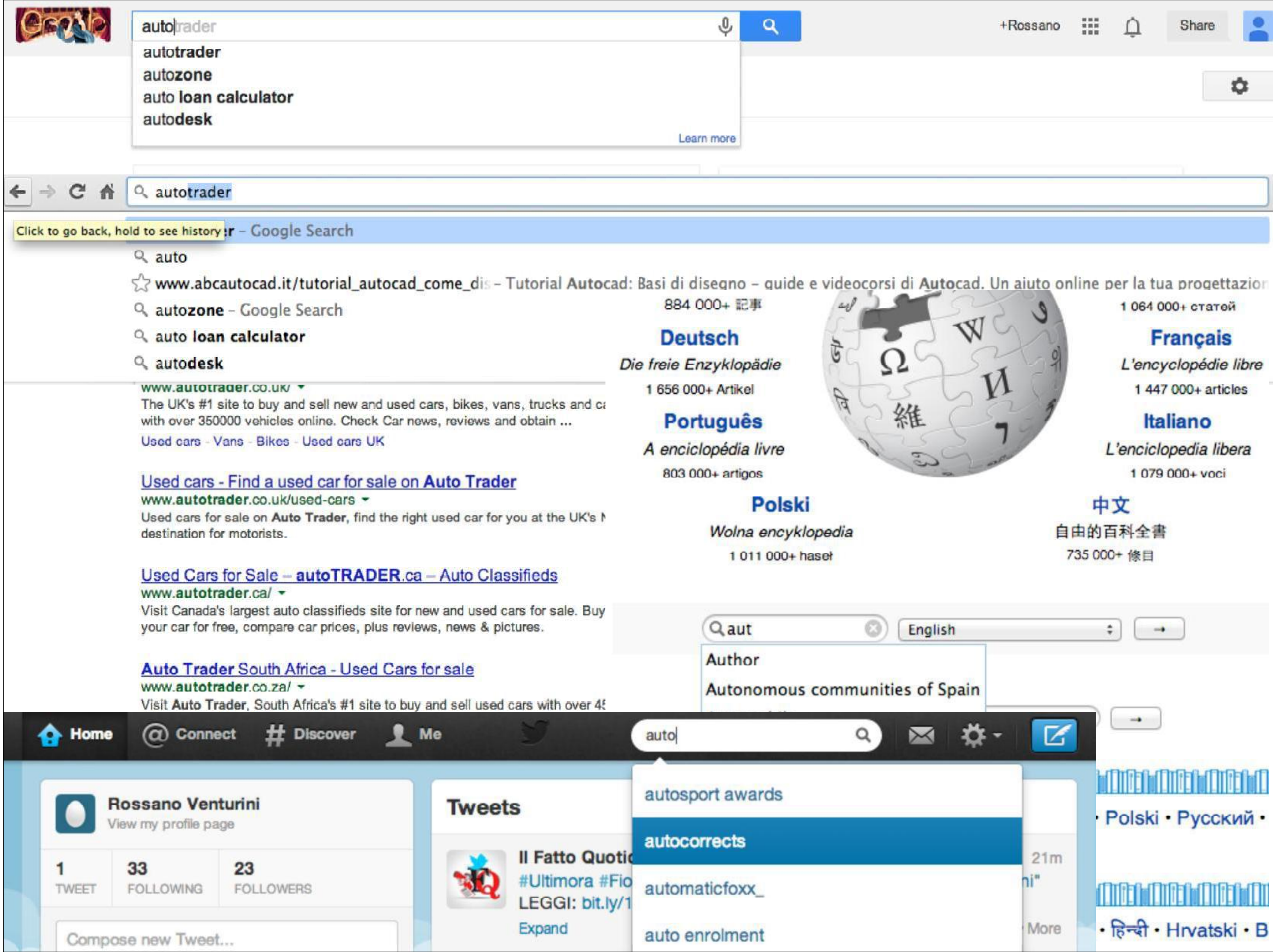

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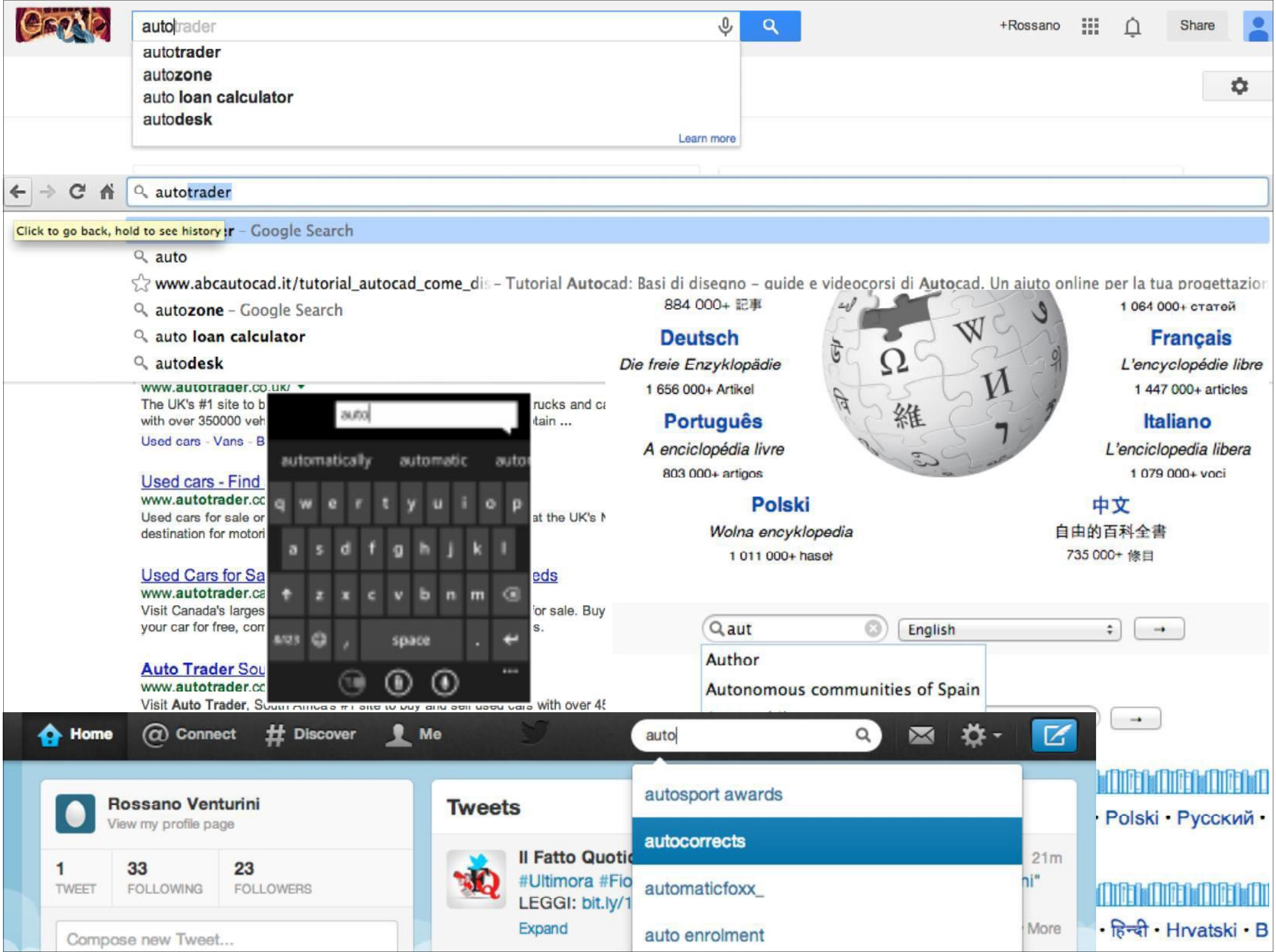

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Dataset?

Dataset?

All the past queries

Dataset? All the past queries

Searches?

Dataset? All the past queries

Searches? Prefix search

Dataset? All the past queries

Searches? Prefix search

Data structure?

Dataset?	All the past queries
Searches?	Prefix search
Data structure?	Trie

Dataset? All the past queries

Searches? Prefix search

Data structure? Trie

How to find top-k efficiently?

Trie

Trie

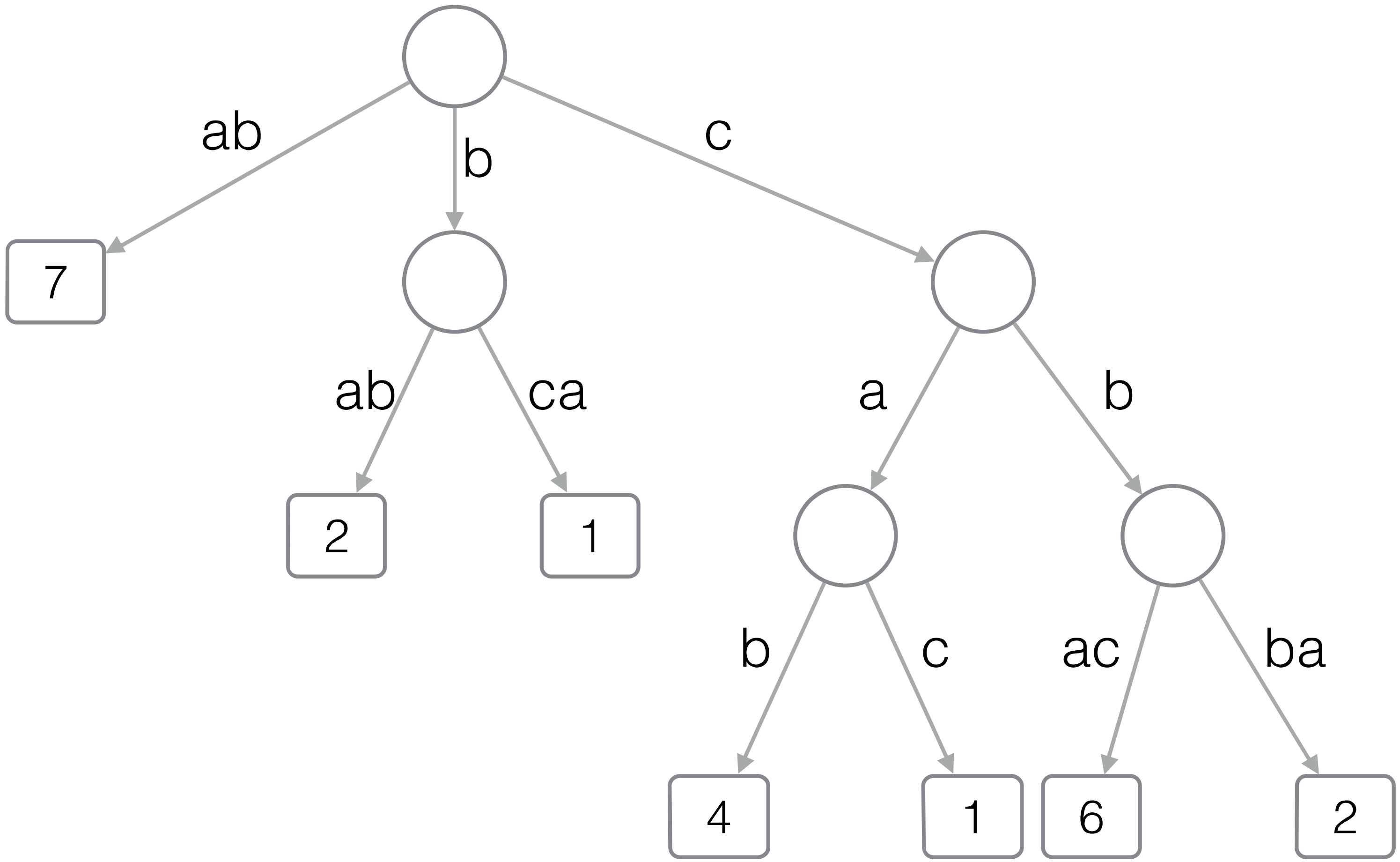
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

Trie

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Trie



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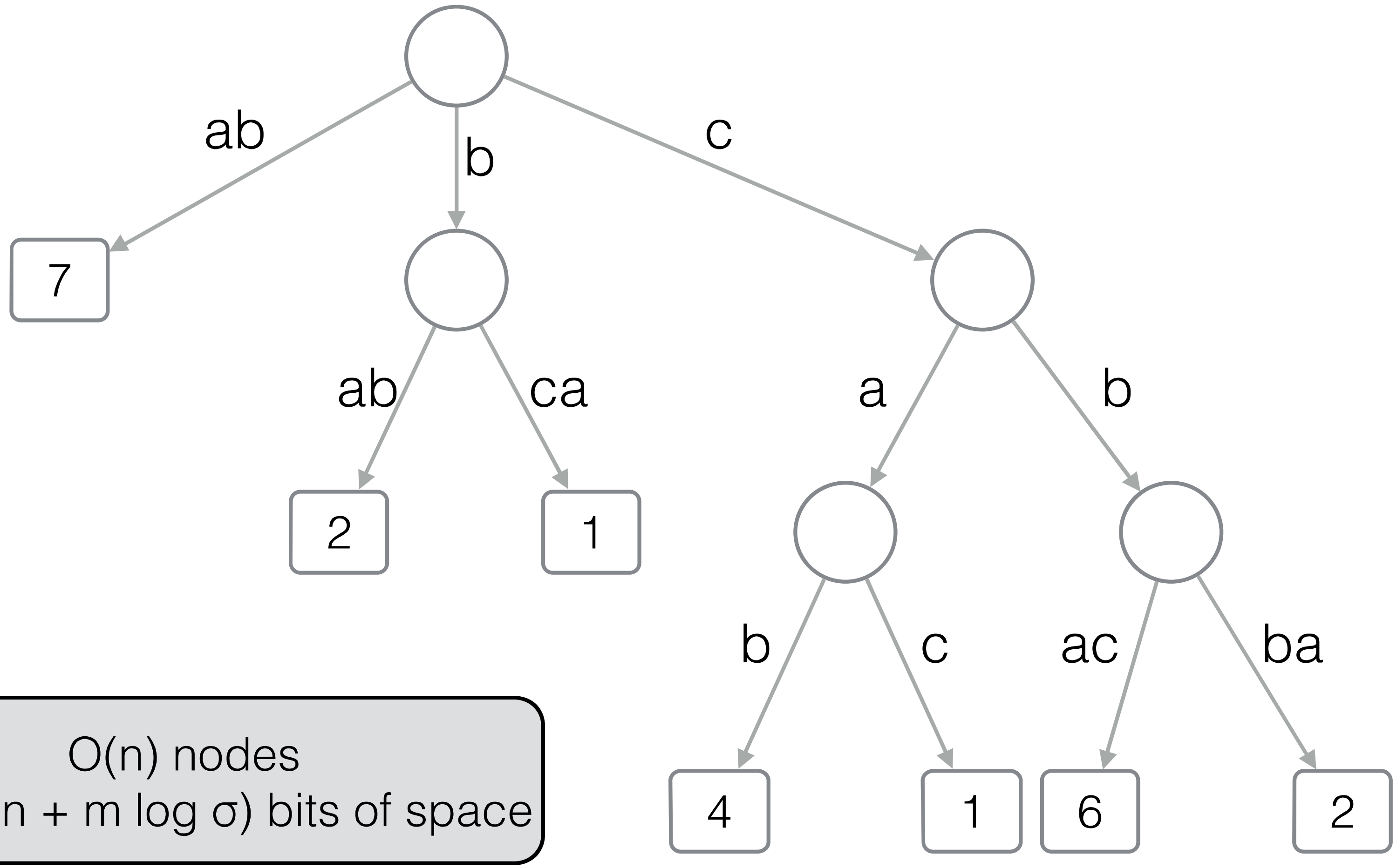

O(n) nodes
O(n log n + m log σ) bits of space

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

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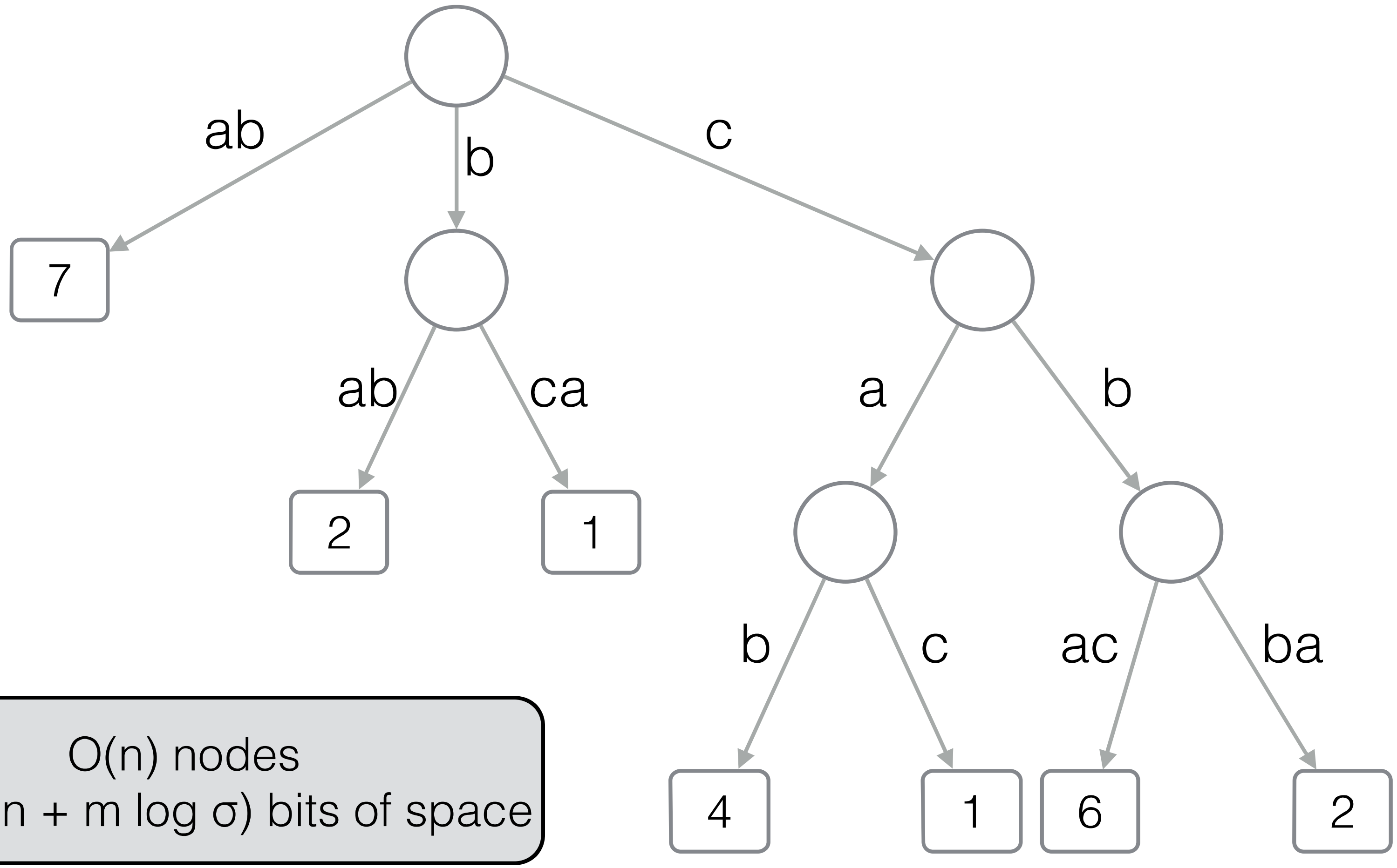


O(n) nodes
O(n log n + m log σ) bits of space

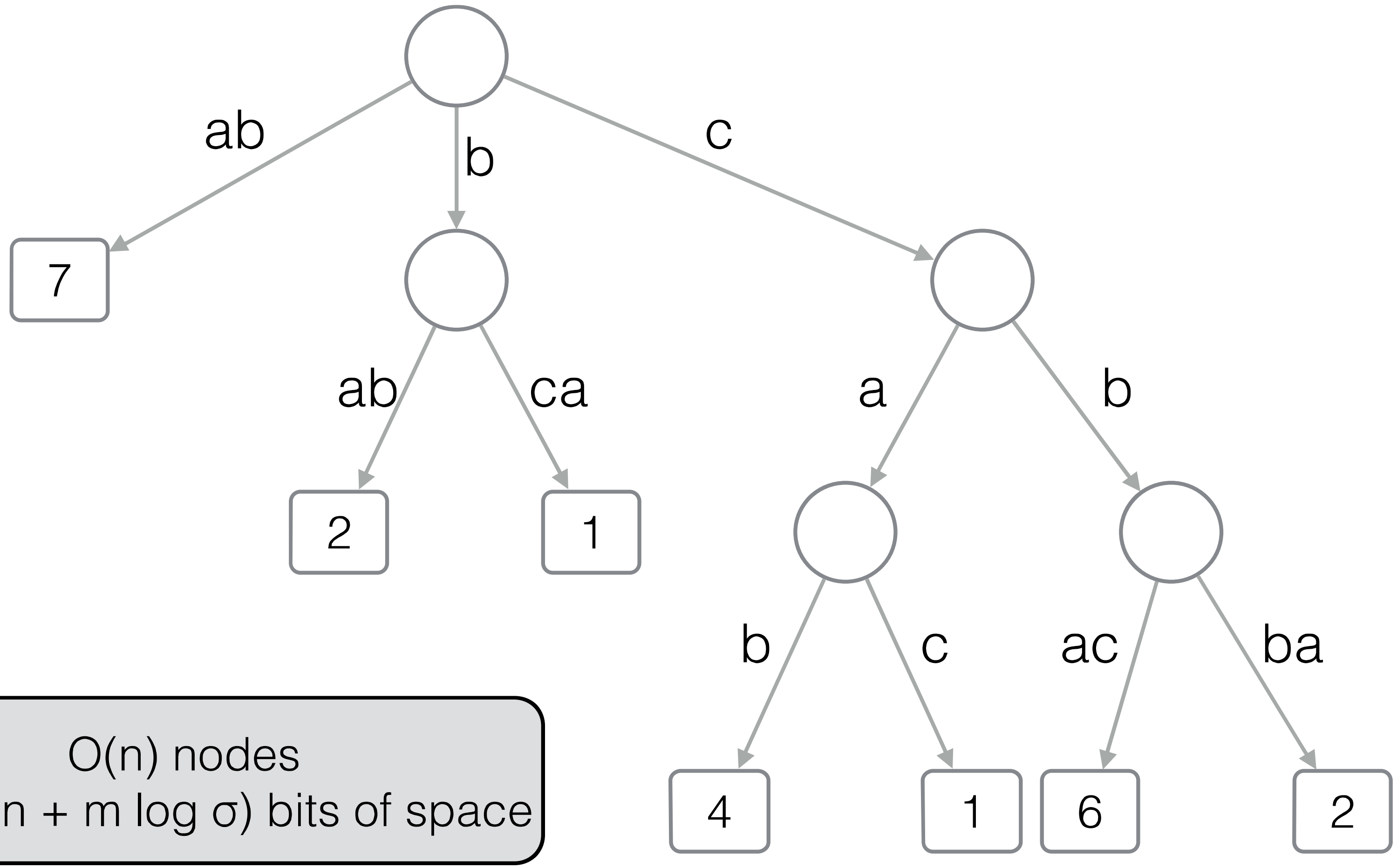
Find all the strings prefixed by any pattern P in O(|P|) time

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Trie

P = c



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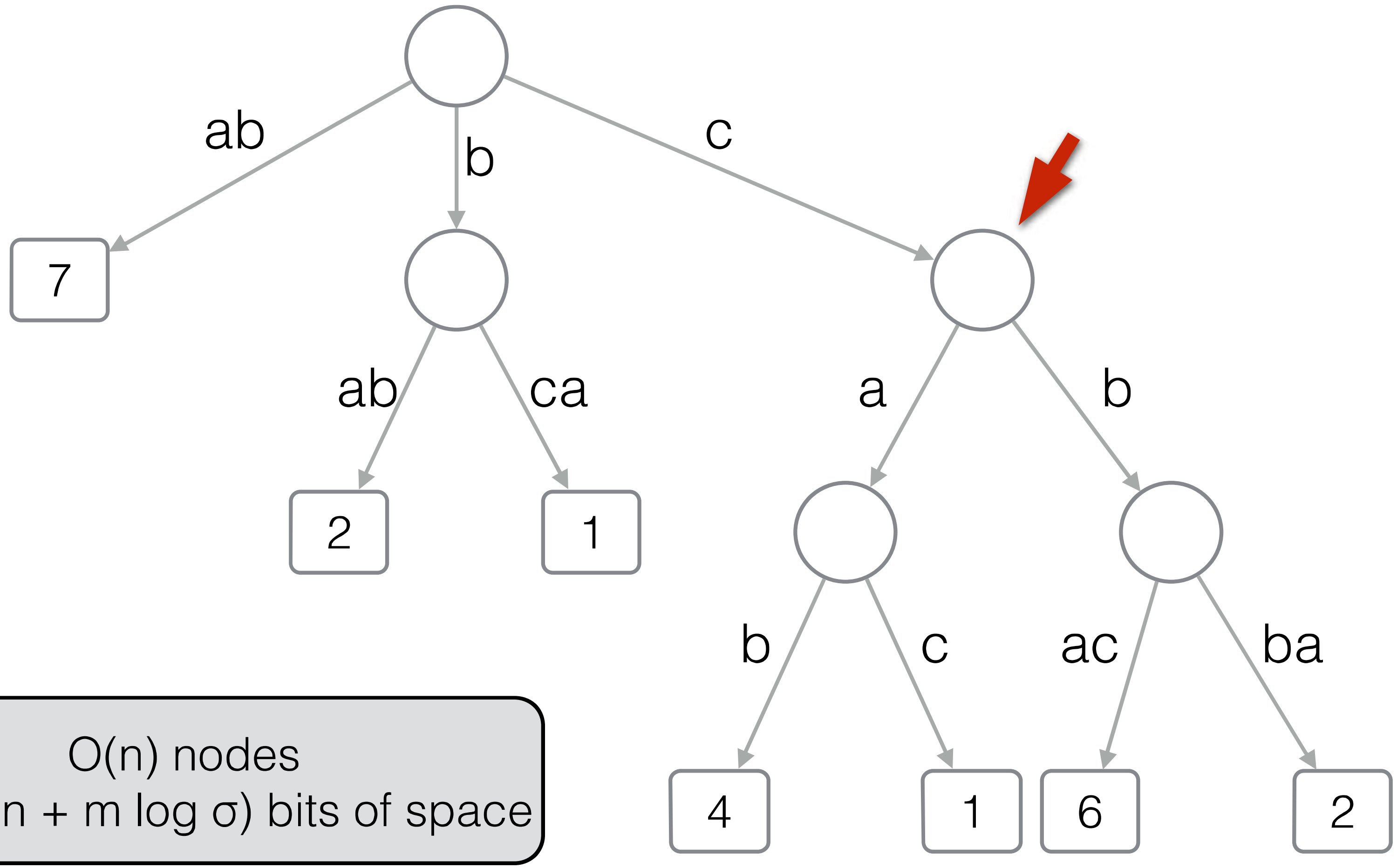

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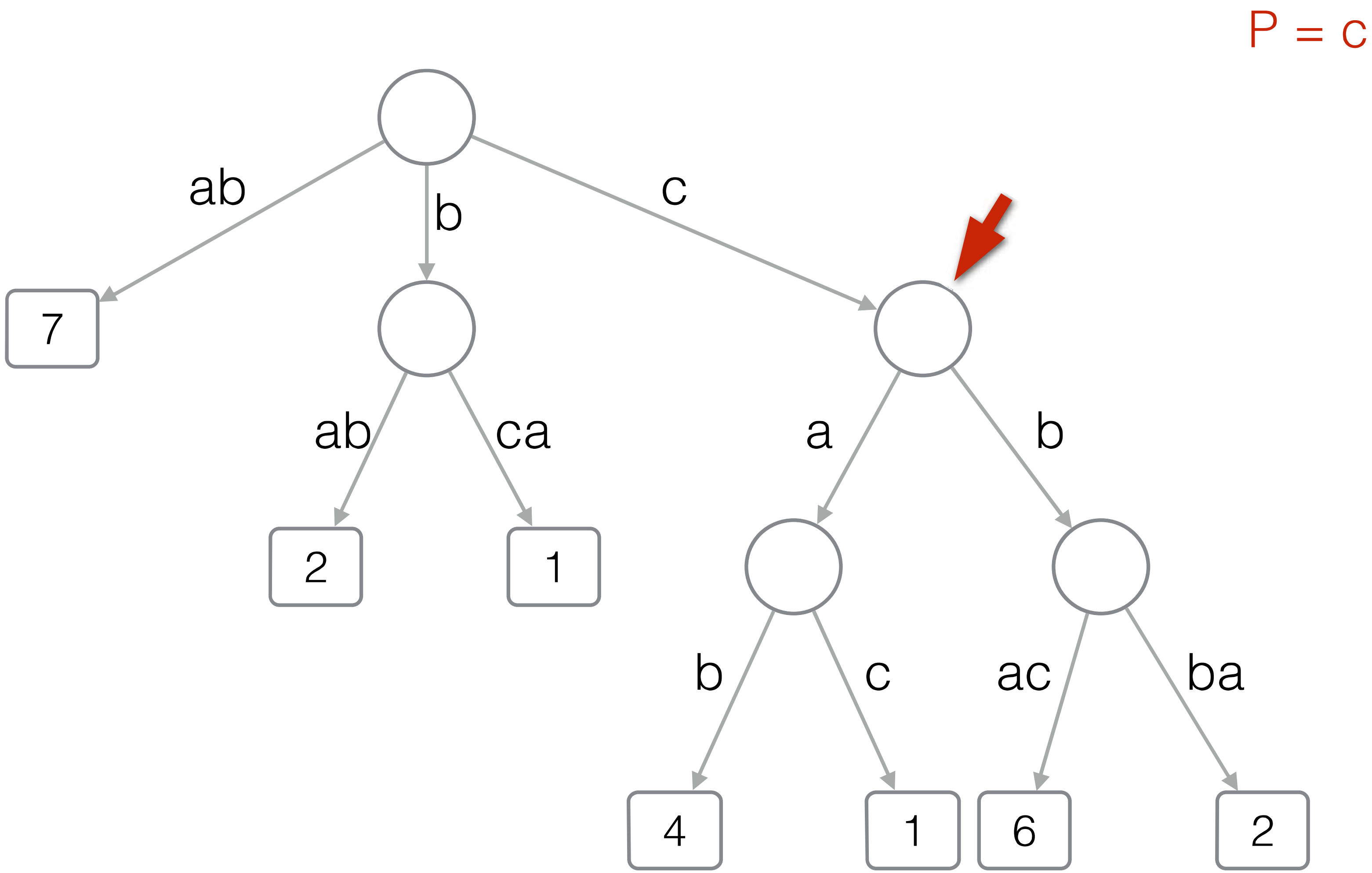
Finding Top-1



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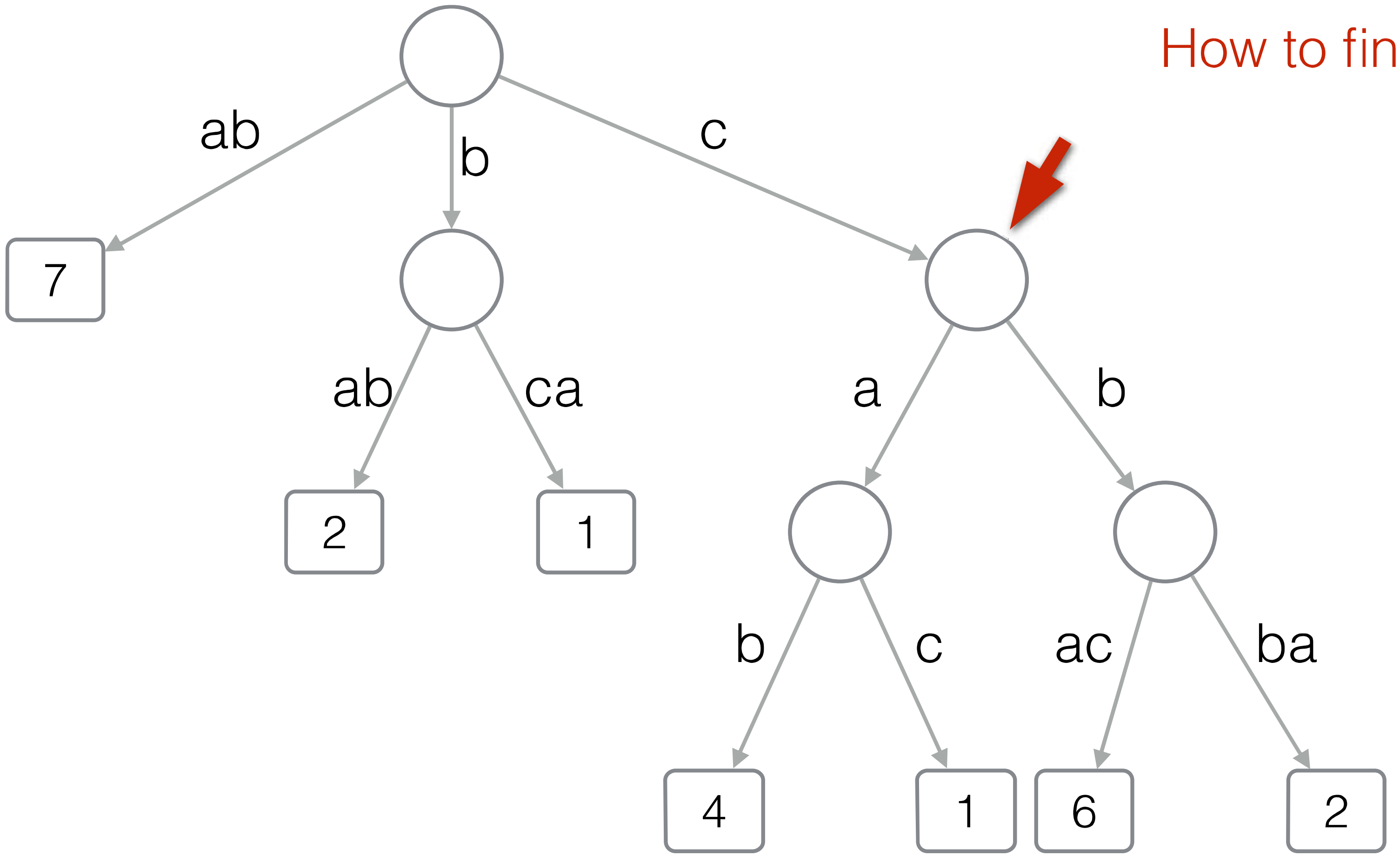
Finding Top-1

P = c

How to find Top-1?



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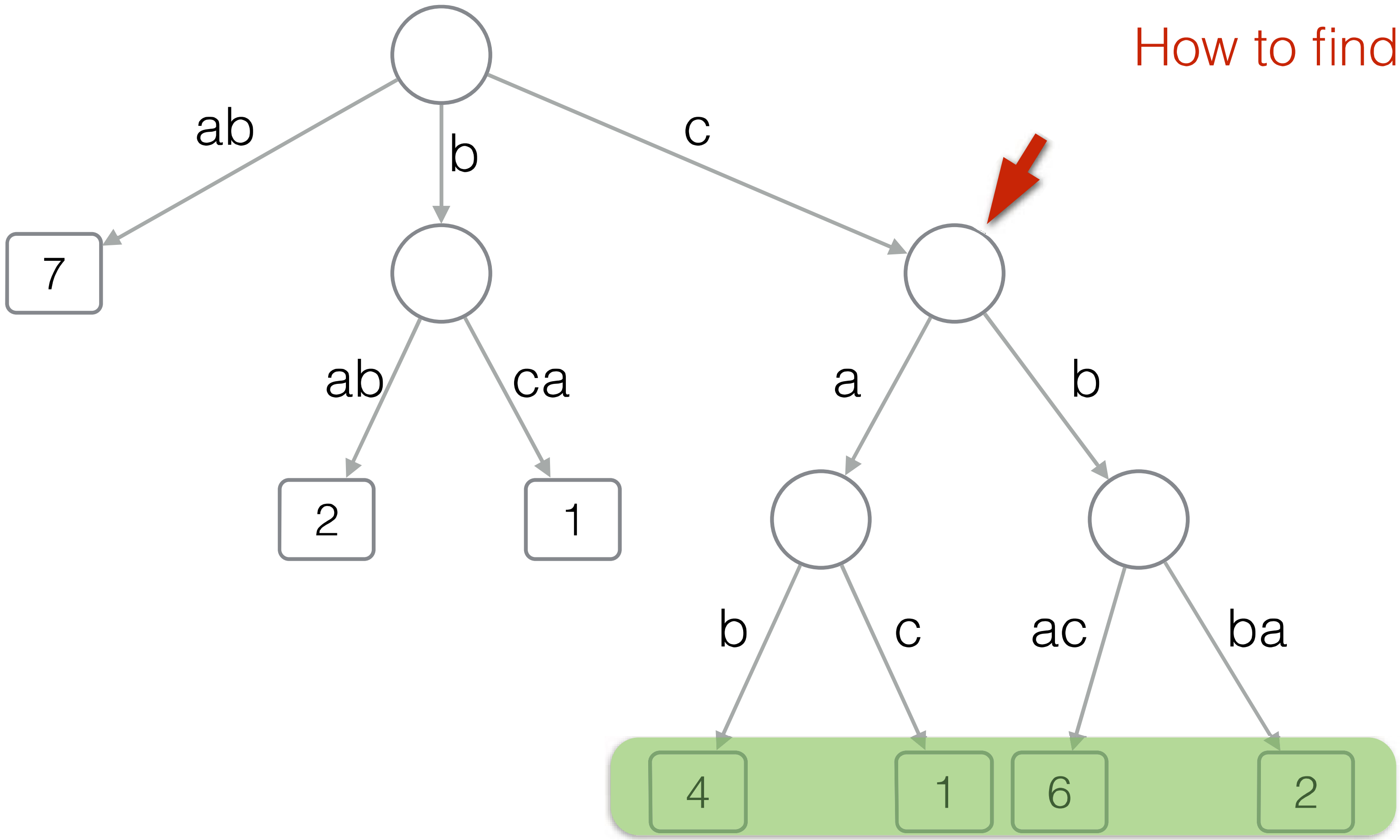

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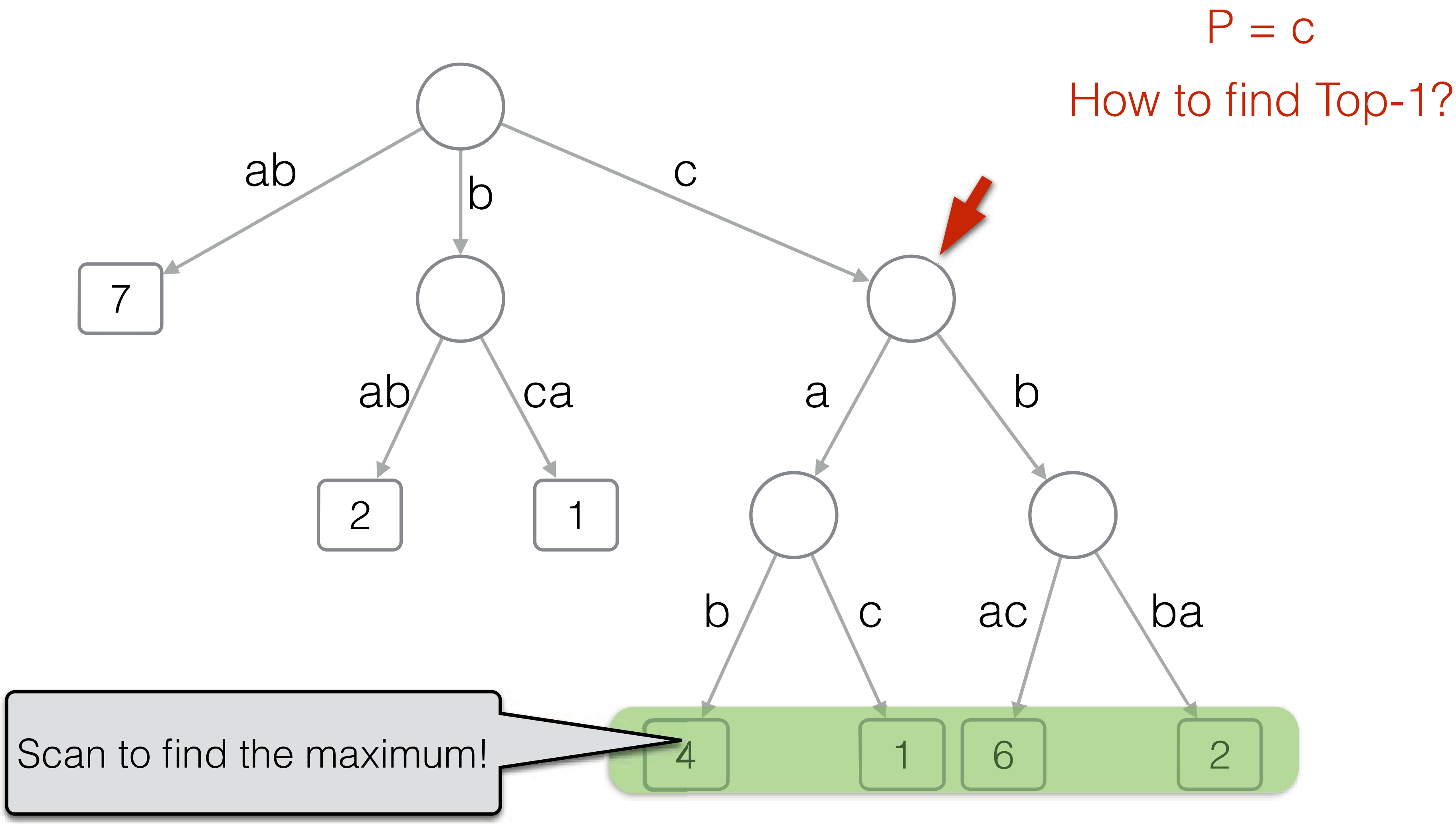

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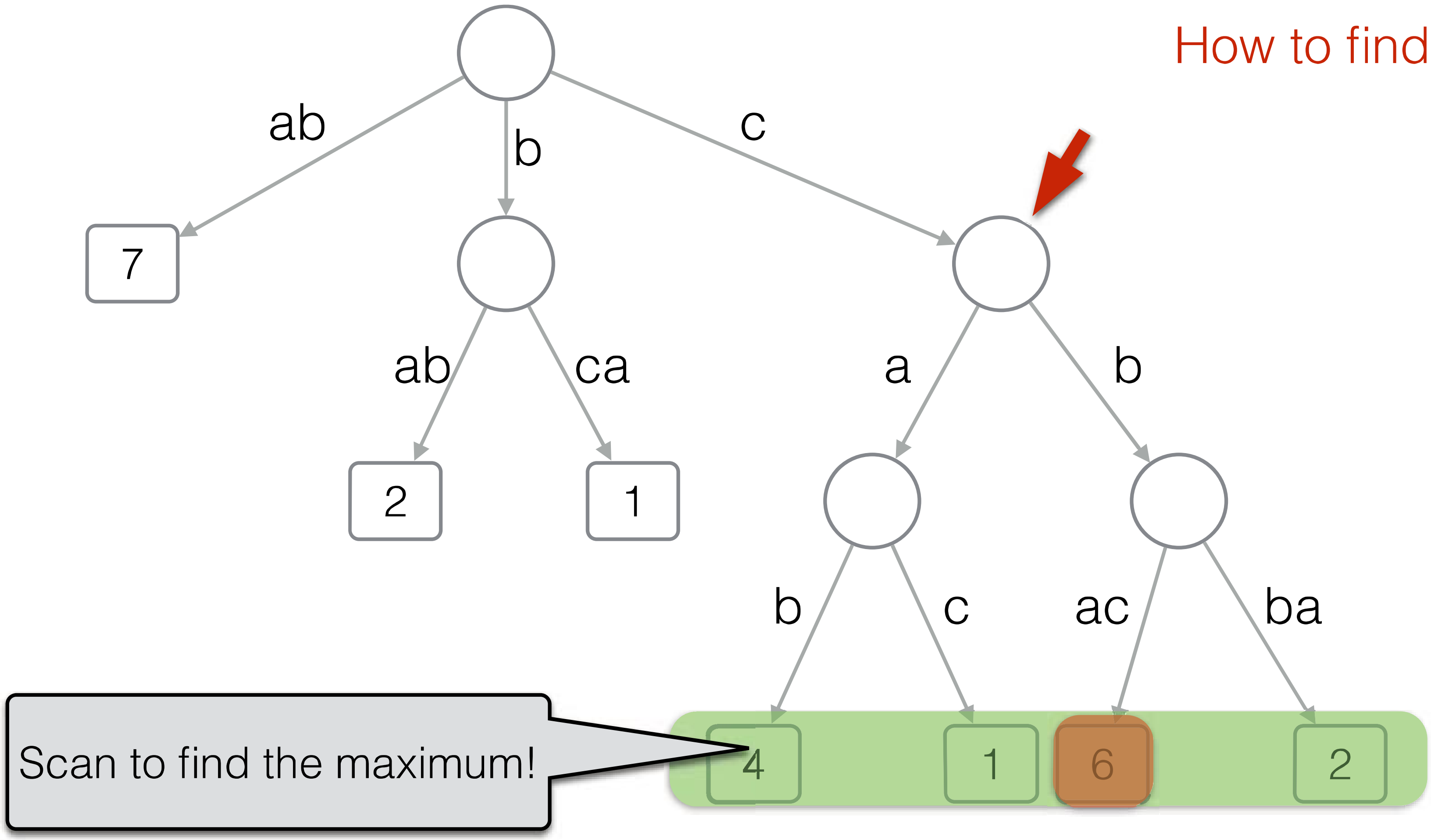

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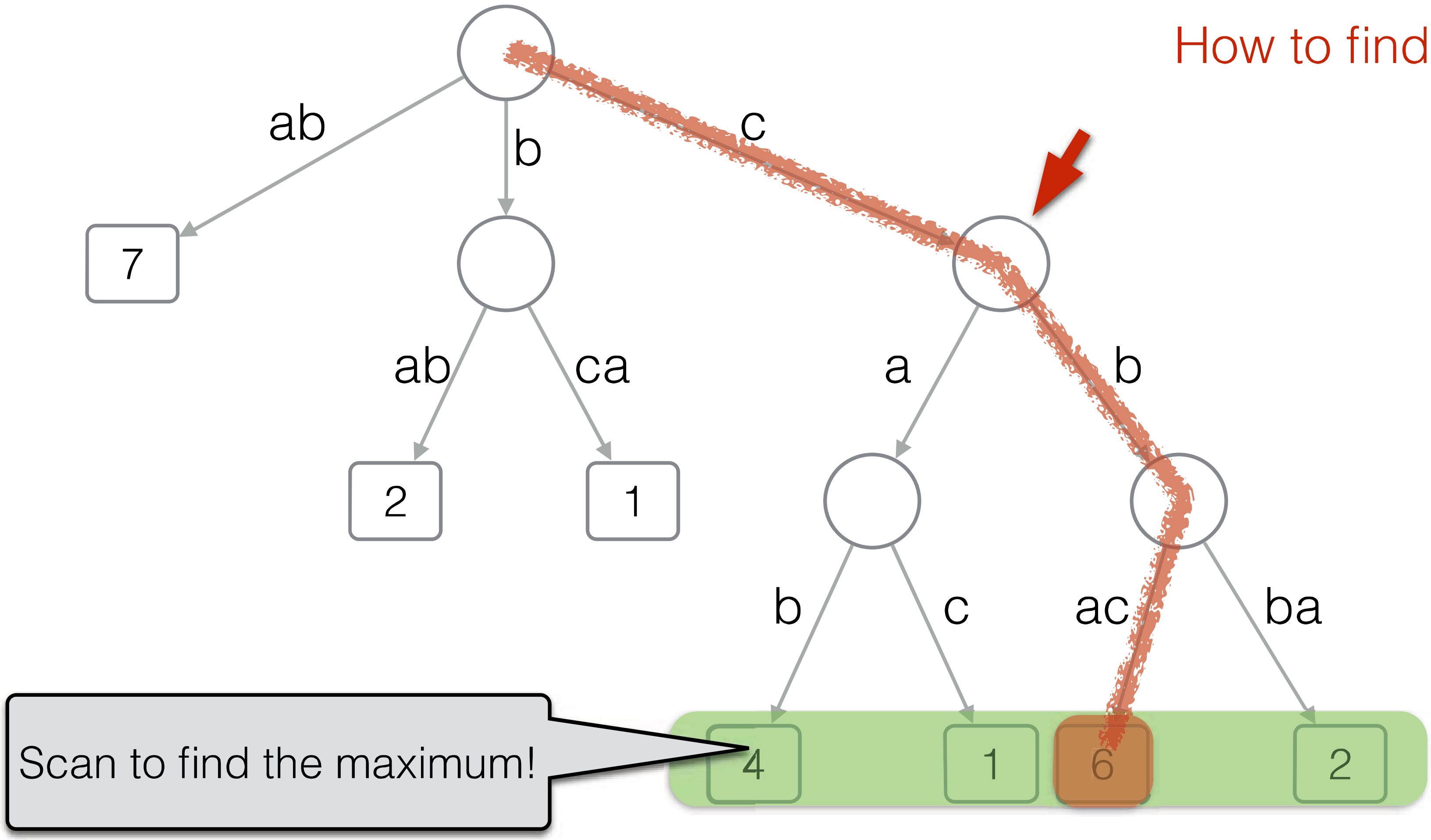

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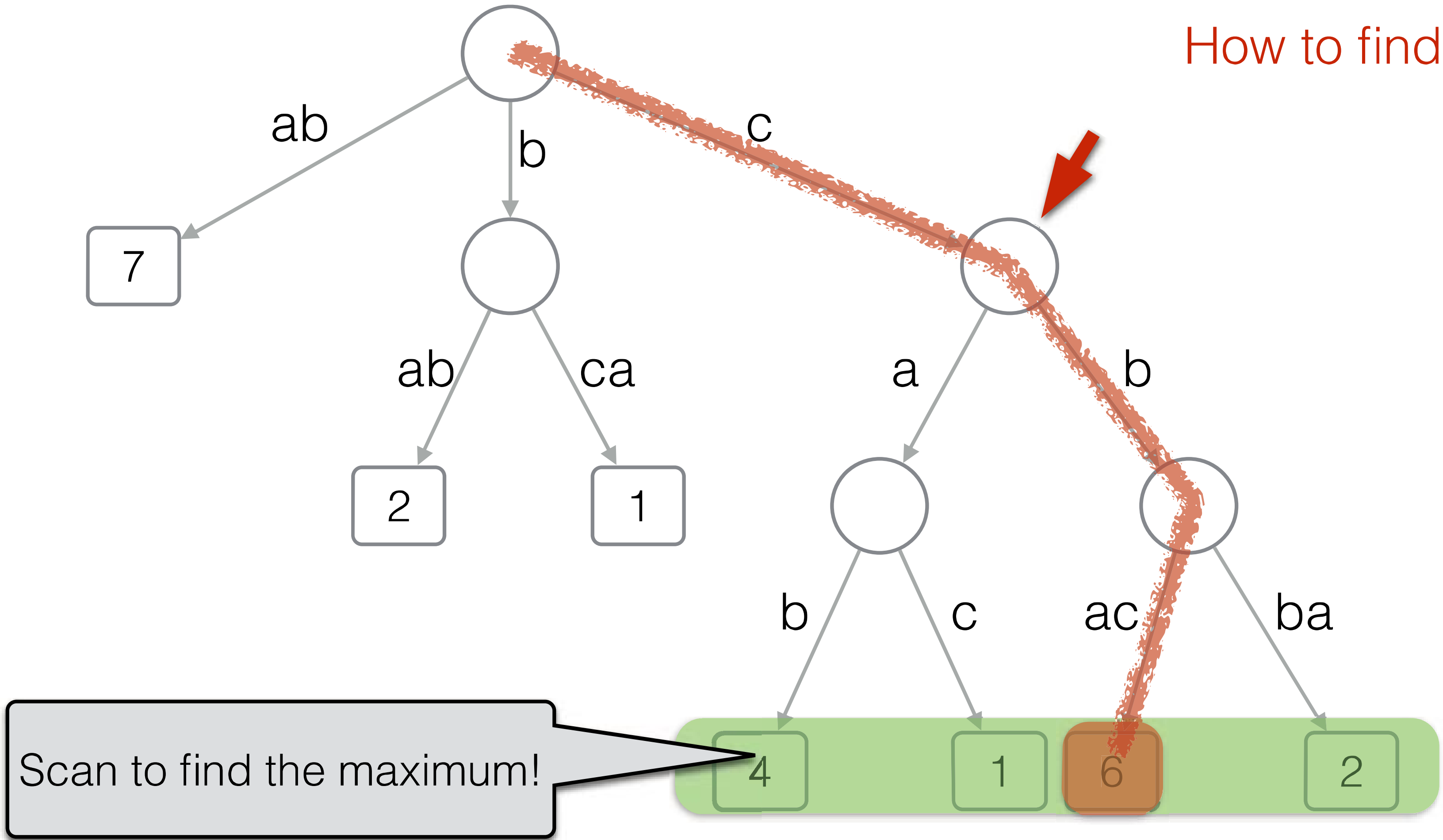

