Spectral ranking algorithms for scientific publications

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Distribution of grants by governmental agencies and universities.

Very delicate problem!!

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  The code should be Open
- Algorithmic efficiency.

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We can represent the citation process as a graph and hence as a binary matrix

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“The only thing worse than being talked about is not being talked about.” [Oscar Wilde]

“I don’t care what you say about me, as long as you say something about me, and as long as you spell my name right.” [George Cohan]

“Don’t pay any attention to what they write about you. Just measure it in inches.” [Andy Warhol]

We can order the matrix by publication year.

$$C = C(y_i, y_j) = \begin{bmatrix} C_{y_1 y_1} & & O \\ C_{y_2, y_1} & C_{y_2 y_2} & \\ \vdots & \ddots & \\ C_{y_k y 1} & \cdots & C_{y_k y_k} \end{bmatrix}$$

Block C_{y_i, y_j} represents the citations of papers published on year y_i to papers published in year y_j

C is (nearly)-block-lower triangular.

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By grouping the papers published by the same journal we can transform the Article citation matrix into a Journal Citation matrix.

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$$P_J(i, j) = 1 \text{ iff paper } p_i \text{ is published on journal } j,$$

Define the Journal Citation Matrix as

$$J_J = P_J^T \, C \, P_J.$$

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Define an annual cross-citation matrix

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Impact Factor

The ISI IF defines the status of a journal for a specific year.

It is defined as the mean number of citations that occurred in the considered year y to articles published in a given journal j during the previous two years.

$$IF(j,y) = \frac{\sum_k (Z_{kj}(y,y-1) + Z_{kj}(y,y-2))}{n(j,y)},$$

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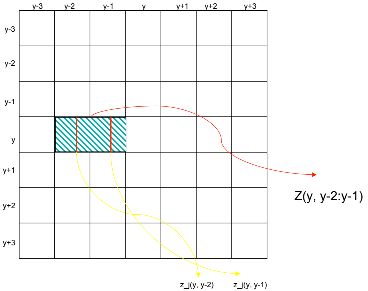

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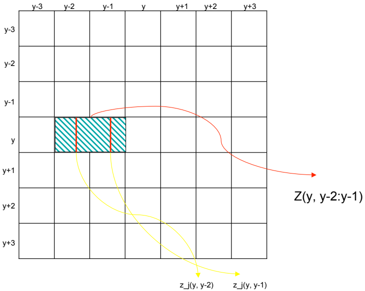

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- The ISI IF is a metric of popularity
- It changes over the time
- It disregards concepts as prestige, reputation, influence or quality.
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Assiomatic Approach

Ignacio Palacios-Huerta and Oscar Volij in *Econometrica* (2004) proposed an assiomatic approach to measure the influence of scholar journals.

A ranking problem is a pair $\langle \mathcal{S}, J_J \rangle$ where $\mathcal{S}$ is the set of journals and J_J is a Journal Citation Matrix within a single discipline.

A ranking method is a function $\Phi : \mathcal{R} \to \Delta$, where $\mathcal{R}$ is the set of all ranking problems, and $\Delta = \left\{ v_j : j \in \mathcal{S}, v_j \geq 0, \sum_{j \in \mathcal{S}} v_j = 1 \right\}$.

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$\Phi(\mathcal{S}, \Lambda J_J) = \Phi(\mathcal{S}, J_J)$, for every non-negative diagonal matrix Λ.

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Φ satisfies homogeneity for two-journal problem if there is $\alpha > 0$, such that for all such problems

$$\frac{\Phi_r(R)}{\Phi_s(R)} = \alpha \frac{J_J(s, r)}{J_J(r, s)}.$$

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$$\frac{\Phi_i(R)}{\Phi_j(R)} = \frac{\Phi_i(R \setminus \{k\})}{\Phi_j(R \setminus \{k\})} \quad \text{for all } i, j \in \mathcal{S} \setminus \{k\}$$

Assiomatic Approach... continue

Theorem:

There is a unique ranking function that satisfies the four properties described, and this is the so called Invariant Method, i.e.

$$\Phi(R) = \mathbf{v} \in \Delta, \quad \text{where } \mathbf{v}^{\mathbf{T}}\mathbf{D}\mathbf{J}_{\mathbf{J}} = \mathbf{v}^{\mathbf{T}},$$

and D is a diagonal matrix so that J_J becomes row-stochastic.

Some Comments:

- Very similar to PageRank, but without dumping factor
- Every journal cites at least another journal.
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They underline that

- The Impact Factor is a metric of popularity
- Weighted PageRank is a metric of prestige

By putting a threshold on the Weighted Page-Rank and on the ISI IF, one can identify Popular Journals versus Prestigious Journals.