O(n) query time :-(

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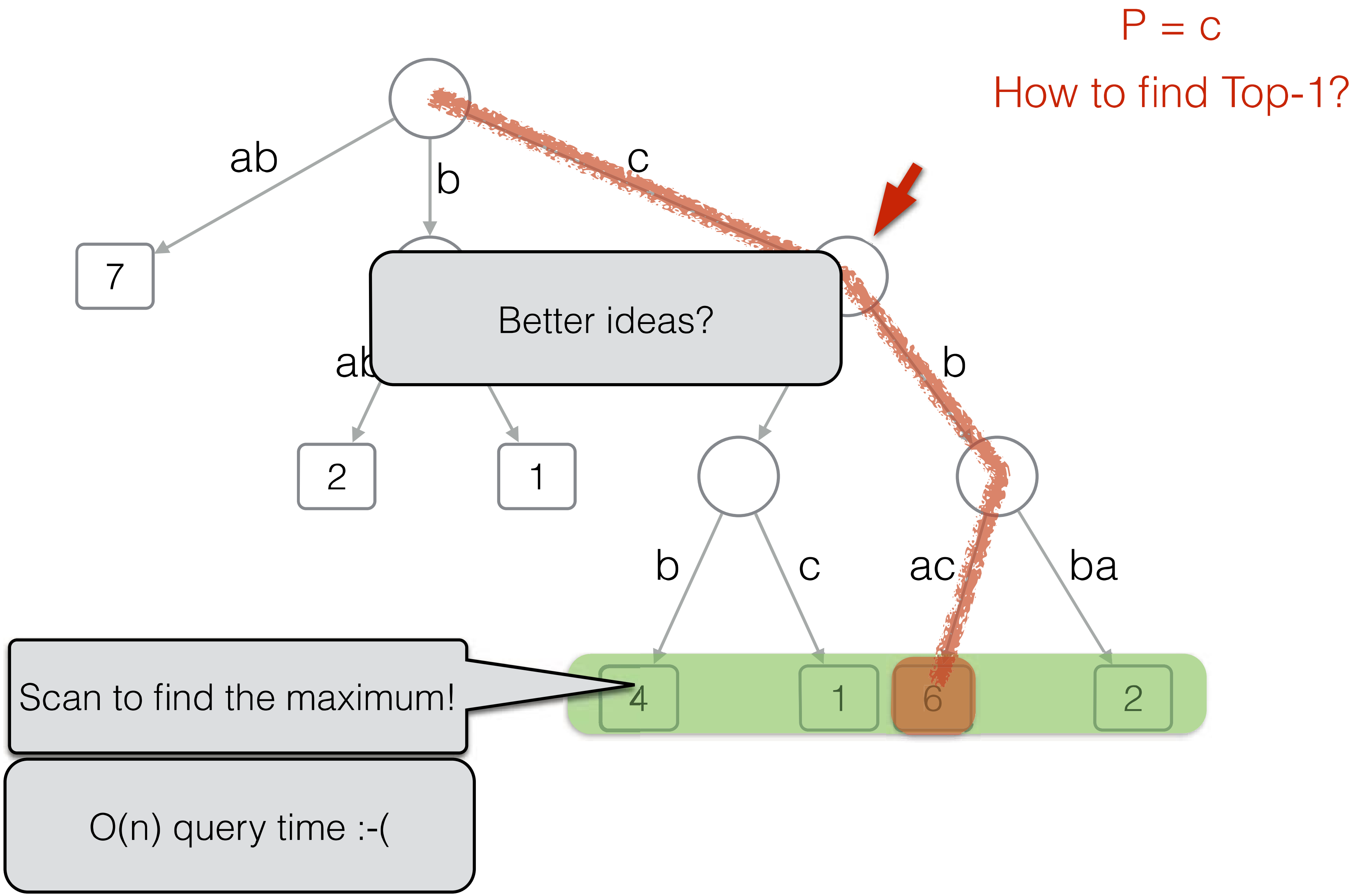
Finding Top-1



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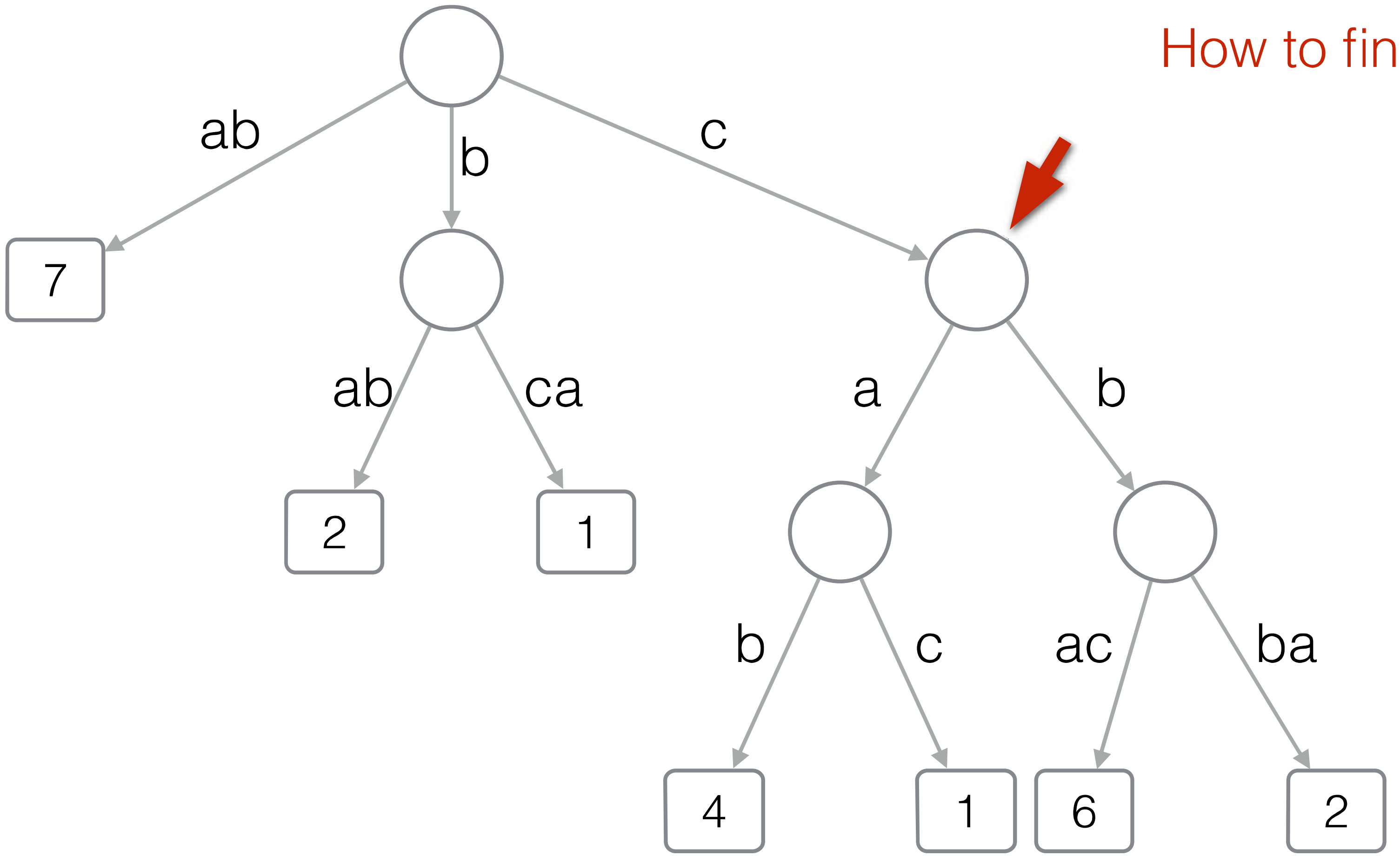

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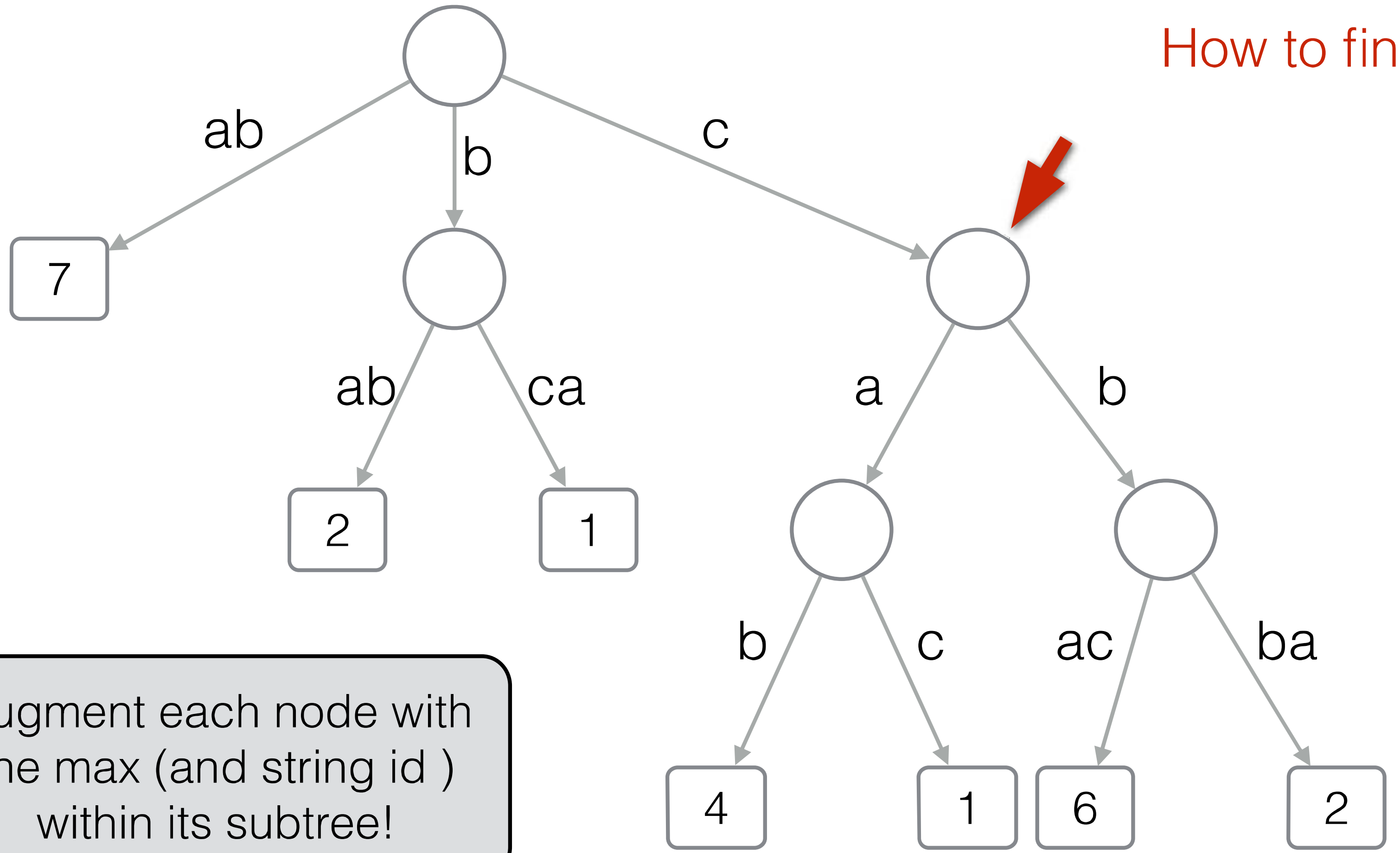

Augment each node with the max (and string id) within its subtree!

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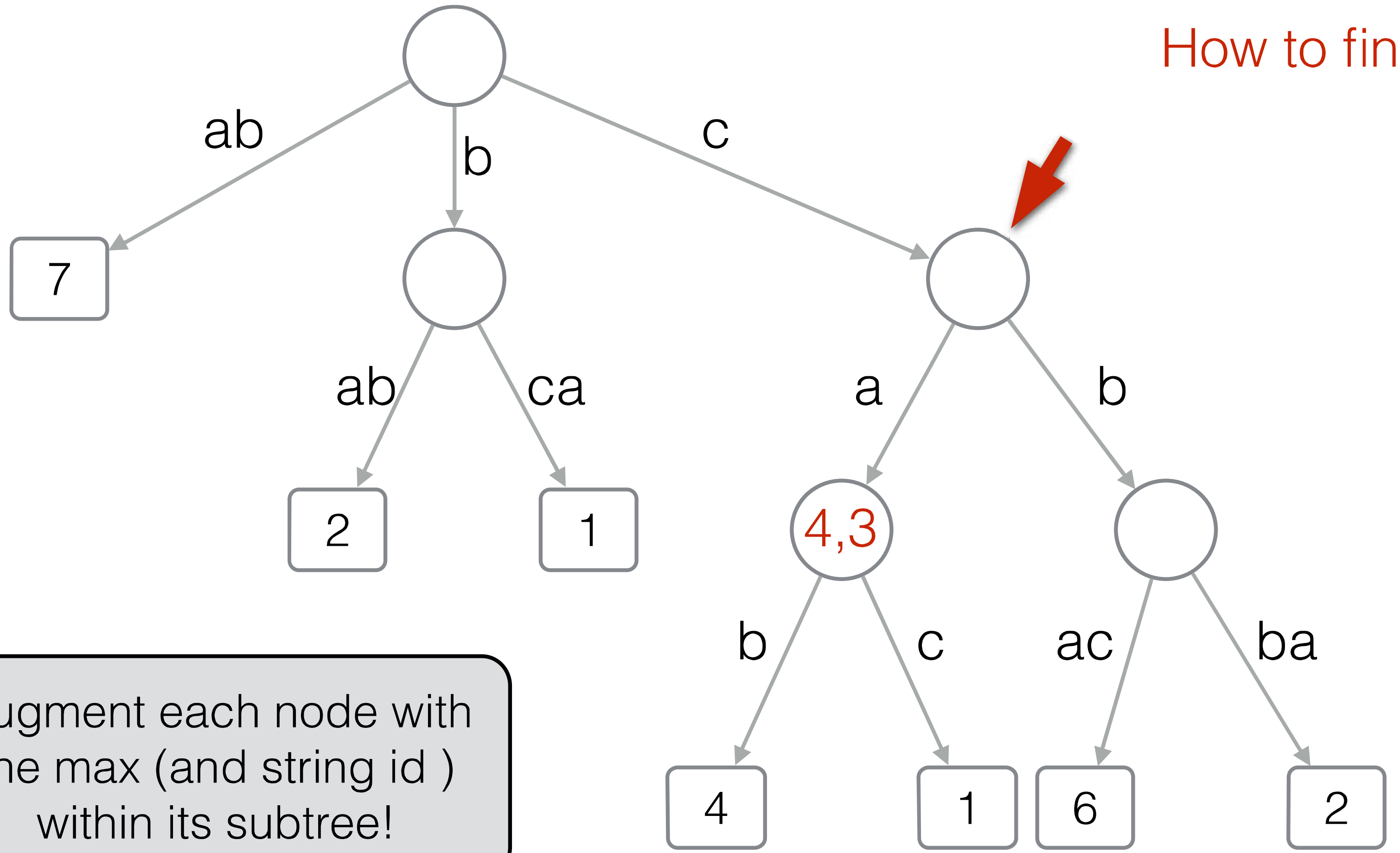

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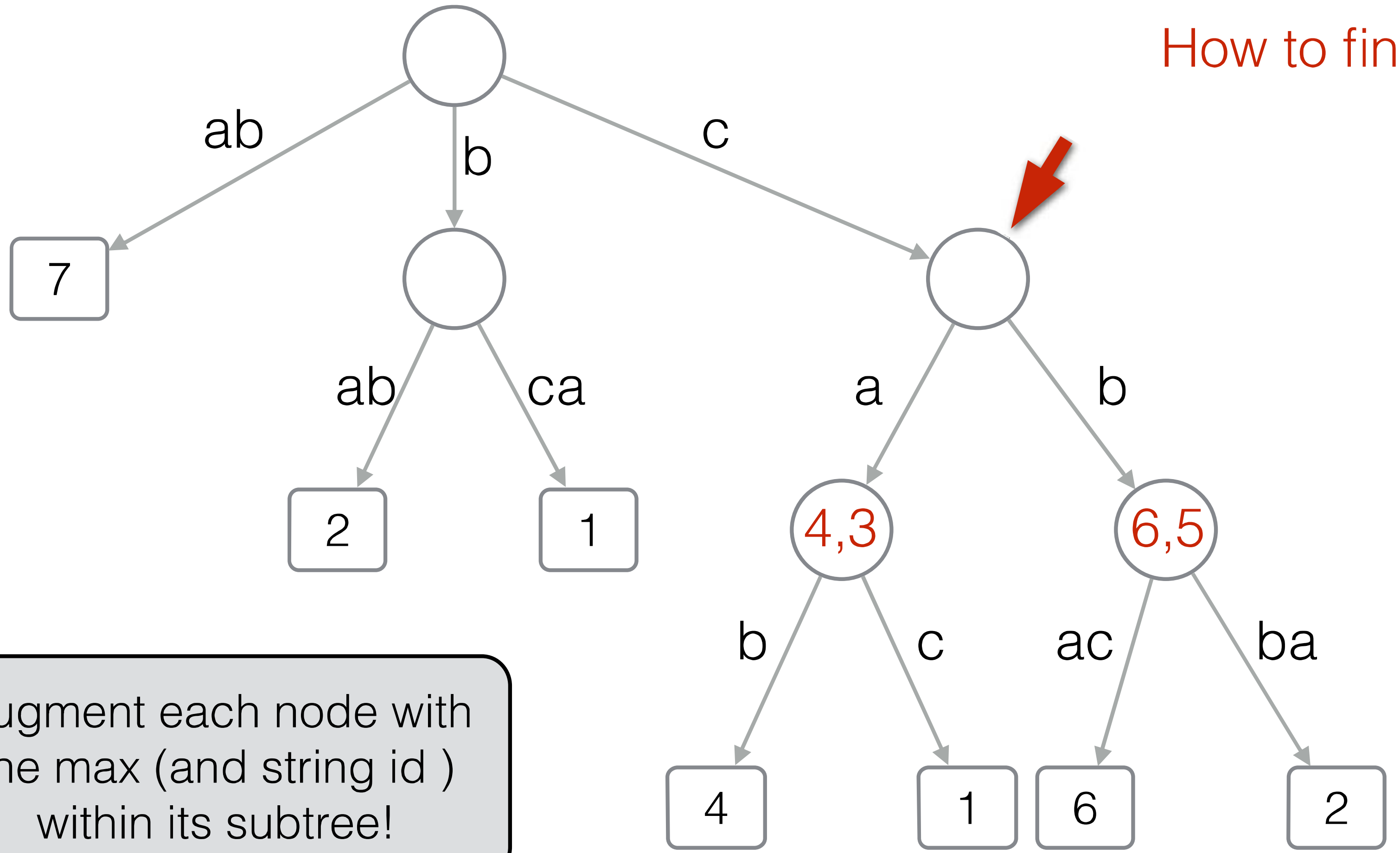

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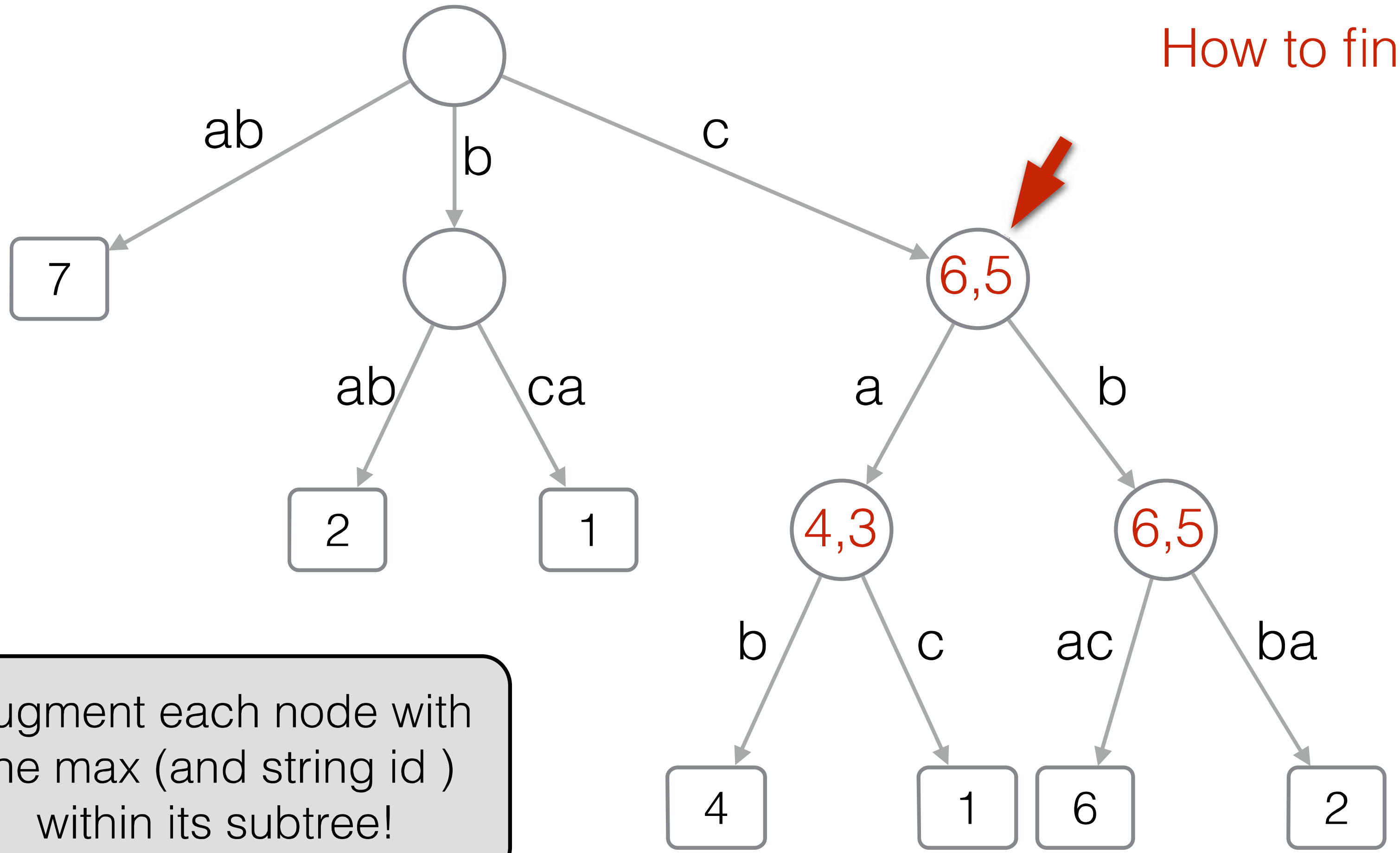

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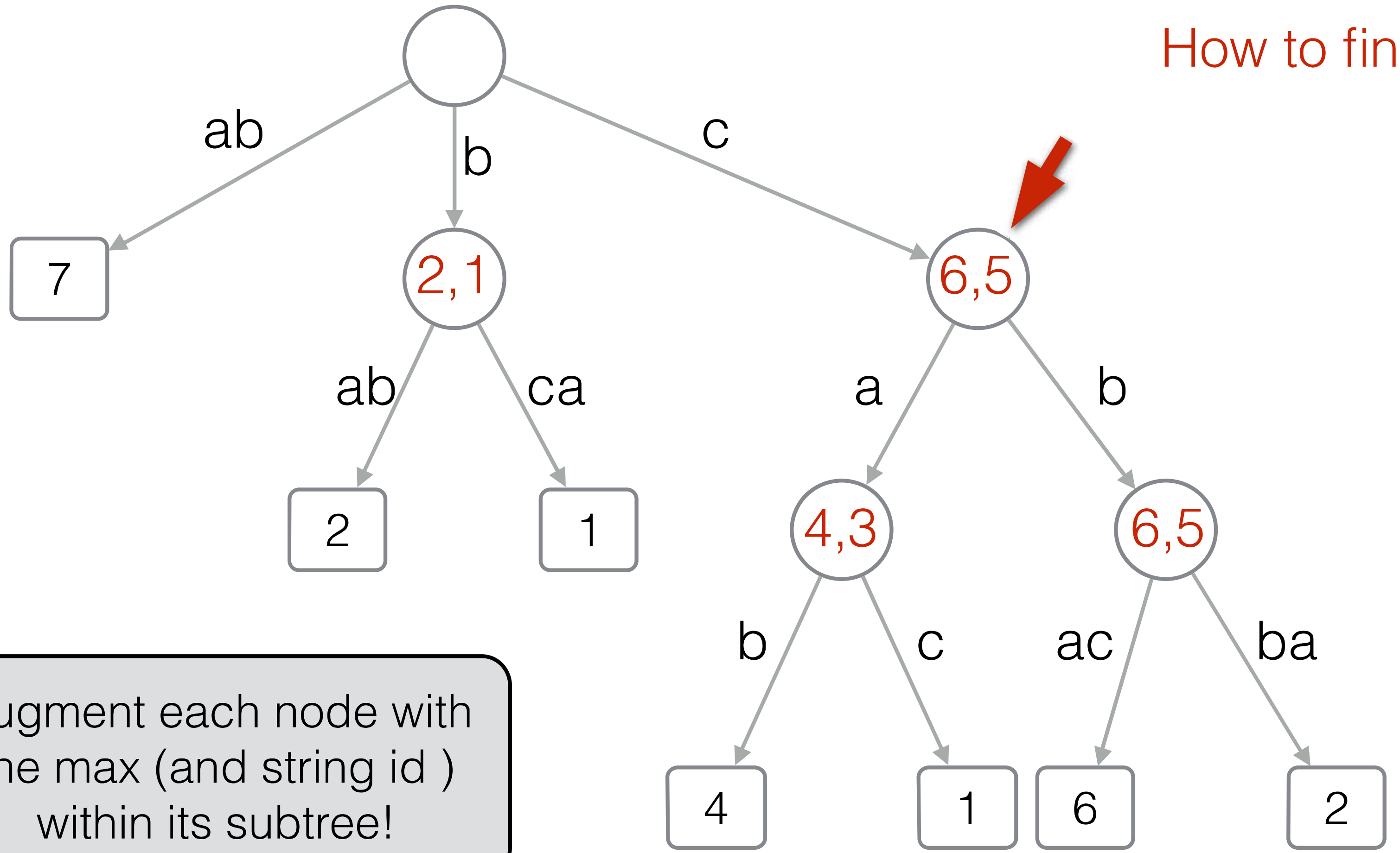

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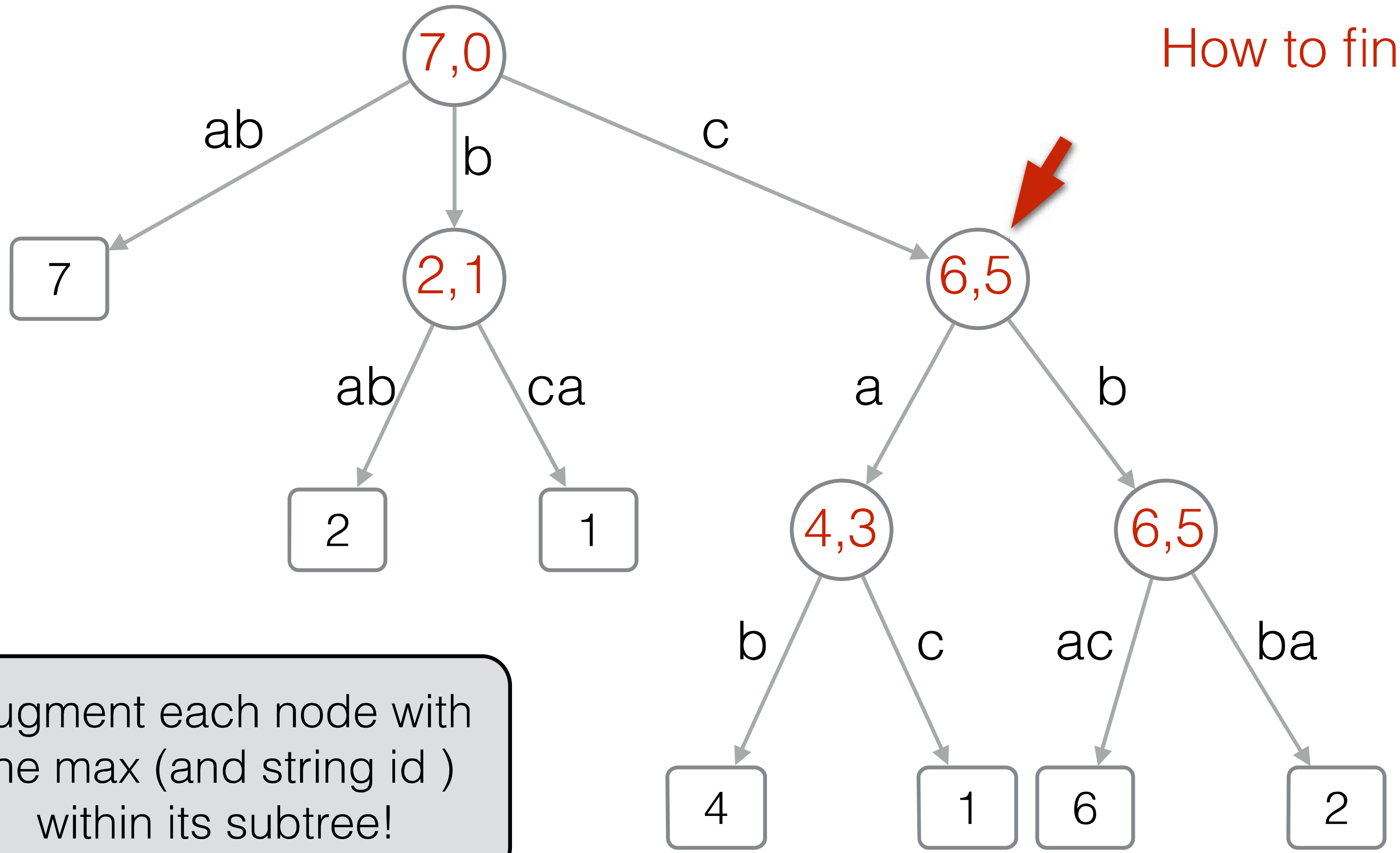

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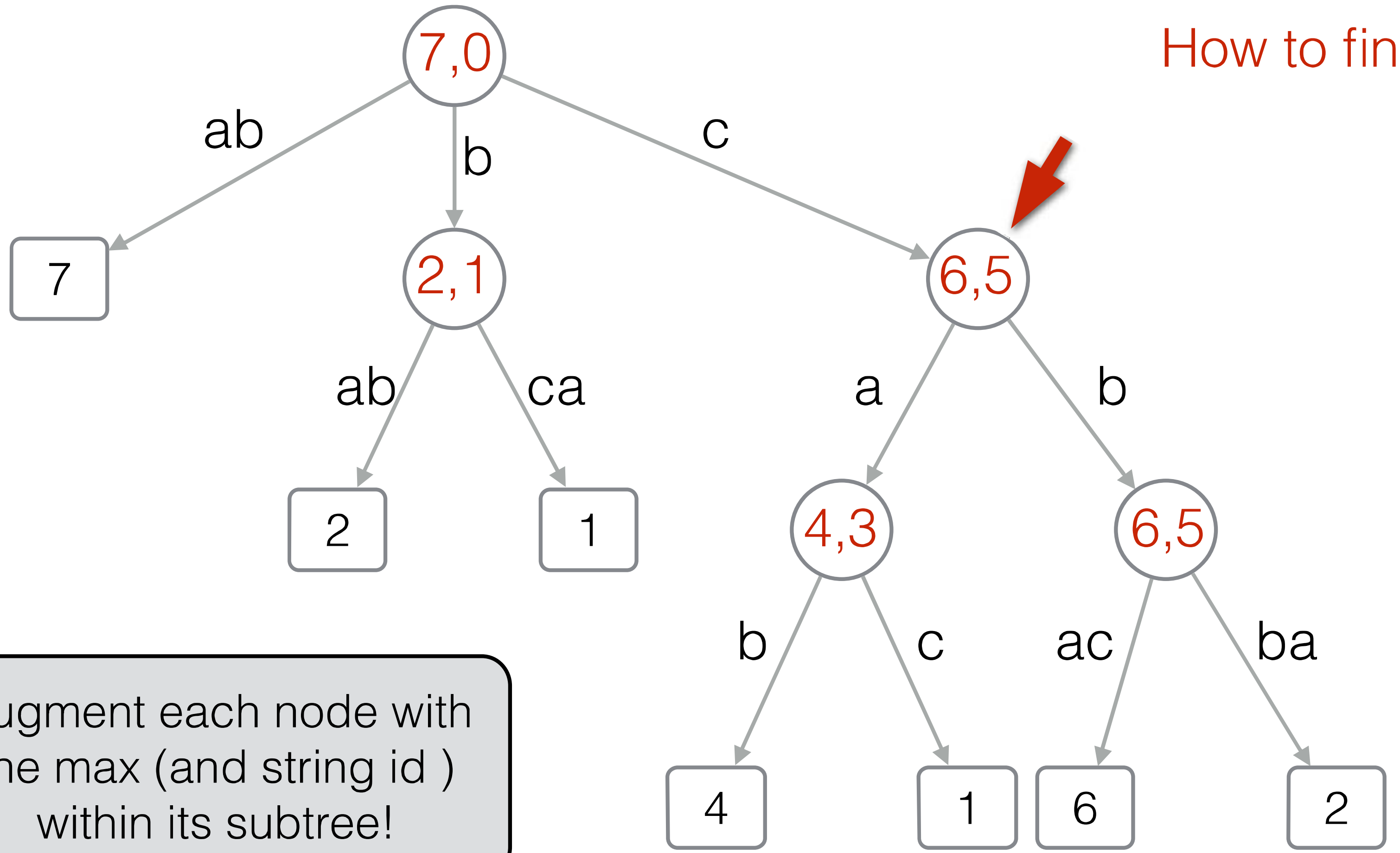


Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

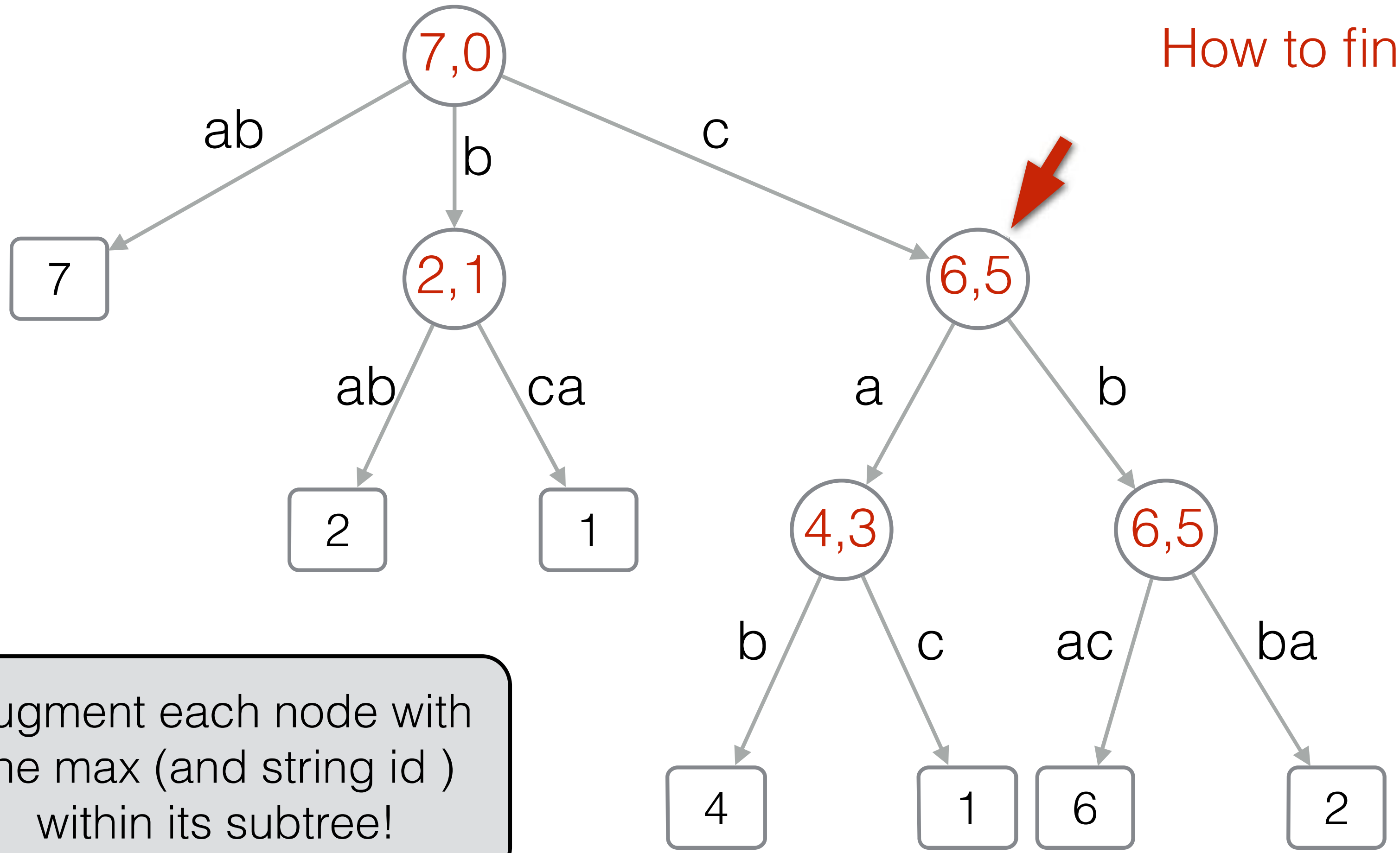
D (1), cab (4), cac (1), cbac (6), cbba (2) }

l length of strings in D

Finding Top-1

P = c

How to find Top-1?



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Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

Solving Top-k?

D

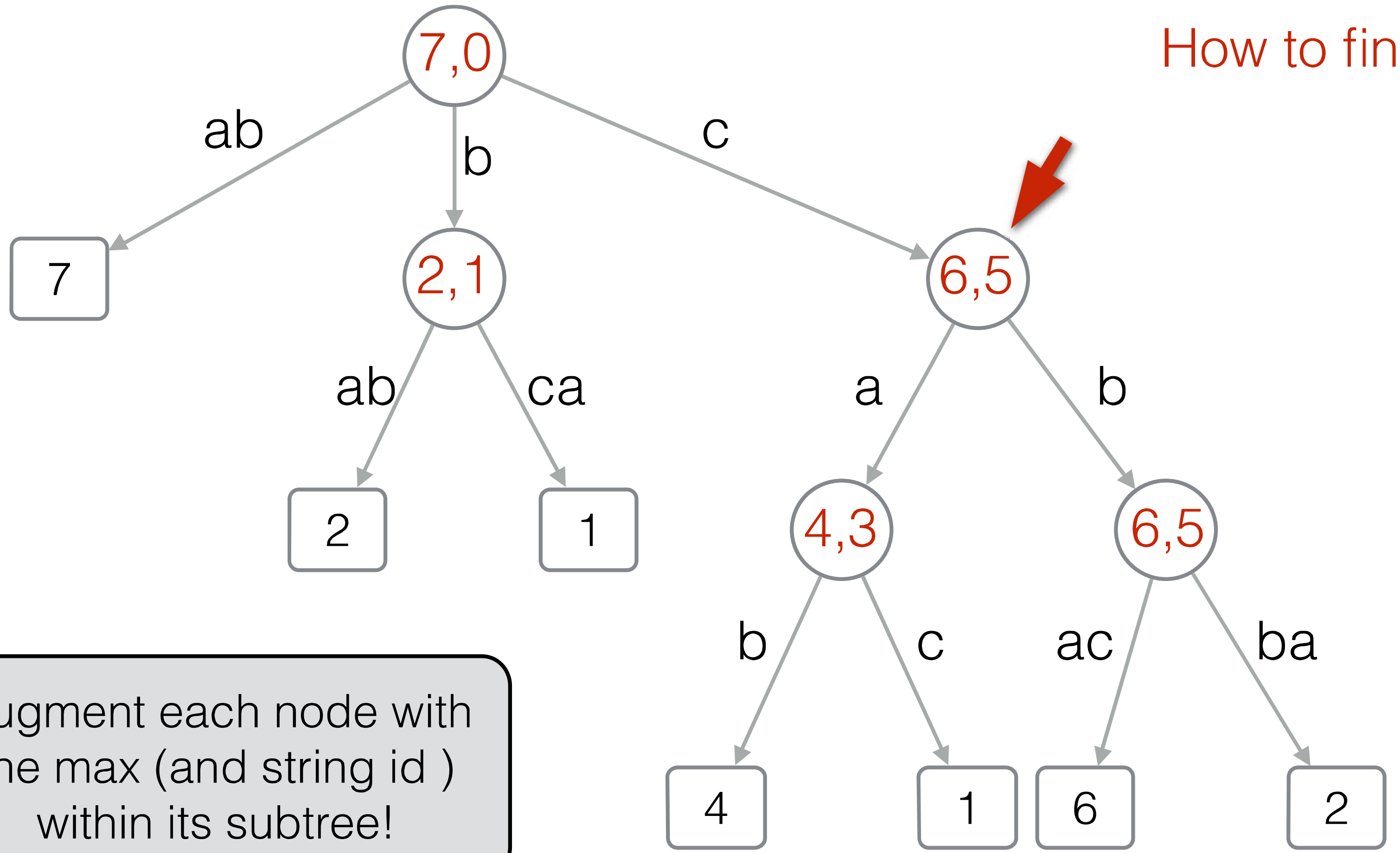
(1), ca

l length of strings in D

Finding Top-1

P = c

How to find Top-1?



Augment each node with the max (and string id) within its subtree!

Preprocessing time: O(n)

Extra space: O(n log n) bits

Query time: O(1)

D ... (1), ca ... l length of strings in D

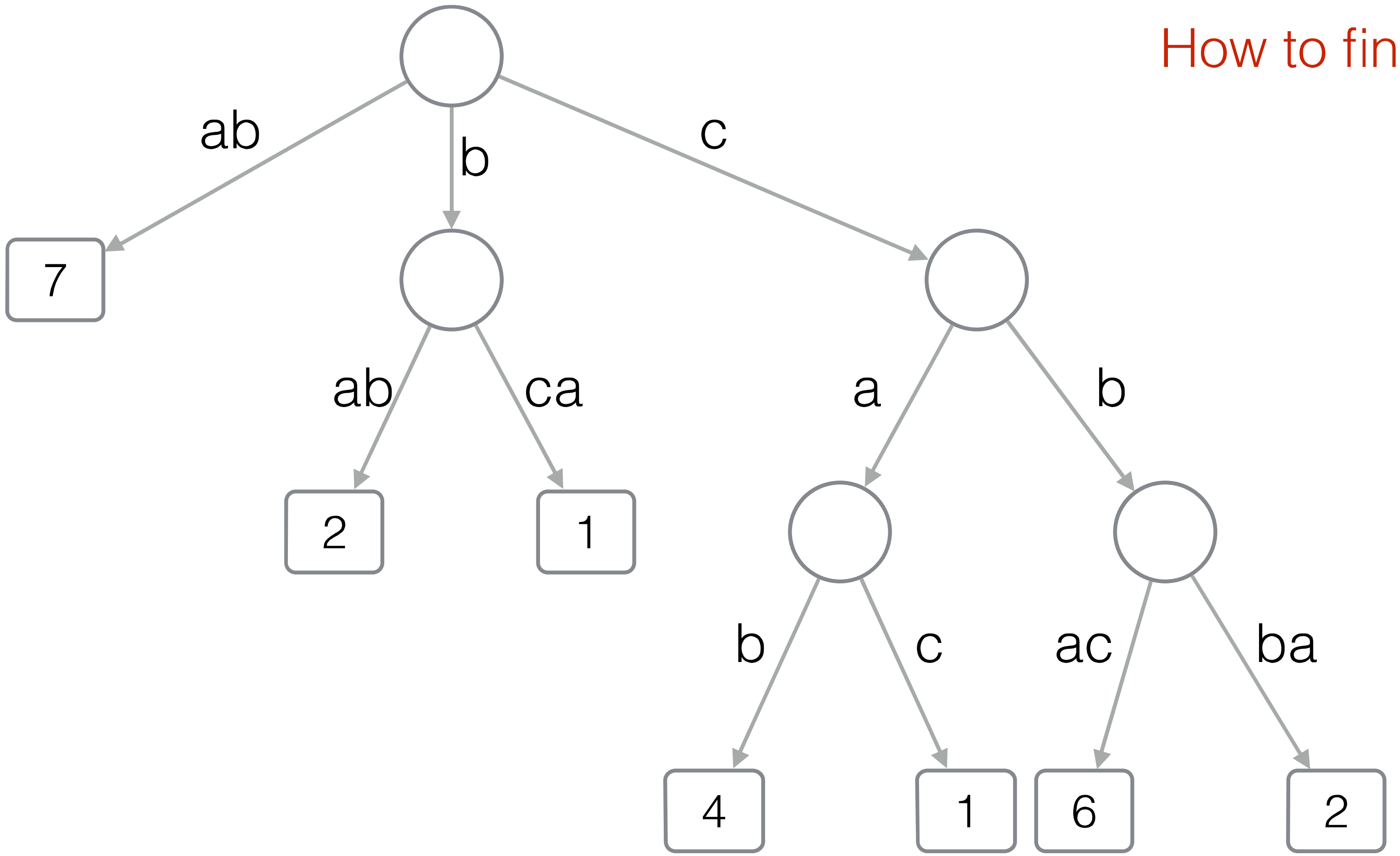
Solving Top-k?

- Extra space: O(k*n*log n) bits :-(
- You must know k at building time! :-(

Finding Top-1

P = c

How to find Top-1?



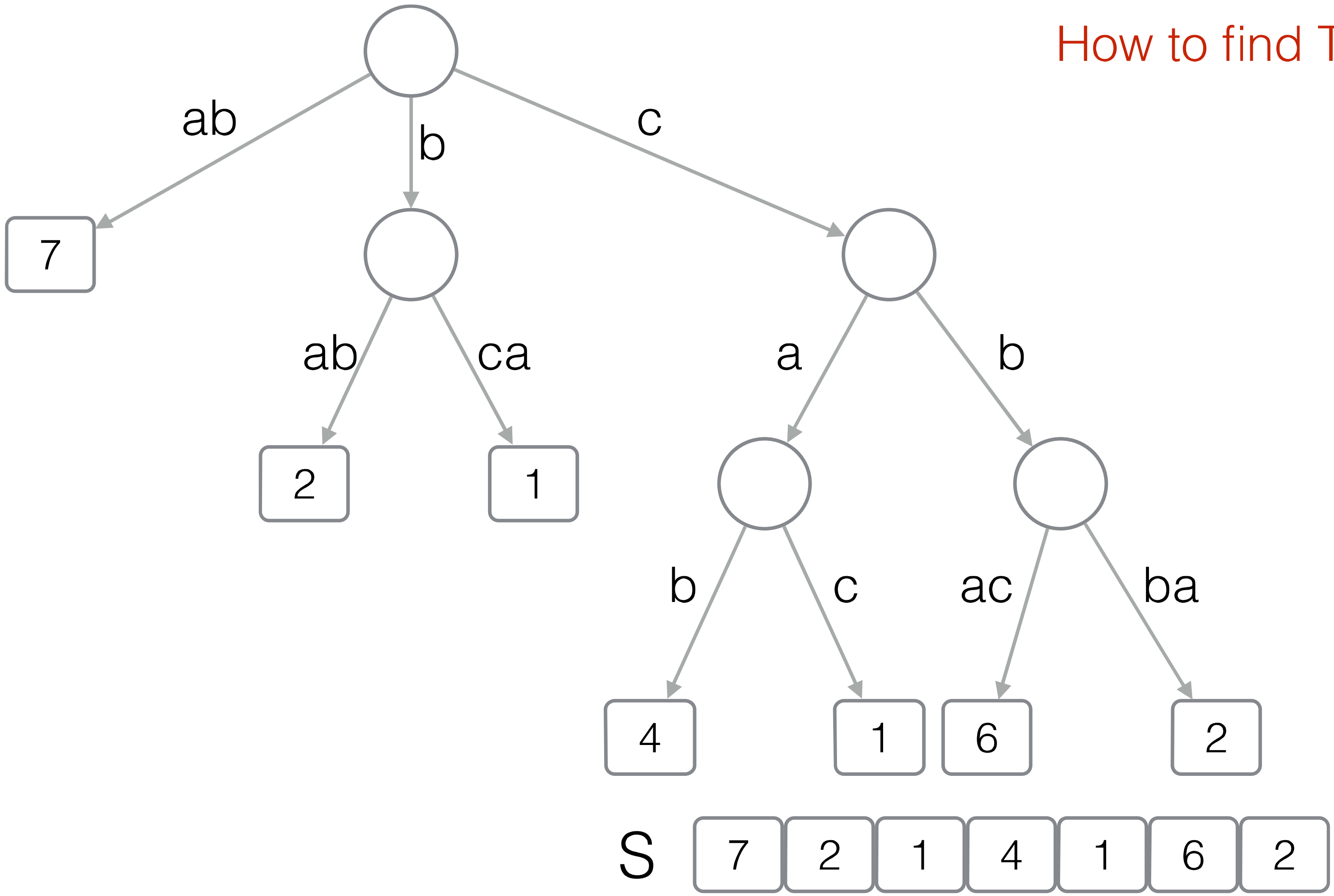
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



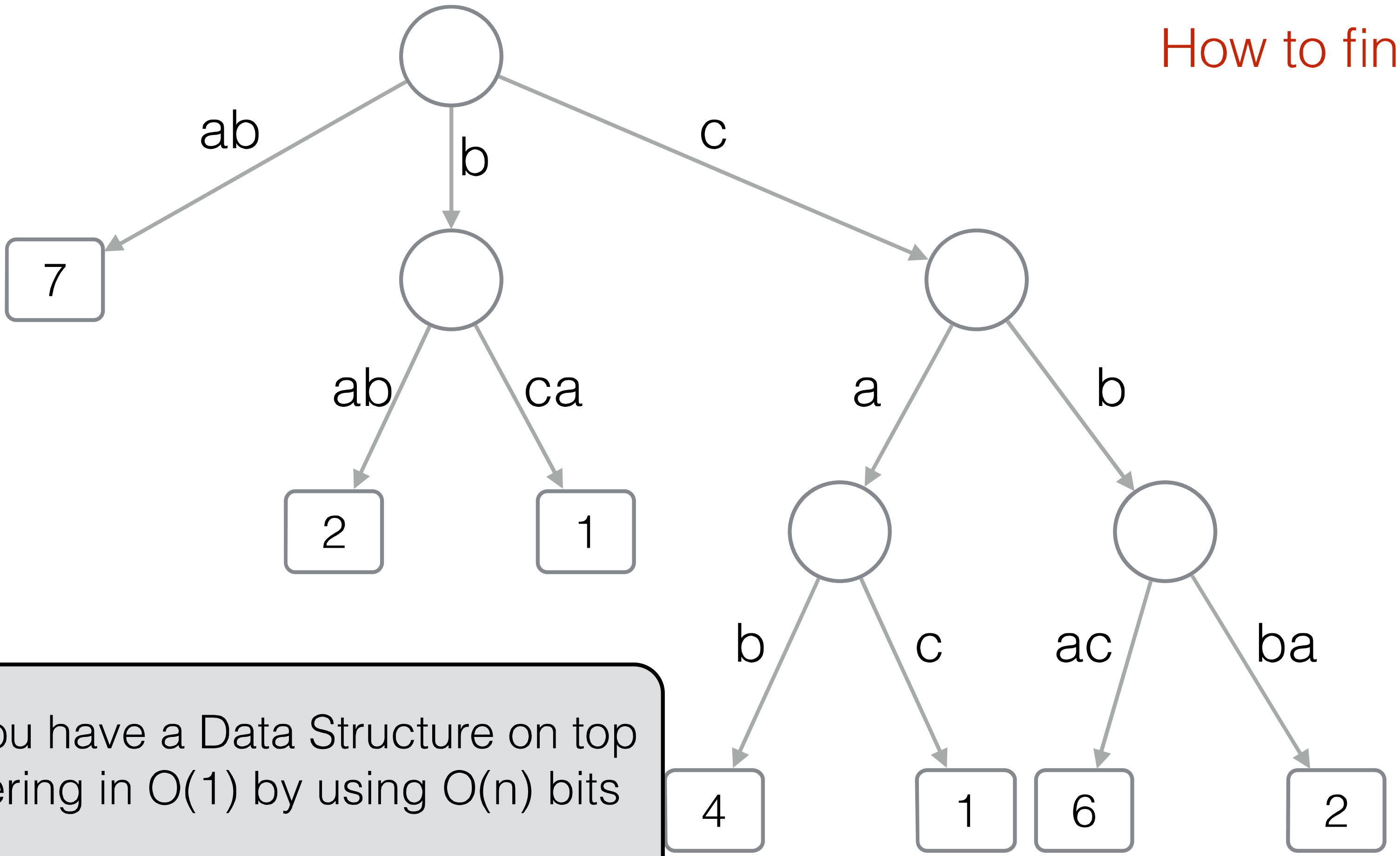
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

S							
	7	2	1	4	1	6	2

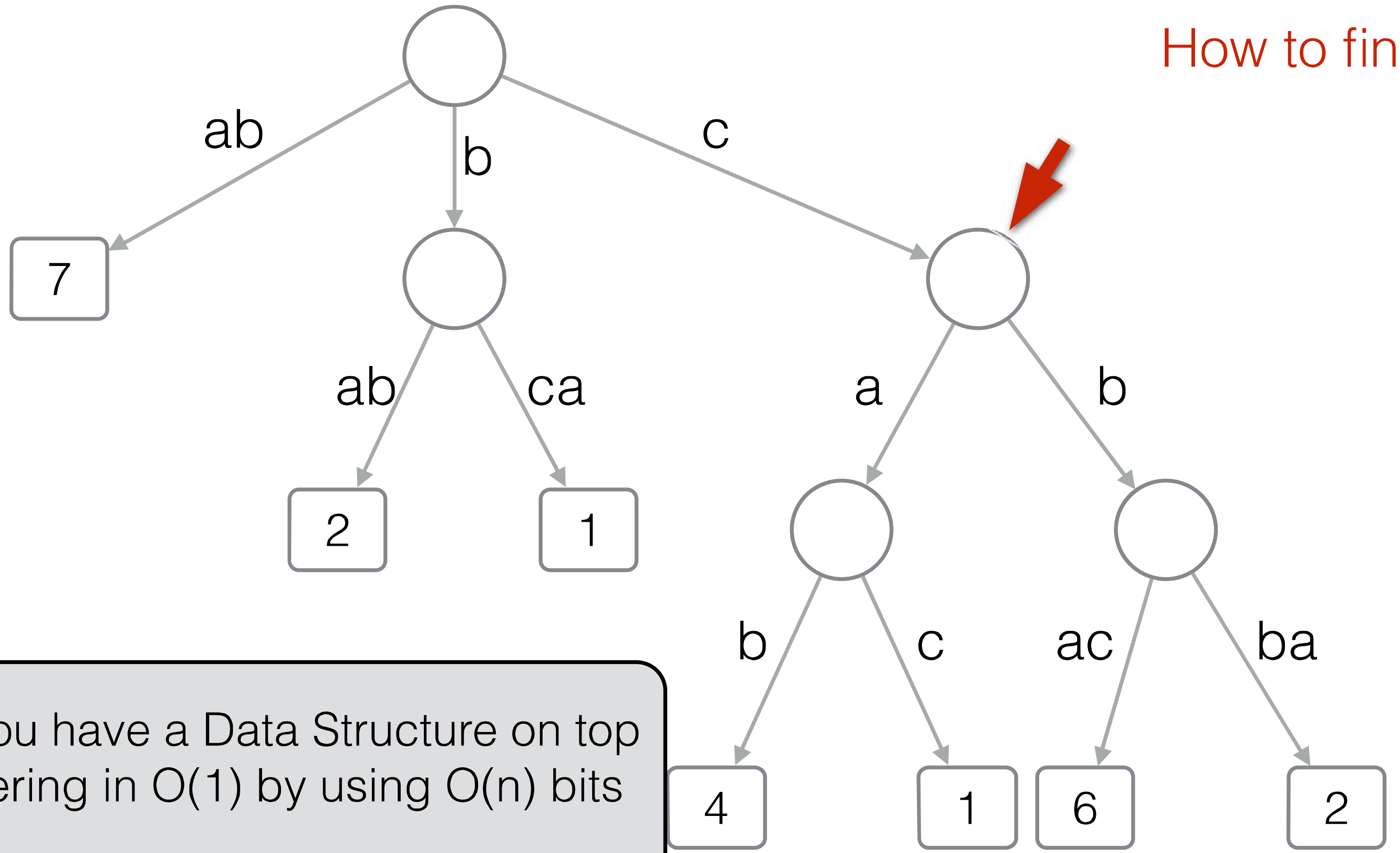
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

S

7	2	1	4	1	6	2

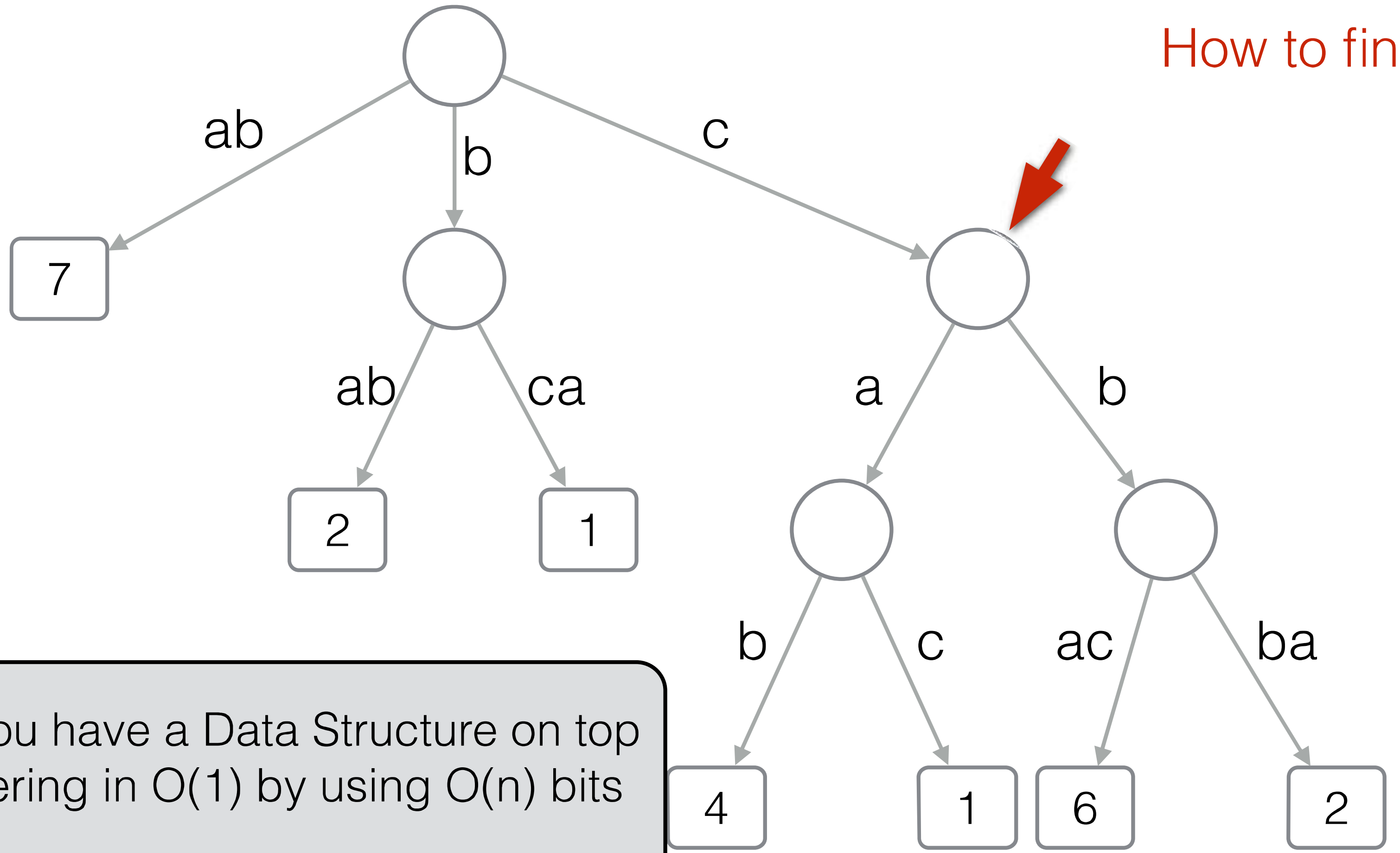
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of the maximum in the range S[i,j]

S							
S	7	2	1	4	1	6	2

RMQ(3,6) = 5

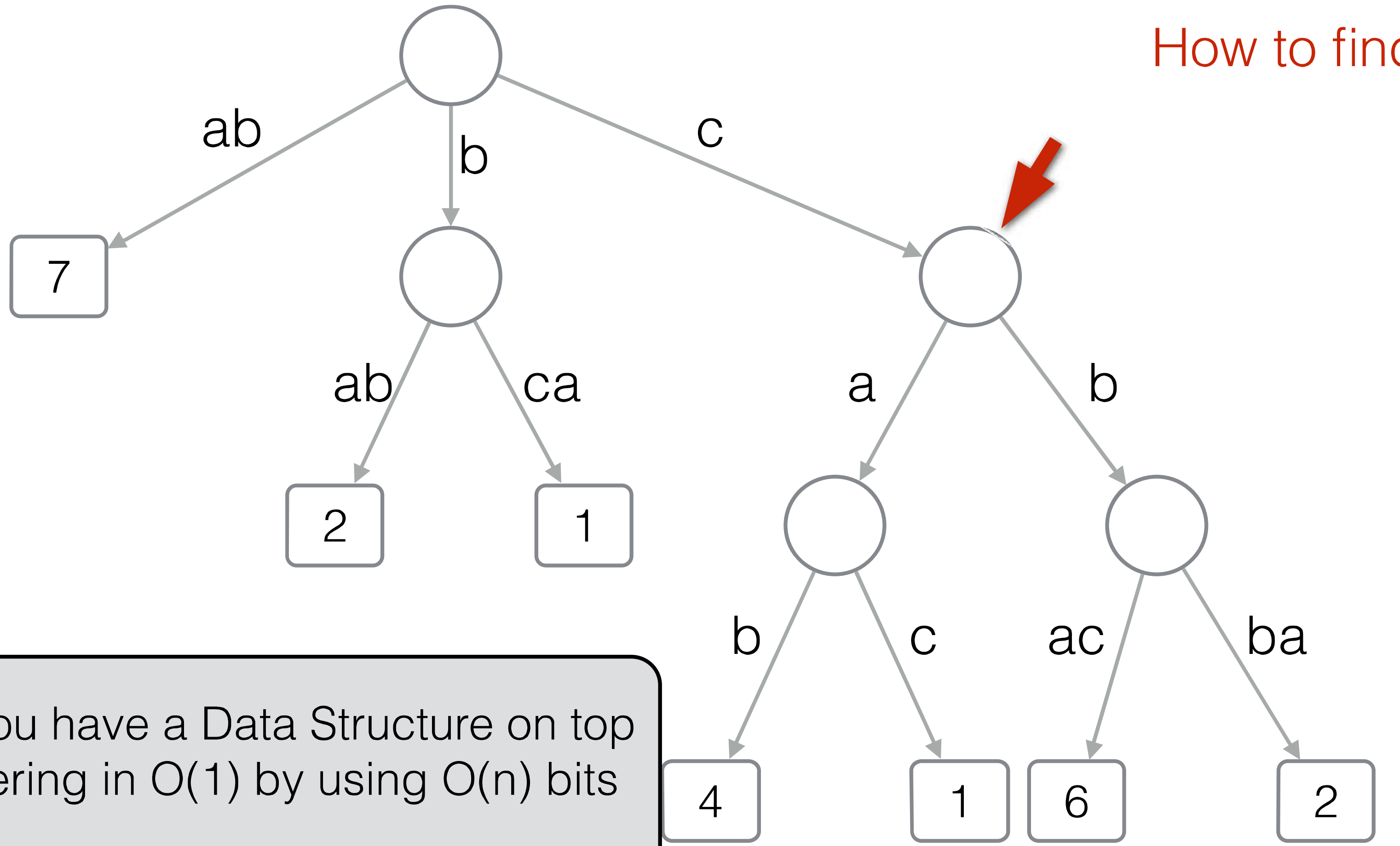
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

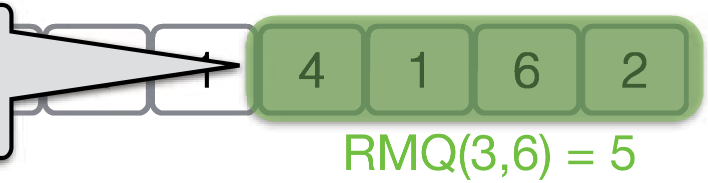
How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?



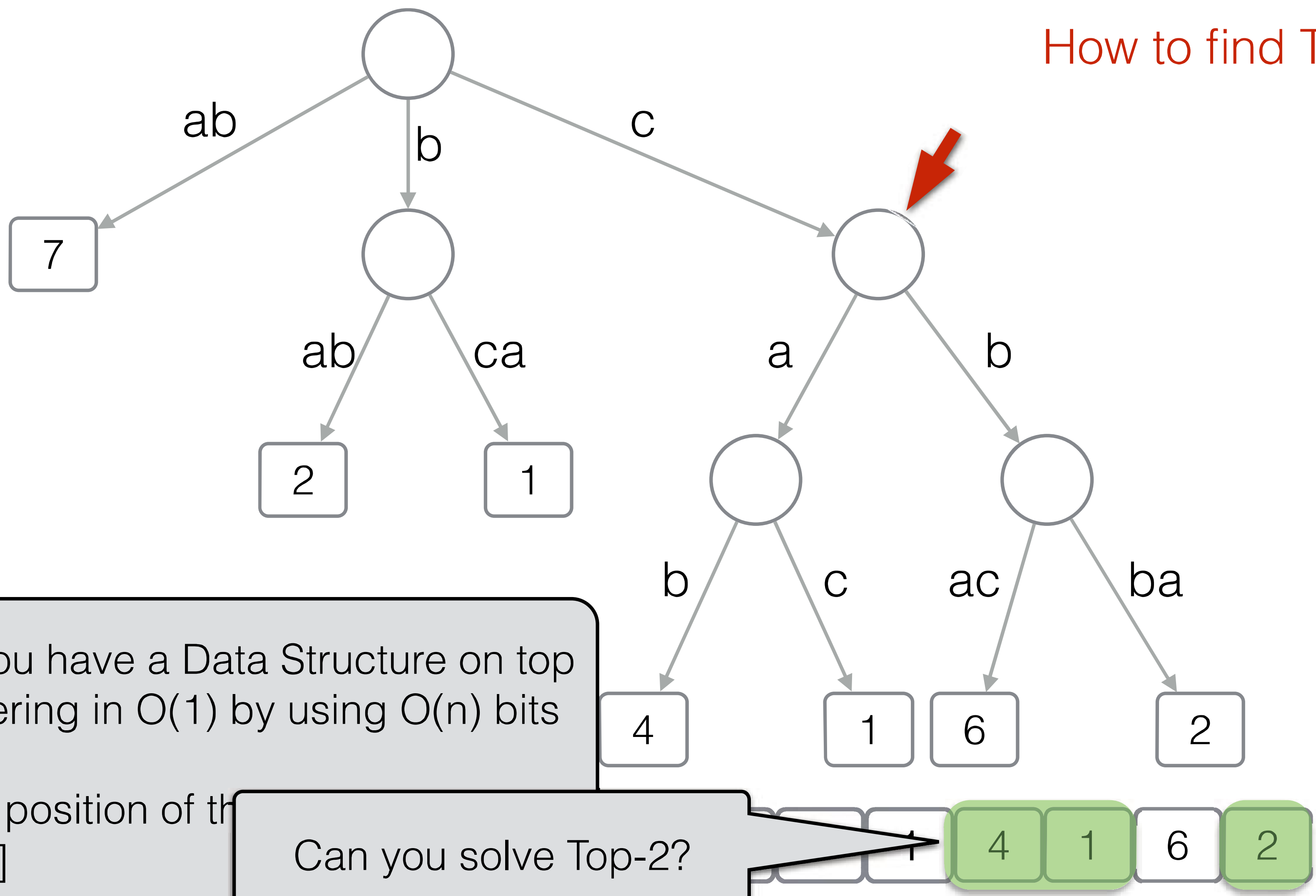
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

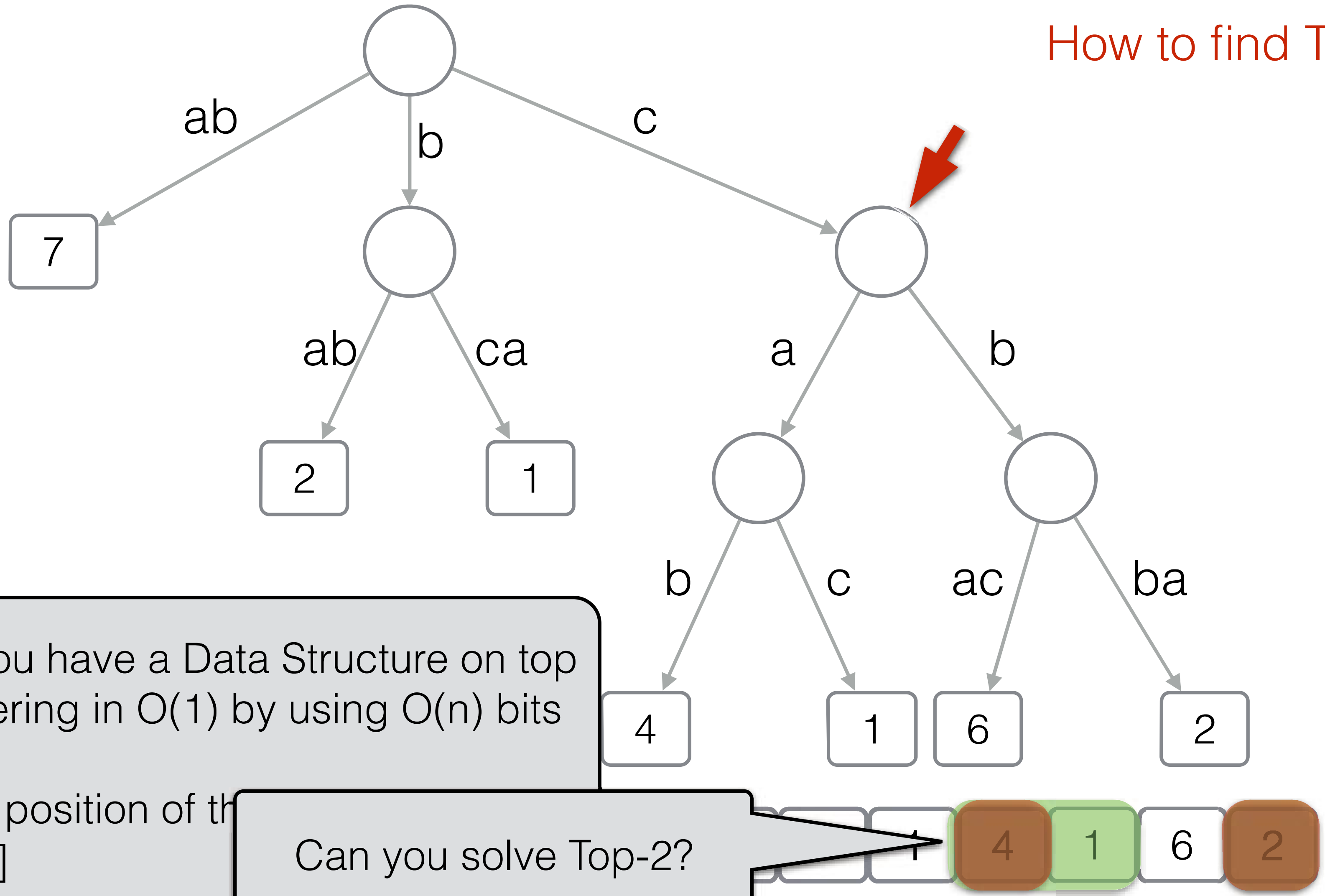
D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Finding Top-1

P = c

How to find Top-1?



Assume you have a Data Structure on top of S answering in O(1) by using O(n) bits

RMQ(i,j) = position of th
range S[i,j]

Can you solve Top-2?

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Range Maximum Query (1)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

$M[i,j] = RMQ(i,j)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

$M[i,j] = RMQ(i,j)$

M	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2												
3												
4												
5												
6												
7												
8												
9												
10												
11												

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (1)

Space: $O(n^2 \log n)$ bits
Query time: $O(1)$

Precompute the answer to any possible query.

There are $O(n^2)$ distinct queries!

$M[i,j] = RMQ(i,j)$

M	0	1	2	3	4	5	6	7	8	9	10	11
0												
1												
2						3						
3												
4												
5												
6												
7												
8												
9												
10												
11												

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i, i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i, i+2^j)$$

M

	0	1	2	3	4
0					
1				?	
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

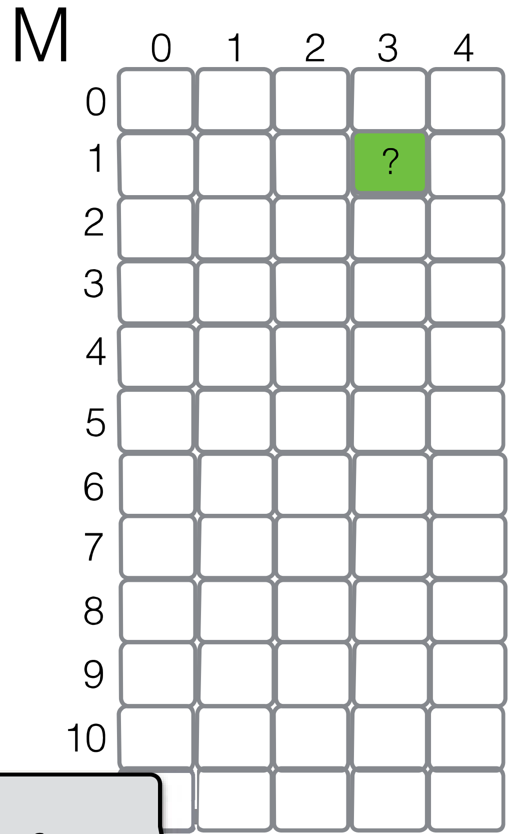
Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i, i+2^j)$$



$9 = 1 + 2^3$

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

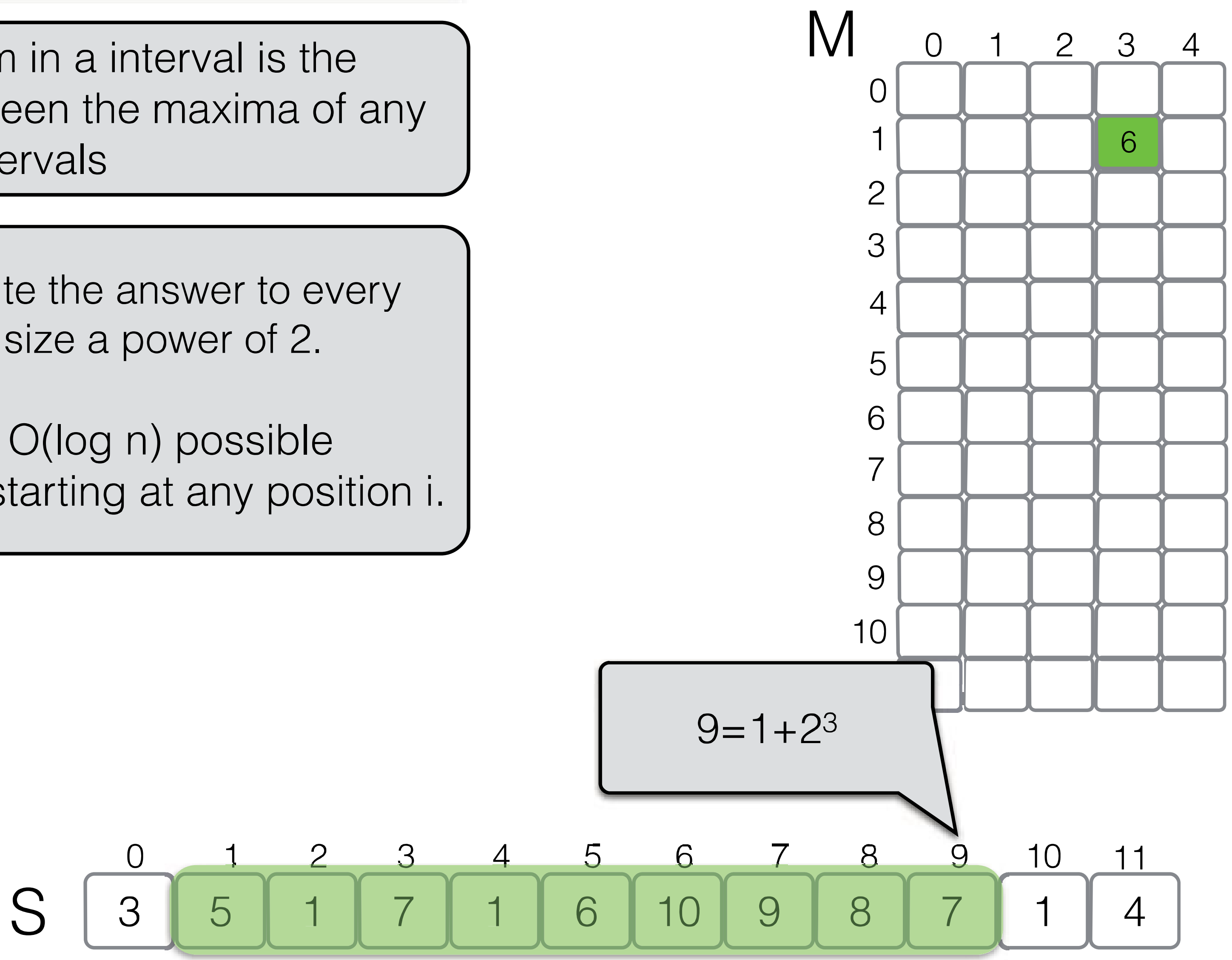
Space: O(n log² n) bits
Query time: O(1)

Maximum in a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are O(log n) possible intervals starting at any position i.

$M[i,j] = RMQ(i, i+2^j)$



Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i, i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

RMQ(1,7) =

$$M[i,j] = RMQ(i, i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

$M[i,j] = RMQ(i,i+2^j)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

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S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

$M[i,j] = RMQ(i, i+2^j)$

Maximum of a interval is the max between the maxima of any its subintervals

M

	0	1	2	3	4
0					
1			3		
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

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S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

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There are $O(\log n)$ possible intervals starting at any position i.

$M[i,j] = RMQ(i,i+2^j)$

M

	0	1	2	3	4
0					
1			3		
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

$$M[i,j] = RMQ(i,i+2^j)$$

M

	0	1	2	3	4
0					
1			3		
2					
3			6		
4					
5					
6					
7					
8					
9					
10					
11					

$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$

S

0	1	2	3	4	5	6	7	8	9	10	11
3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (2)

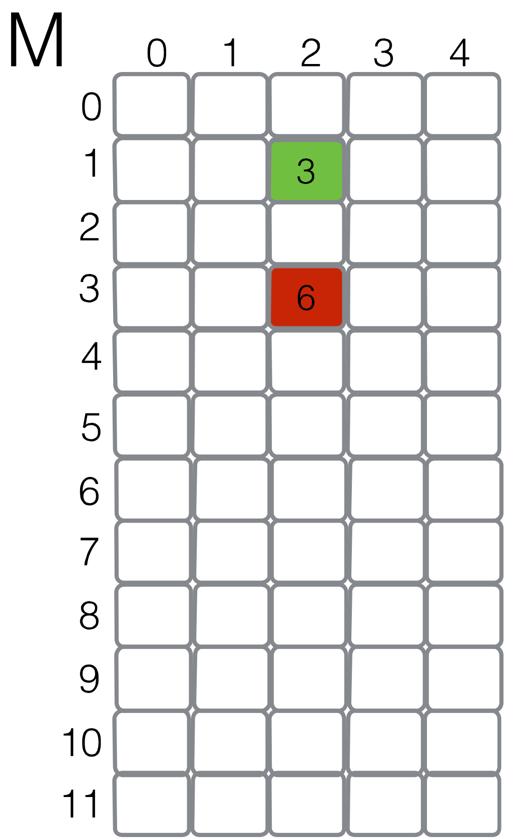
Space: $O(n \log^2 n)$ bits
Query time: $O(1)$

Maximum of a interval is the max between the maxima of any its subintervals

Precompute the answer to every interval of size a power of 2.

There are $O(\log n)$ possible intervals starting at any position i.

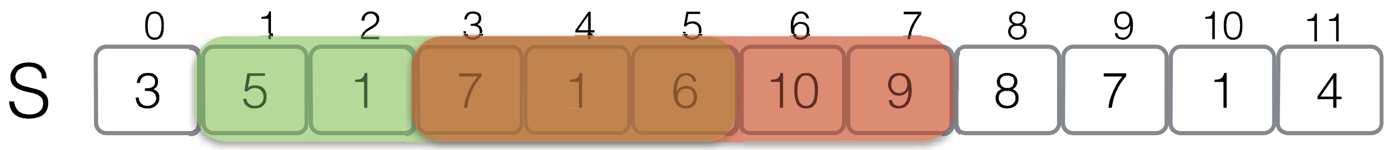
$$M[i,j] = RMQ(i, i+2^j)$$



$$RMQ(1,7) = \text{argmax}(S[M[1,1+2^2]], S[M[7-2^2,7]]) = 6$$

$$RMQ(i,j) = \text{argmax}(S[M[i,i+2^{len}]], S[M[j-2^{len},j]])$$

where $len = \lfloor \log (j-i+1) \rfloor$



Range Maximum Query (3)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

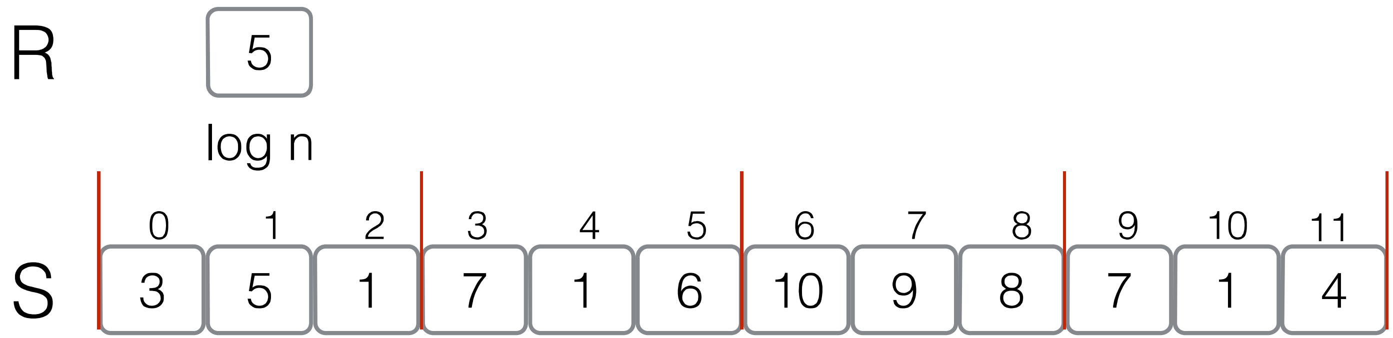
R

log n

	0	1	2	3	4	5	6	7	8	9	10	11
S	3	5	1	7	1	6	10	9	8	7	1	4

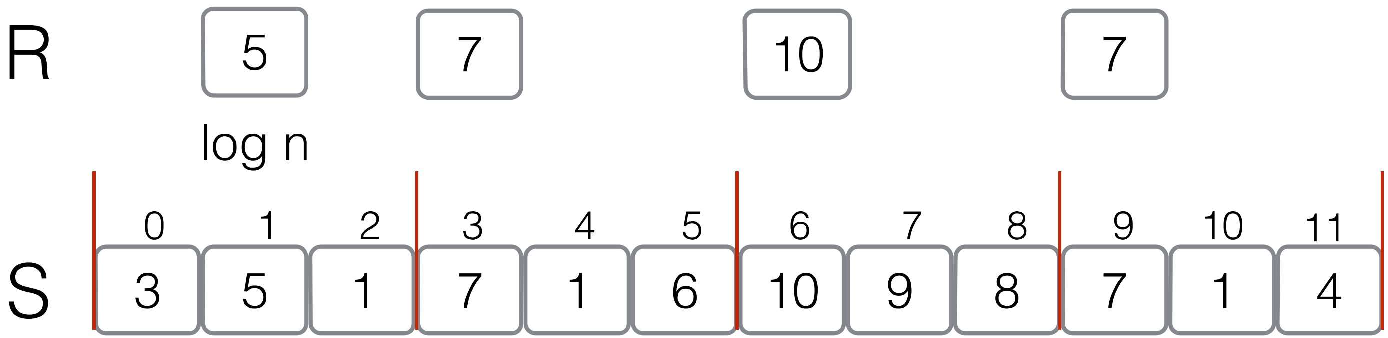
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)



Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

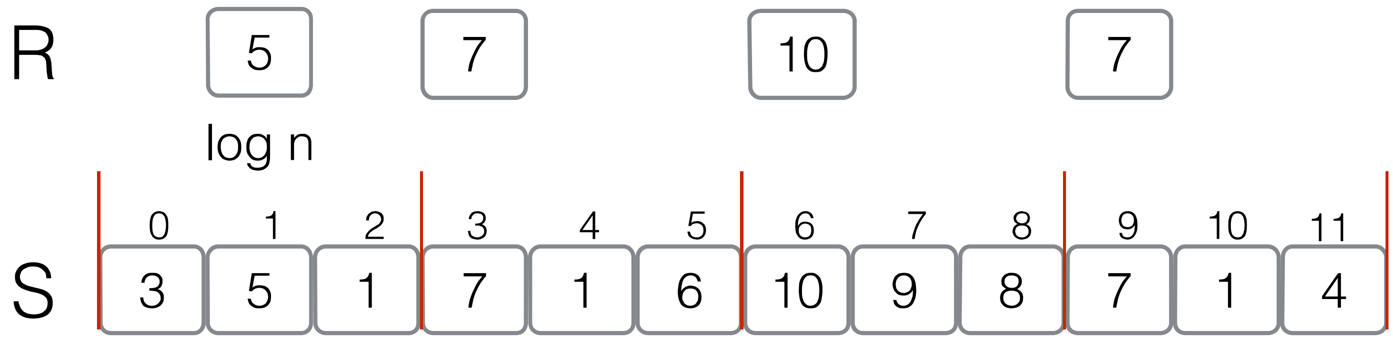


Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: ? bits
Query time: O(1)

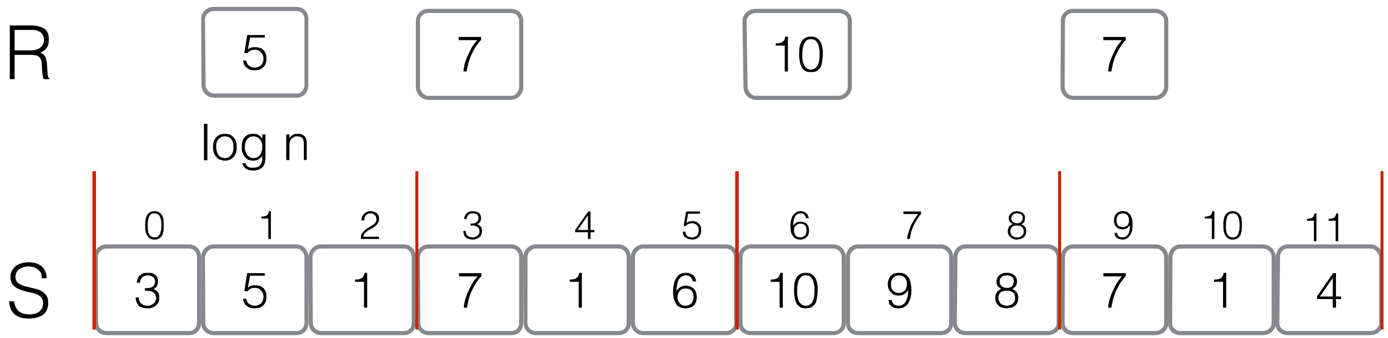


Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)



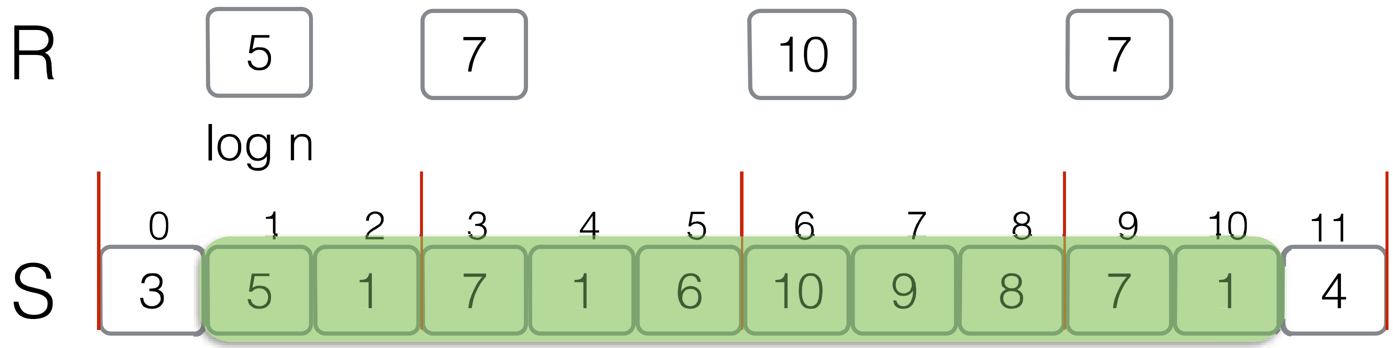
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



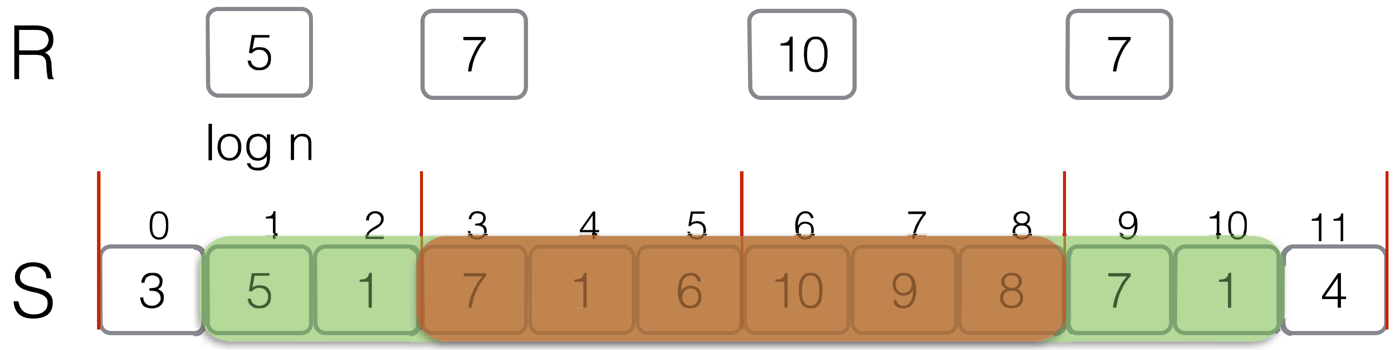
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



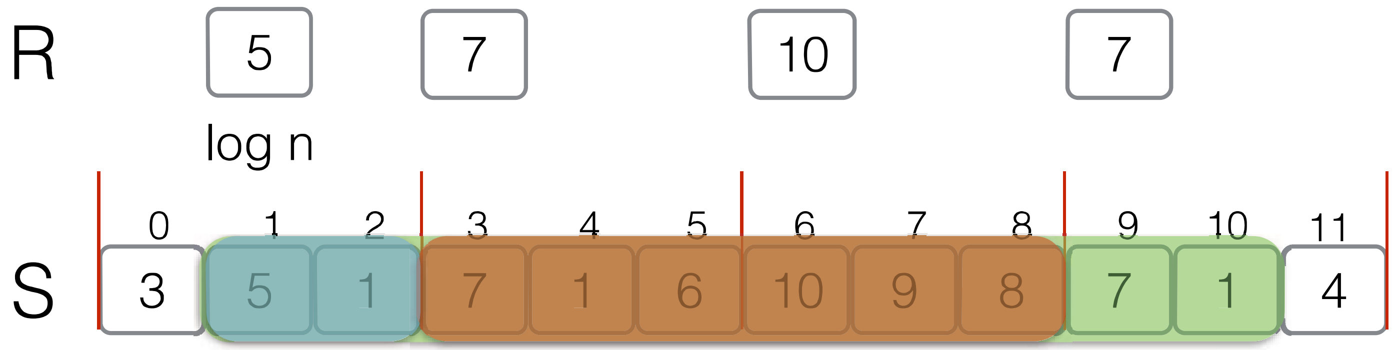
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



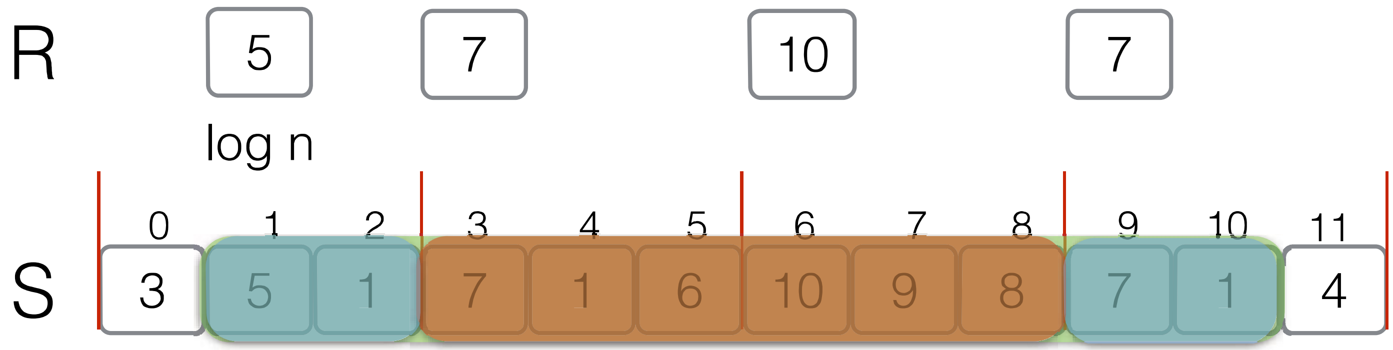
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



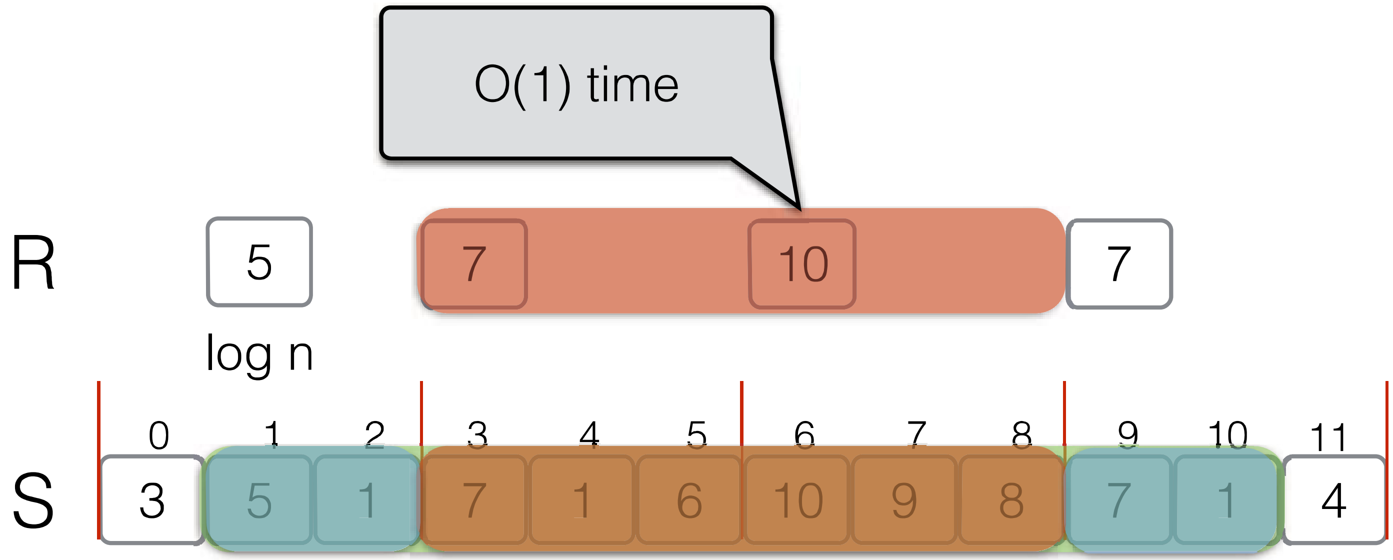
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



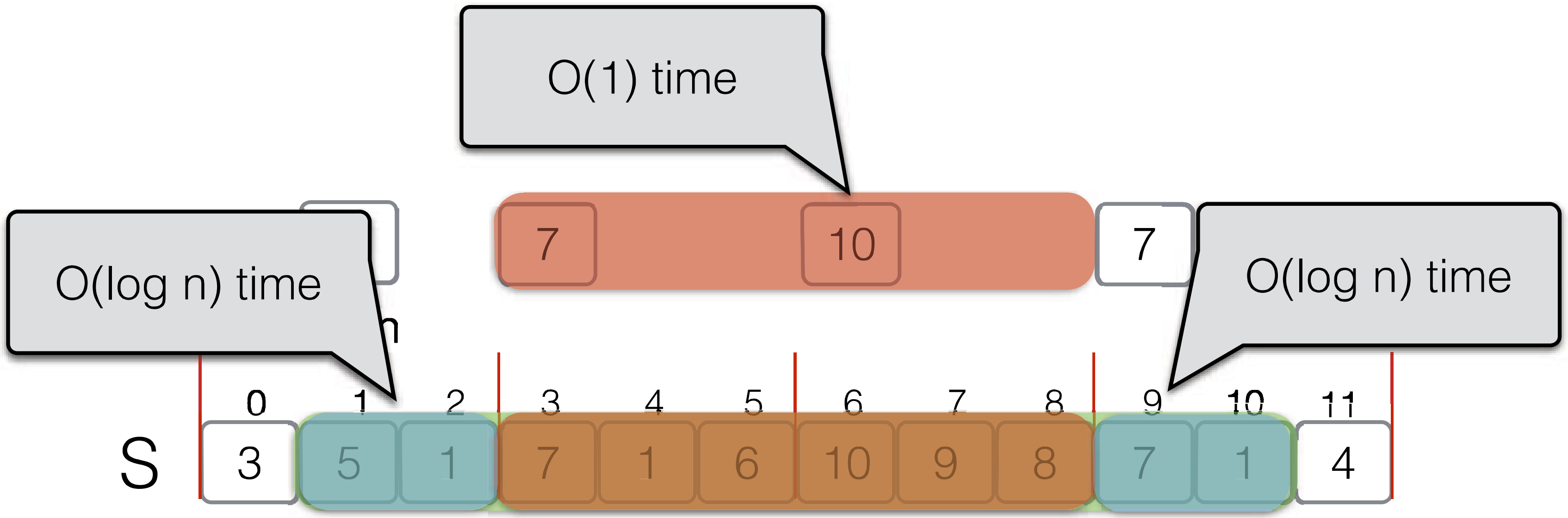
Range Maximum Query (3)

Space: O(n log n) bits
Query time: O(log n)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



Range Maximum Query (3)

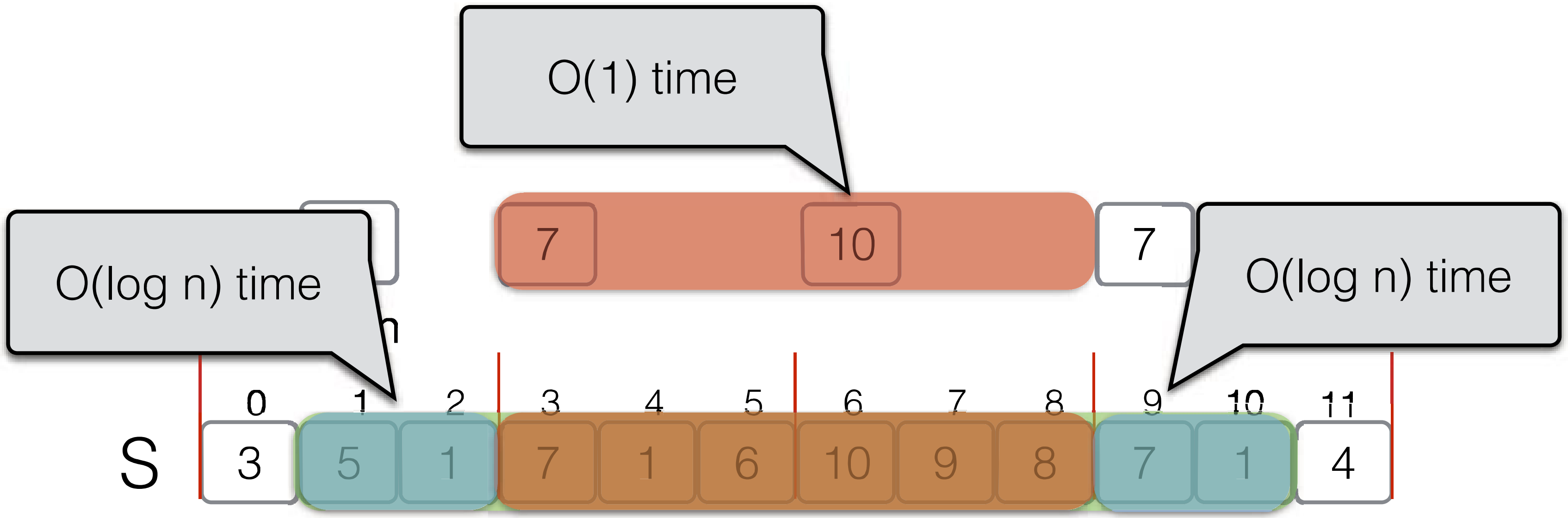
Space: O(n log n) bits
Query time: O(log n)

Space: O(n log n) bits
Query time: O(1)

Use the previous solution on R!

Space: O(n log n) bits
Query time: O(1)

RMQ(1,10) = ?



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Use the previous solution on R!