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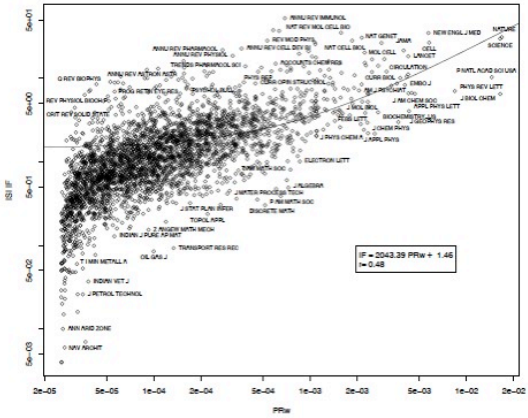

NATURE
SCIENCE
CELL
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JAMA
NEW ENGL J MED
IF = 2043.39 PRw + 1.46
r= 0.48
2e–05
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$$Y(j) = IF(j) \times PR_w(j).$$

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The Eigenfactor Method uses the Page-Rank approach for ranking journals.

It considers a d-year cross-citation matrix for year y as follows:

$$M(y,d) = \sum_{k=1}^{d} Z(y-k, y),$$

where

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where $A = \mathbf{a}\mathbf{e}^T$, and a_j=article in journal j/(total articles).

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Mutual reinforcement between papers, journals, authors

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Many matrices play a role in our problem:

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The impact of co-authorship should be marginal respect to that of papers

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The matrix

We come up with the following block-matrix

$$M = \begin{bmatrix} J_J & J_A & J_P \\ A_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}$$

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The matrices in the first column contribute to the ranking of journals

The matrix

We come up with the following block-matrix

$$M = \begin{bmatrix} J_J & J_A & J_P \\ A_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}$$

The matrices in the second column contribute to the ranking of authors

The matrix

We come up with the following block-matrix

$$M = \begin{bmatrix} J_J & J_A & J_P \\ A_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}$$

The matrices in the third column contribute to the ranking of papers

The matrix

The matrices involved for the journals rank

$$M = \begin{bmatrix} P_J^T P_P P_J & J_A & J_P \\ P_A^T P_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}$$

The matrix

The matrices involved for the journals rank

$$M = \begin{bmatrix} P_J^T P_P P_J & J_A & J_P \\ P_A^T P_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix}$$

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Let $\pi = [\pi_J, \pi_A, \pi_P]$ be the vector of the ranking scores of journals, authors and papers.

We can compute π with an iterative scheme using a matrix "derived" from M.

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We can compute π with an iterative scheme using a matrix "derived" from M.

What does "derived" mean?

The matrix M

We have to weight M in order to emphasize the role of some matrices rather than others in the computation of the ranks.

$$M(w) = \begin{bmatrix} w(1,1)\,J_J & w(1,2)\,J_A & w(1,3)\,J_P \\ w(2,1)\,A_J & w(2,2)\,A_A & w(2,3)\,A_P \\ w(3,1)\,P_J & w(3,2)\,P_A & w(3,2)\,P_P \end{bmatrix}$$

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$$\begin{aligned} \pi_J^{(k+1)} &= \pi_J^{(k)} w(1,1)\,J_J + \pi_A^{(k)} w(2,1)\,A_J + \pi_P^{(k)} w(3,1)\,P_J \\ \pi_A^{(k+1)} &= \pi_J^{(k)} w(1,2)\,J_A + \pi_A^{(k)} w(2,2)\,A_A + \pi_P^{(k)} w(3,2)\,P_A \\ \pi_P^{(k+1)} &= \pi_J^{(k)} w(1,3)\,J_P + \pi_A^{(k)} w(2,3)\,A_P + \pi_P^{(k)} w(3,3)\,P_P \end{aligned}$$

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π is the eigenvector corresponding to the dominant eigenvalue of M.

$$\pi^T M = \lambda \pi^T$$

The matrix M is non-negative, but not necessarily irreducible or primitive.

Perron-Frobenius guarantees only that there exist a $\lambda = \rho(M)$ and a corresponding non-negative eigenvector.

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This model has two major problems

- We have to add semantic to the model

  Some of the matrices should be normalized by row others by column

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We introduce some normalization.

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Some proposals:

- Subtract the diagonal to the diagonal blocks J_J, A_A to avoid self-citations
- Invariance with respect to then number of authors

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Fixing the math: a first possibility

- Make every block row-stochastic, and choose the matrix of the weights w also row-stochastic.
- The matrix $\widetilde{M}(w)$ becomes then globally row-stochastic.
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- We do not force $M(w)$ to be stochastic, but we remain more "close" to the intuition behind the model

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- M is not necessarily irreducible or primitive.
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A possible solution

- To force M to be irreducible and primitive consider the matrix

$$\widehat{M} = \begin{bmatrix} M & \varepsilon\,\mathbf{e} \\ \varepsilon\,\mathbf{e}^T & \varepsilon \end{bmatrix}$$

- $\widehat{M}$ is irreducible and primitive
- Under suitable hypothesis

$$\mathbf{y}(\varepsilon) = \hat{\pi} + \varepsilon\,\frac{\|\pi\|_1}{\lambda_1}\,\mathbf{e}_n + O\left(\varepsilon^2\right)$$

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These methods are not time aware!