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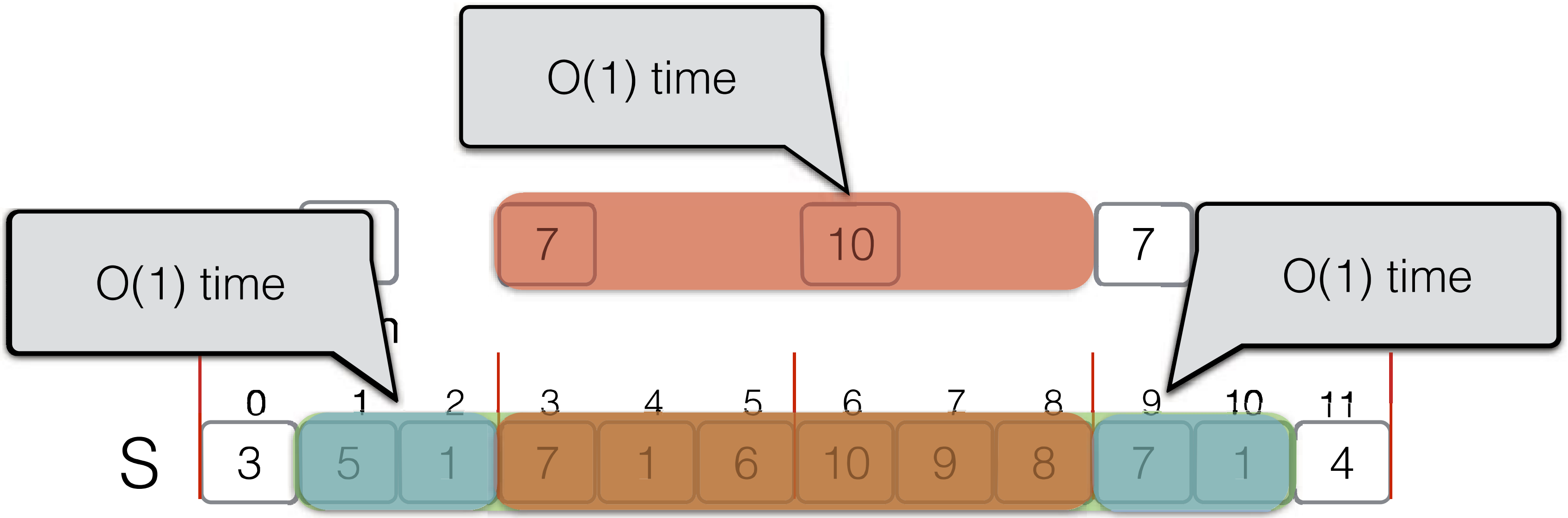

Range Maximum Query (3)

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Space: O(n ~~log~~ n) bits
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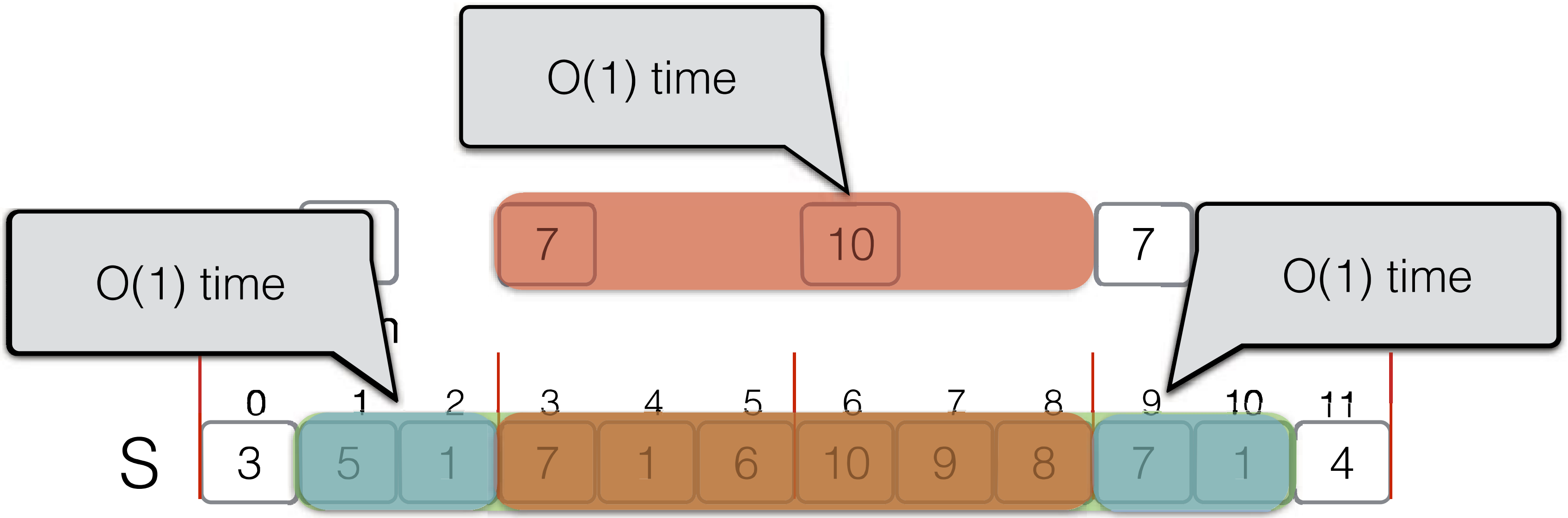
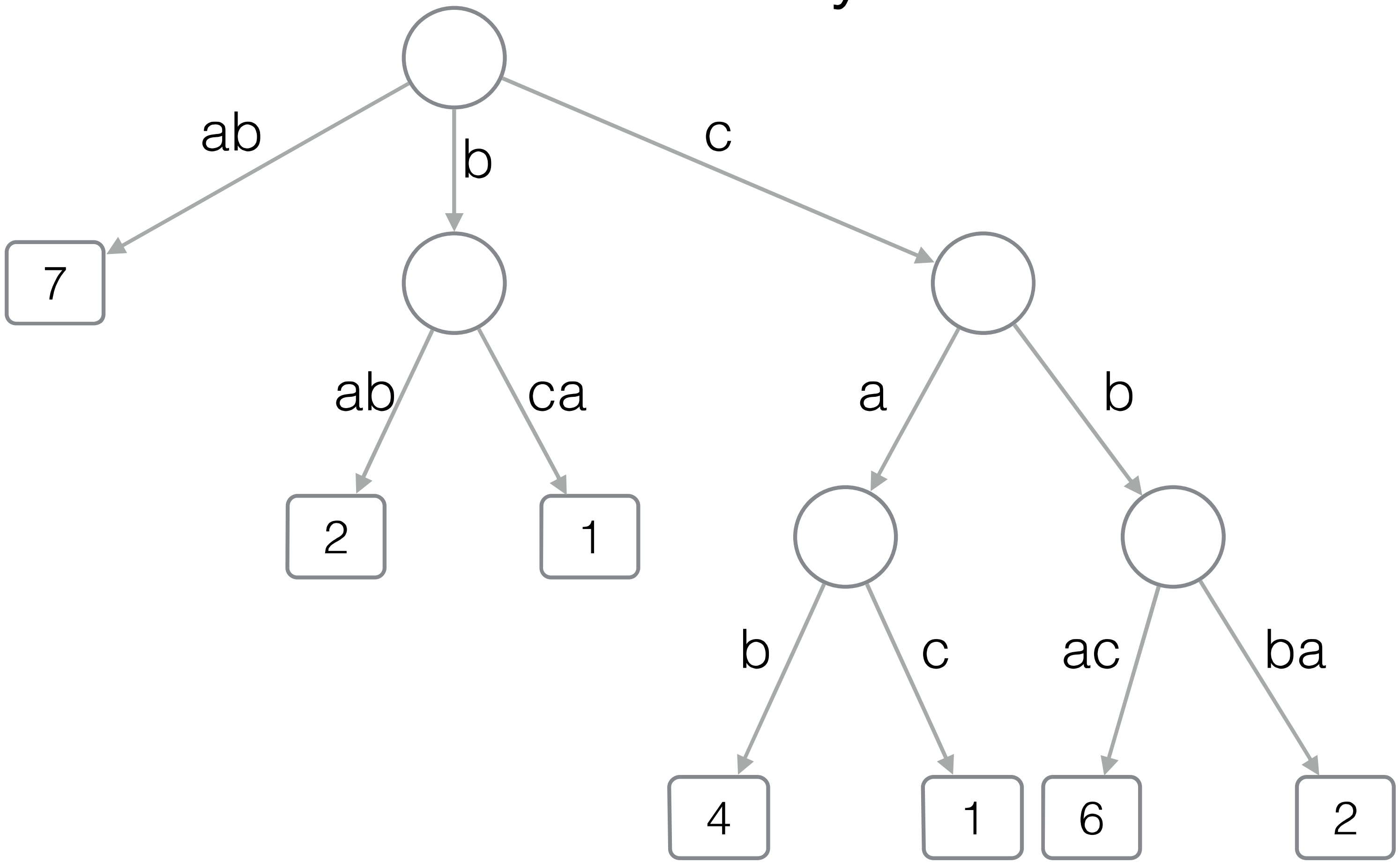


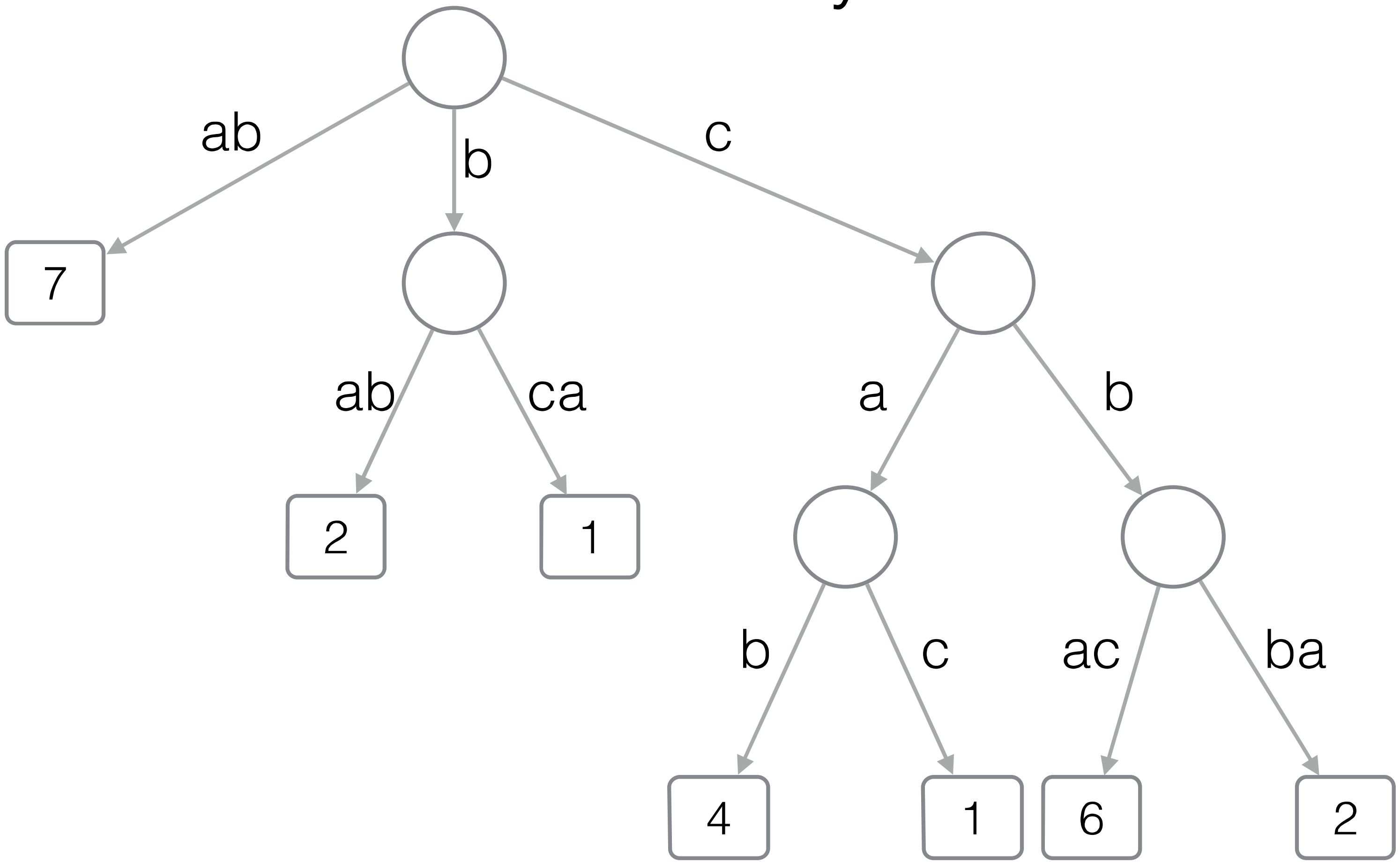
Summary



D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node "prefixed" by P

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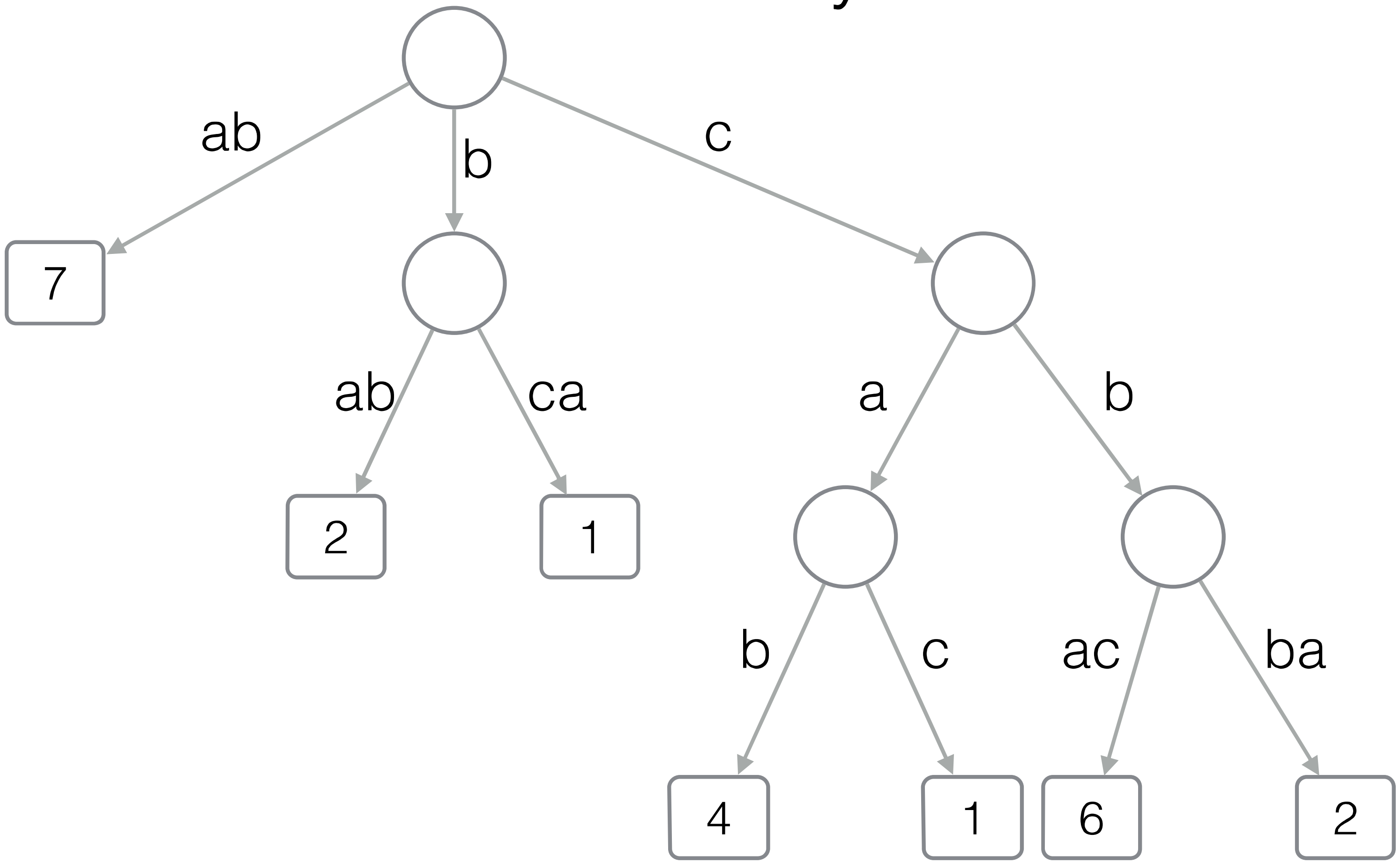
Summary



Find the node "prefixed" by P

O(|P|) time

D = { ab (7), bab (2), bca (1), cab (4), cac (1), cbac (6), cbba (2) }

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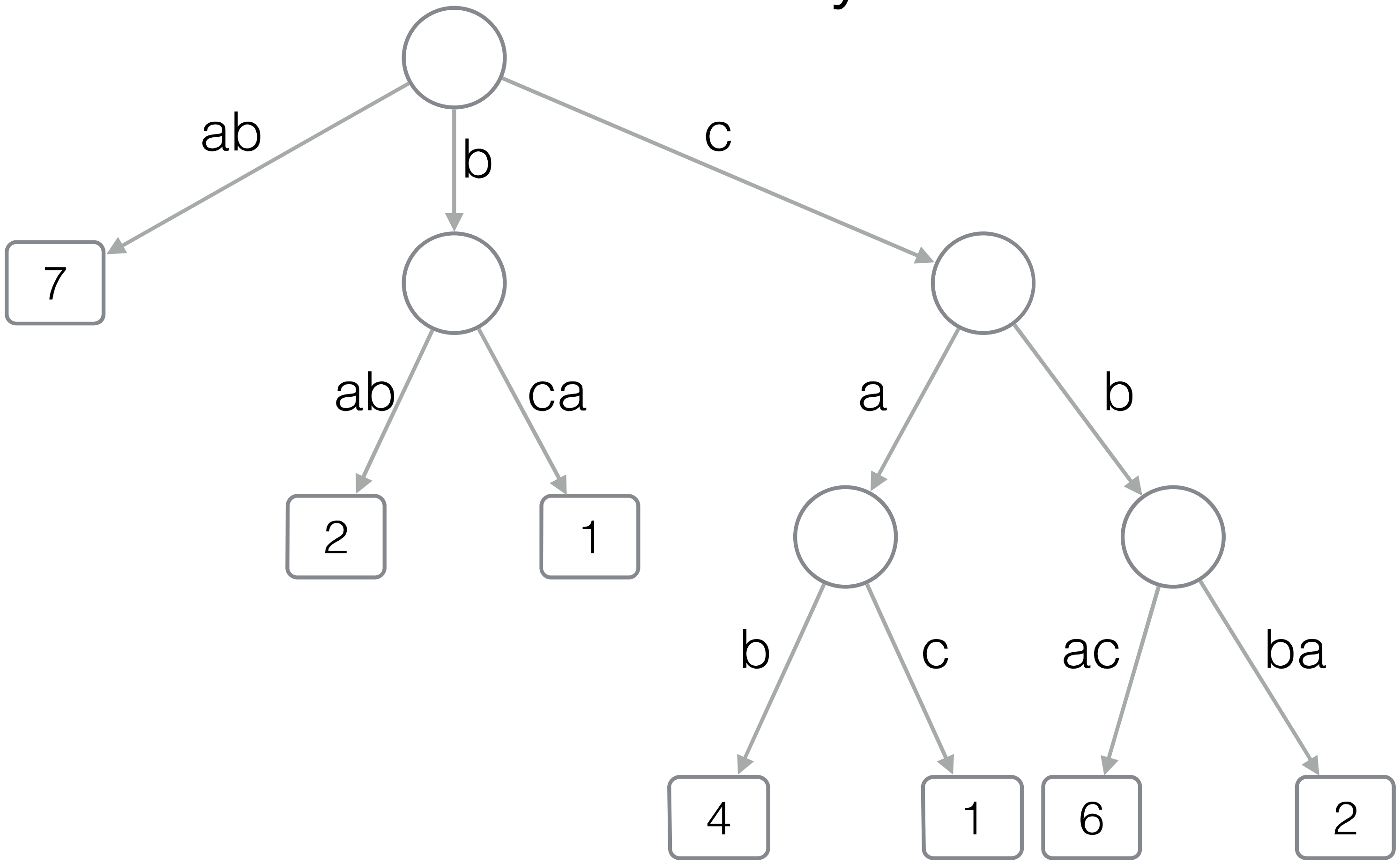
Summary



Find the node “prefixed” by P

O(|P|) time

O(m log σ + n log m) bits

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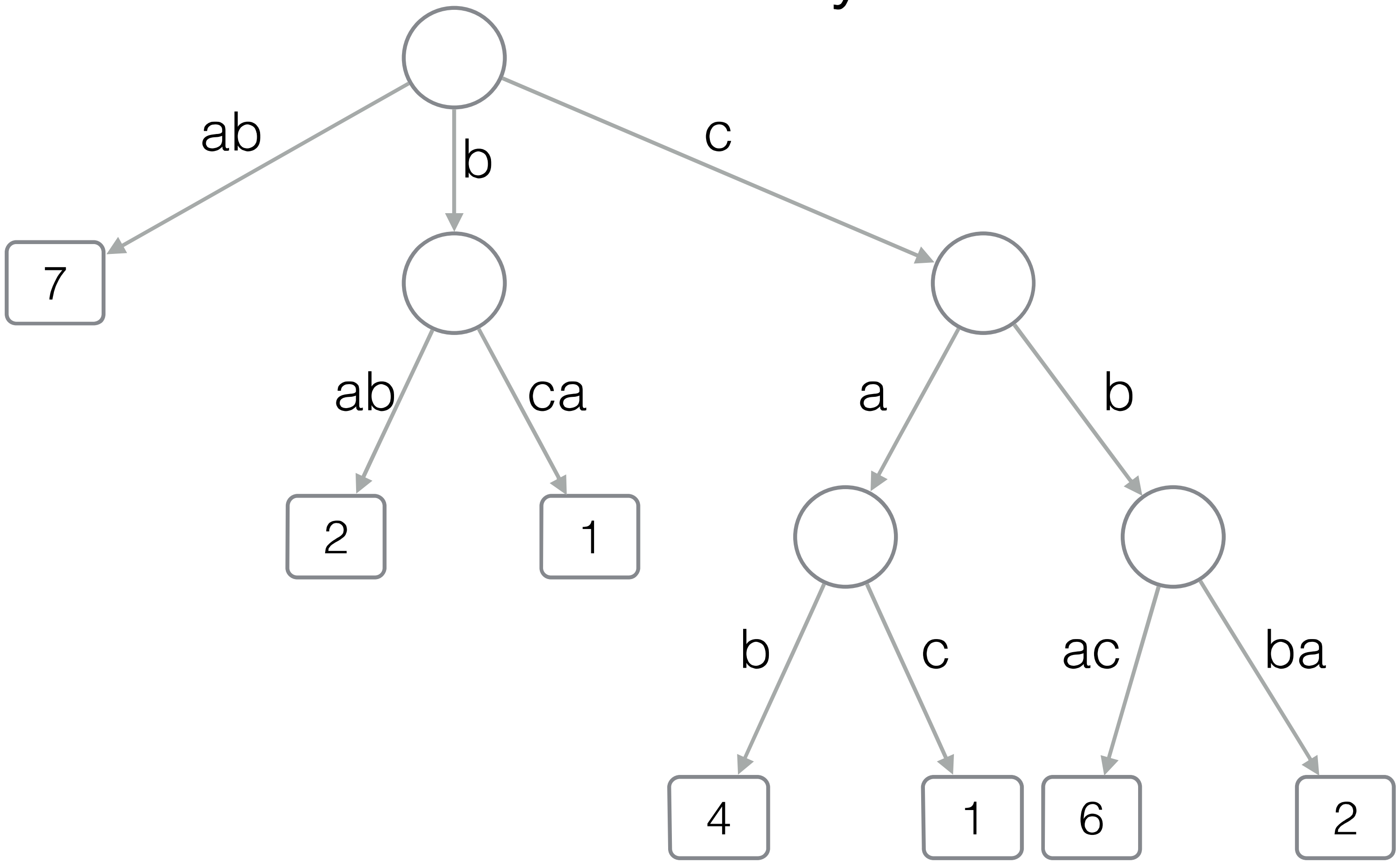
Summary



Find the node "prefixed" by P

O(|P|) time

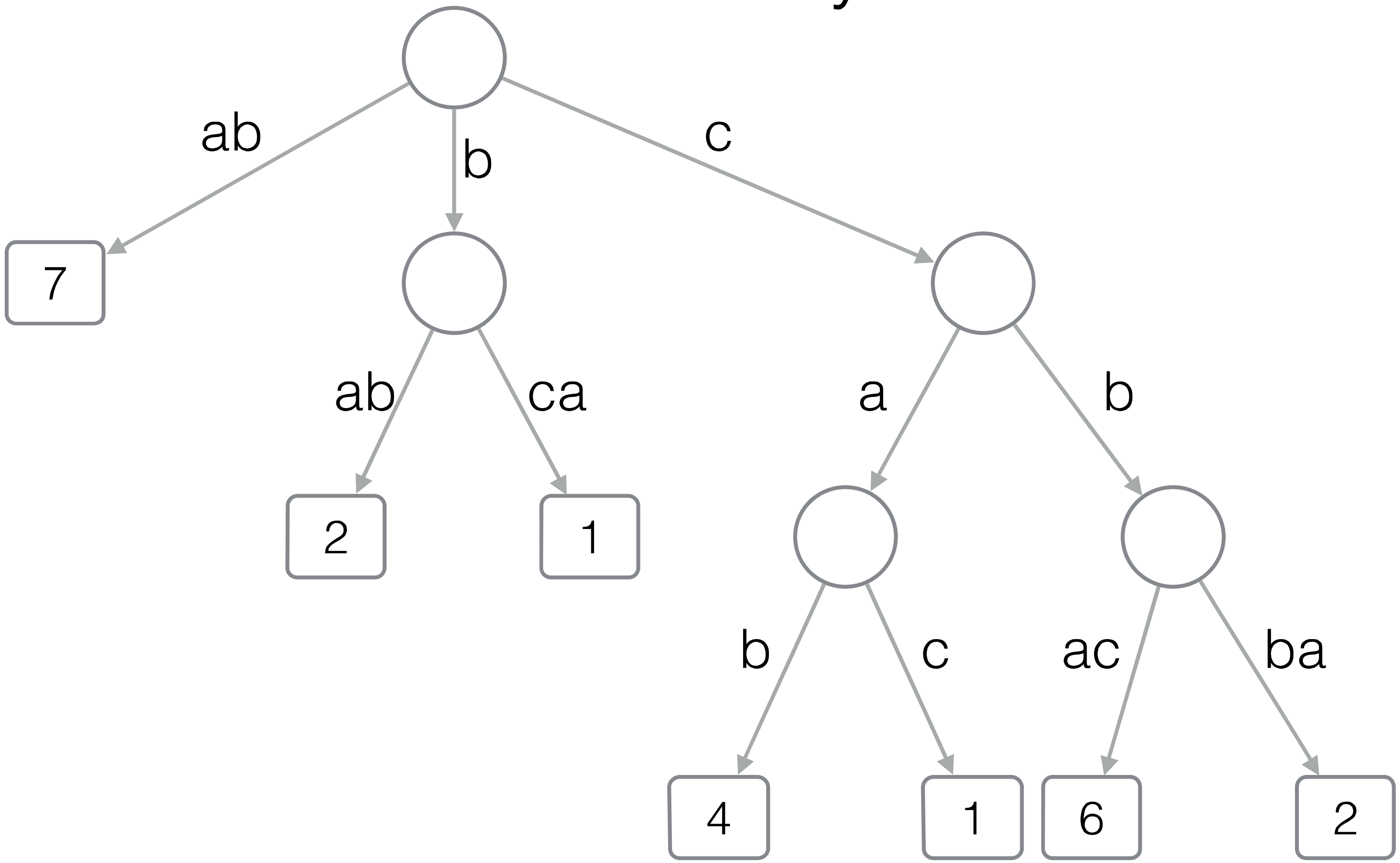
O(m log σ + n log m) bits

Compute the top-k strings

a (1), cab (4), cac (1), cbac (6), cbba (2) }

n = |D|, m total length of strings in D

Summary



Find the node "prefixed" by P

O(|P|) time

O(m log σ + n log m) bits

Compute the top-k strings

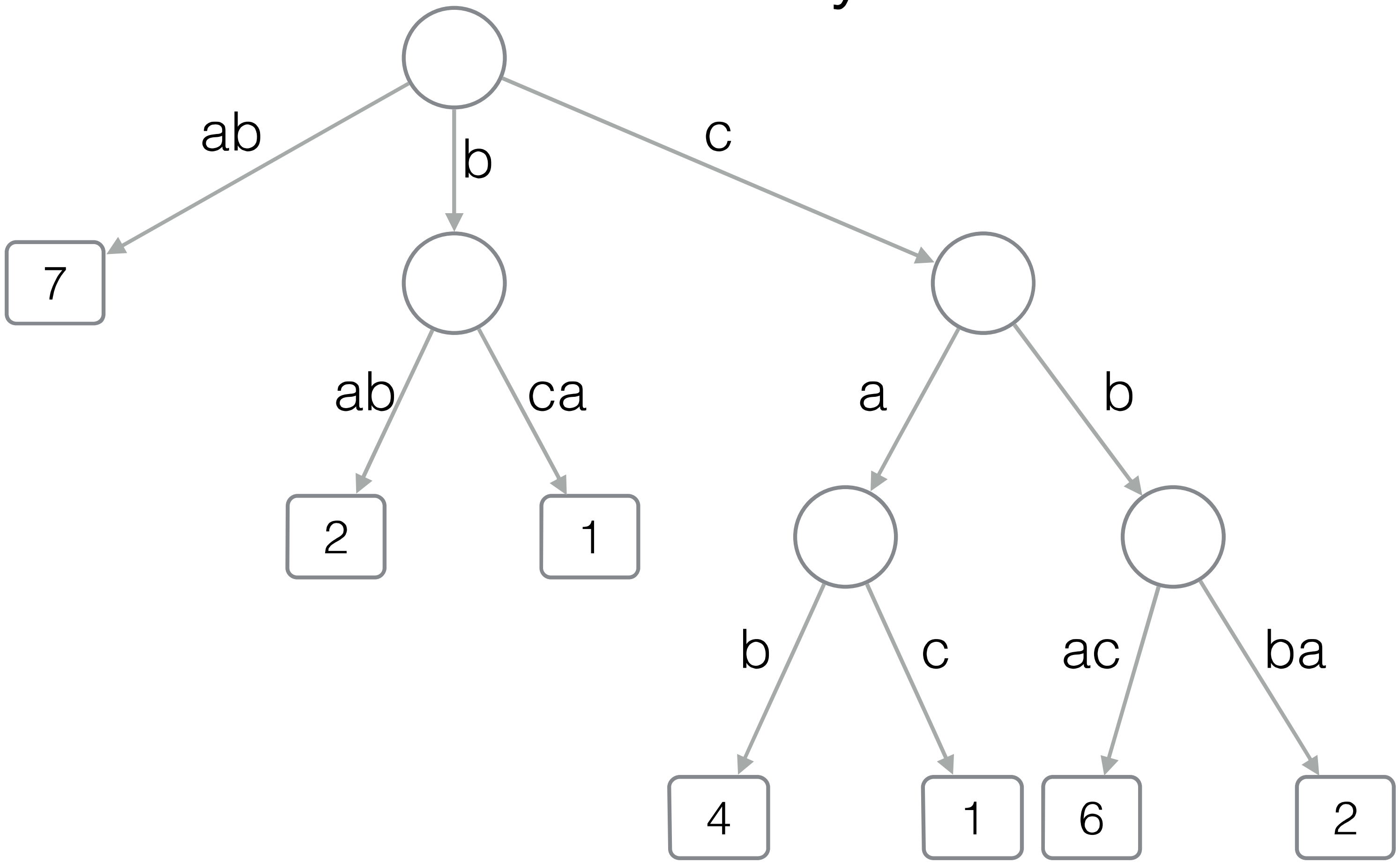
a

O(k log k) time

cbac (6), cbba (2) }

n = |D|, m total length of strings in D

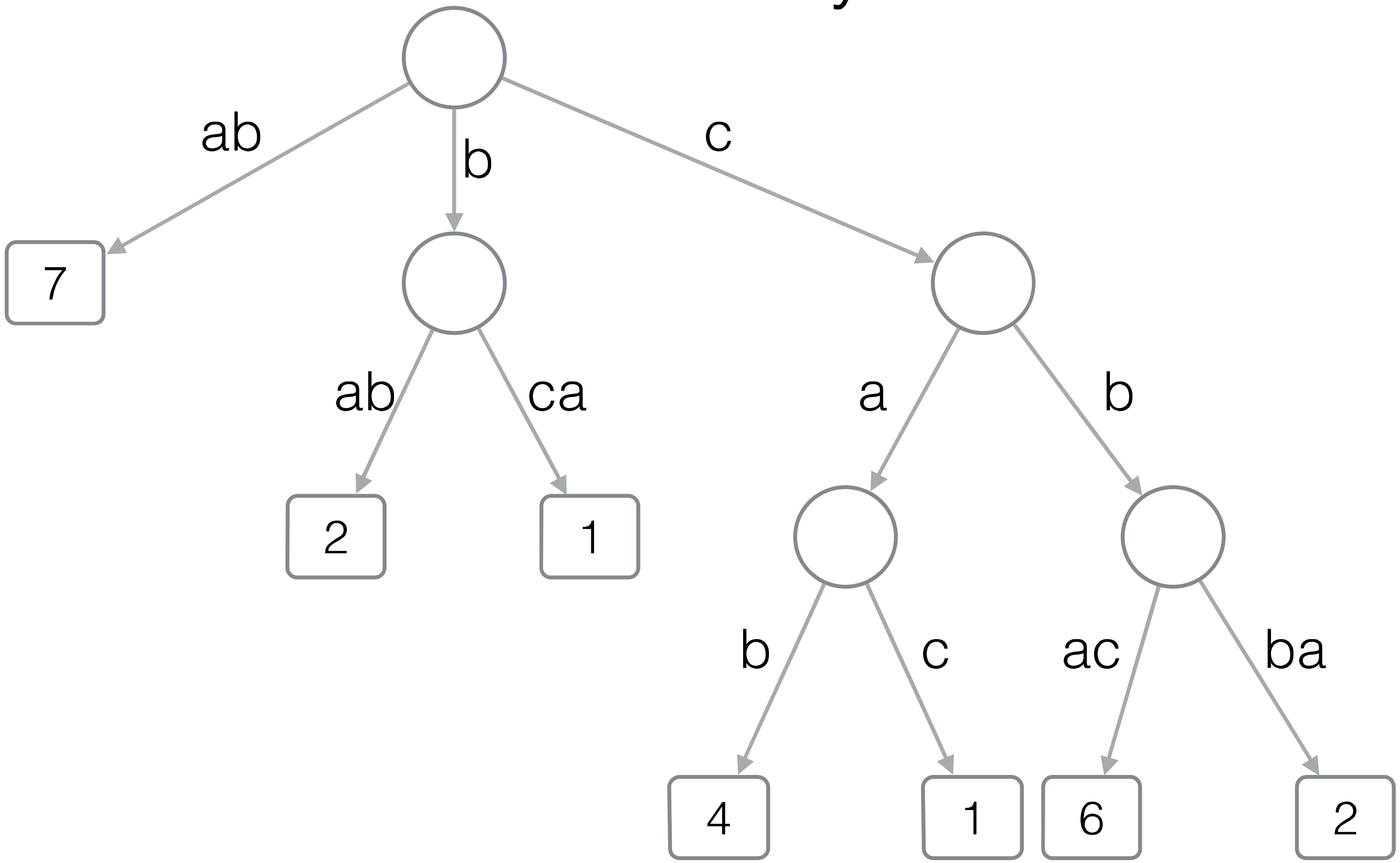
Summary



Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



Find the node “prefixed” by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

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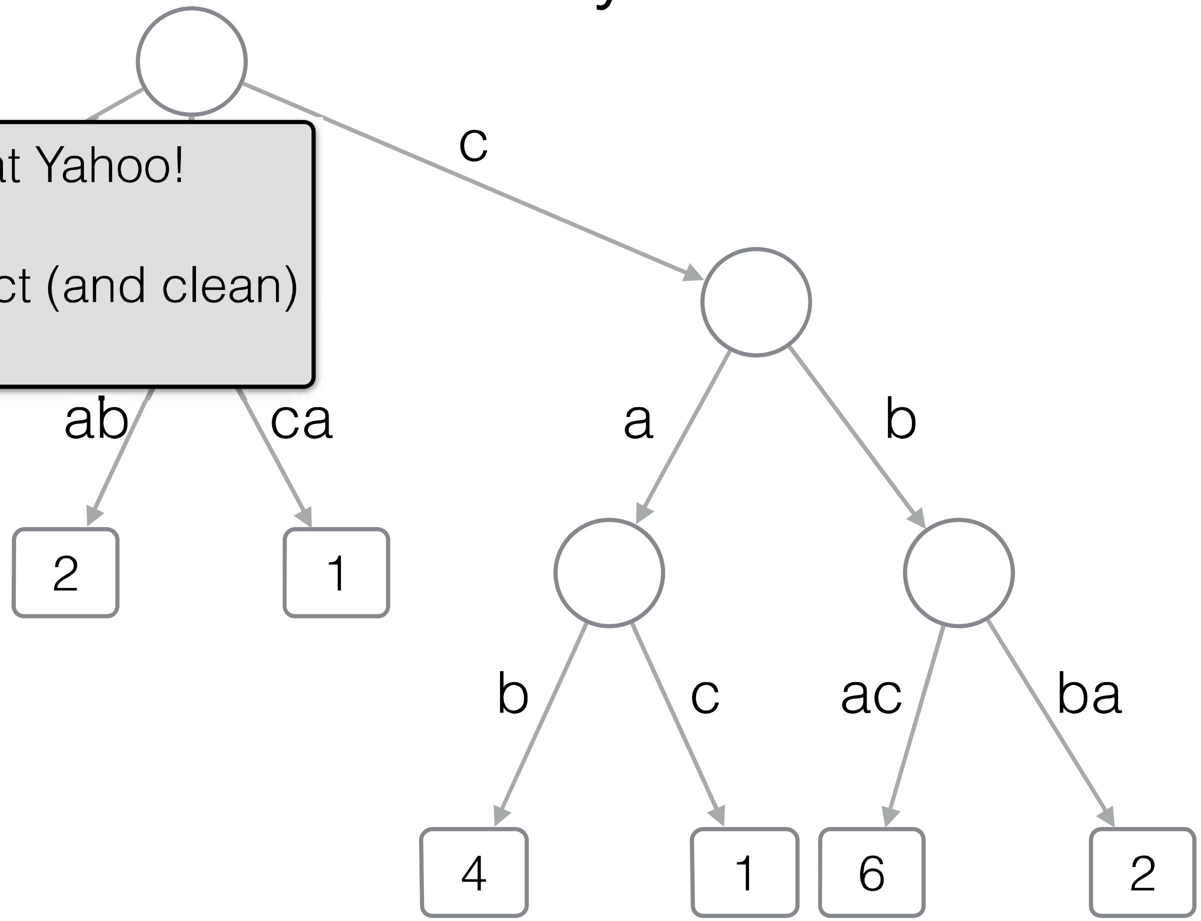
Summary



3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

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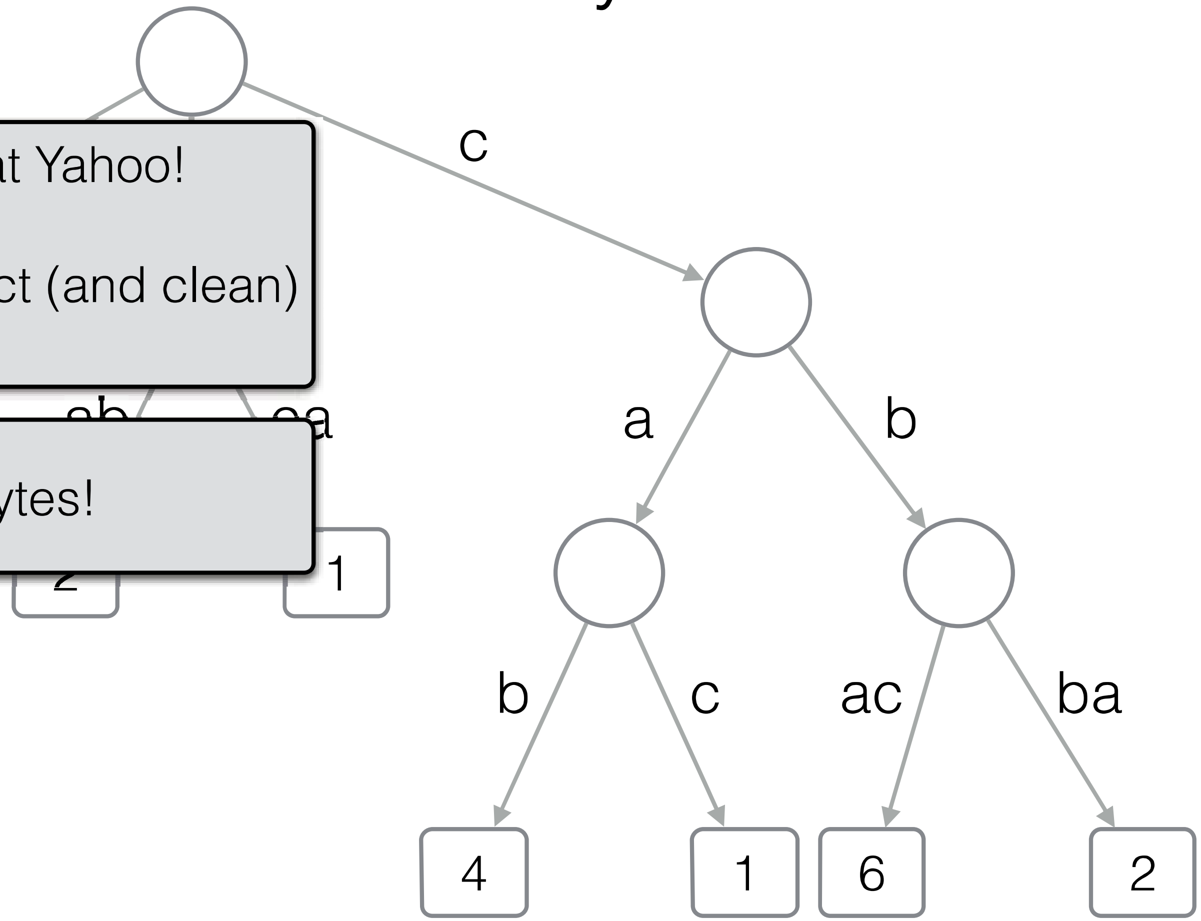
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Summary



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Trie requires ≈50 Gbytes!

Find the node "prefixed" by P	O(|P|) time	O(m log σ + n log m) bits
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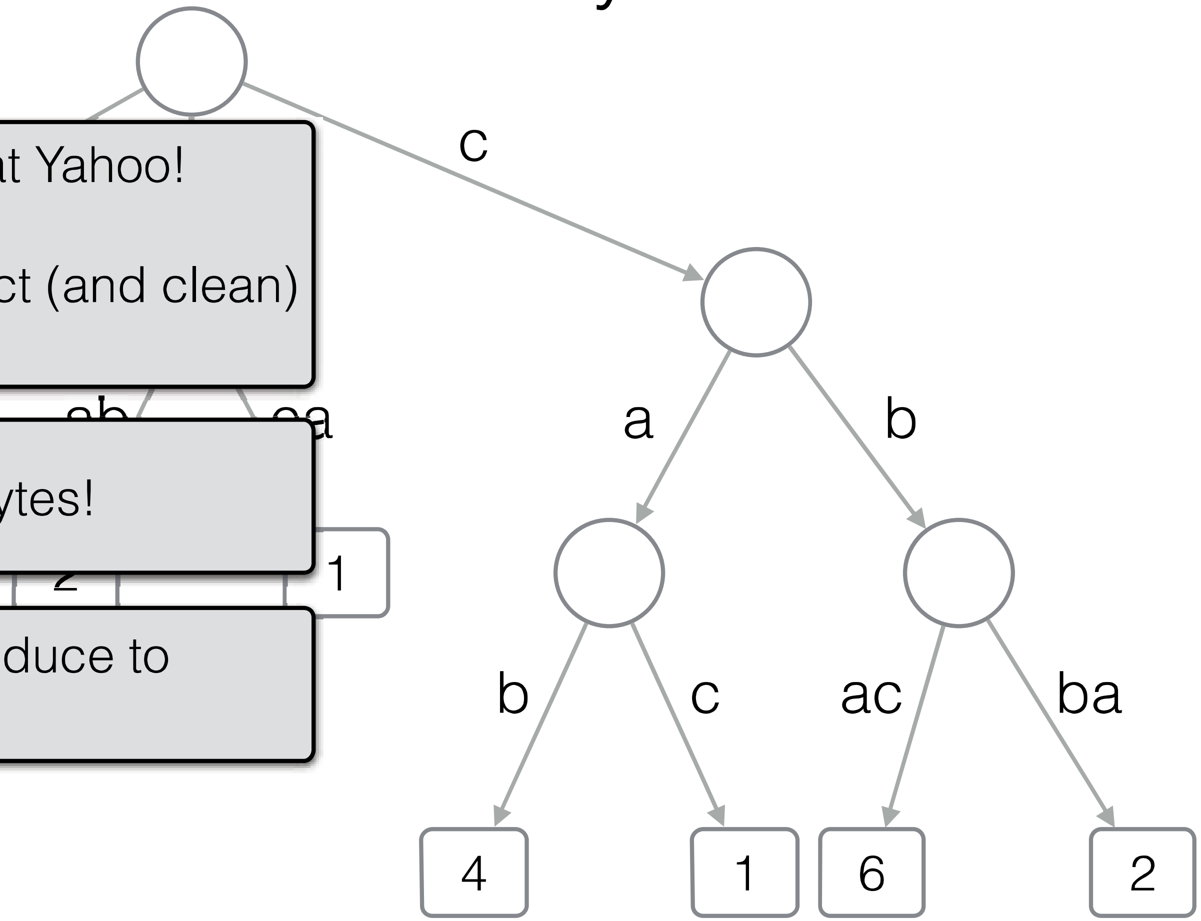
Summary



3 months query log at Yahoo!

≈600 million of distinct (and clean) queries

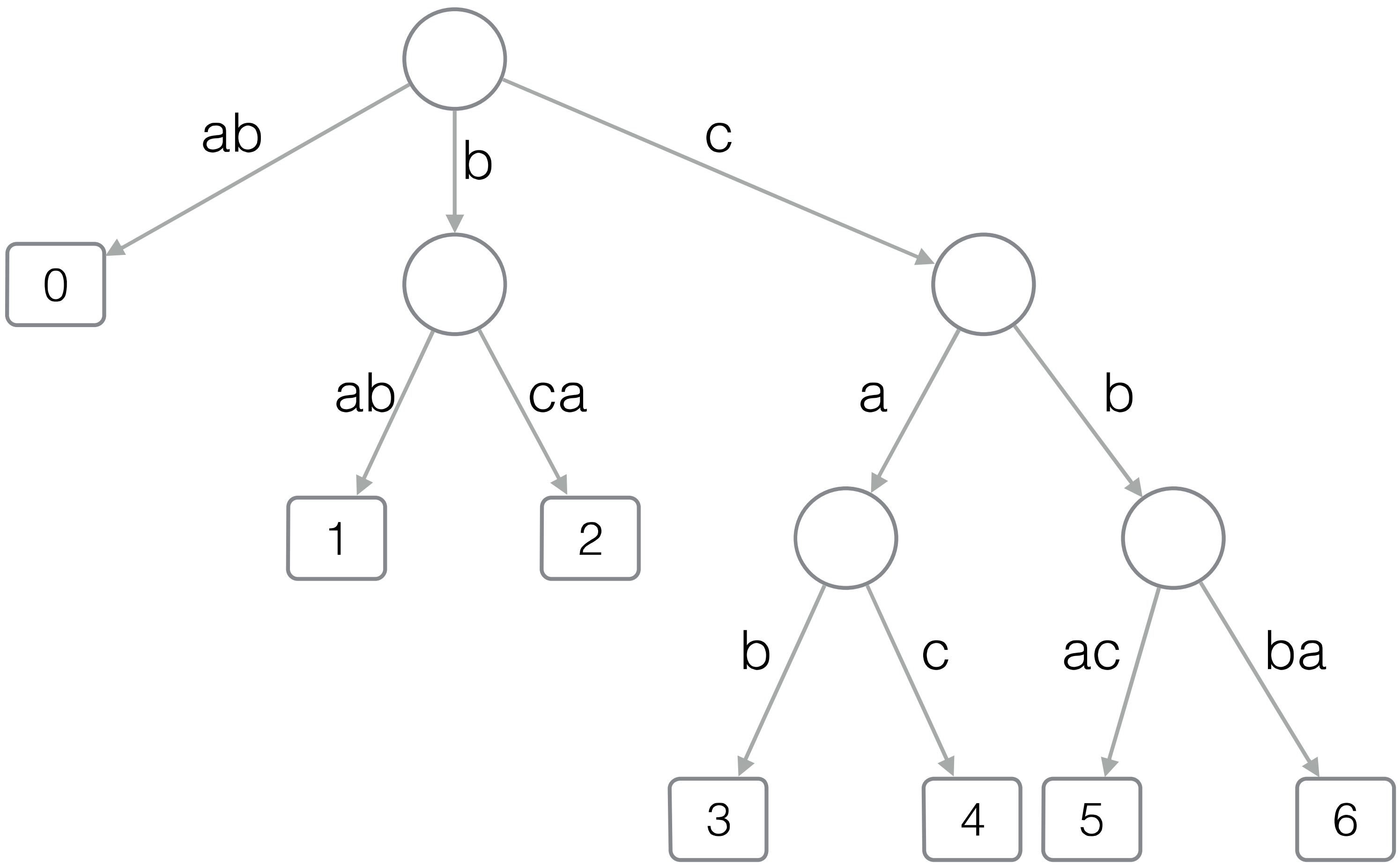
Trie requires ≈50 Gbytes!

We will see how to reduce to ≈5 Gbytes!

Find the node “prefixed” by P	O(|P|) time	O(m log σ + n log m) bits
Compute the top-k strings	O(k log k) time	O(n) bits

n = |D|, m total length of strings in D

Patricia trie



D = { ab, bab, bca, cab, cac, cbac, cbba }

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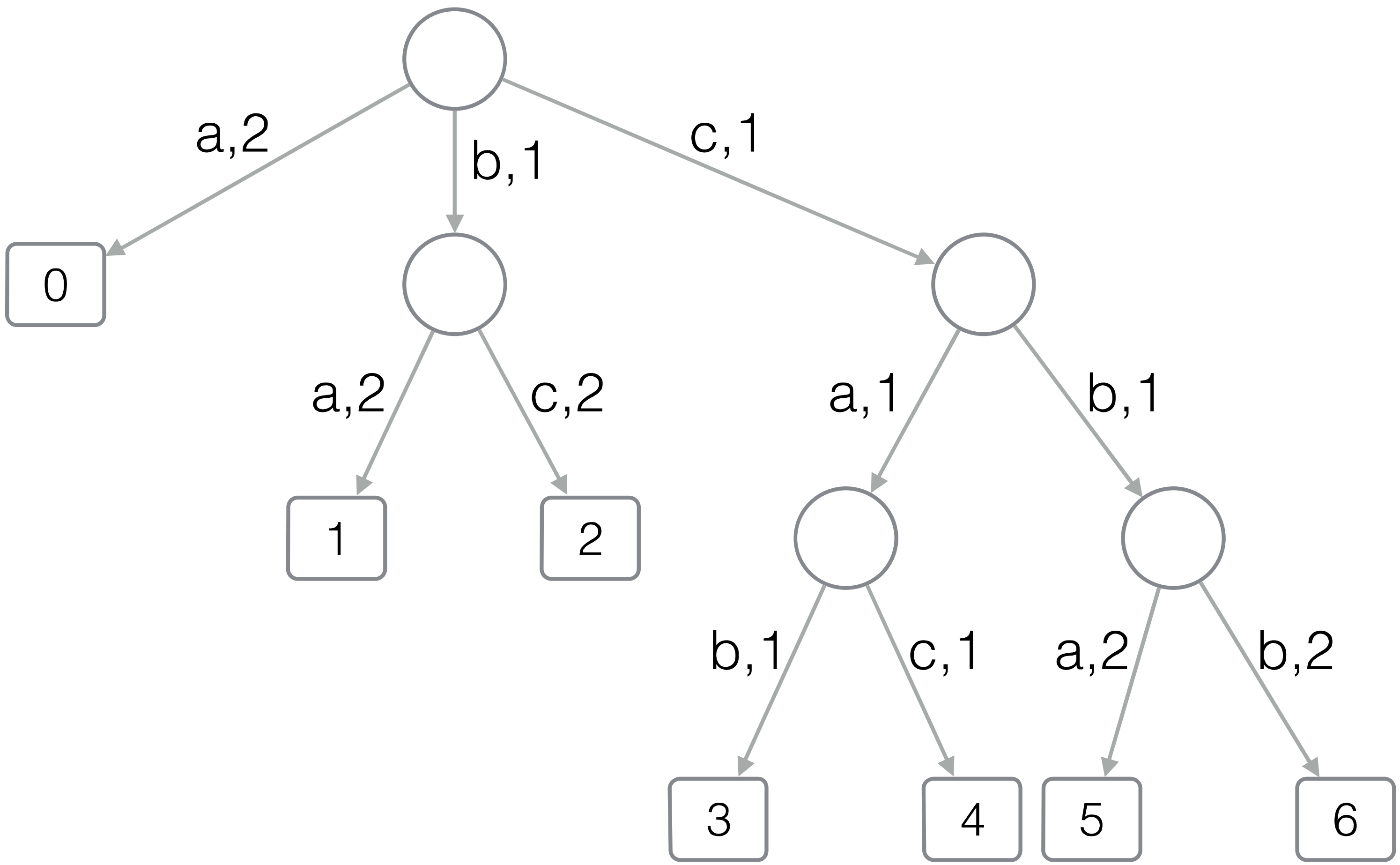

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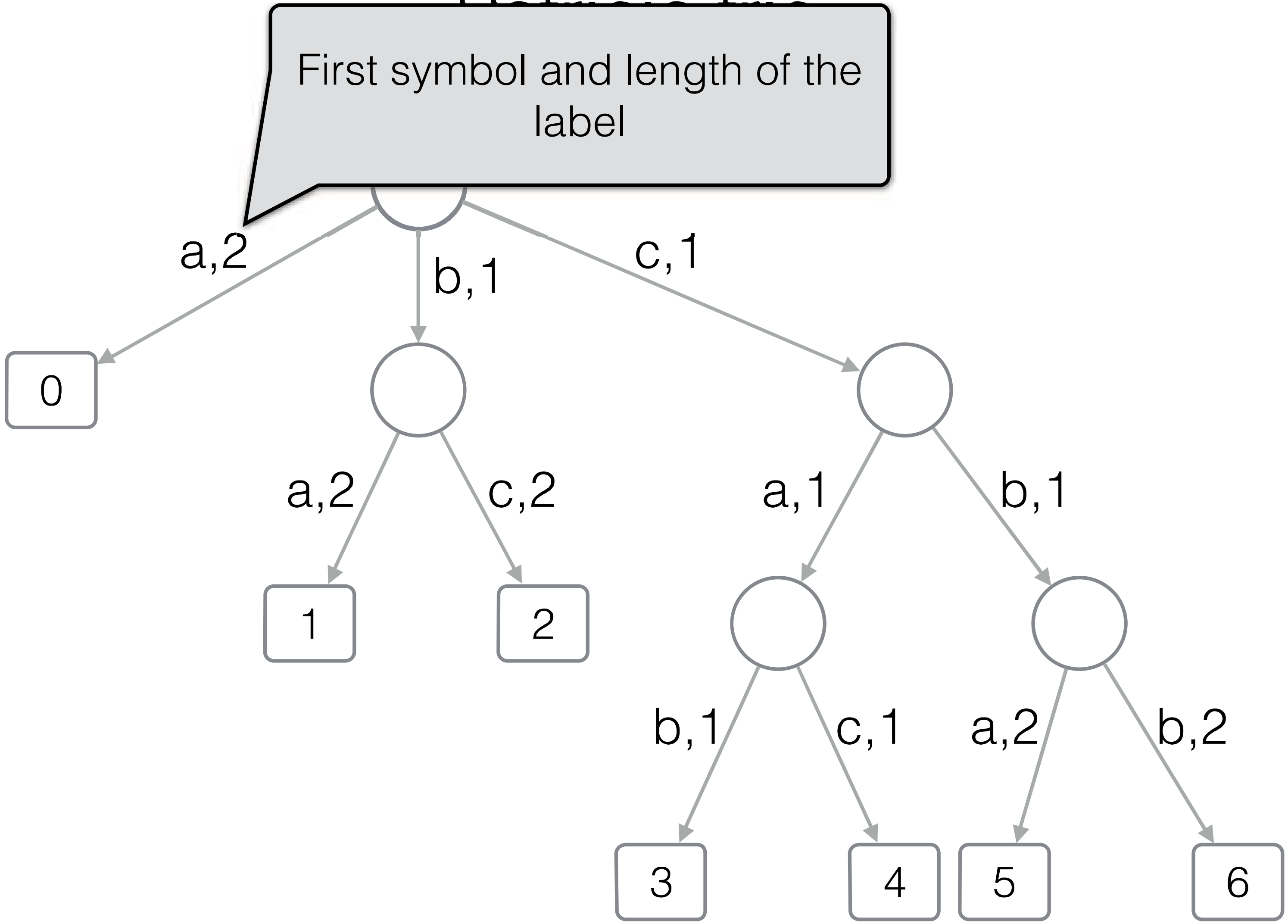

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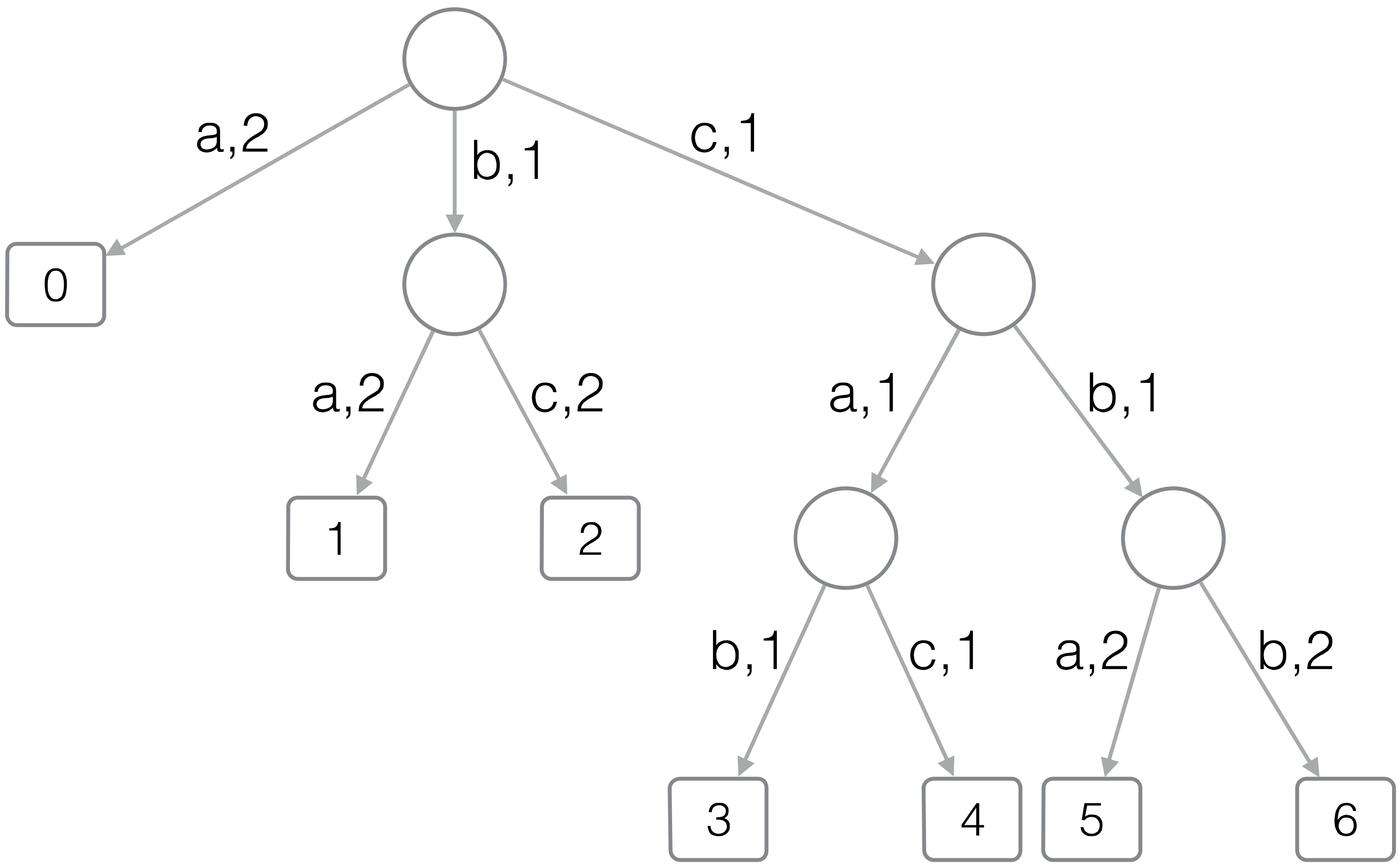

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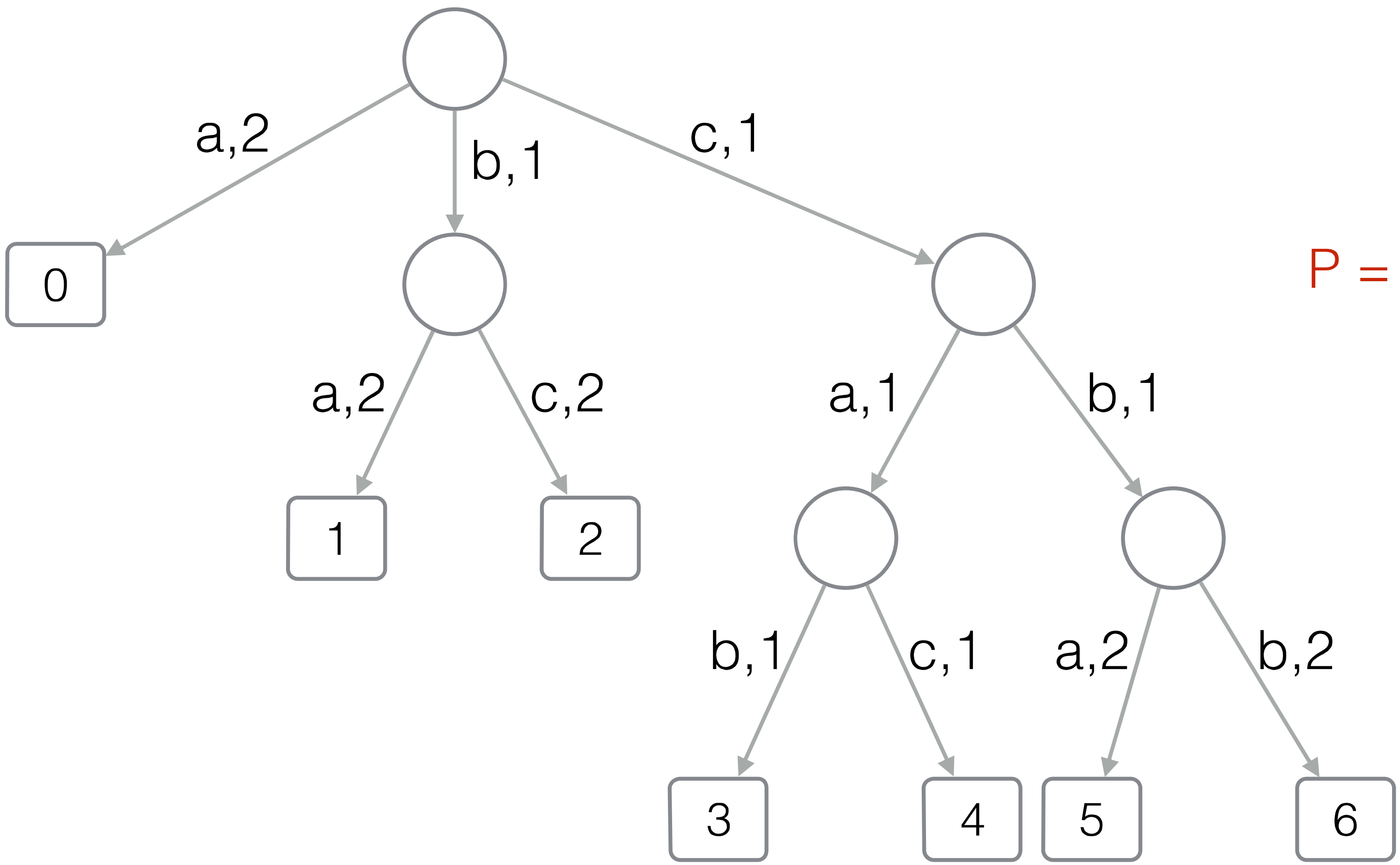

P = cba

D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D

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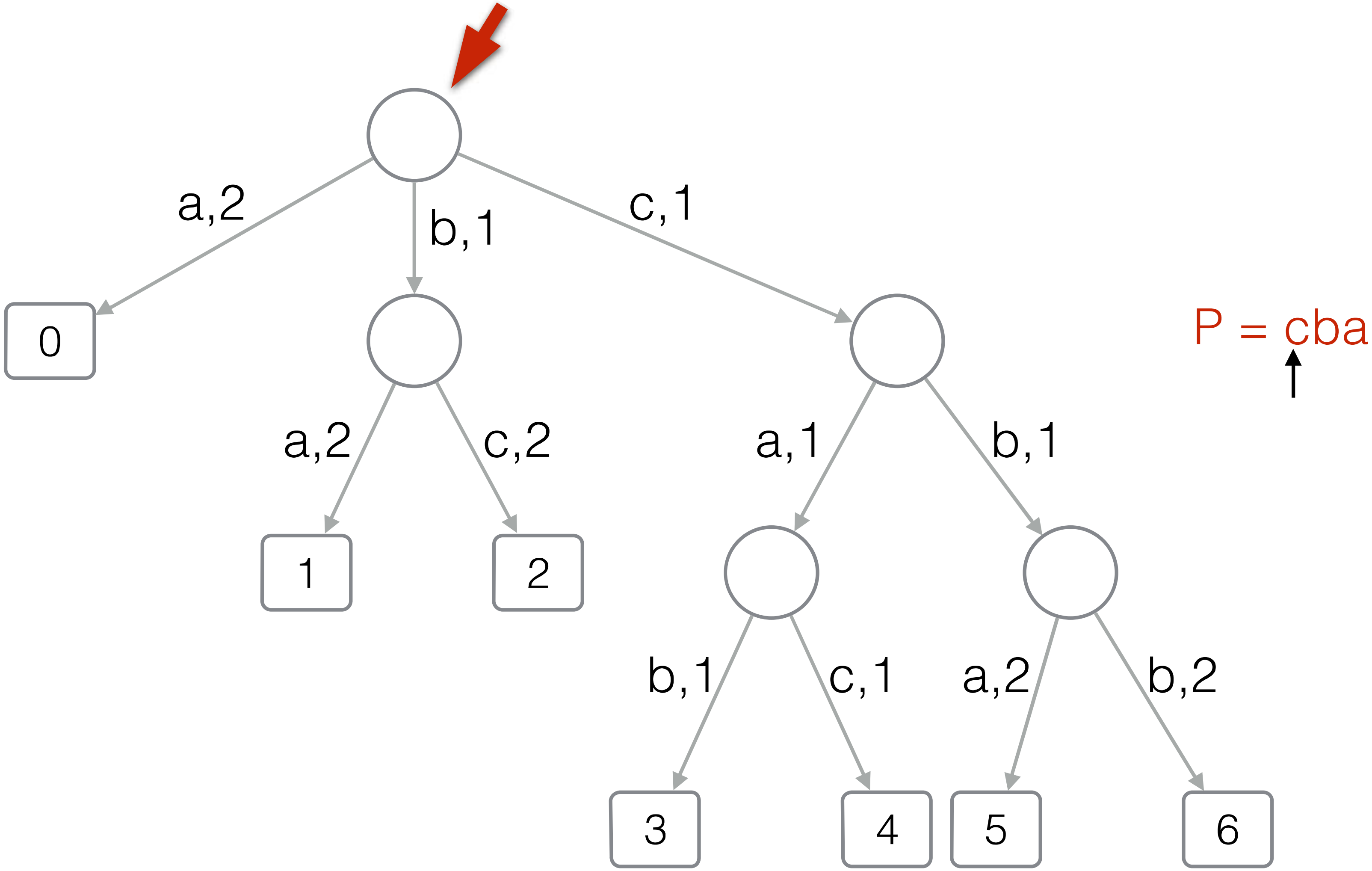

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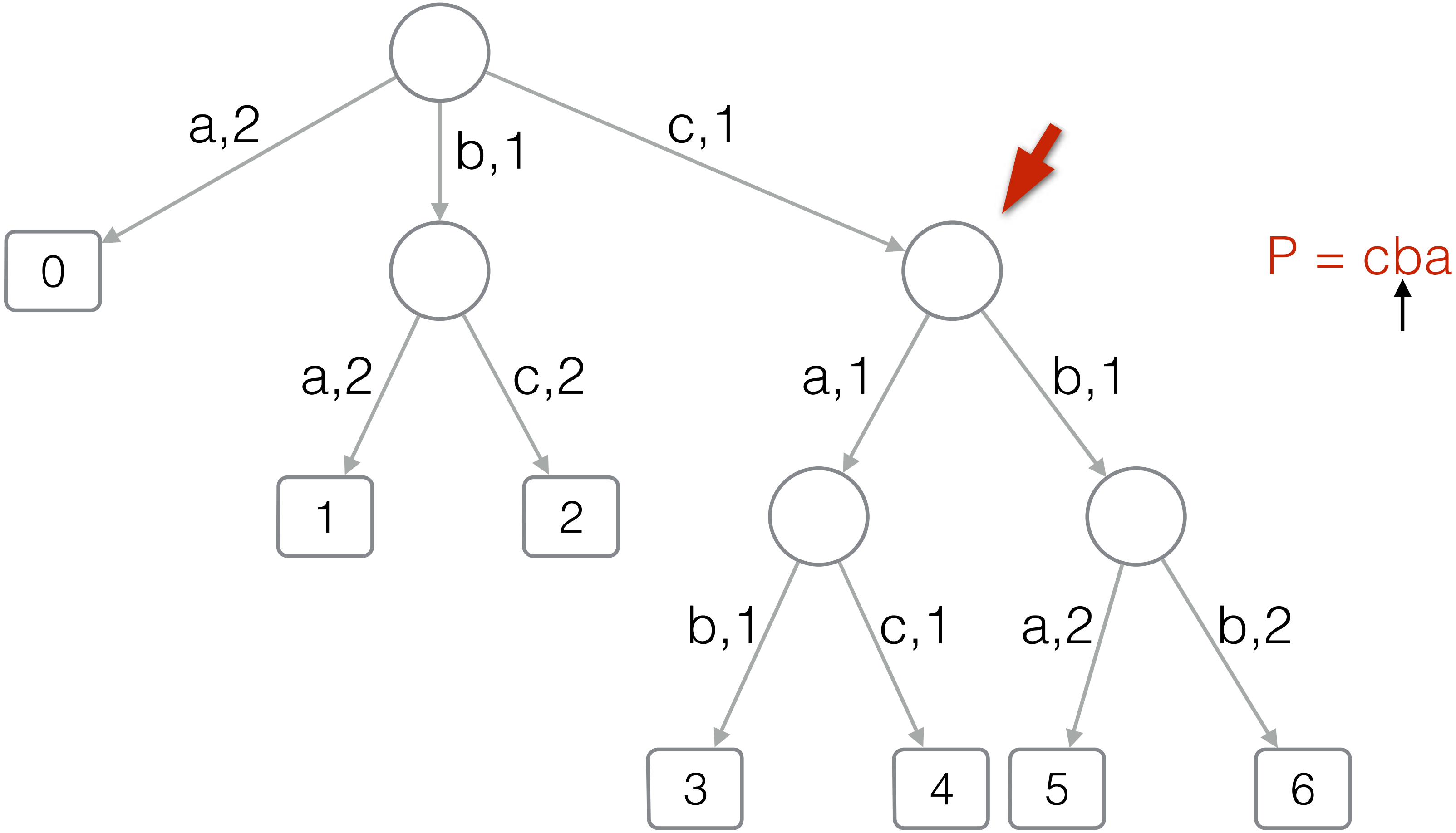

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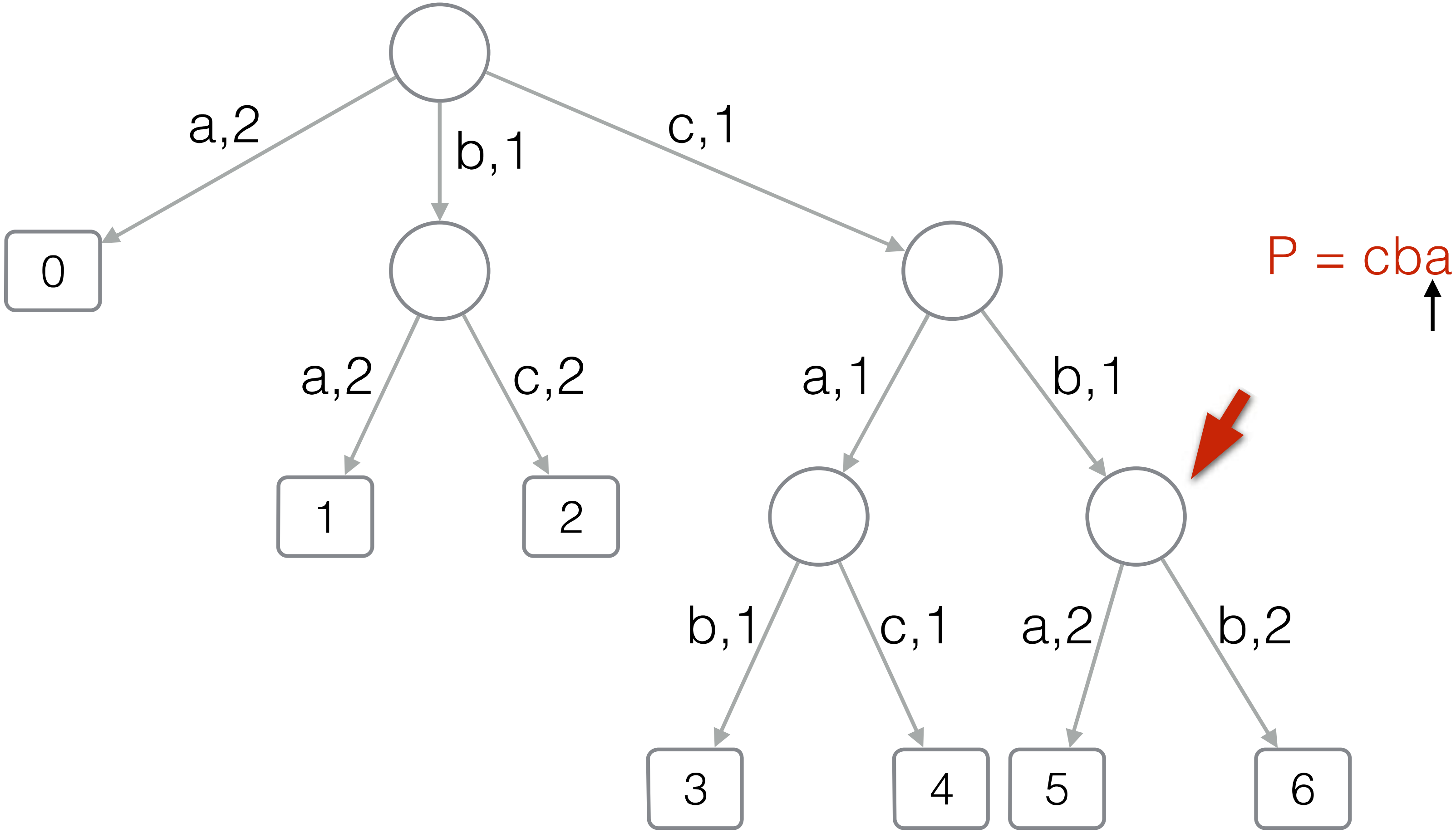

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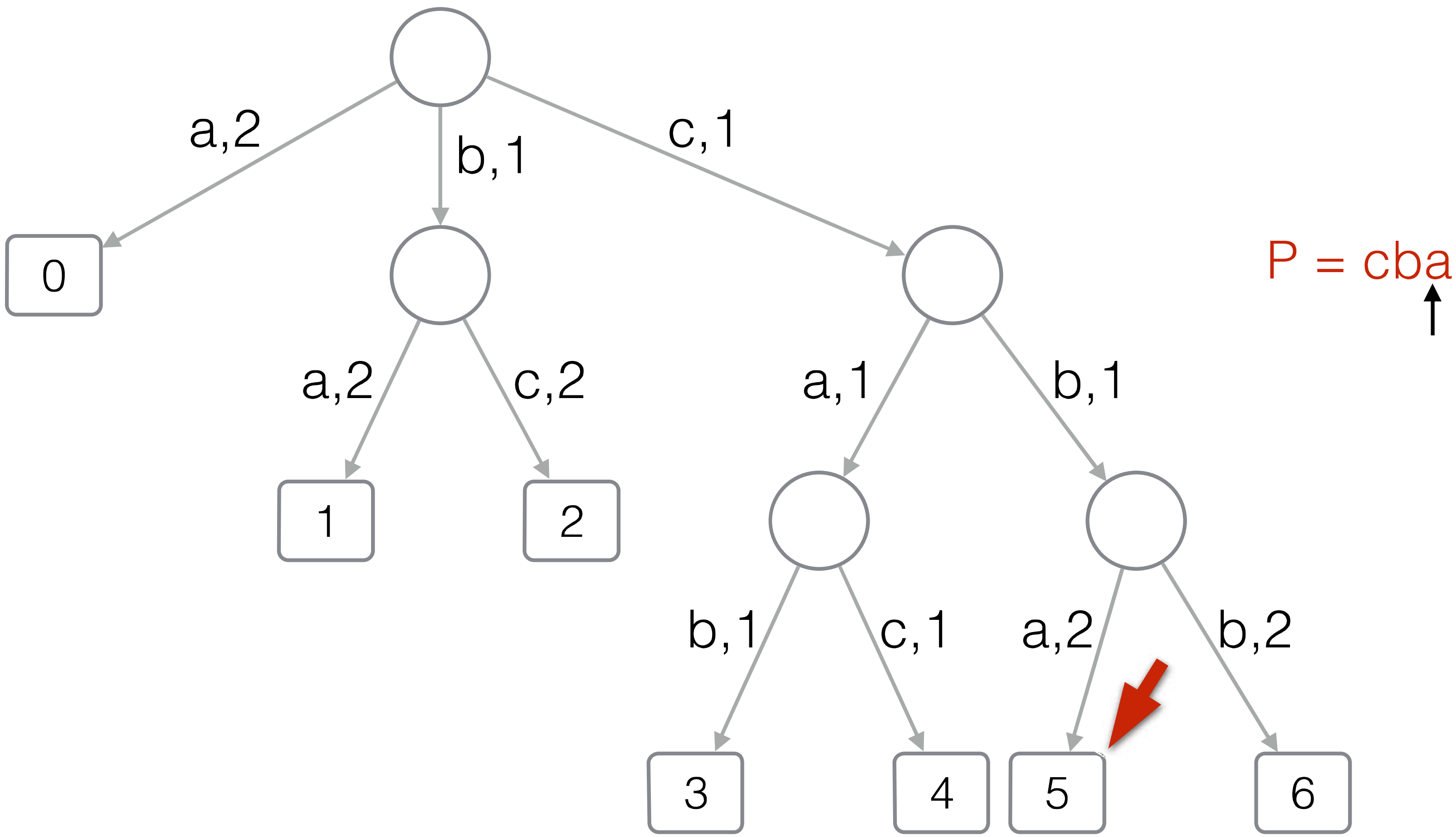

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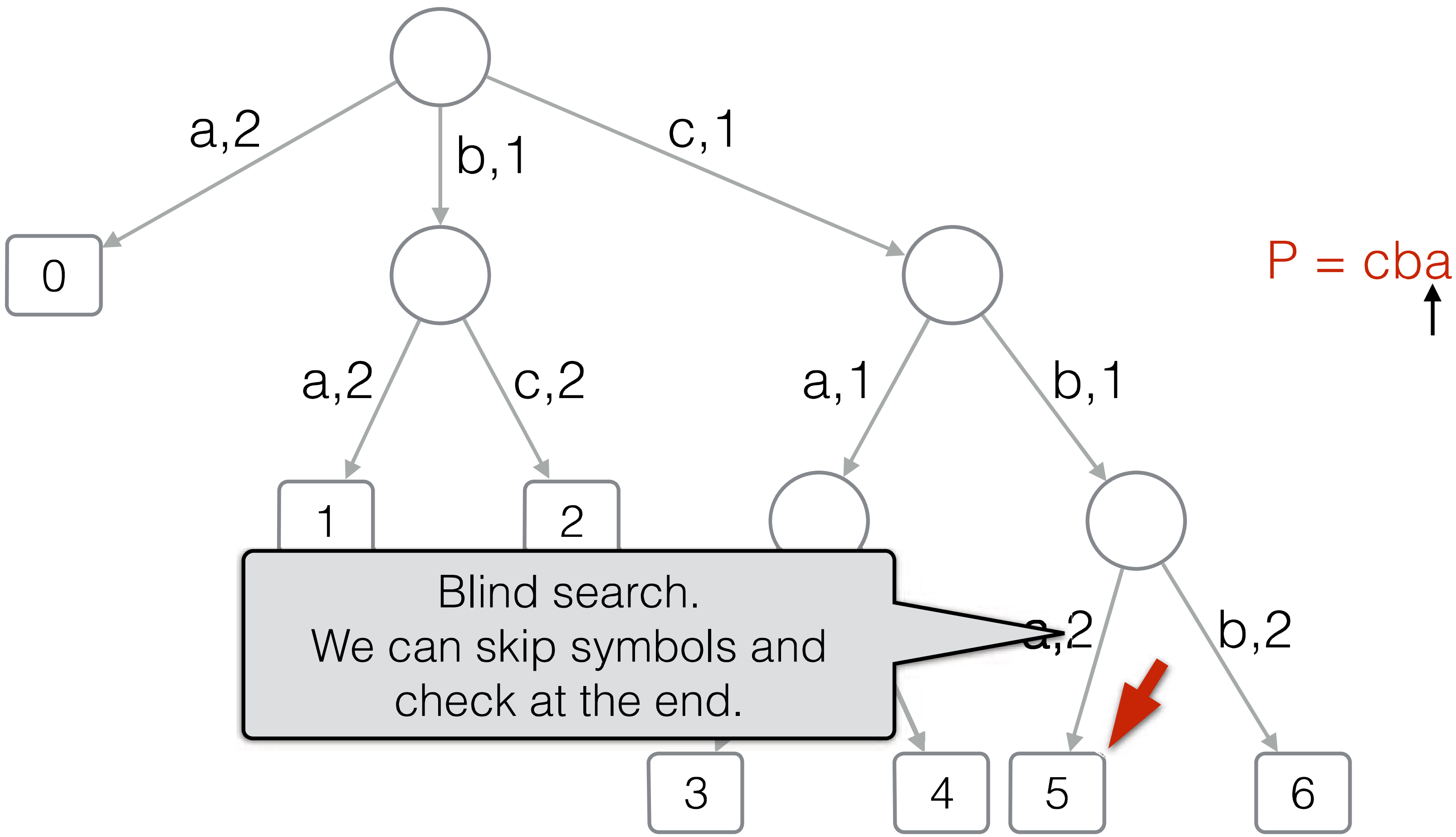

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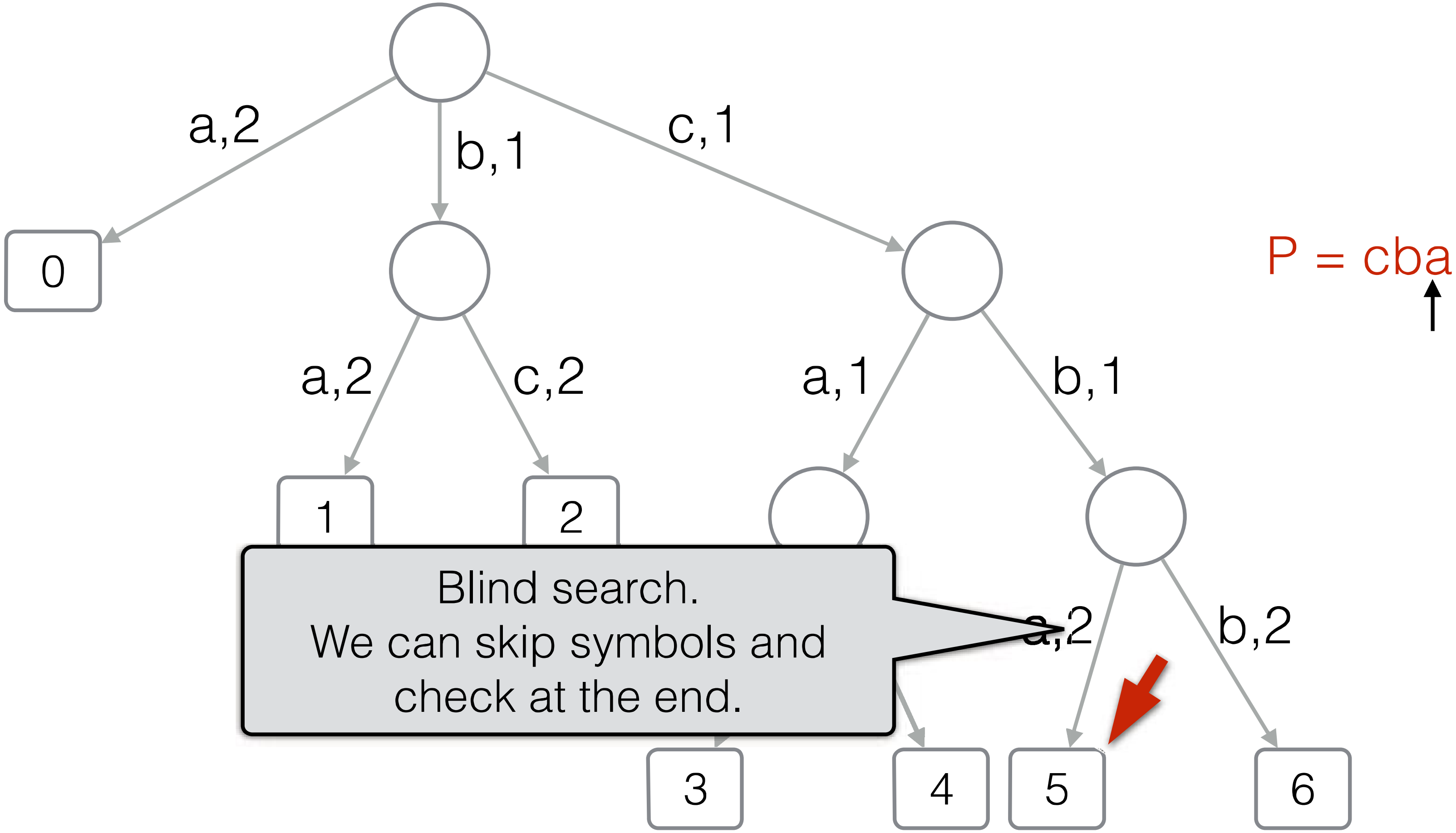


Blind search.
We can skip symbols and
check at the end.

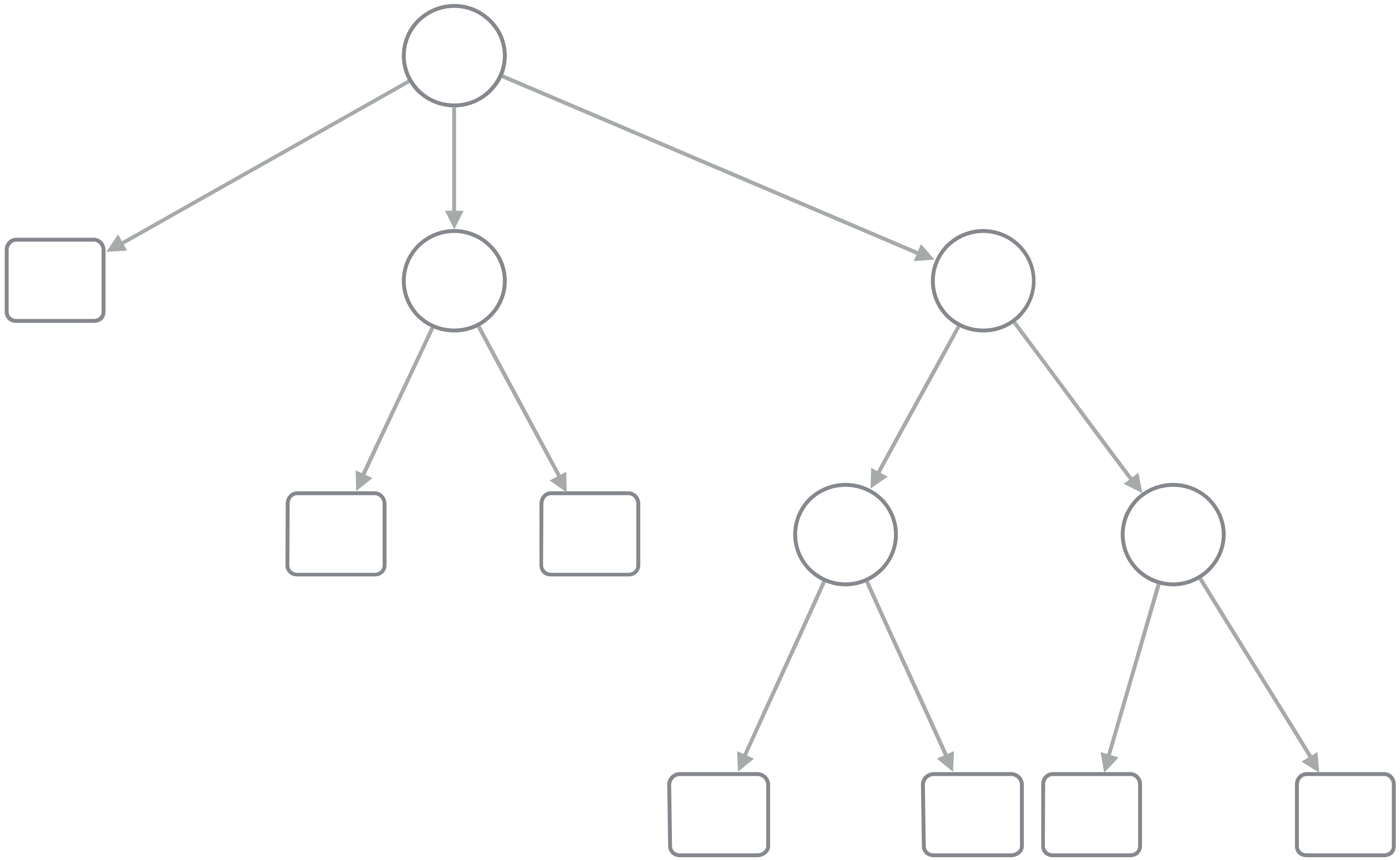
O(|P|) time

O(n log m + m log σ) bits

n = |D|, m total length of strings in D

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

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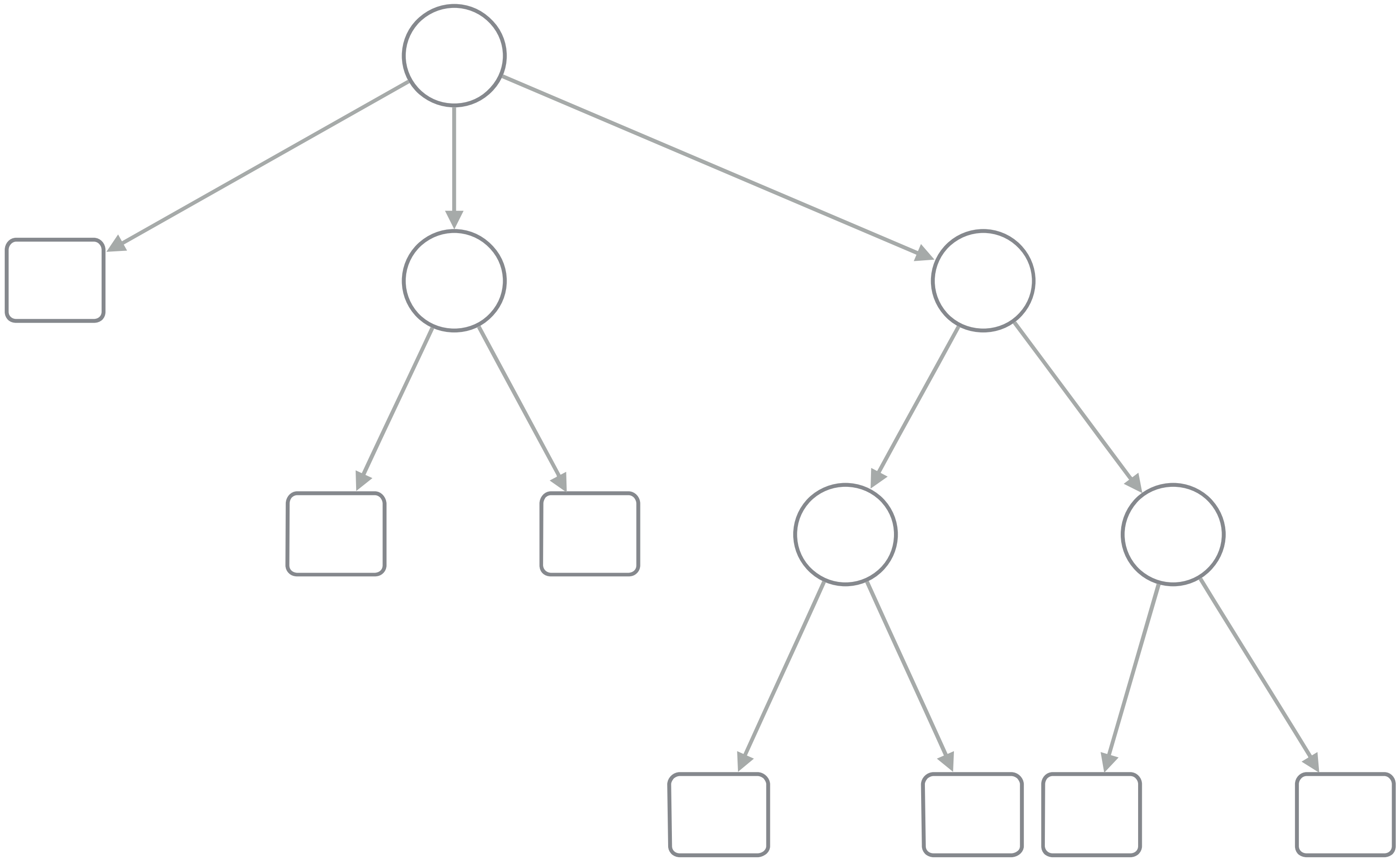

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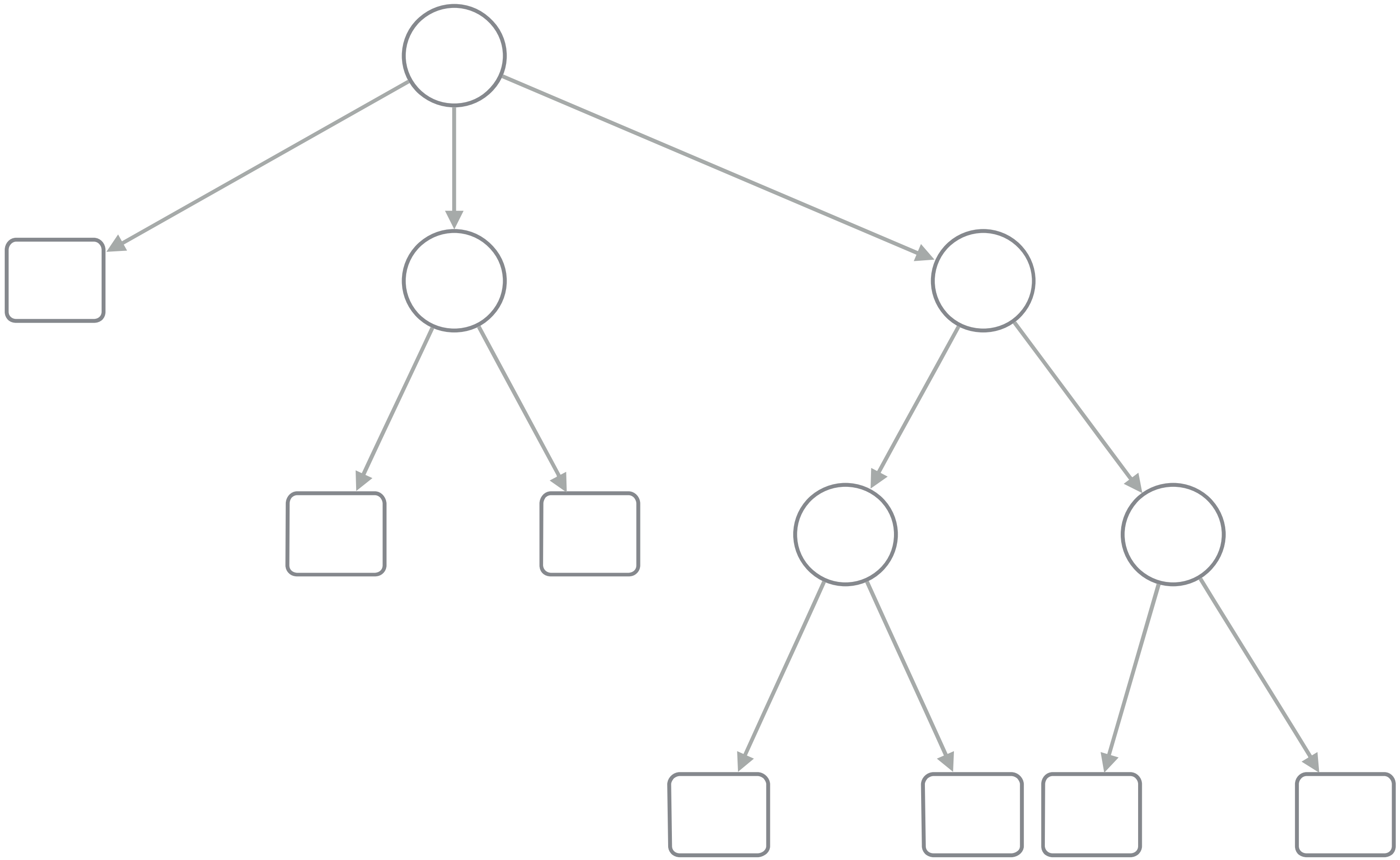

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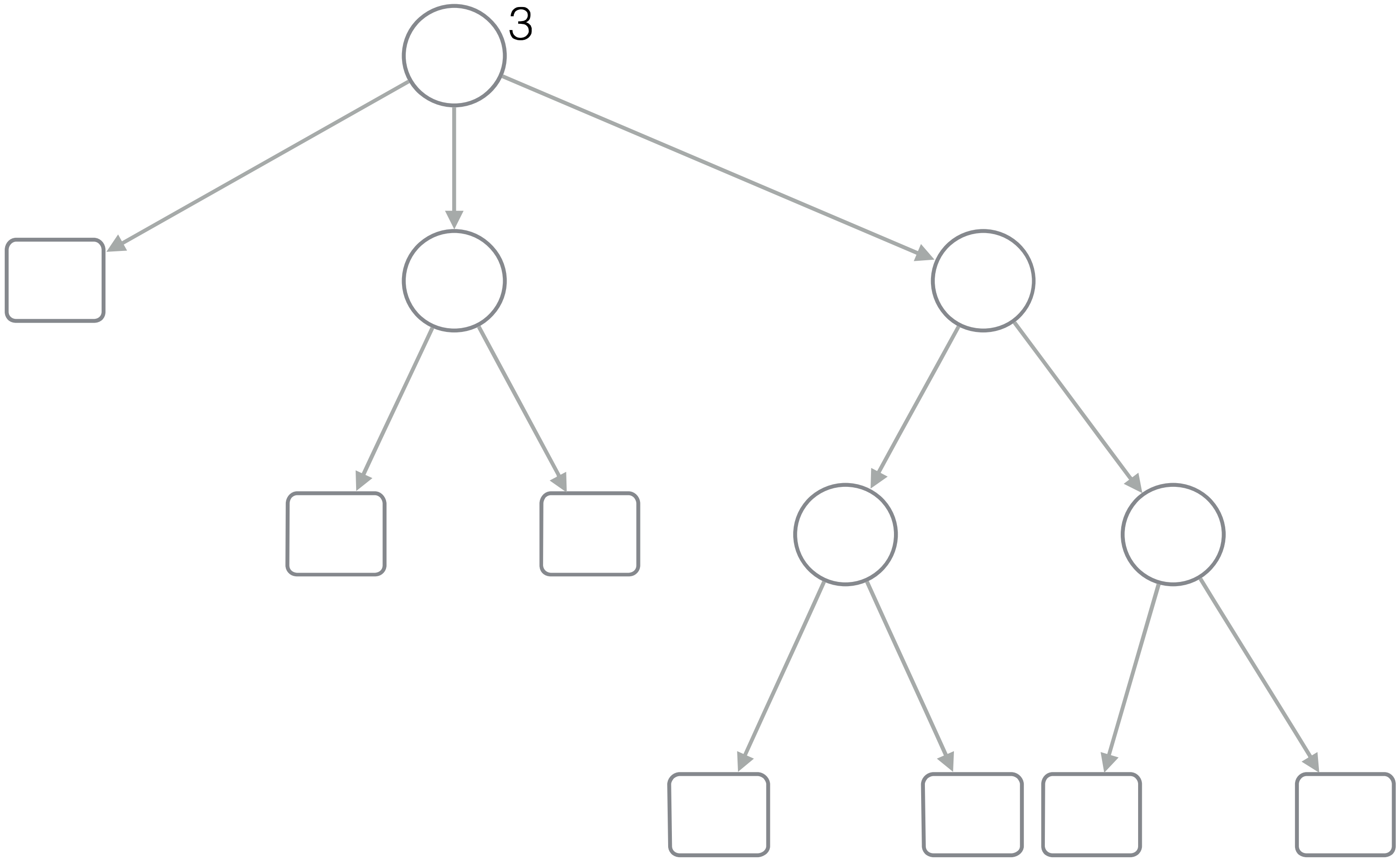

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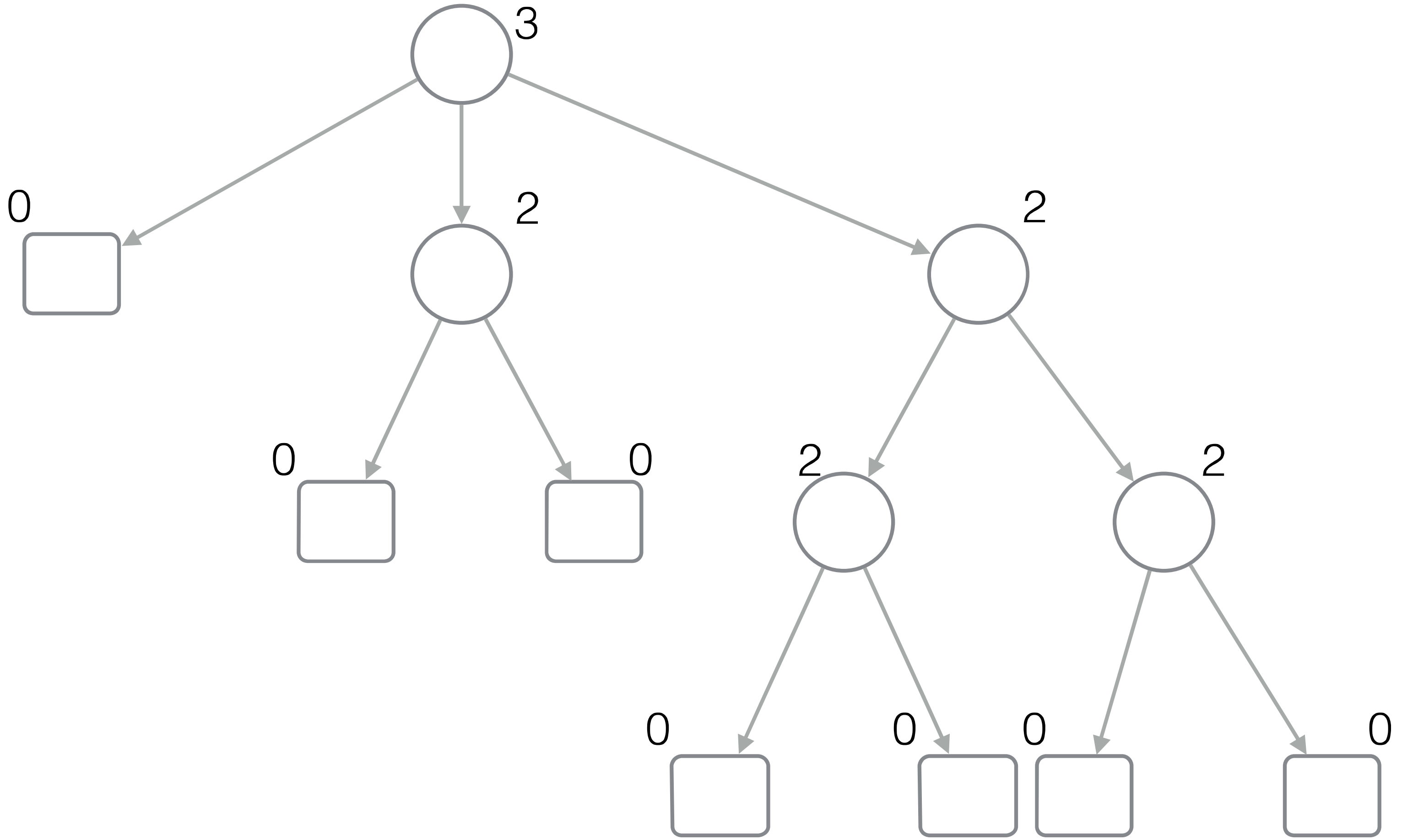

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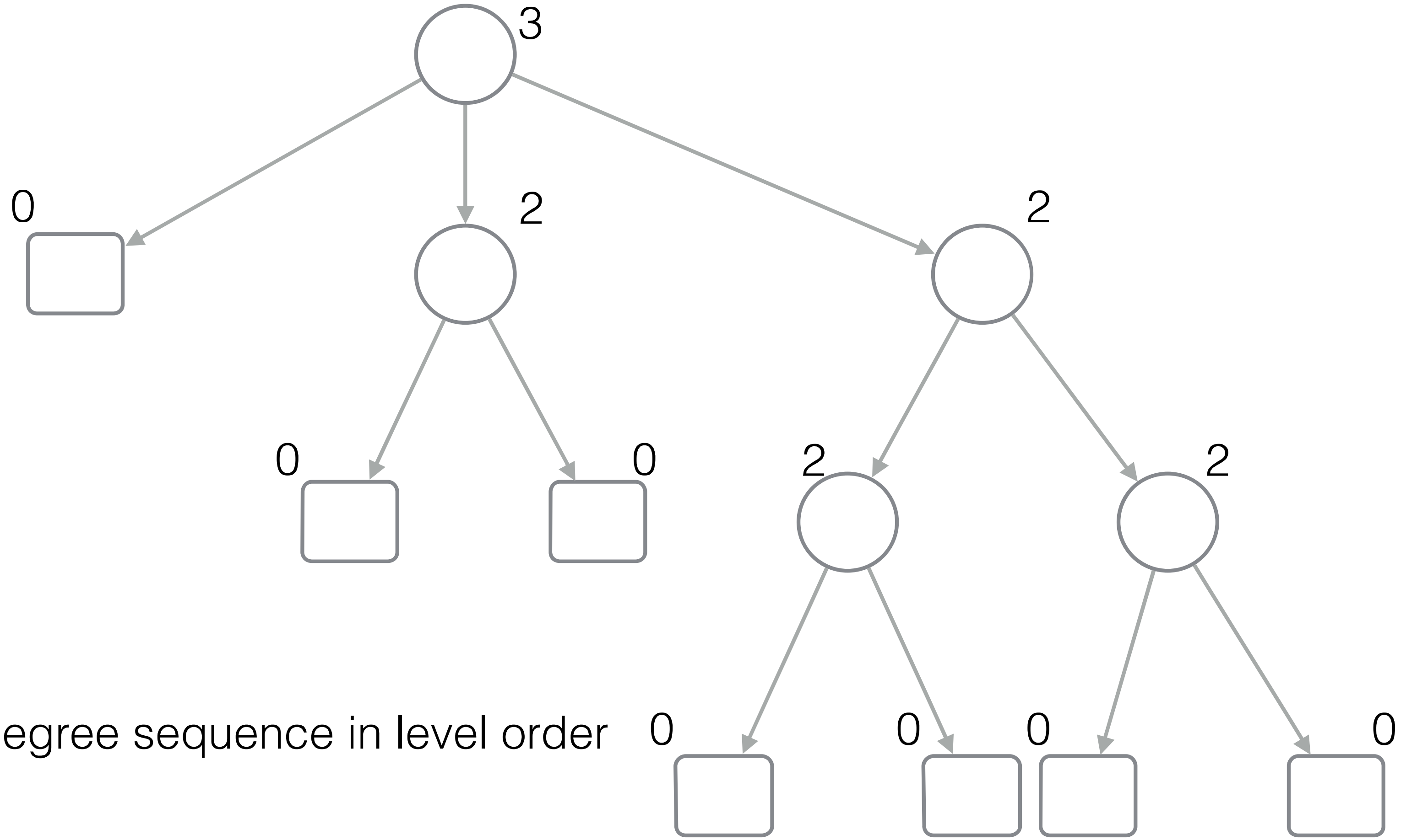