- Newly published papers do not have yet received enough citations
- Their rank is destined to be low
- The same for junior researchers whose rank remains lower than that of senior researchers

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A proposal for mostly recent cited papers

If we do not require the matrix to be row stochastic, we can scale the citation matrix P_P with an exponential decay function.

The importance of a paper decays over the time

A paper not cited recently looses its importance

An old paper that is cited recently increases its importance

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Experimental results

- We apply our ideas to a set of more than 300.000 papers, 120.000 authors and 3500 math. journals.
- The references have been crawled from the AMS MathSciNet database, starting from the 2007 indexed papers.
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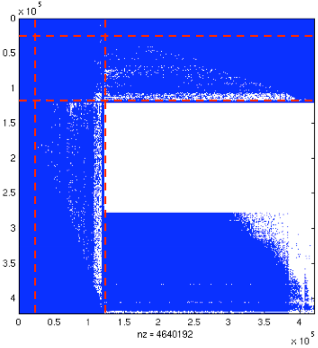

Top Journals

Journal	num. cit	num. pap.	IF
Trans. AMS	22796	5247	0.820
Inventiones Mathematicae	21181	2481	1.659
Annals of Mathematics	19365	2193	2.426
Proc. AMS	16722	6045	0.513
Comm. Math. Phys	21997	3748	2.077
J. Algebra	15059	4457	0.568
Duke Math. J.	11939	2161	1.409
Mathematische Annalen	11248	2670	0.902
J. Functional Analysis	13778	2437	0.866
Comm. on Pure Appl. Math.	12111	1227	2.031

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Top authors - time aware method

Author	num. cit	num. pap.
Lions, Pierre-Louis (FM)	2641	199
Erdös, Paul	1358	377
Bourgain, Jean (FM)	1019	156
Simon, Barry	1502	198
Shelah, Saharonh	972	333
Brezis, Haïm	1698	127
Lustzig, George	1145	87
Caffarelli, Luis	1288	131
Yau, Shing Tung (FM)	1571	136
Connes, Alain (FM)	1114	79
Arnold, Vladimir	551	90

Top papers

paper	pos.	cit.
Crandall, Ishii, Lions, P. L. **Bull. AMS** (1992)	2	221
Gidas, Ni, Nirenberg, **Comm. Math. Phys.** (1979)	1	256
Saad, Schultz, **SISC** (1986)	3	197
Brézis, Nirenberg, **Comm. Pure Appl. Math** (1983)	4	187
Kazhdan, Lusztig, **Invent. Math.** (1979)	11	136
Ambrosetti, Rabinowitz, **J. Func. Anal.** (1973)	5	170
Bar-Natan, **Topology** (1995)	18	128
Hironaka, **Ann. of Math.** (1964)	19	127
Simon, **Ann. Mat. Pura Appl.** (1987)	10	138
Kontsevich, Manin, **Comm. Math. Phys.** (1994)	61	98

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paper	pos.	cit.
Crandall, Ishii, Lions, P. L. **Bull. AMS** (1992)	2	221
Kauffman, Louis **European J. Combin** (1999)	310	60
Ambrosetti, Rabinowitz, **J. Func. Anal.** (1973)	5	170
Khovanov, Mikhail, **Duke Math. J.** (2000)	1355	34
Brézis, Nirenberg, **Comm. Pure Appl. Math** (1983)	4	187
Gidas, Ni, Nirenberg, **Comm. Math. Phys.** (1979)	1	256
Aronszajn, **Trans. AMS** (1950)	82	89
Culler, Gordon, Luecke, Shalen **Ann. of Math.**(1987)	43	105
Simon, **Ann. Mat. Pura Appl.** (1987)	10	138
Jones, **Invent. Math** (1983)	7	146

Top papers - time aware method

paper	pos.	cit.
Kirkpatrick, Gelatt, Vecchi- Simulated Annealing	2	1337
R. Bryant - BDD	1	1636
Rivest, Shamir, Adleman - Public Key criptography	3	1218
Geusebroek, Smeulders, van de Weijer (gauss filtering)	10	834
Sally Floyd, Van Jacobson (TCP/IP)	4	1125
Diffie, Hellman- Cryptography	31	553
Ousterhout - Tcl and the Tk Toolkit	8	913
Harel - Complex Systems	6	1042
Elman- Neural Networks	26	589
Jones - VDM	23	609

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