Write the degree sequence in level order

Succinct representation of trees (1)

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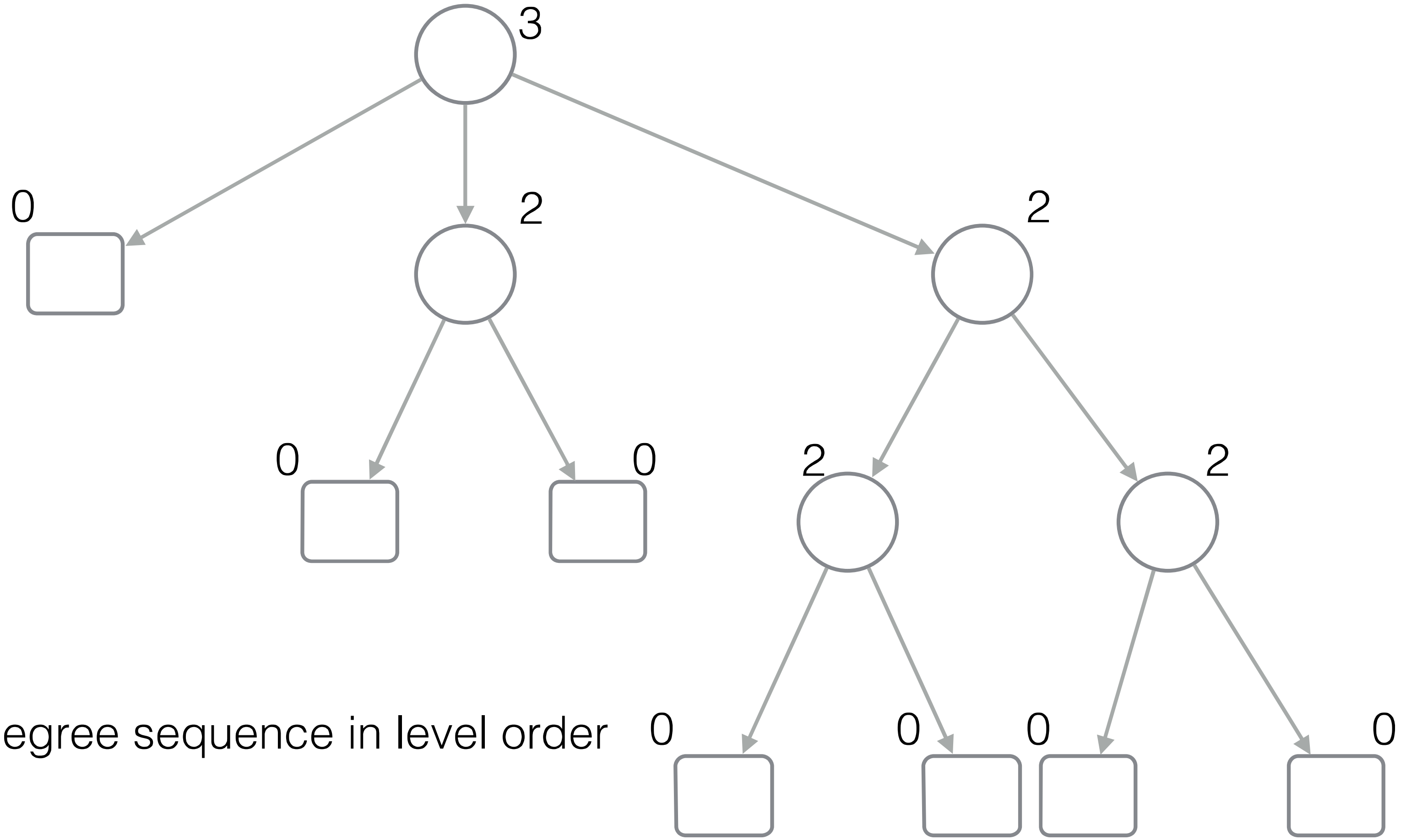

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D 3

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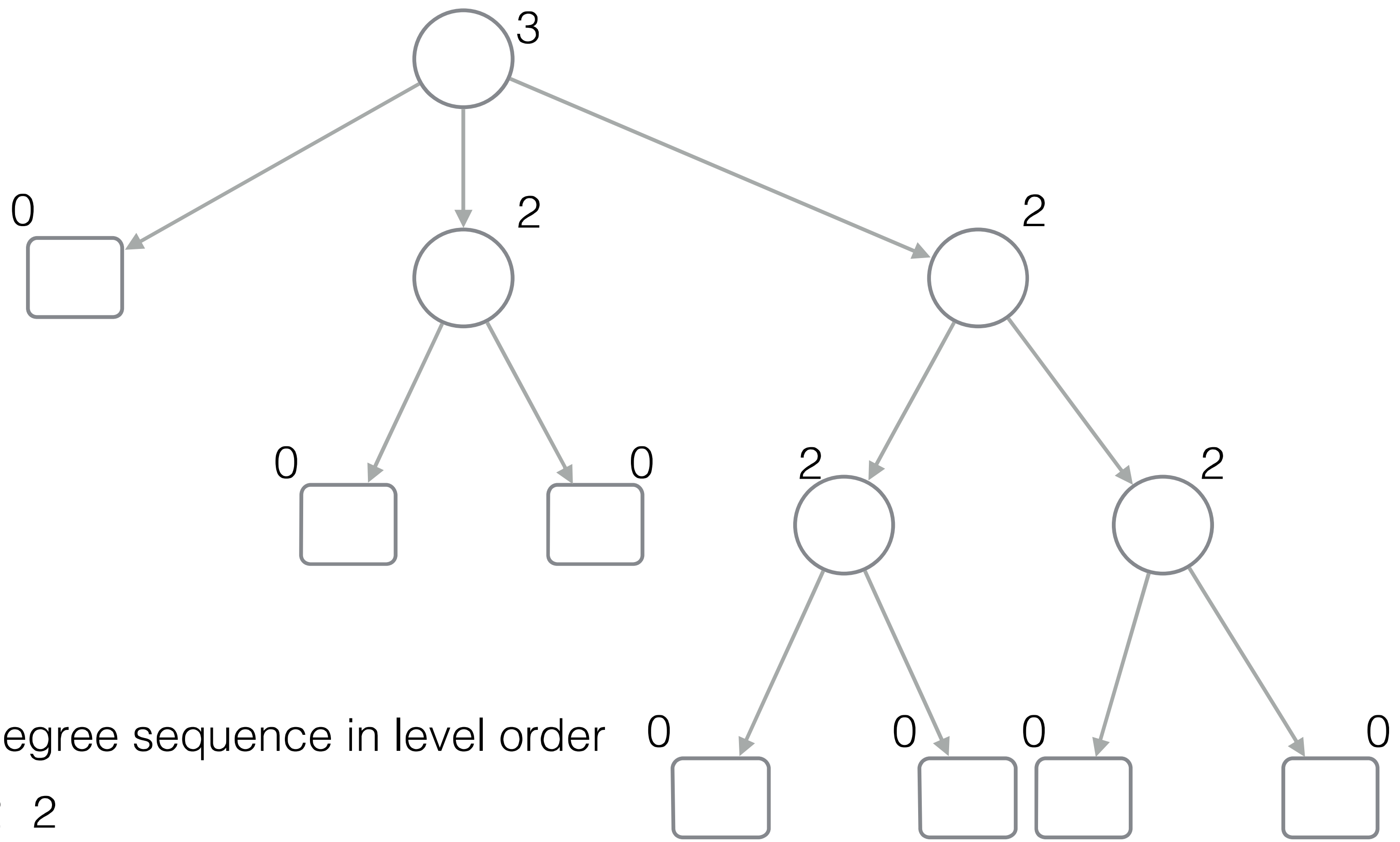

Write the degree sequence in level order

D 3 0 2 2

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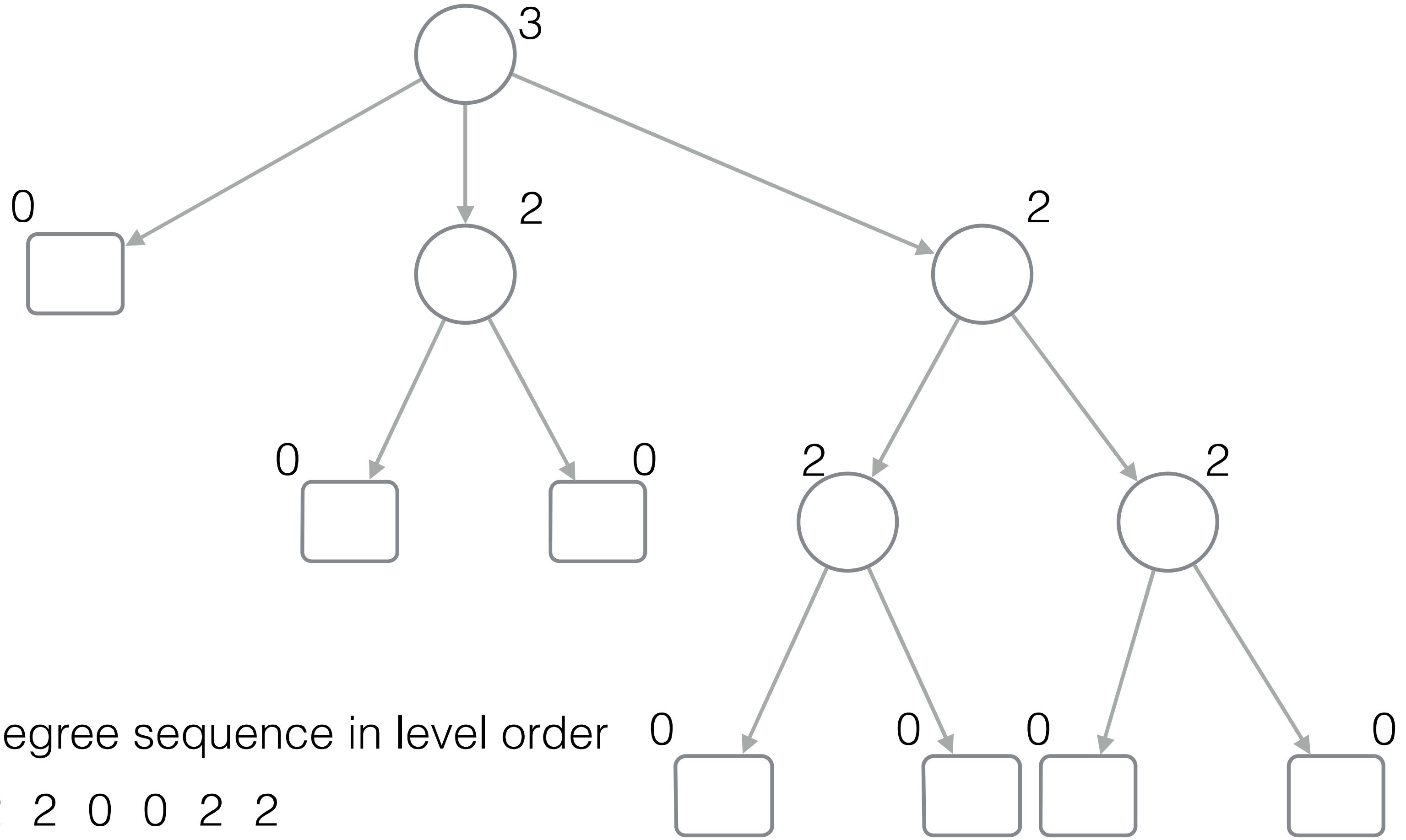

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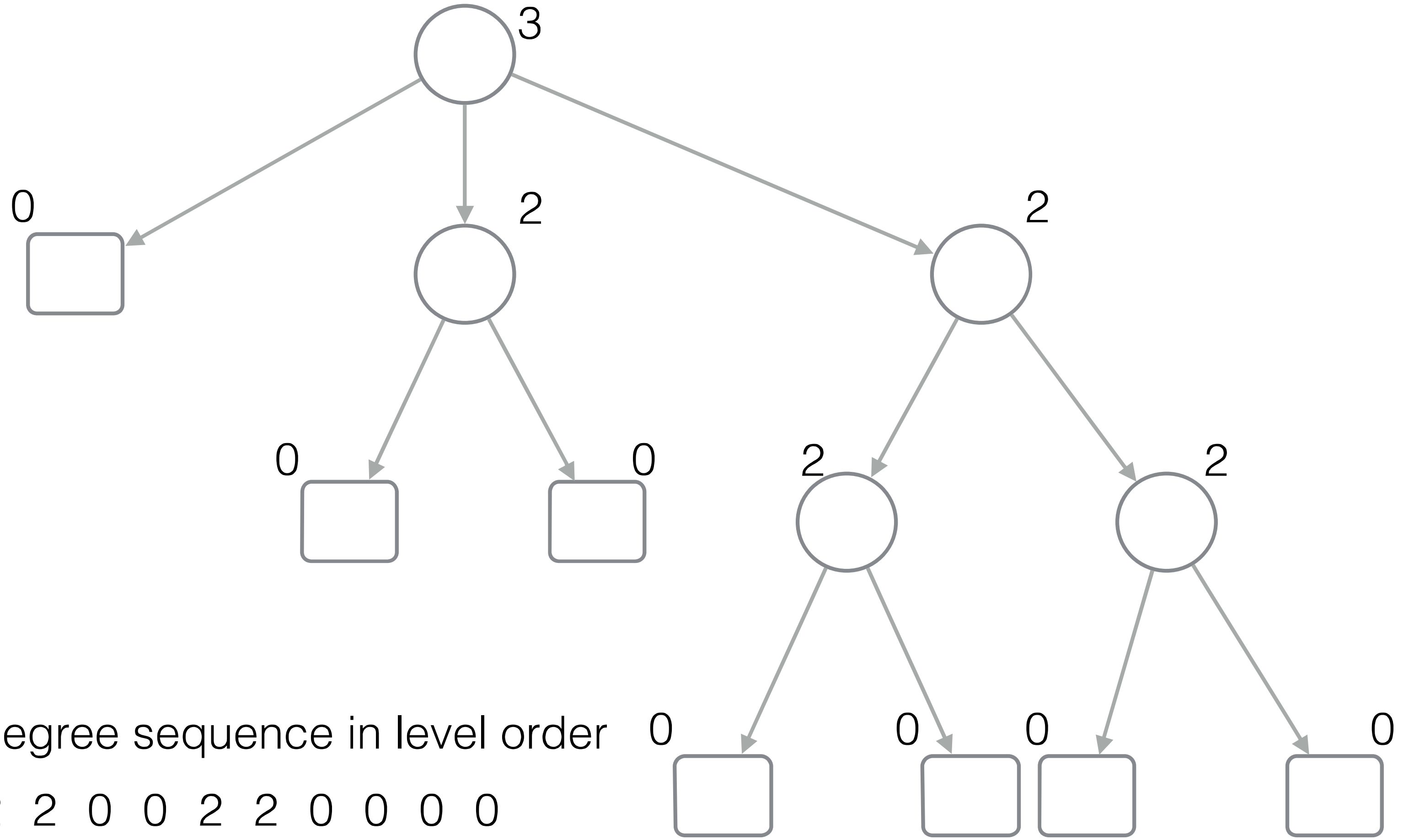

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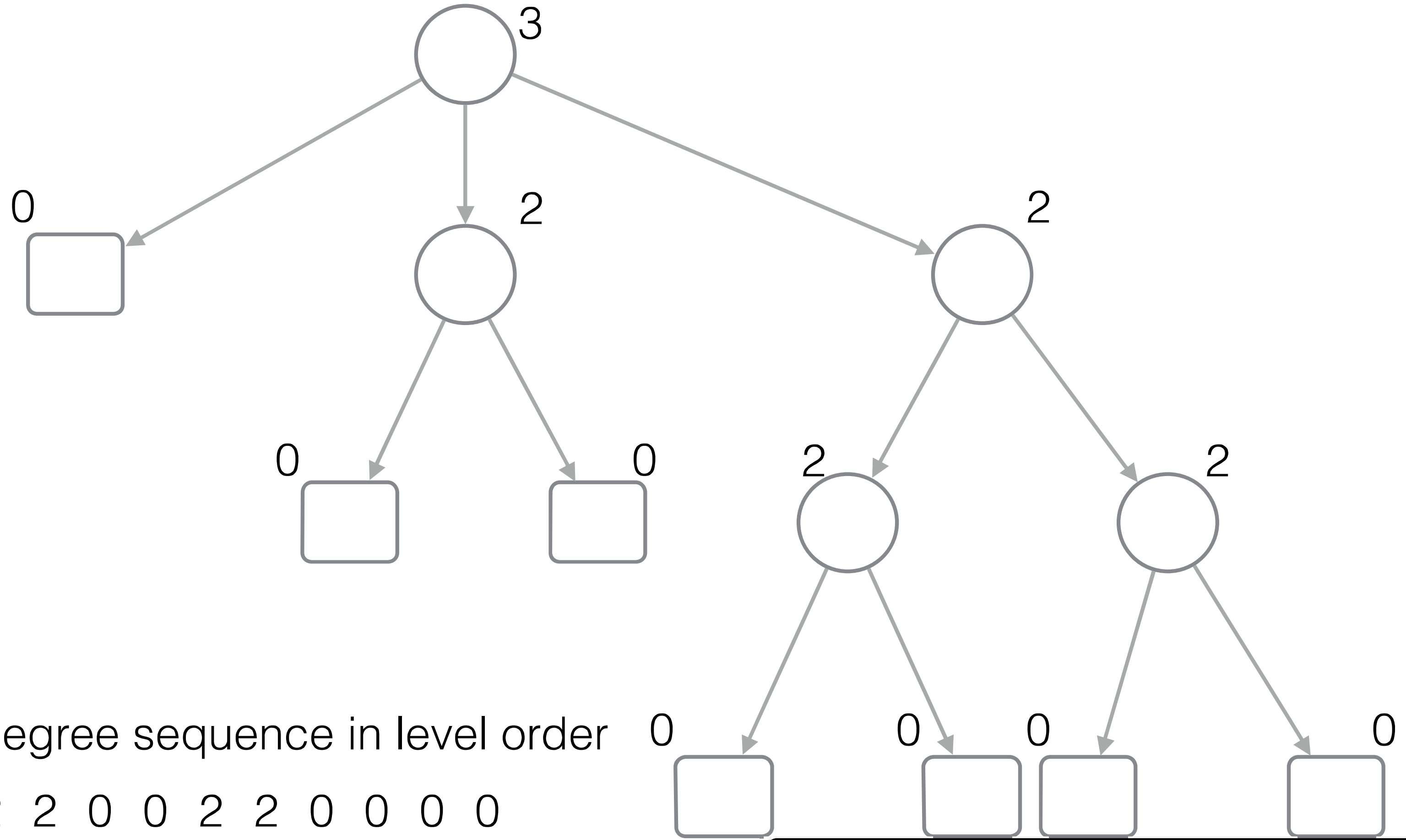


Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

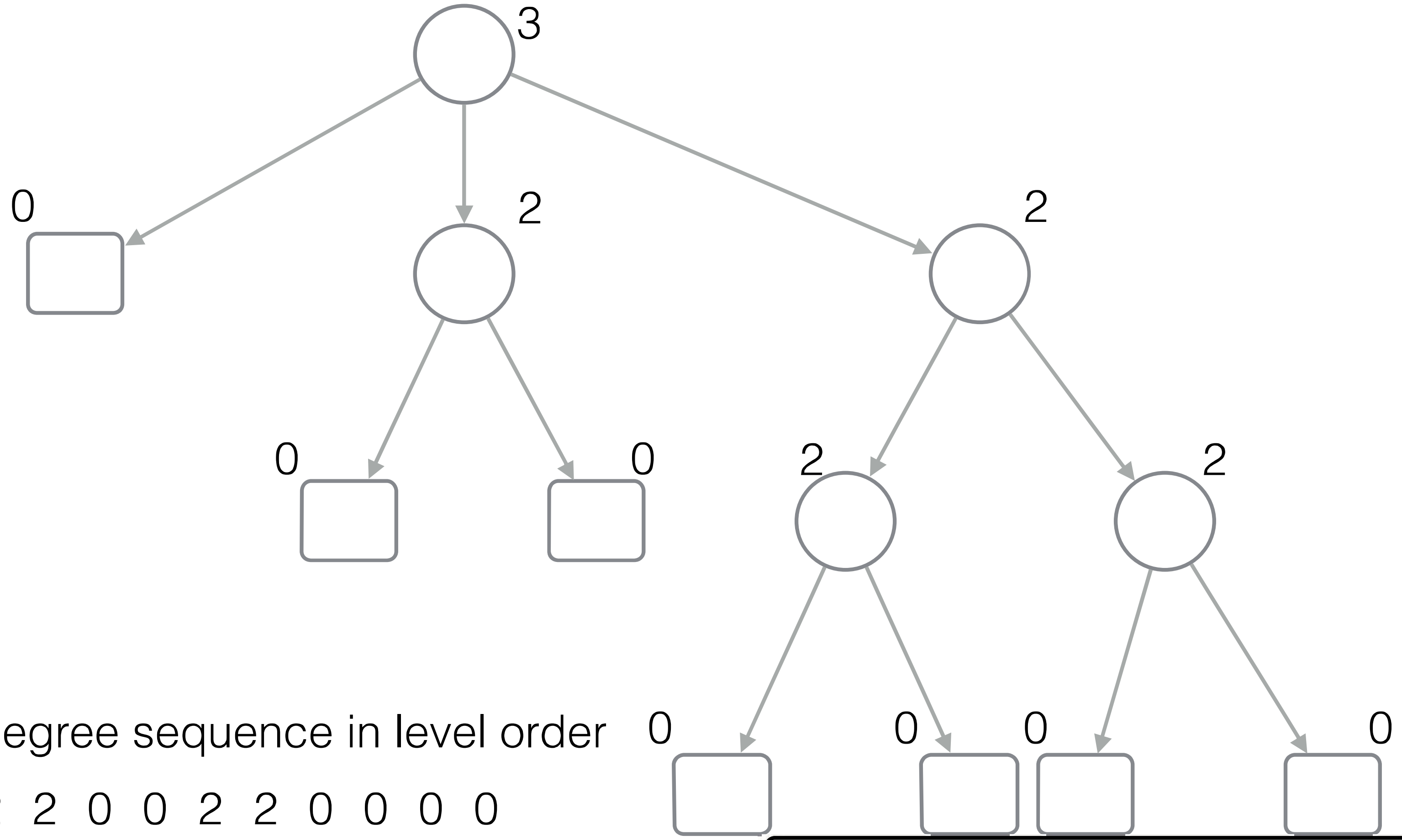
A tree is uniquely determined by the degree sequence

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

A tree is uniquely determined by the degree sequence

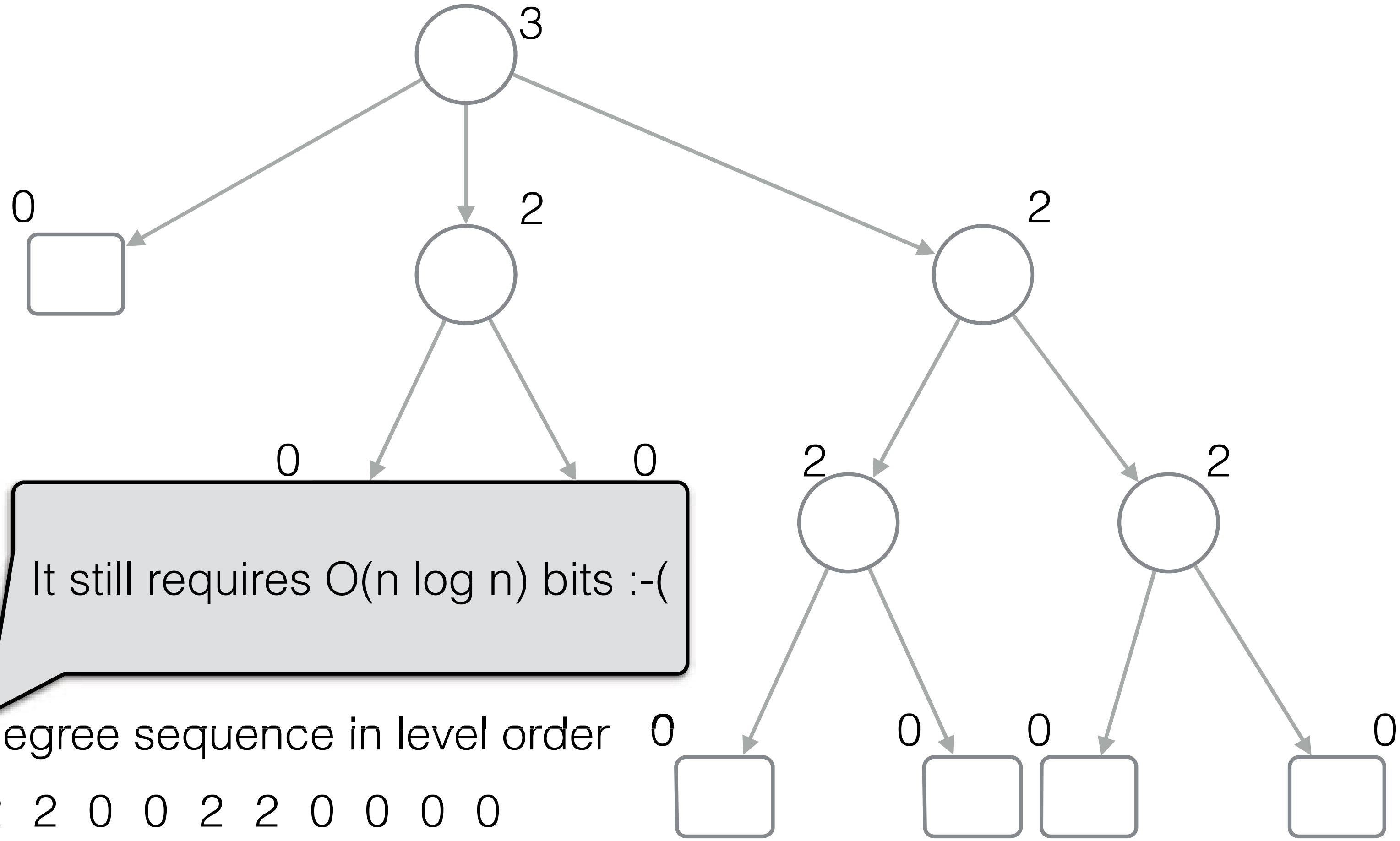
How reconstruct the tree?

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

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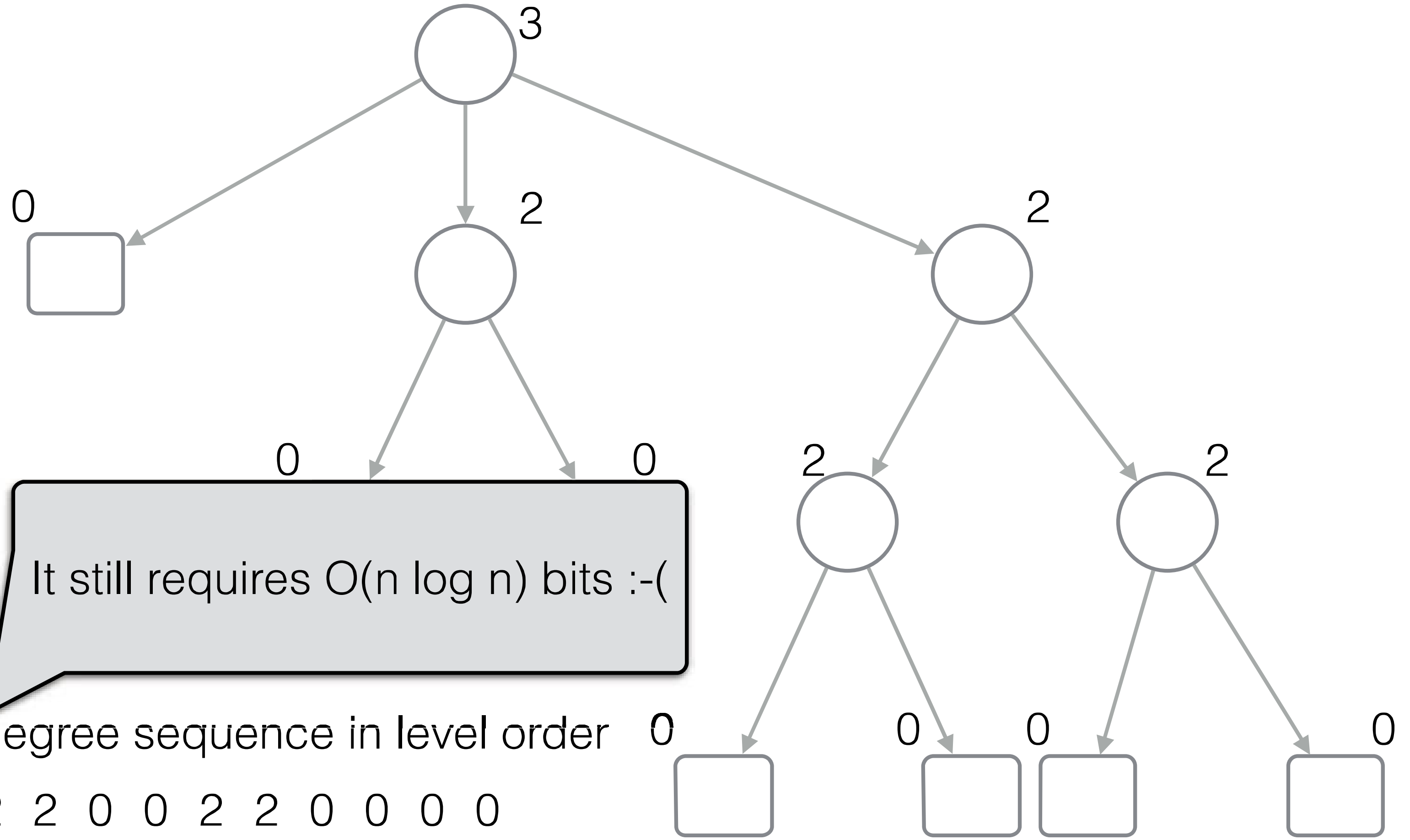

It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

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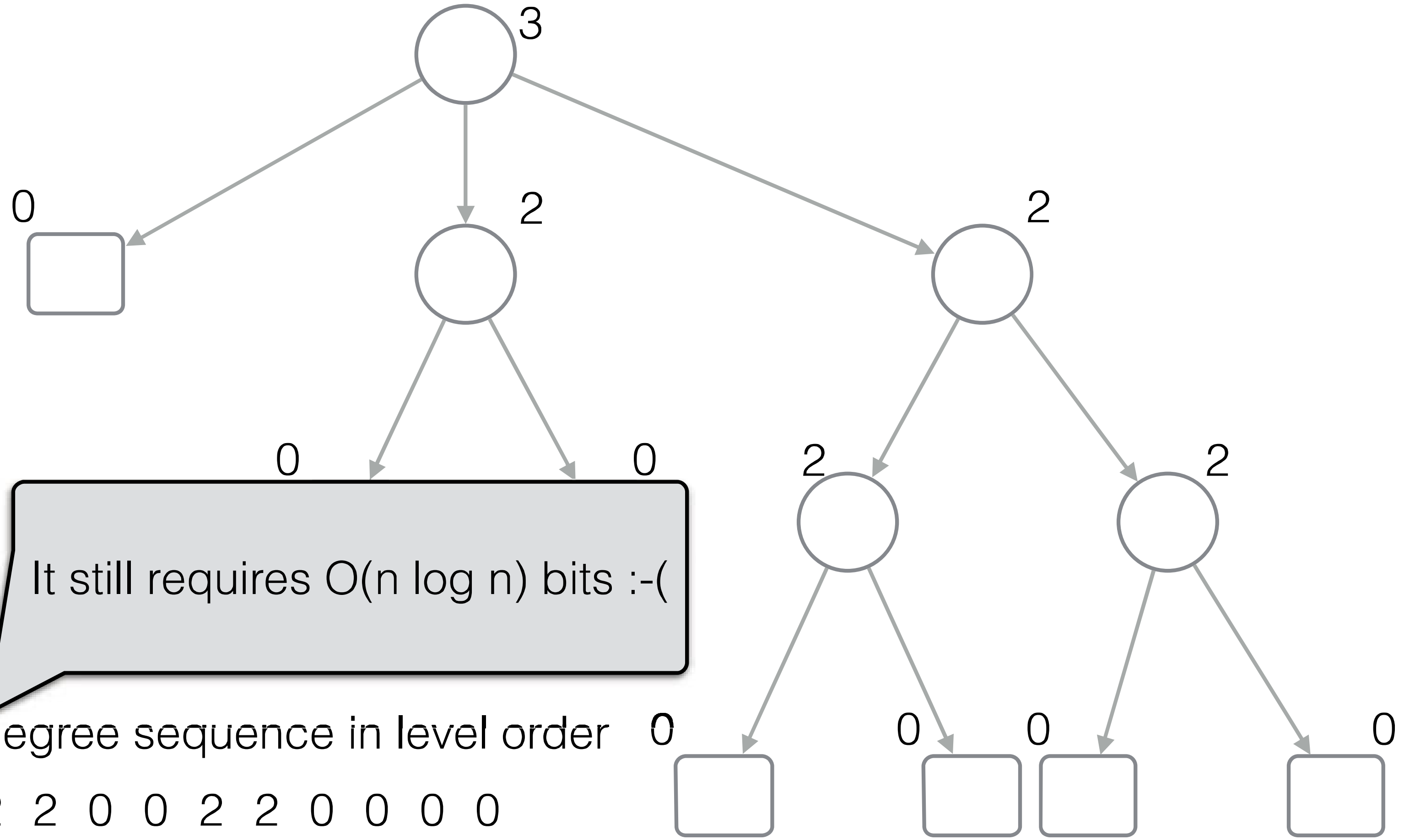


It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

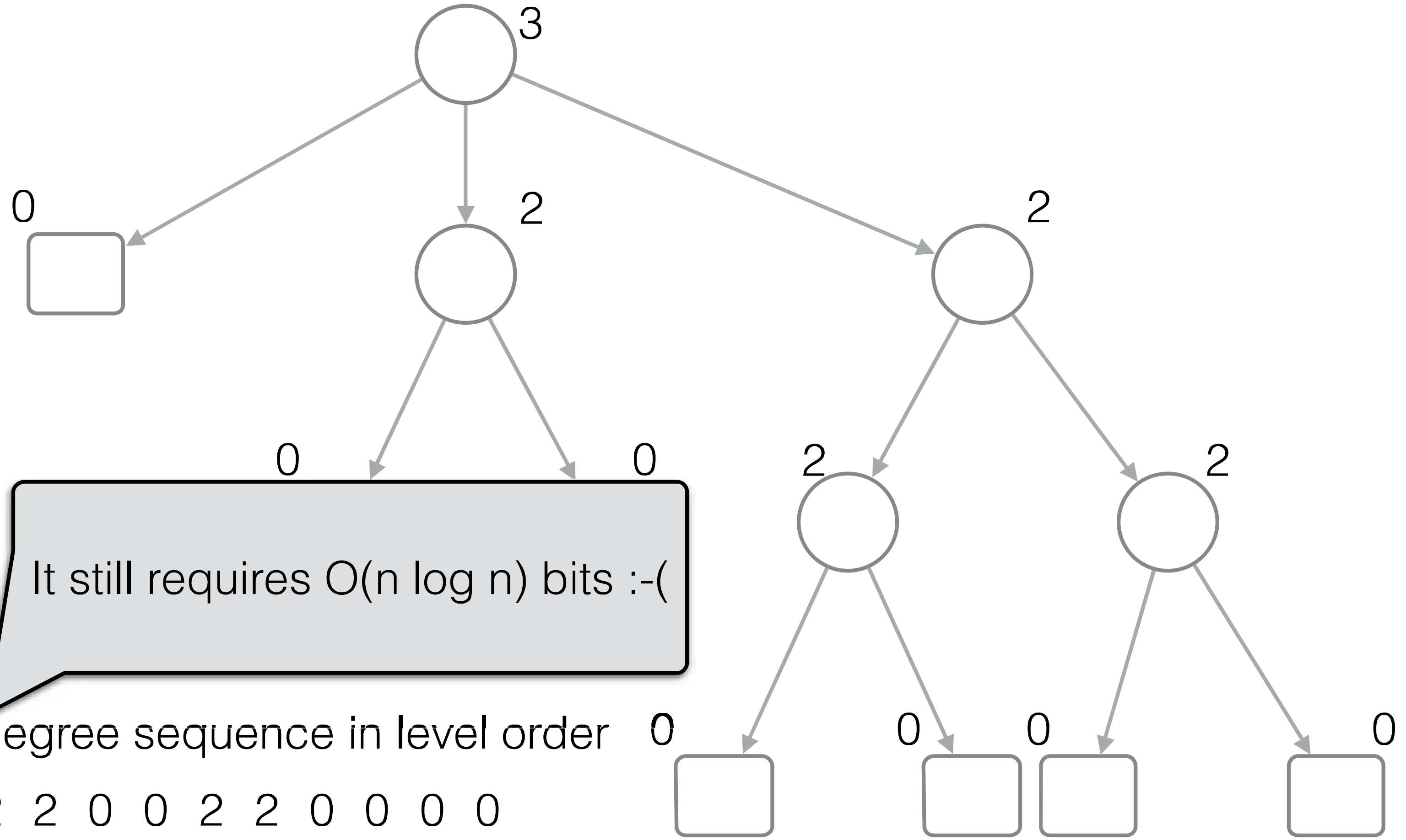
B

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

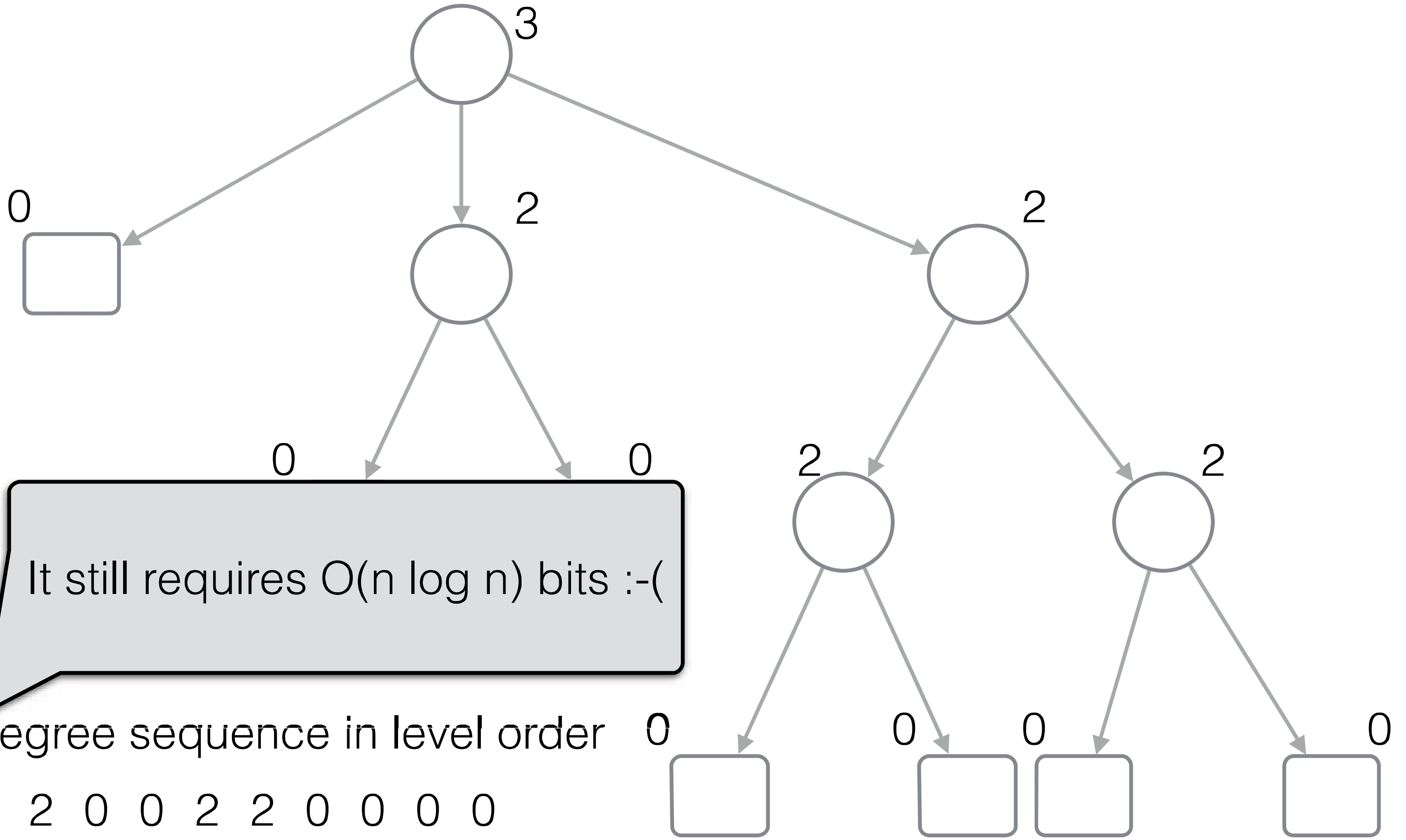
B 1110

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

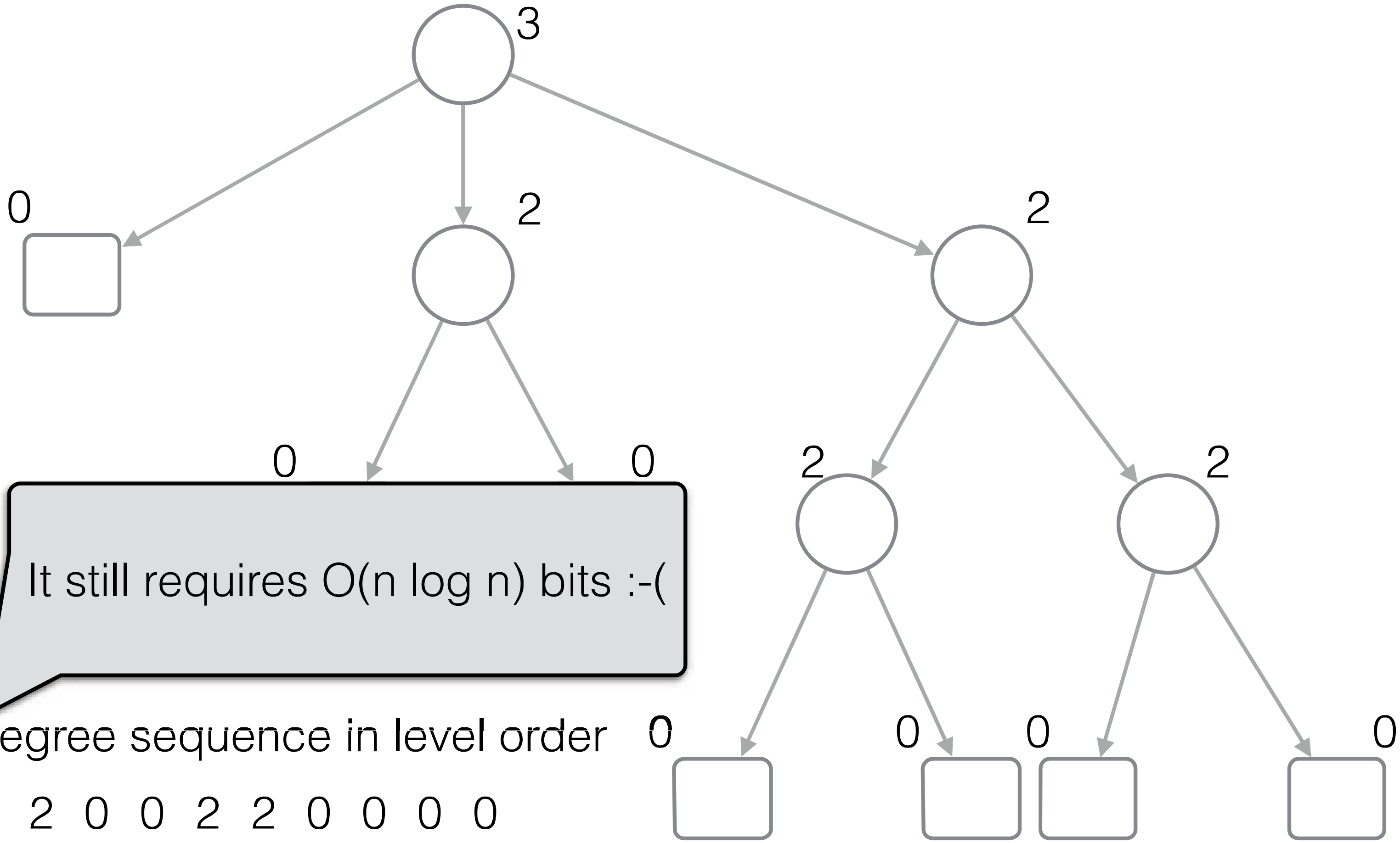
B 1110 0 110 110 0 0 110 110 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degree sequence in level order

D 3 0 2 2 0 0 2 2 0 0 0 0

Solution: write them in unary

B 1110 0 110 110 0 0 110 110 0 0 0 0

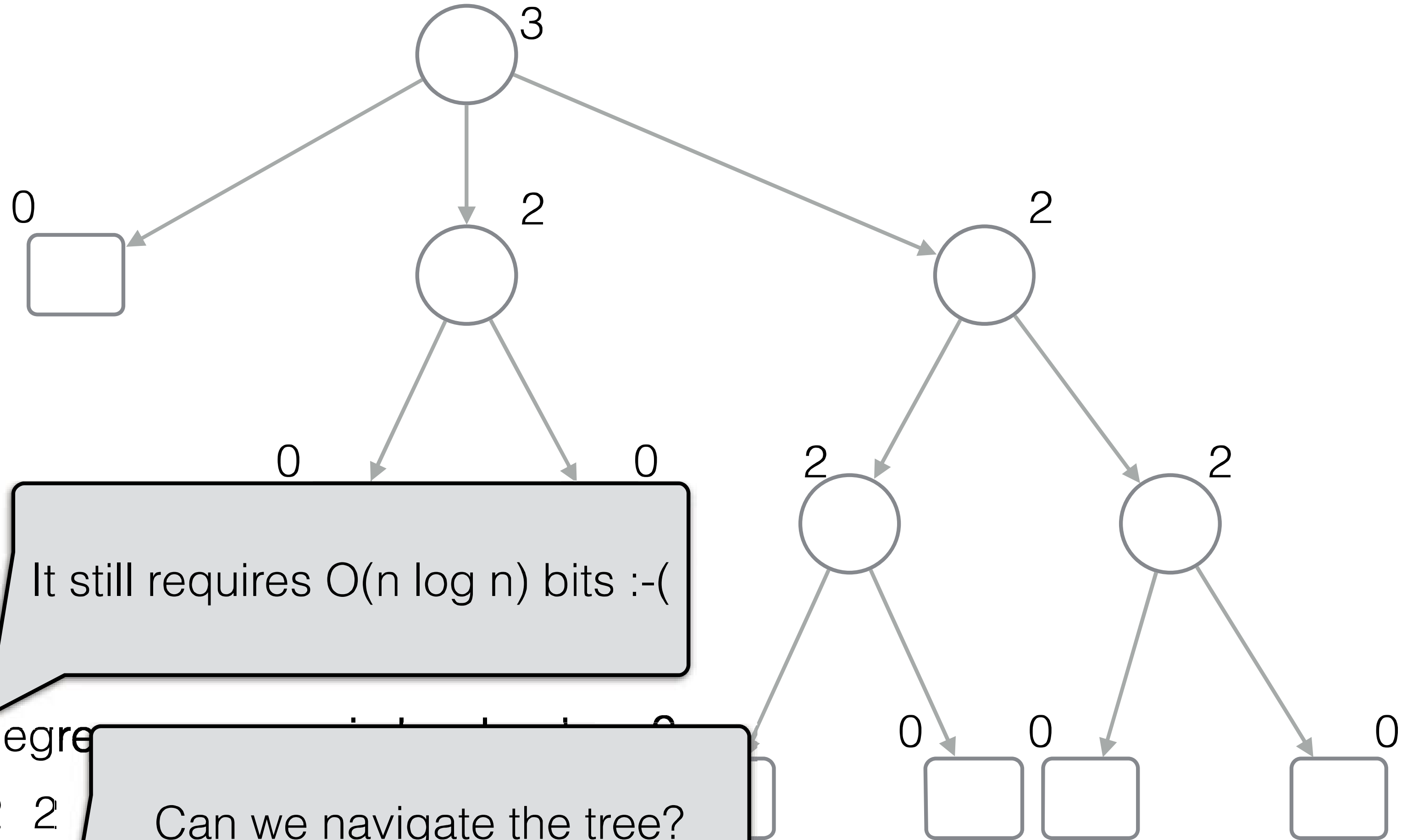
B takes 2n - 1 bits!
For each node we have a 0 and a 1
(but the root)

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

Trivial: O(n log n) bits

Best: 2n bits



It still requires O(n log n) bits :-(

Write the degre

D 3 0 2 2

Can we navigate the tree?

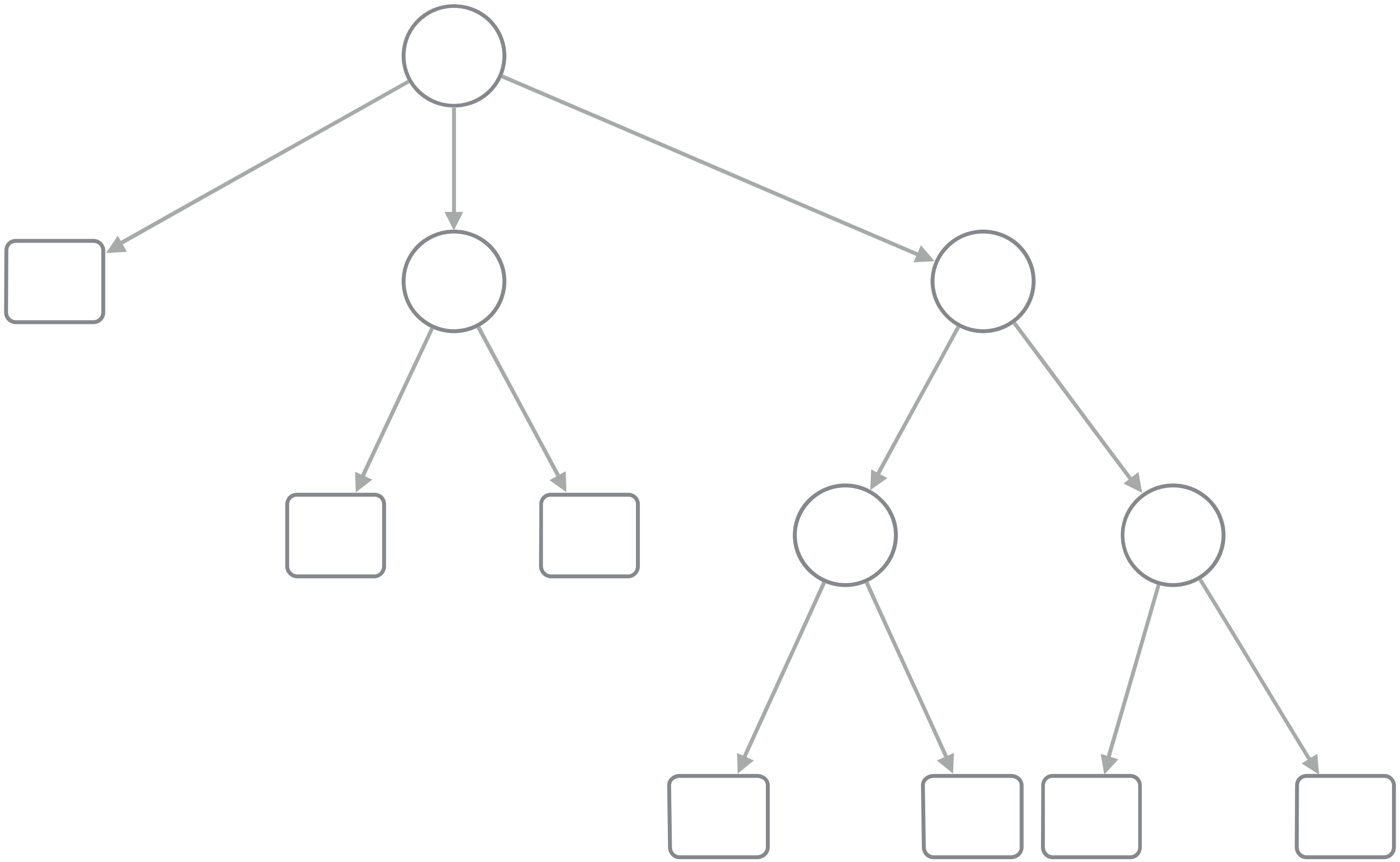
Solution: write ... em in unary

B 1110 0 110 110 0 0 110 110 0 0 0 0

B takes 2n - 1 bits!
For each node we have a 0 and a 1
(but the root)

Succinct representation of trees (1)

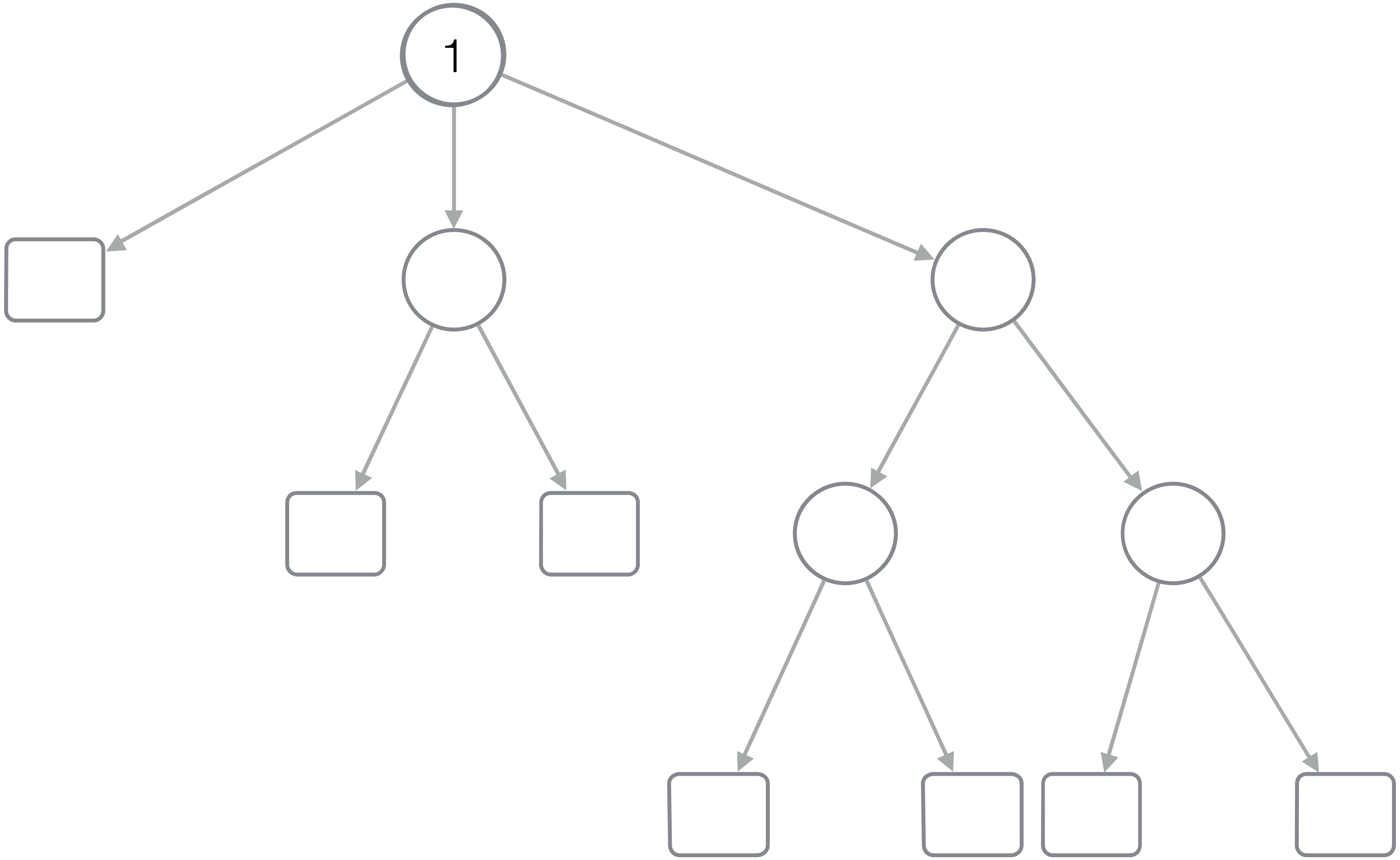
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

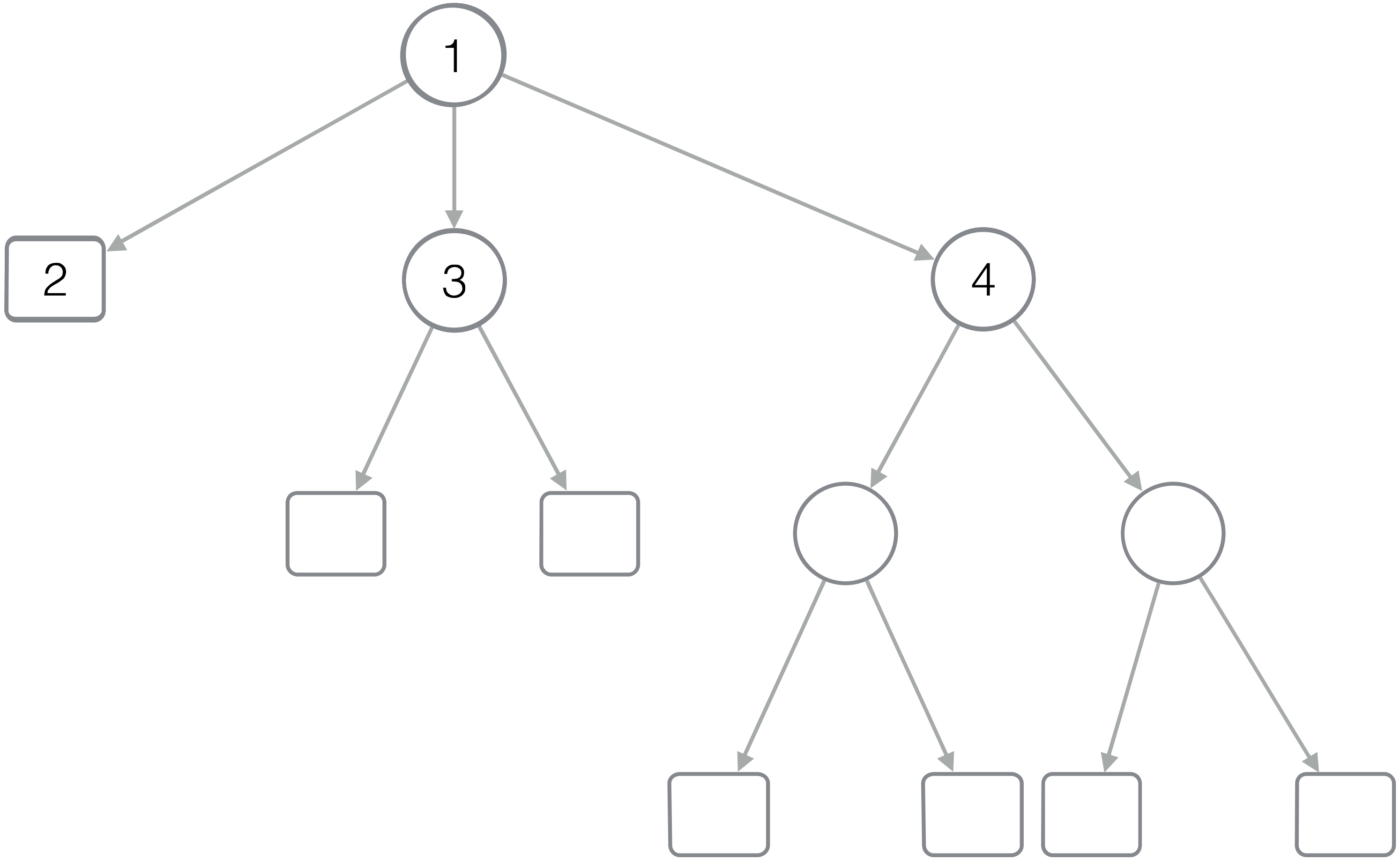
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

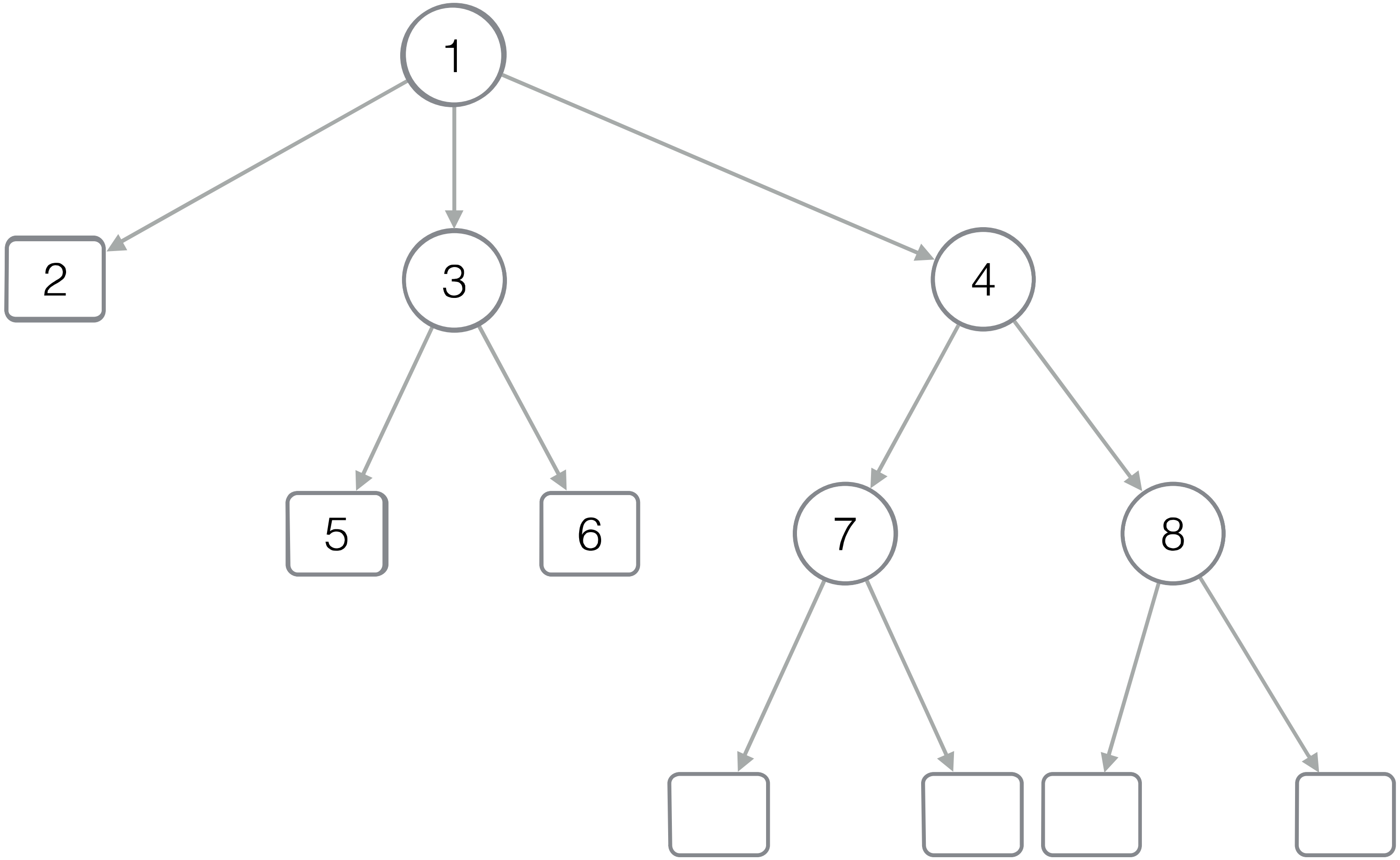
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

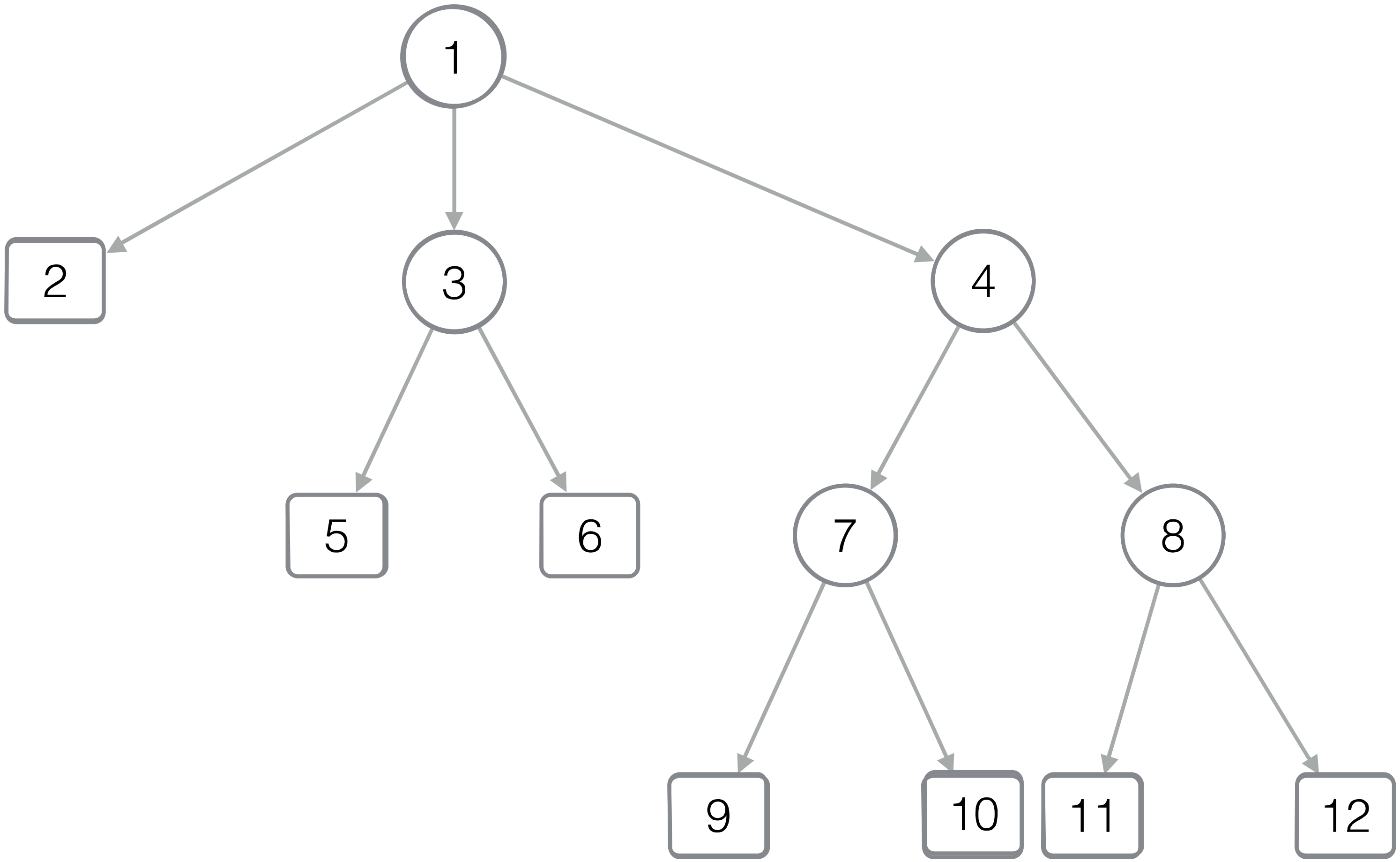
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

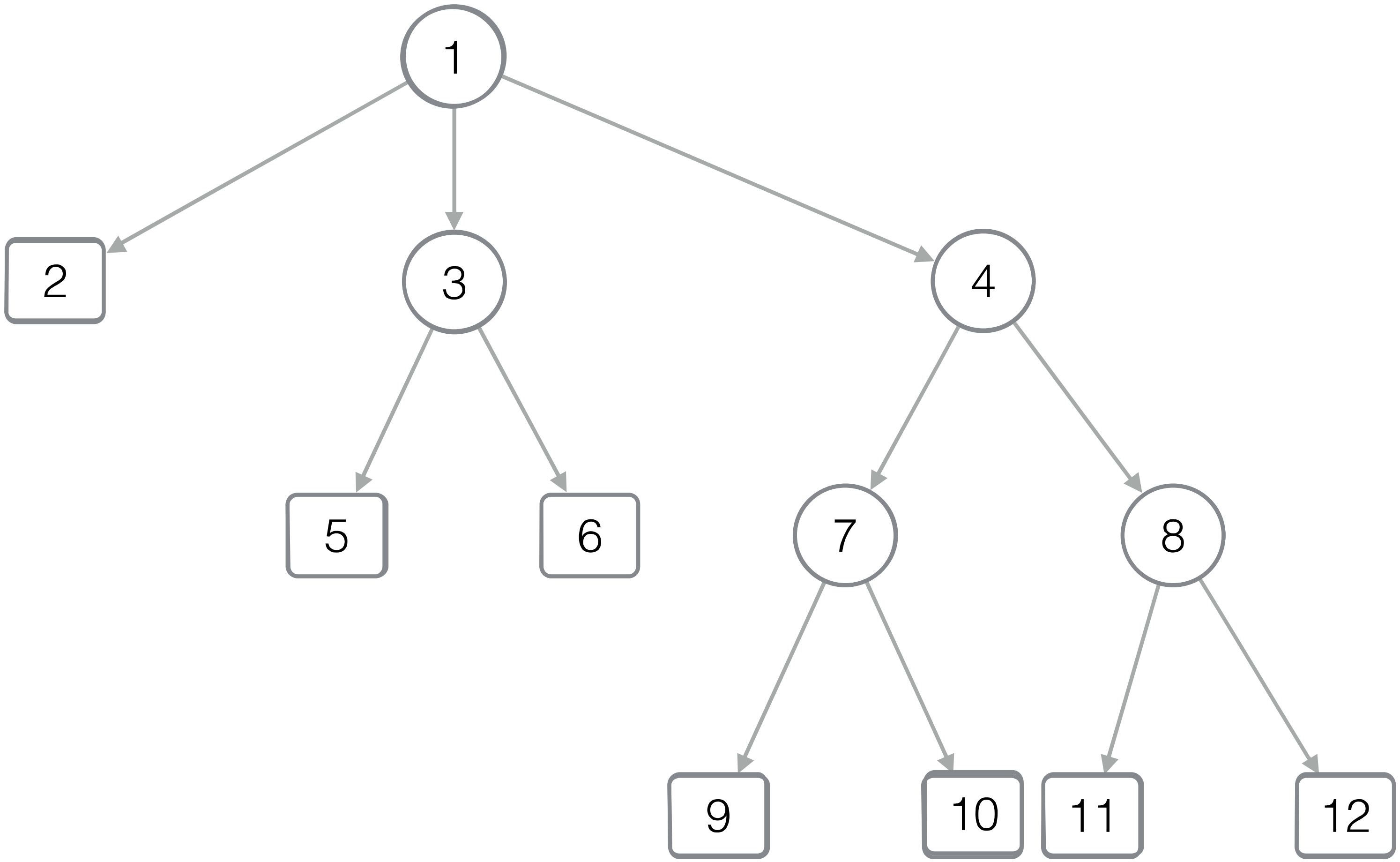
[LOUDS - Level-order unary degree sequence]



B 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

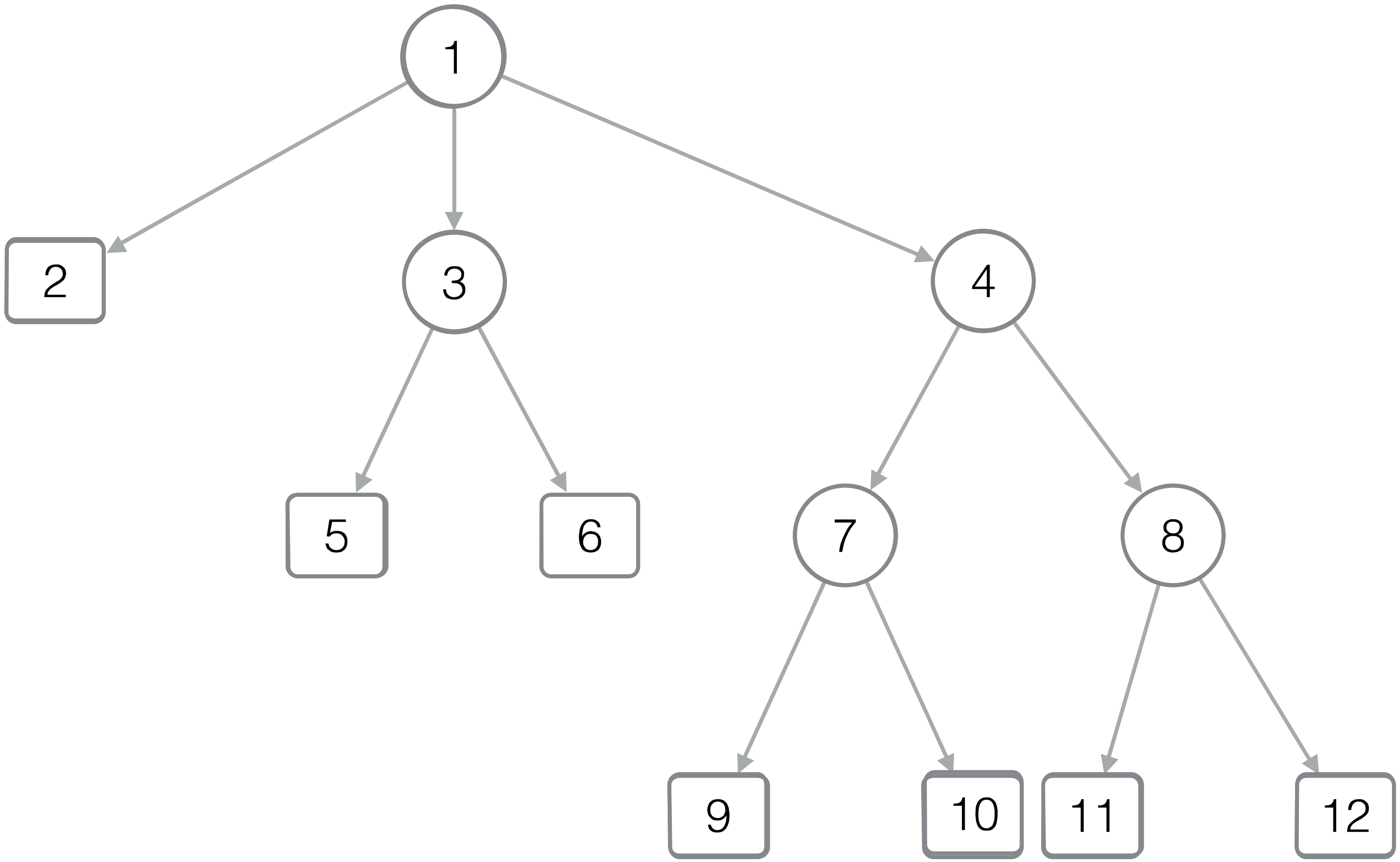
[LOUDS - Level-order unary degree sequence]



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

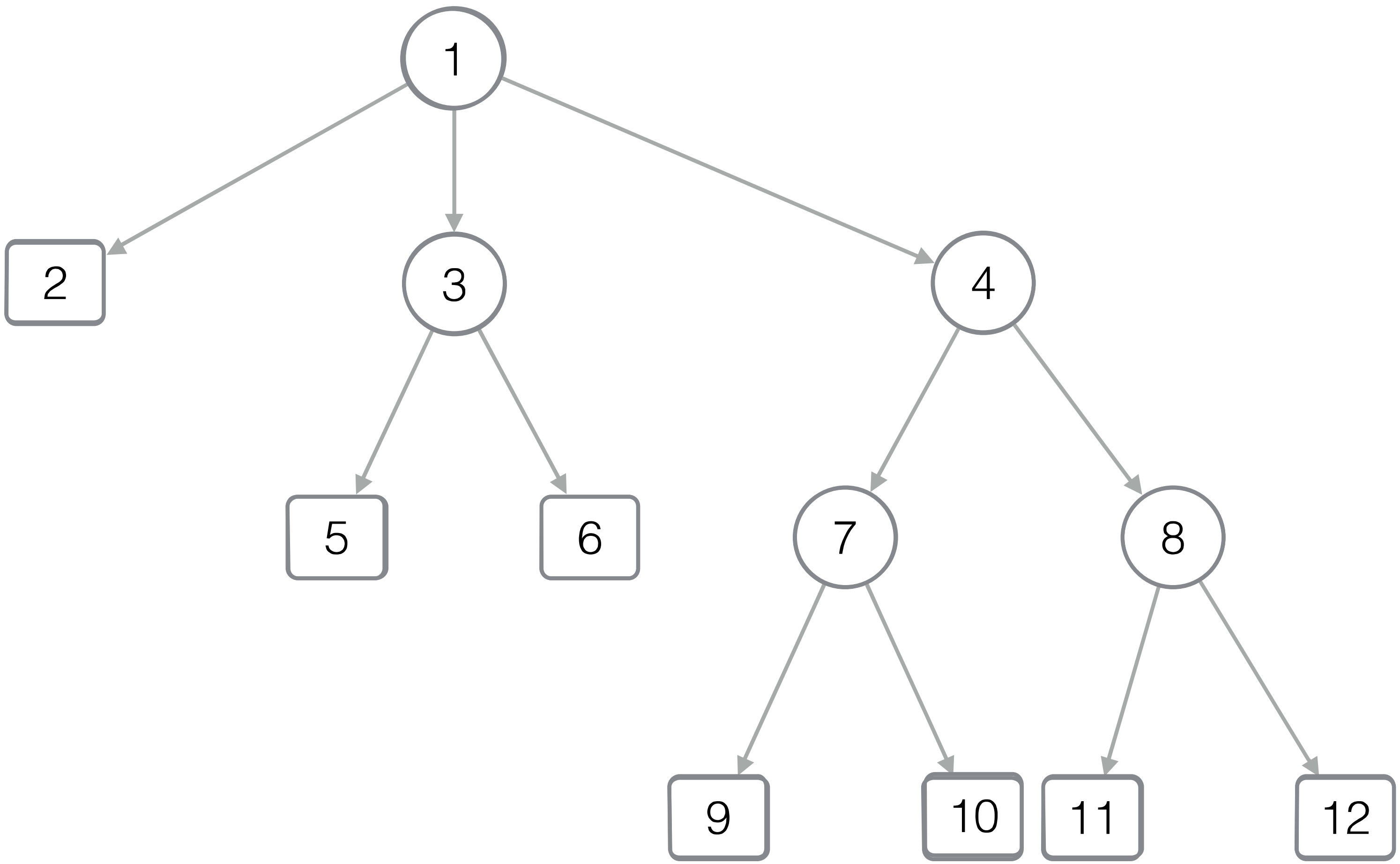


B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



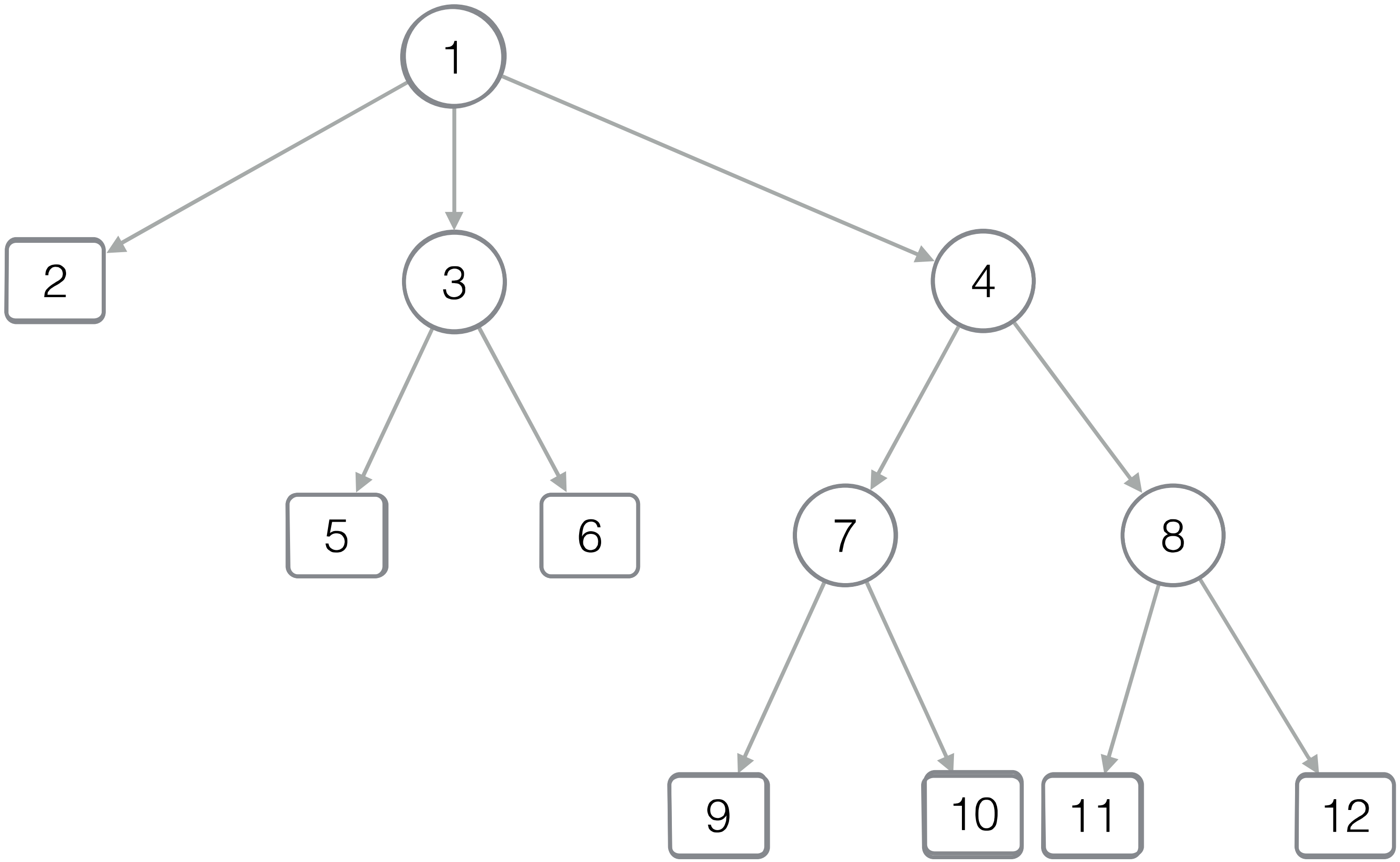
B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

pos(x) =



pos(5)

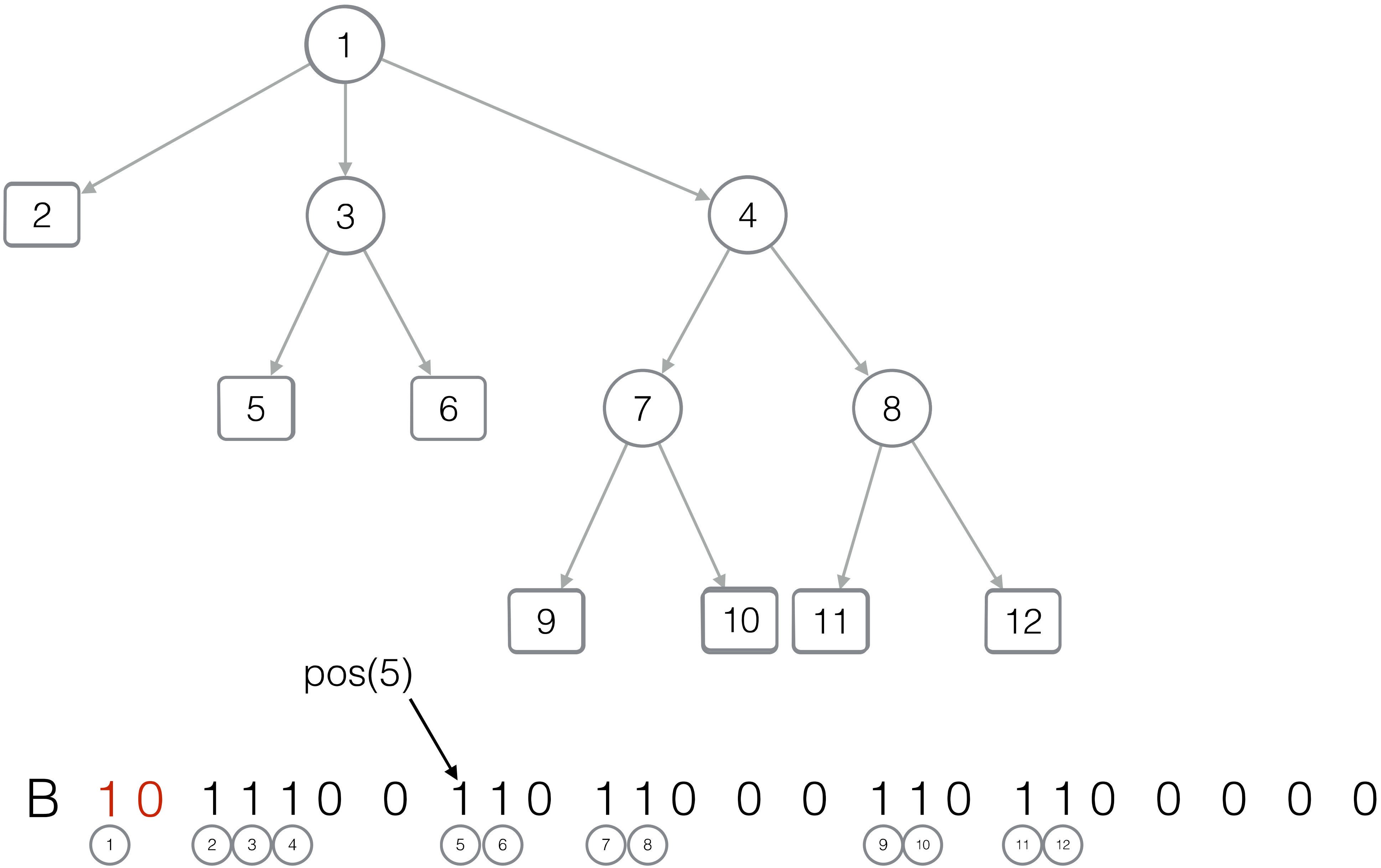
B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$\text{pos}(x) = \text{Select}_1(x)$

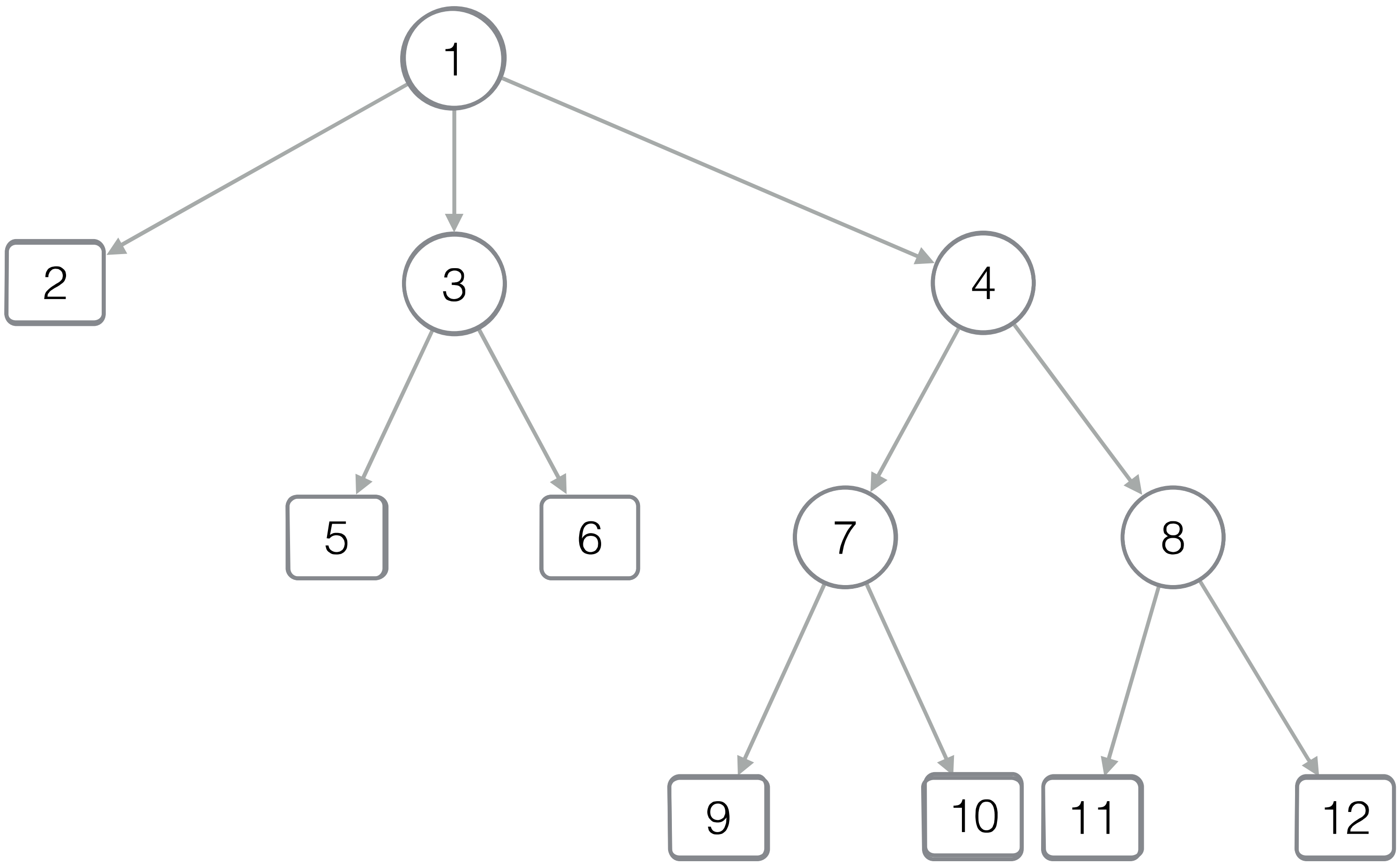


Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

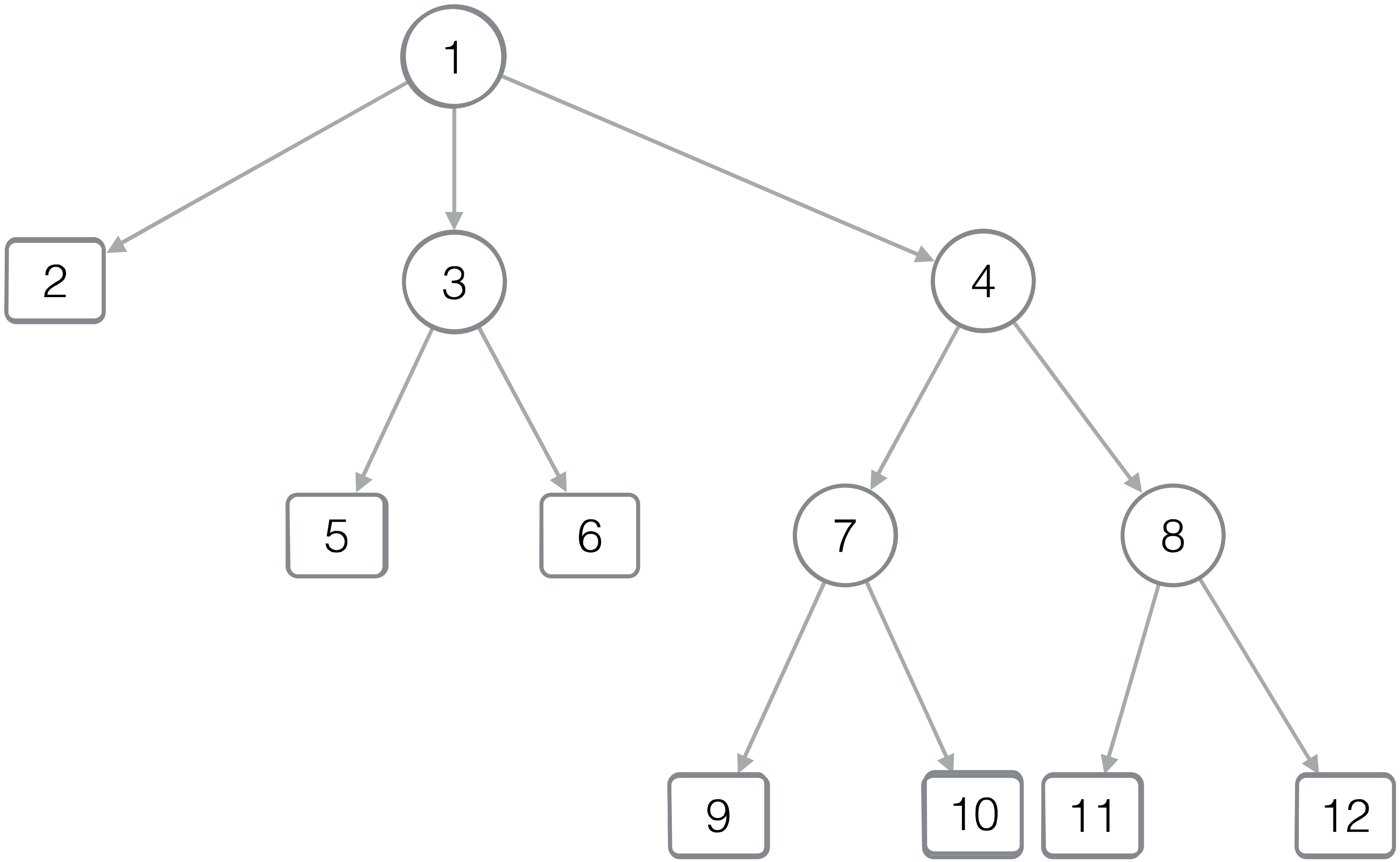
[LOUDS - Level-order unary degree sequence]

$\text{pos}(x) = \text{Select}_1(x)$

$\text{firstChild}(x) = ?$

$y = \text{Select}_0(x)+1$

// start of x's children in B



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

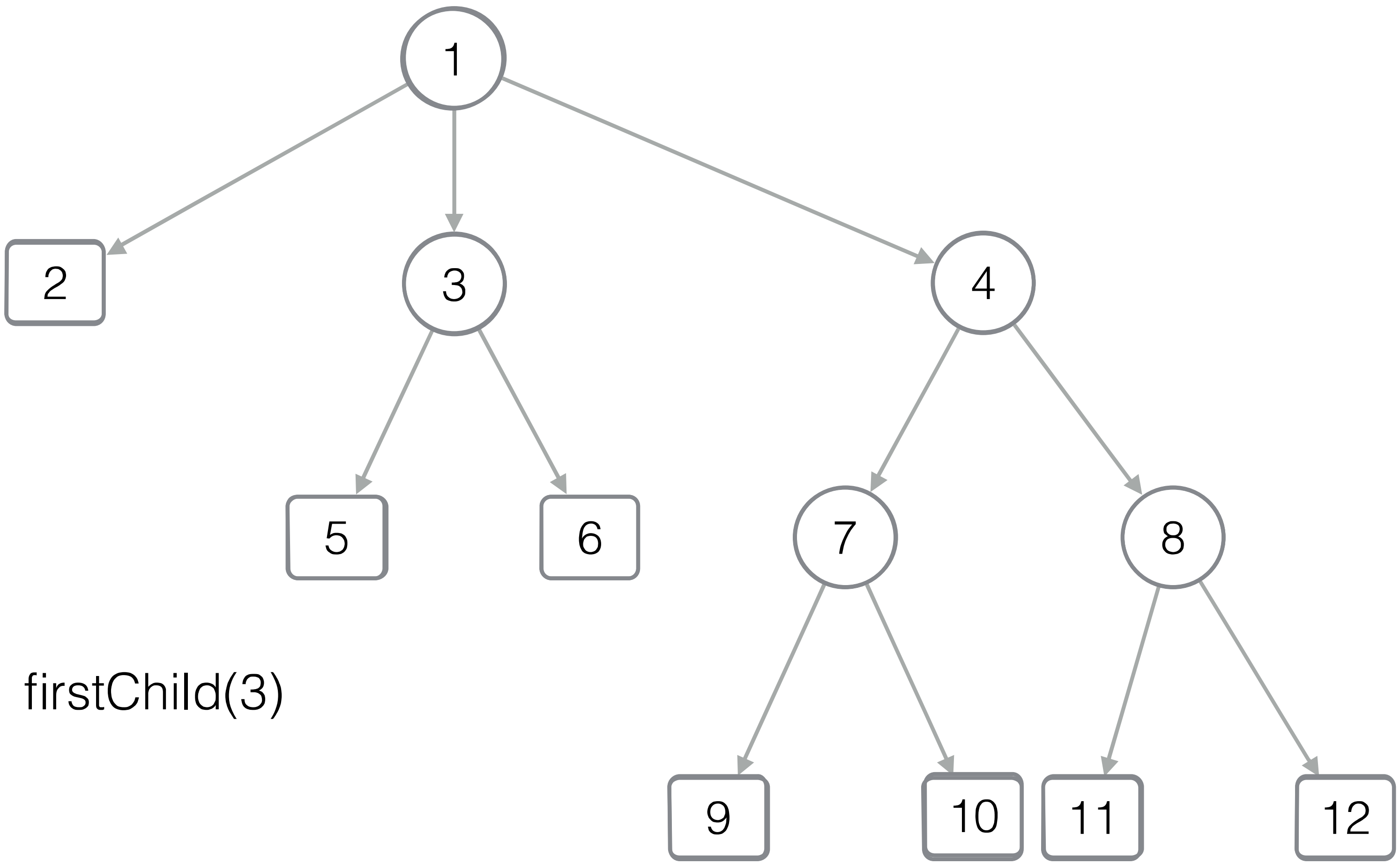
[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B



firstChild(3)

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

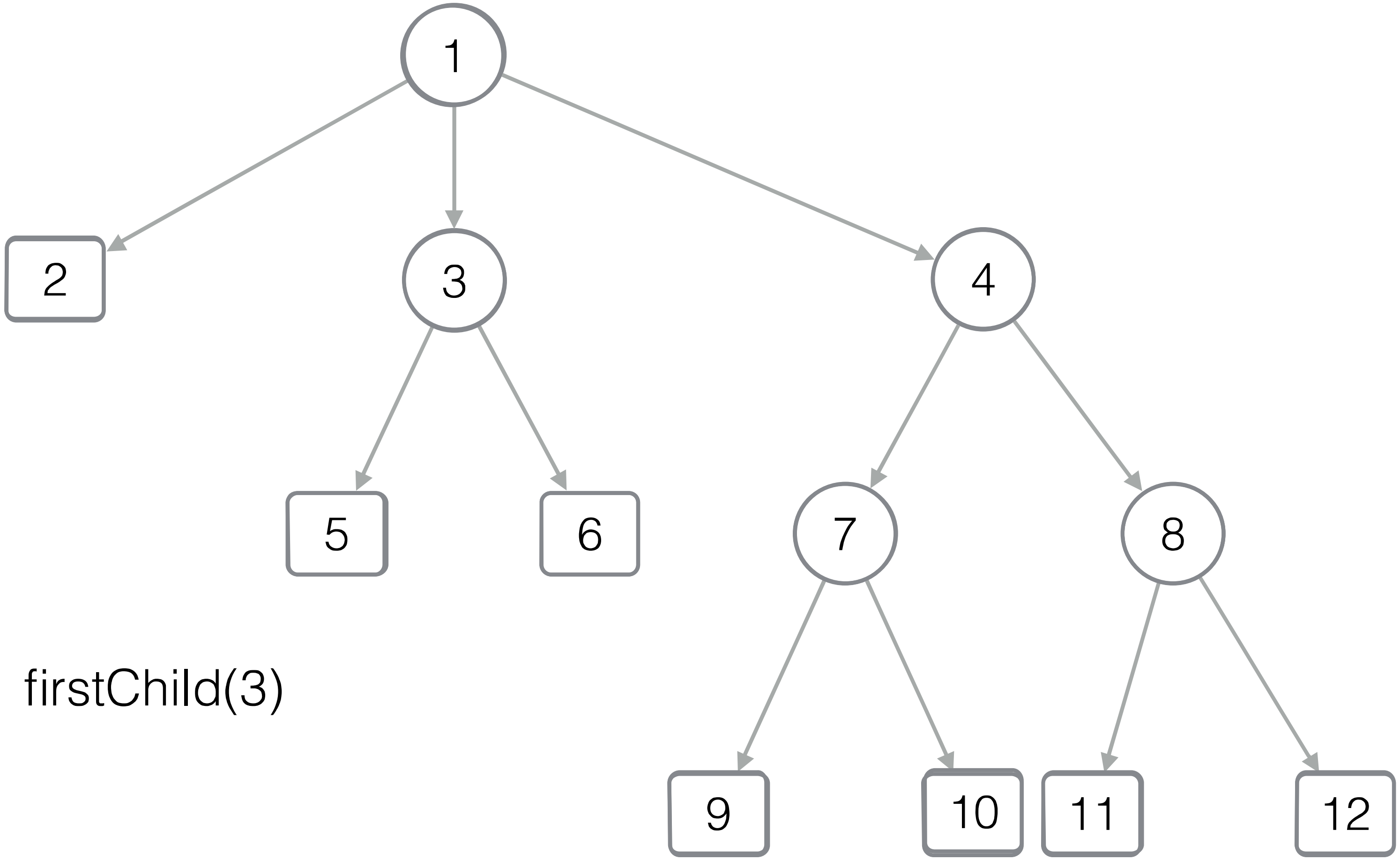
[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B



firstChild(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

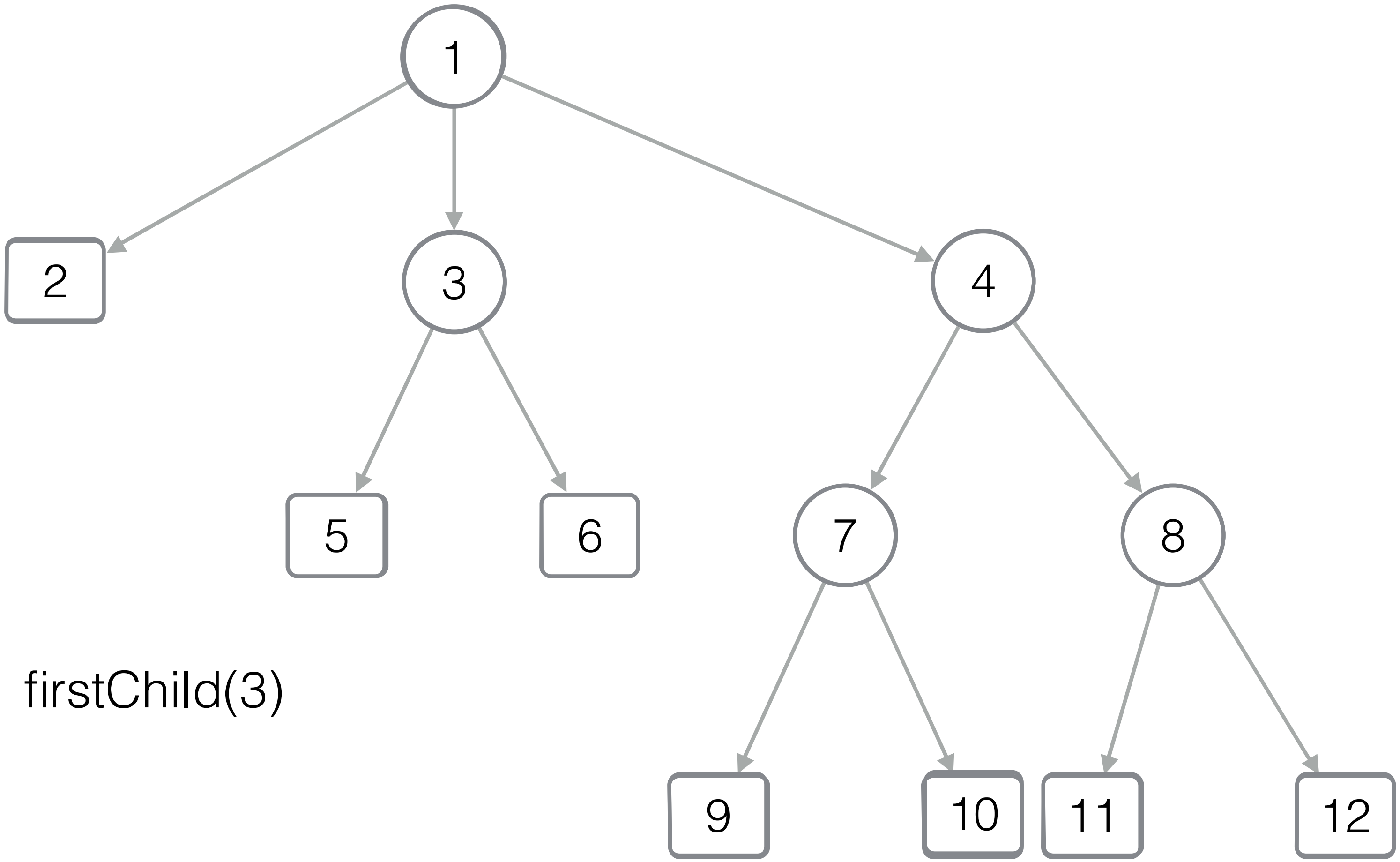
$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if B[y] == 0

return -1 // is a leaf



firstChild(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

pos(x) = $Select_1(x)$

firstChild(x) = ?

y = $Select_0(x)+1$

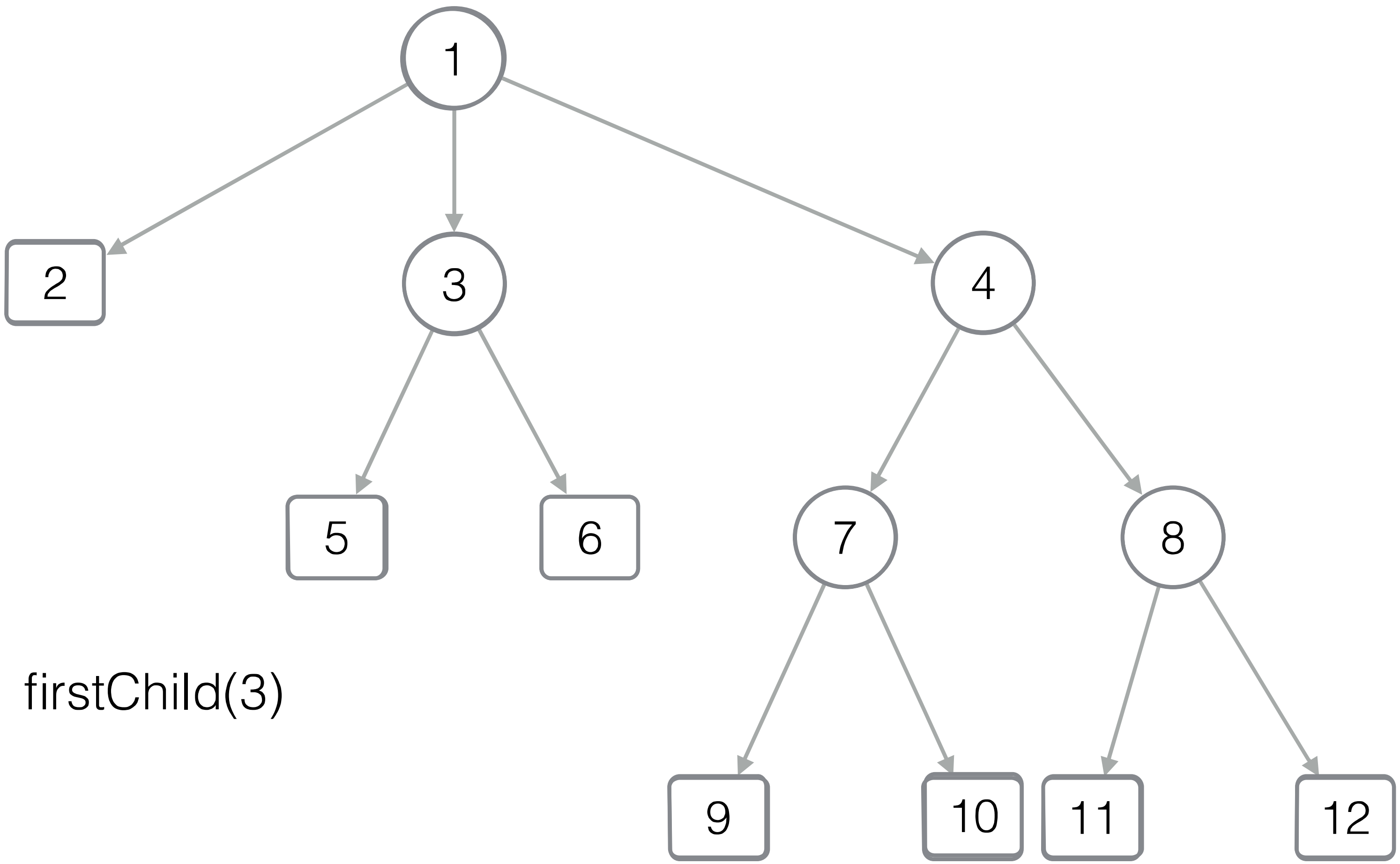
// start of x's children in B

if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$



firstChild(3)

y= $Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

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$firstChild(x) = ?$

$y = Select_0(x)+1$

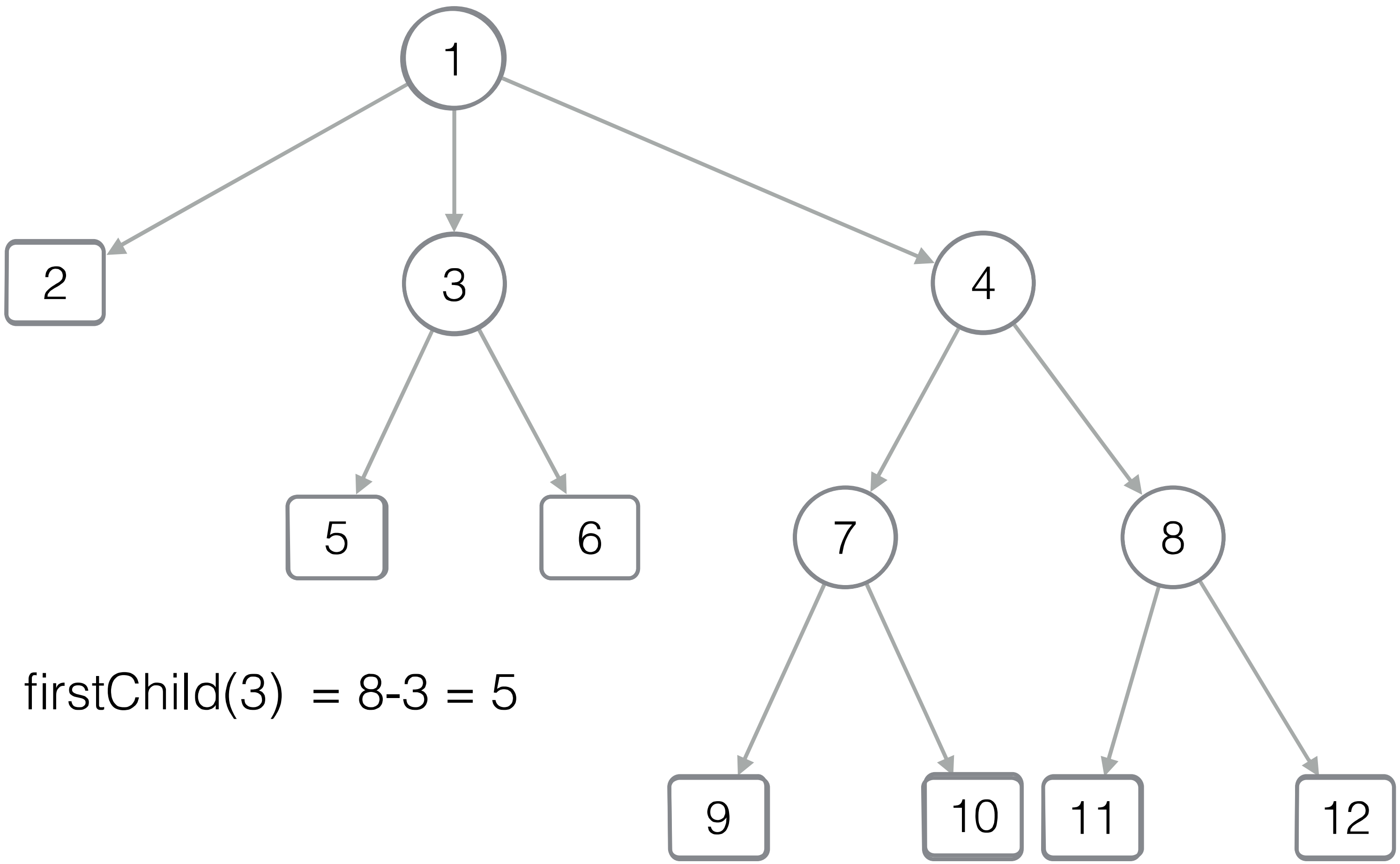
// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

else

return $y-x$ // $Rank_1(y)$



firstChild(3) = 8-3 = 5

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

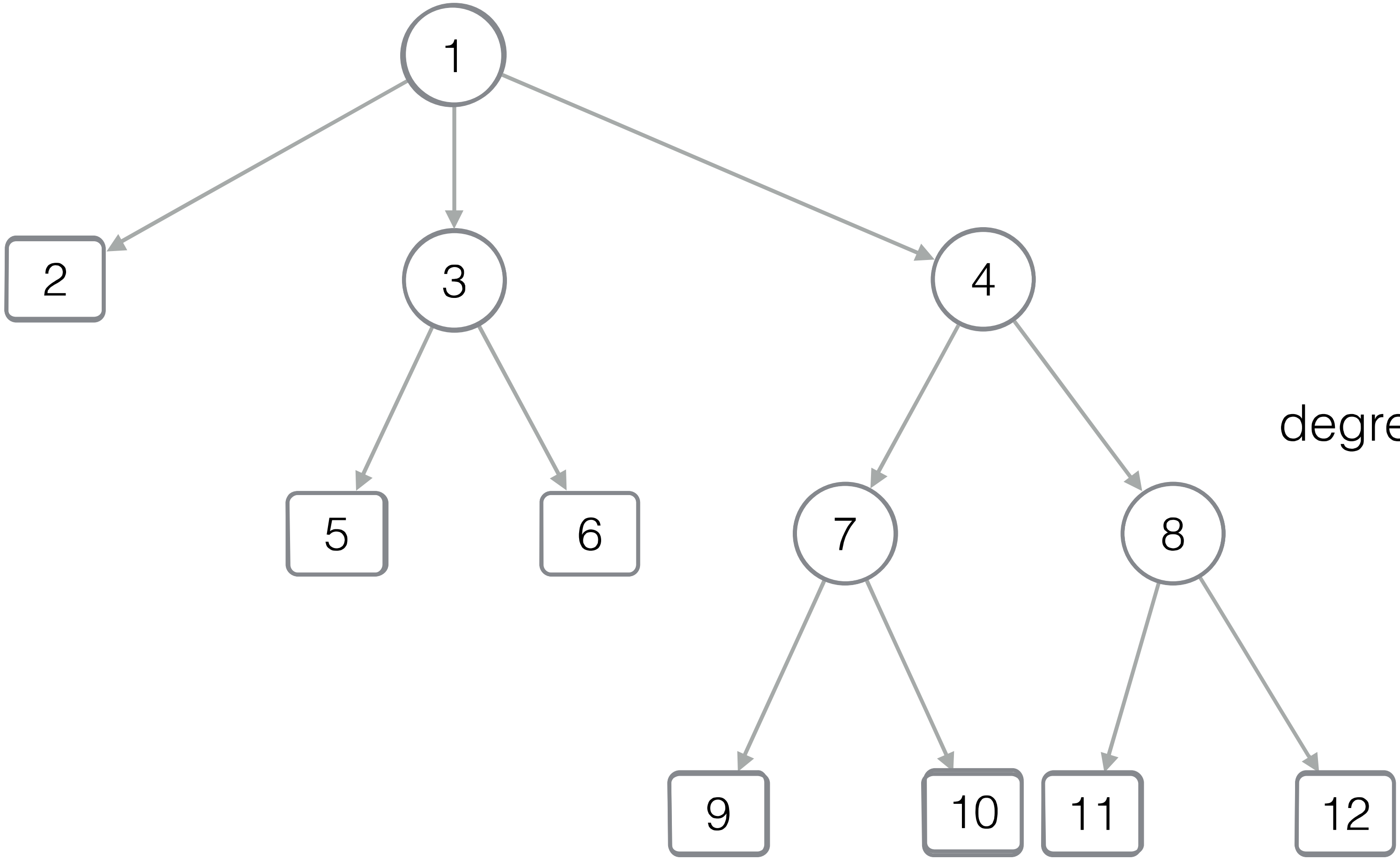
if $B[y] == 0$

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$



$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

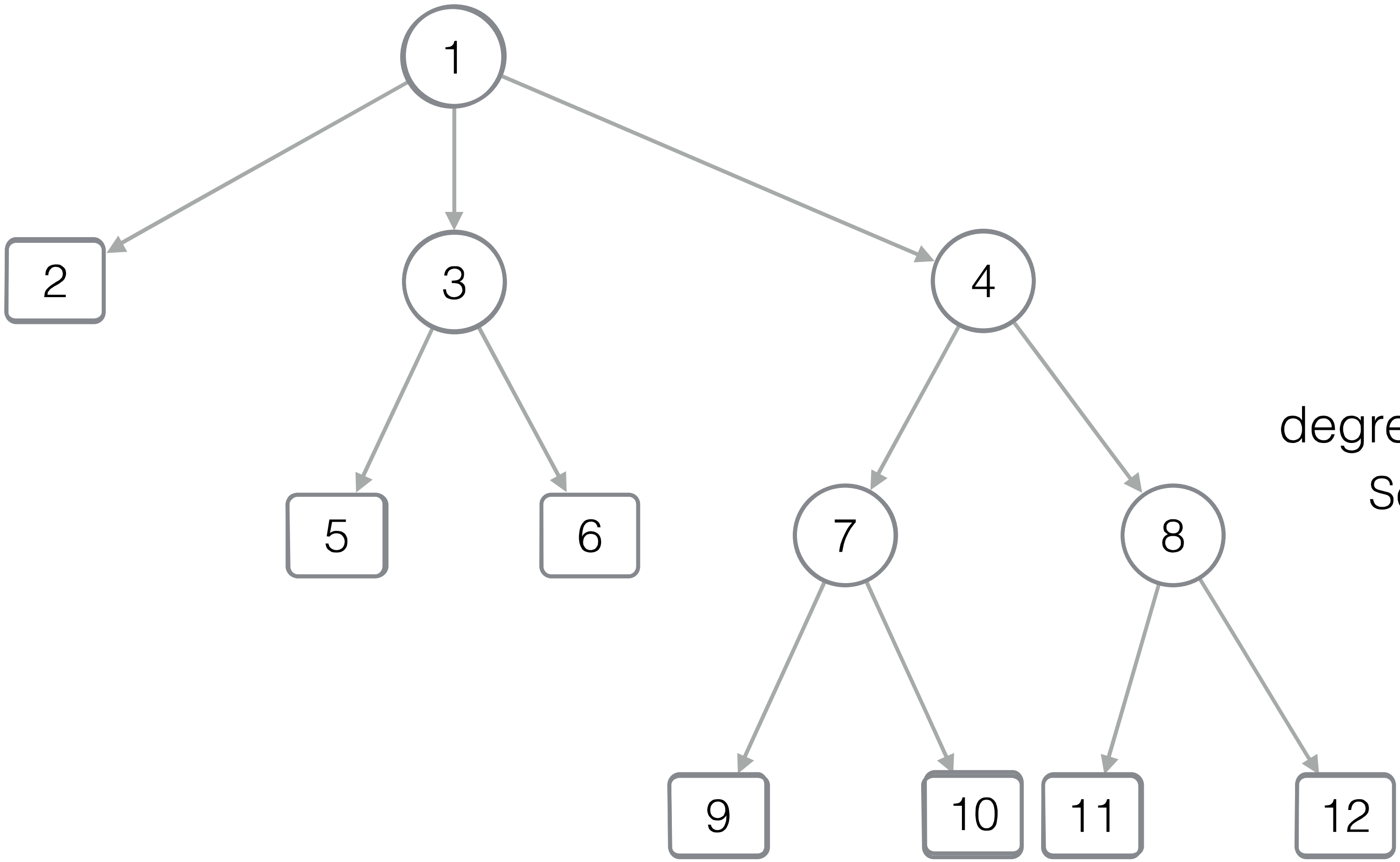
return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

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$firstChild(x) = ?$

$y = Select_0(x)+1$

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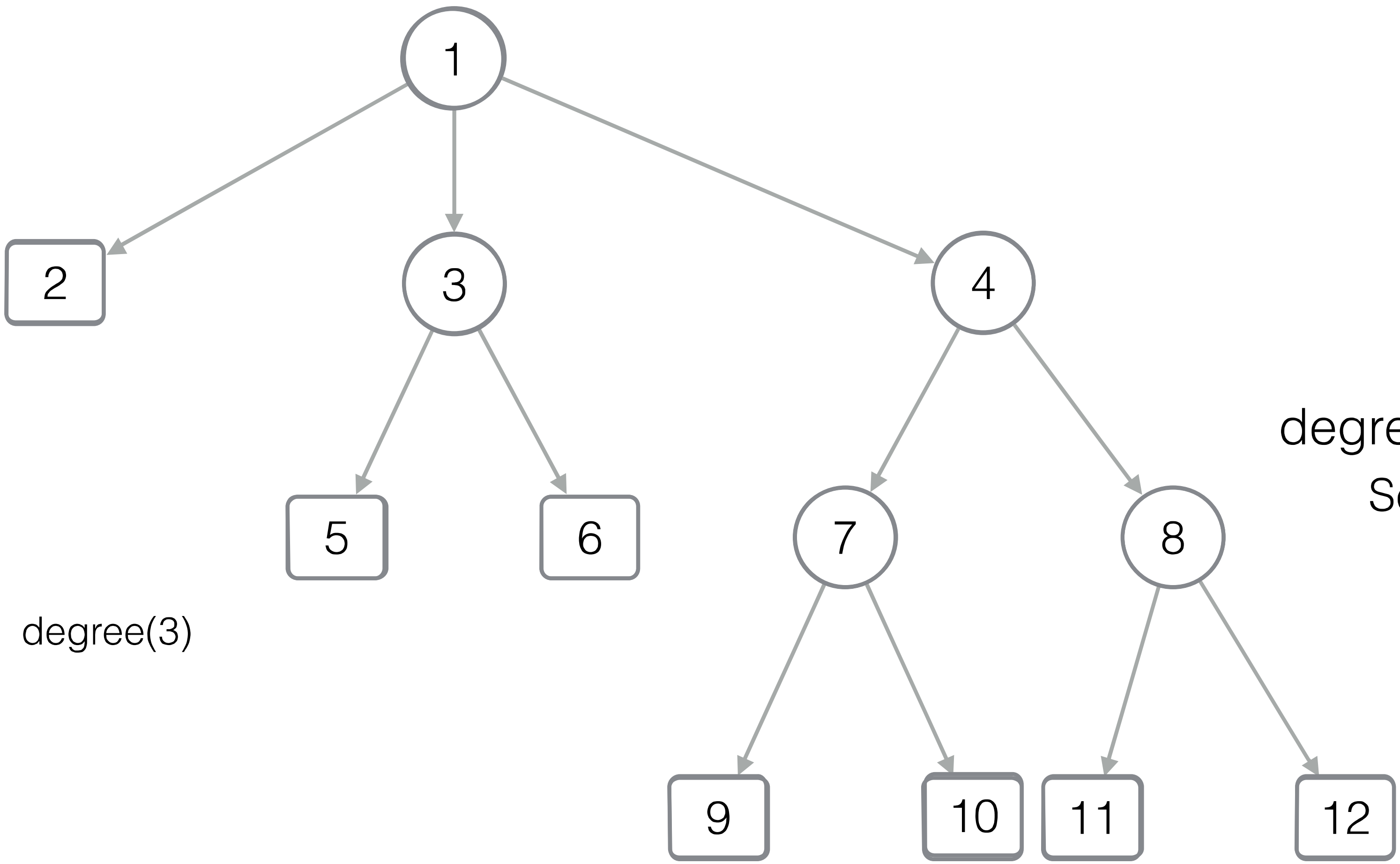
$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



degree(3)

$y = Select_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

pos(x) = $\mathrm{Select}_1(x)$

firstChild(x) = ?

$y = \mathrm{Select}_0(x)+1$

// start of x's children in B

if B[y] == 0

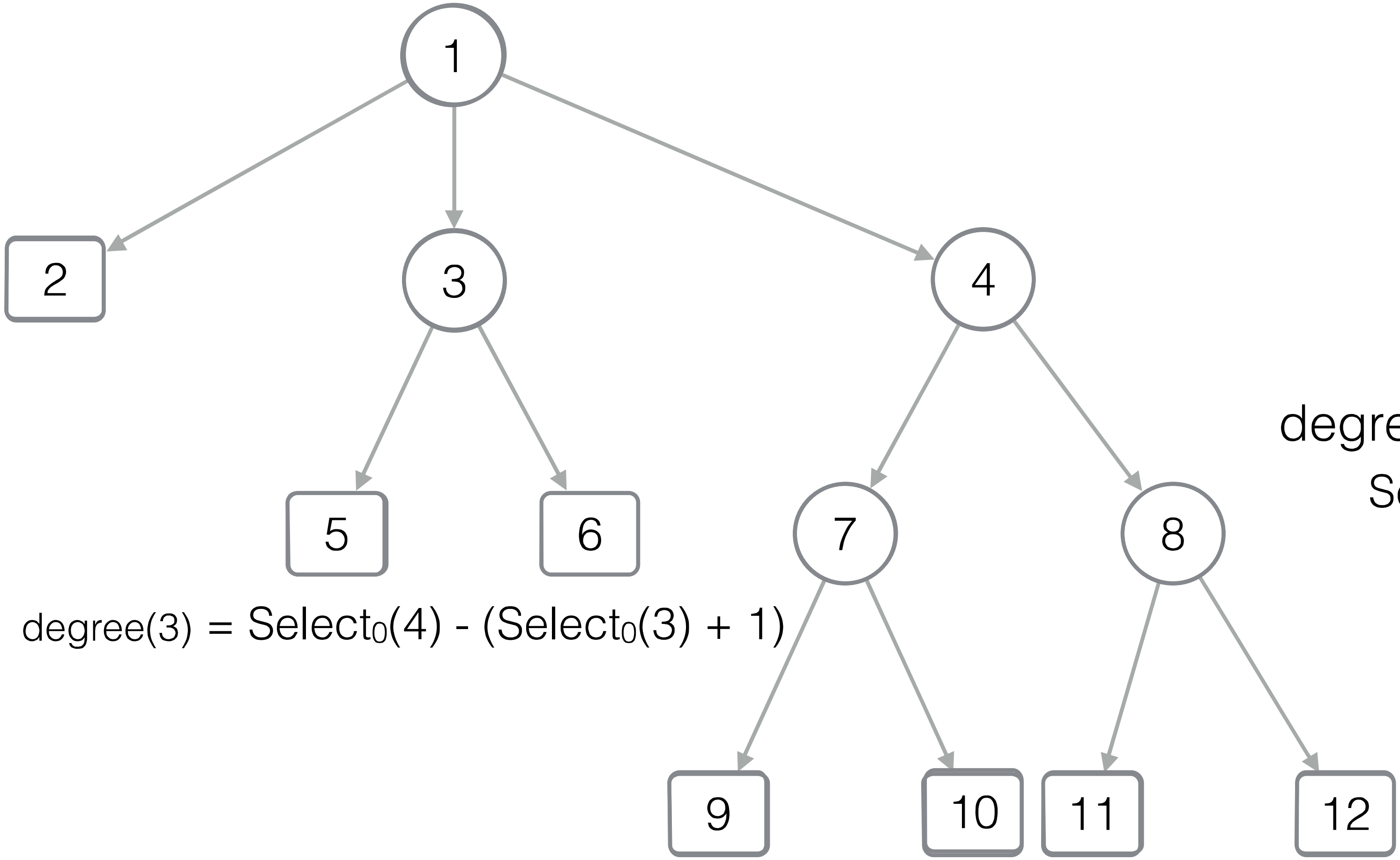
return -1 // is a leaf

else

return y-x // $\mathrm{Rank}_1(y)$

degree(x) = ?

$\mathrm{Select}_0(x+1) - (\mathrm{Select}_0(x) + 1)$



degree(3) = $\mathrm{Select}_0(4) - (\mathrm{Select}_0(3) + 1)$

$y = \mathrm{Select}_0(3)+1=8$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

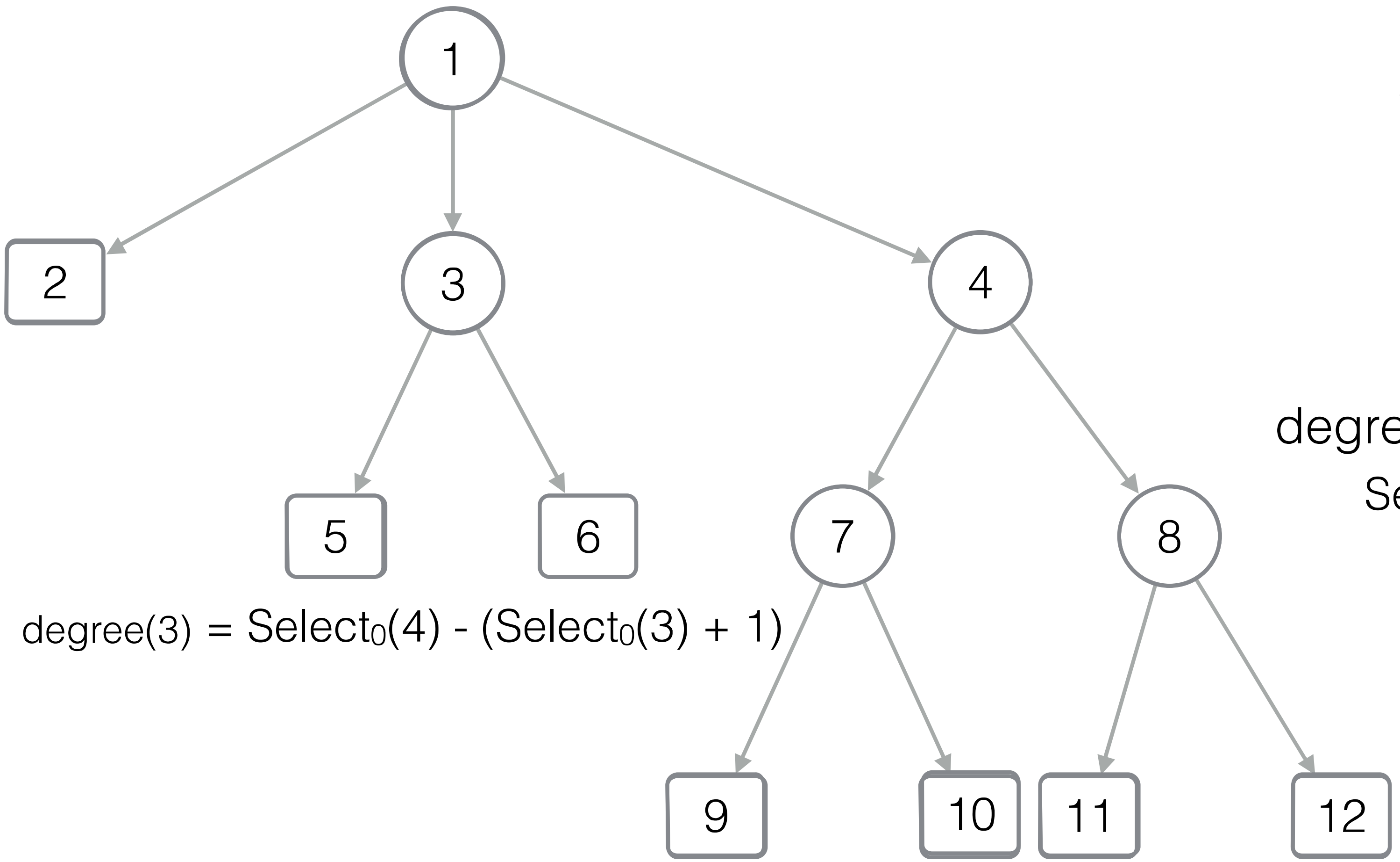
return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$



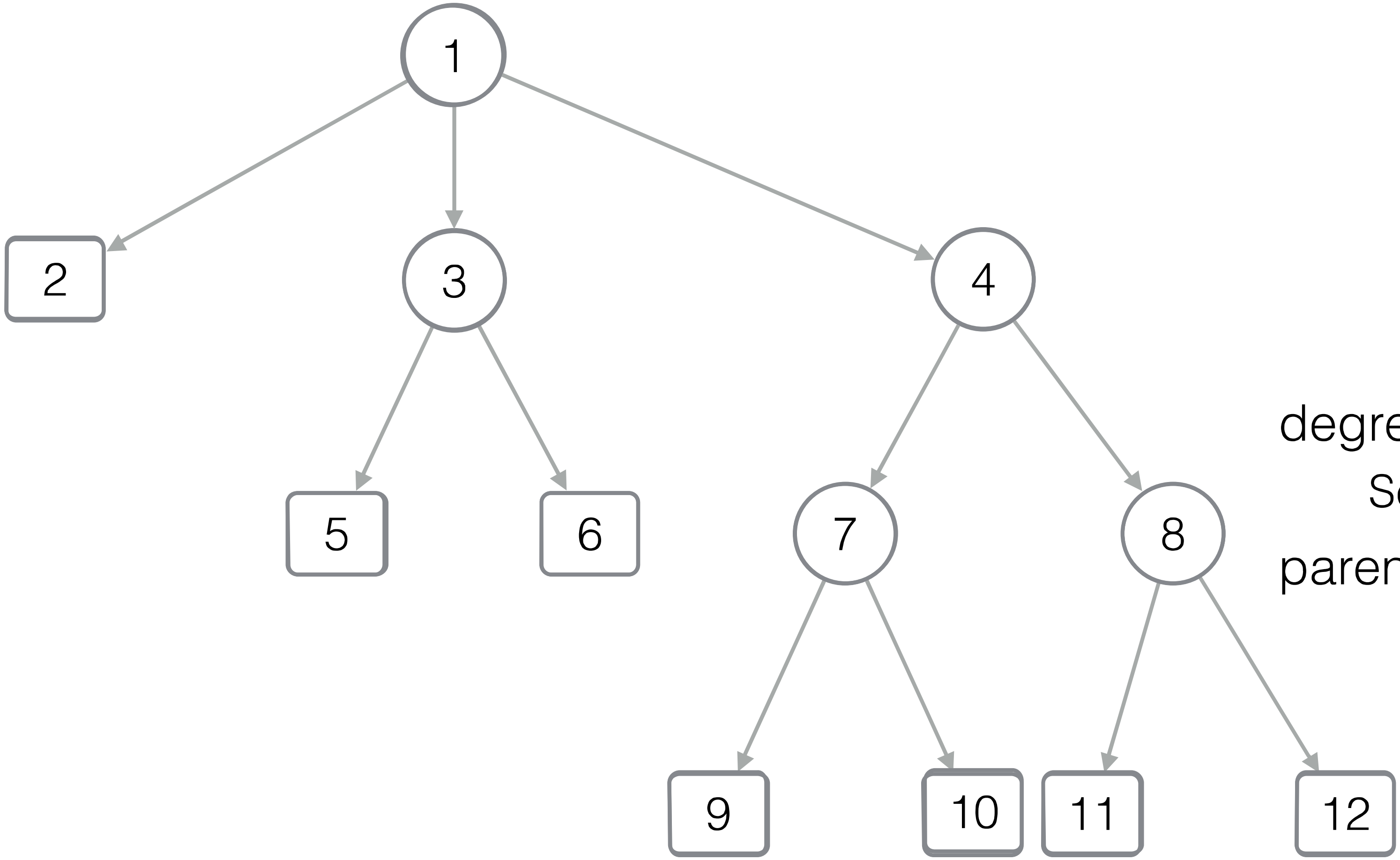
$y = Select_0(3)+1=8$ $Select_0(4)=10$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

firstChild(x) = ?

$y = Select_0(x)+1$

// start of x's children in B

if B[y] == 0

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

degree(x) = ?

$Select_0(x+1) - (Select_0(x) + 1)$

parent(x) =

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

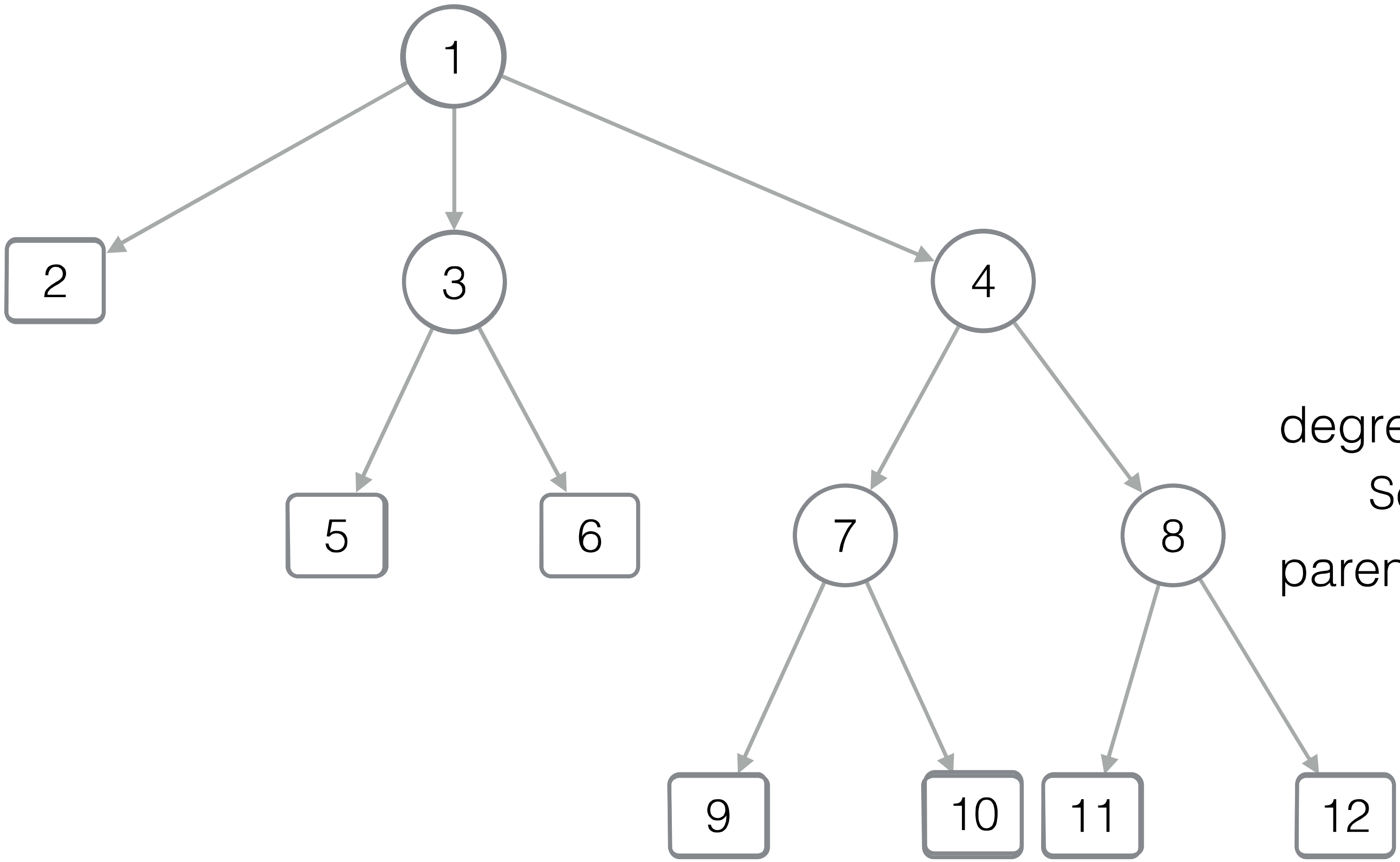
else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$



B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]

$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

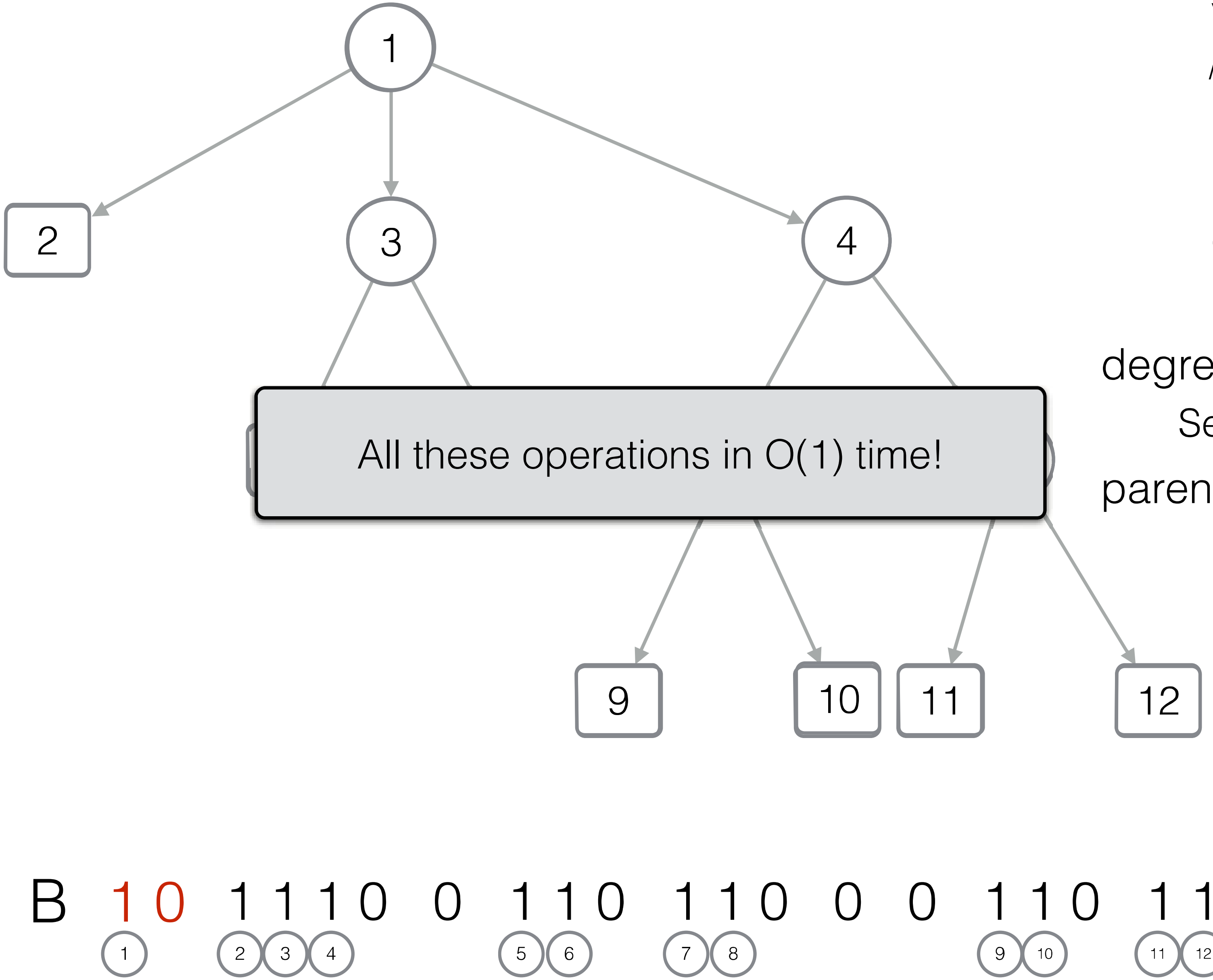
else

return y-x // $Rank_1(y)$

$degree(x) = ?$

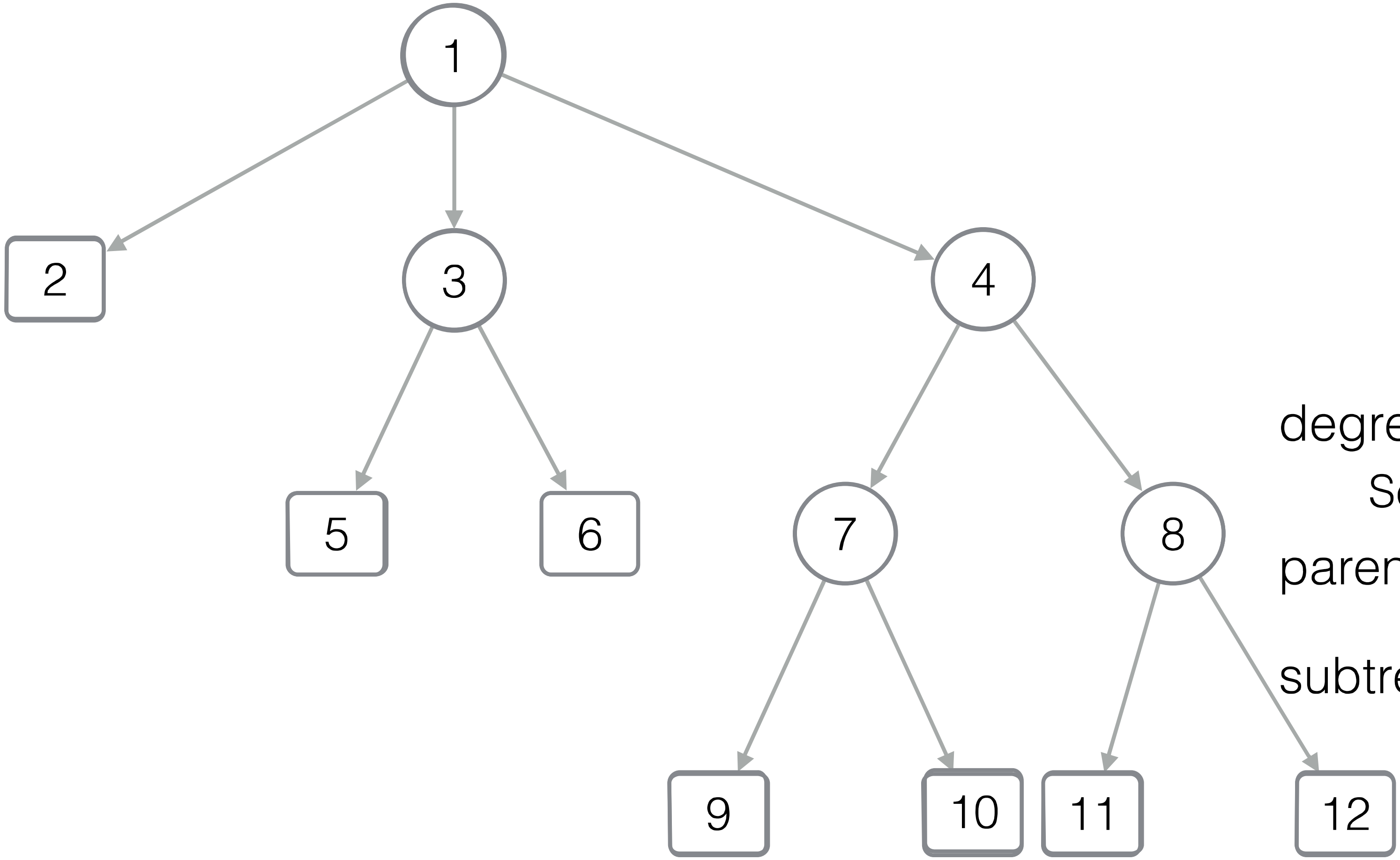
$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$



Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

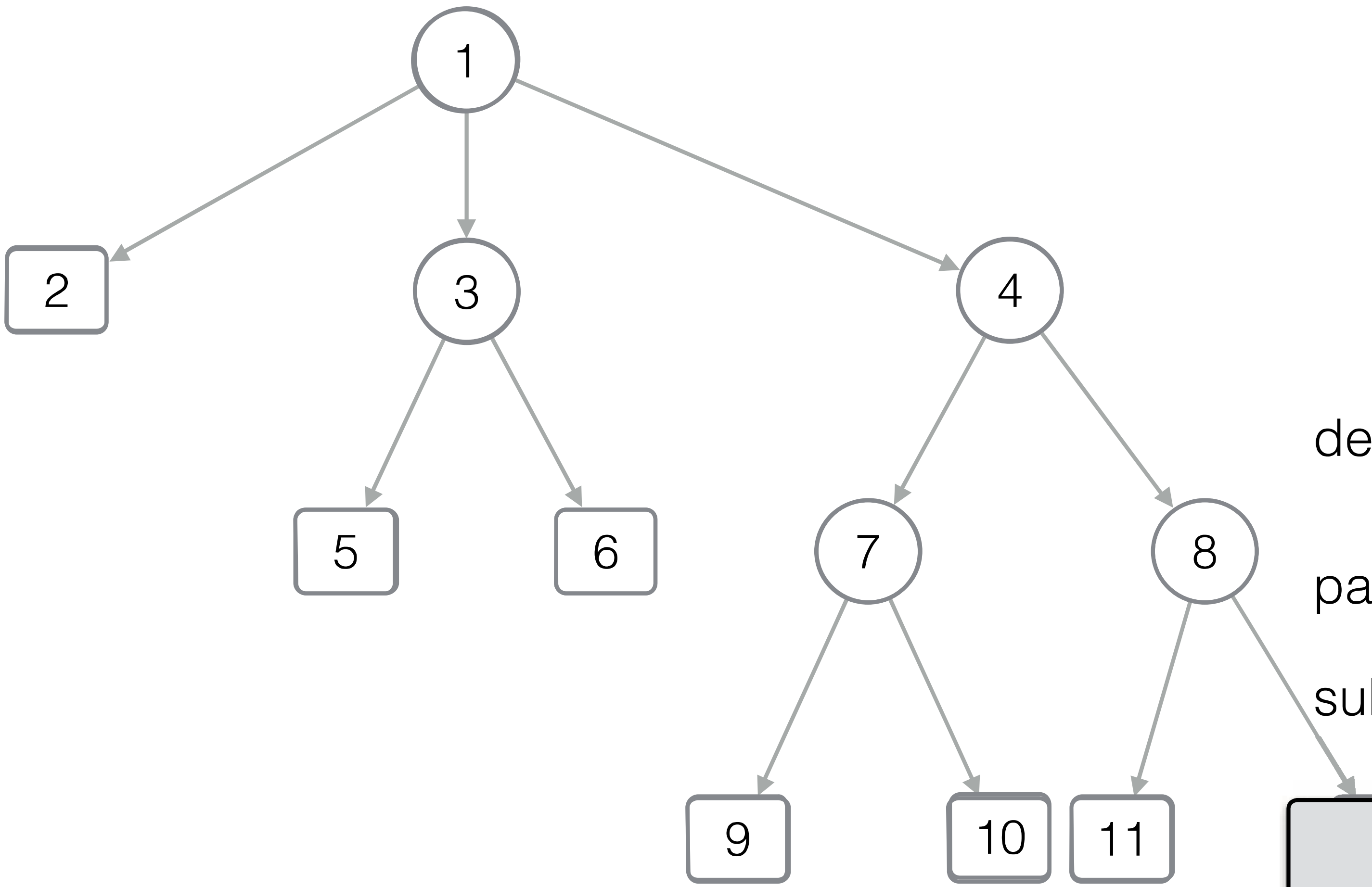
$subtreeSize(x) = ?$

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Succinct representation of trees (1)

[LOUDS - Level-order unary degree sequence]



$pos(x) = Select_1(x)$

$firstChild(x) = ?$

$y = Select_0(x)+1$

// start of x's children in B

if $B[y] == 0$

return -1 // is a leaf

else

return y-x // $Rank_1(y)$

$degree(x) = ?$

$Select_0(x+1) - (Select_0(x) + 1)$

$parent(x) = Rank_0(pos(x))$

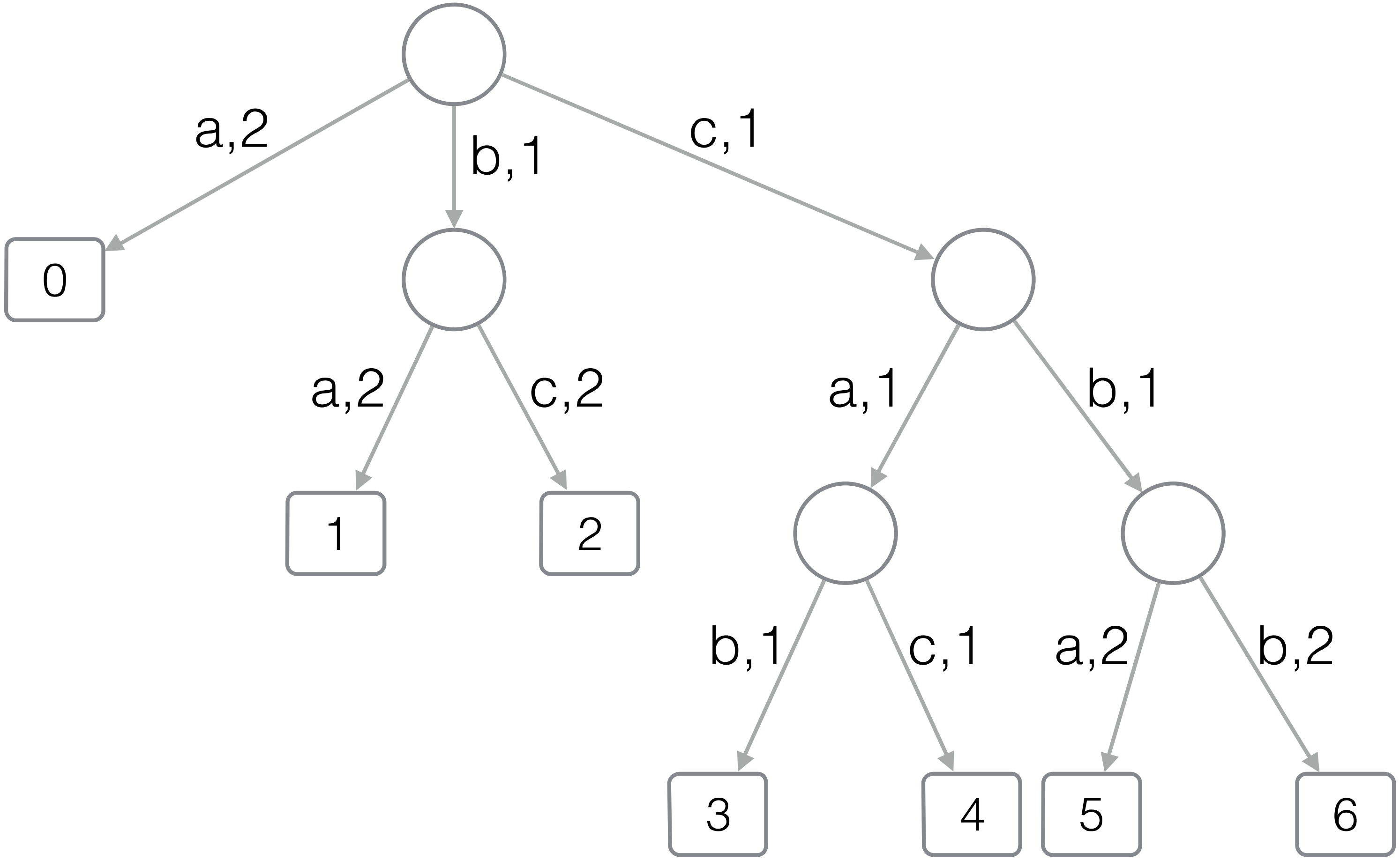
$subtreeSize(x) = ?$

Not efficient!
Nodes of the subtree are spread in B

B 1 0 1 1 1 0 0 1 1 0 1 1 0 0 0 1 1 0 1 1 0 0 0 0 0

1 2 3 4 5 6 7 8 9 10 11 12

Patricia trie



We store: The bit vector B (2n+1 bits),
its Rank/Select data structure (o(n) bits),
the n char labels (O(n log |Alphabet|) bits)
the n int-labels spread over [1,m], using Elias-Fano in n log m/n + 2n bits

Total space is (m+n) log |Alphabet| + n log m/n + O(n) bits
THUS
avoiding the term n log m bits

D = { ab, bab, bca, cab, cac, cbac, cbba }

n = |D|, m total length of strings in D