Spectral ranking algorithms for scientific publications

Gianna M. Del Corso
joint work with Dario A. Bini and Francesco Romani

Dipartimento di Informatica, Università di Pisa, Italy
The Problem

Research evaluation is a very hot topic.

Distribution of grants by governmental agencies and universities.

Very delicate problem!!
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- **Equity.** The evaluation parameters have to be equal for everyone
- Being Transparent. The evaluation parameters have to be public and well known a priori.
- The code should be Open
- Algorithmic efficiency.
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Notation

**Citations** are the basis of most attempts to assess scholarly impact.

We can represent the citation process as a graph and hence as a binary matrix

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“The only thing worse than being talked about is not being talked about.” [Oscar Wilde]

“I don’t care what you say about me, as long as you say something about me, and as long as you spell my name right.” [George Cohan]

“Don’t pay any attention to what they write about you. Just measure it in inches.” [Andy Warhol]
We can order the matrix by publication year.

\[ C = C(y_i, y_j) = \begin{bmatrix}
  C_{y_1y_1} & C_{y_2y_1} & \cdots & O \\
  C_{y_2y_1} & C_{y_2y_2} & \cdots & \\
  \vdots & \vdots & \ddots & \\
  C_{y_ky_1} & \cdots & C_{y_ky_k}
\end{bmatrix} \]

Block \( C_{y_i,y_j} \) represents the citations of papers published on year \( y_i \) to papers published in year \( y_j \).

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By grouping the papers published by the same journal we can transform the Article citation matrix into a Journal Citation matrix.

Let

\[ P_J(i, j) = 1 \text{ iff paper } p_i \text{ is published on journal } j, \]

Define the Journal Citation Matrix as

\[ J_J = P_J^T C P_J. \]

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The Journal Citation matrix considers cross-citations between journals over the years.

Define an annual cross-citation matrix

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We can construct \( Z \) from the blocks of \( C(y_i, y_j) \)

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Impact Factor

The ISI IF defines the status of a journal for a specific year.

It is defined as the mean number of citations that occurred in the considered year $y$ to articles published in a given journal $j$ during the previous two years.

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IF(j, y) = \frac{\sum_k (Z_{kj}(y, y - 1) + Z_{kj}(y, y - 2))}{n(j, y)},
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The ISI IF is a metric of popularity.

- It changes over the time.
- It disregards concepts as prestige, reputation, influence or quality.
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Invariant Method

Assiomatic Approach


A ranking problem is a pair $\langle S, J_J \rangle$ where $S$ is the set of journals and $J_J$ is a Journal Citation Matrix within a single discipline.

A ranking method is a function $\Phi : \mathcal{R} \rightarrow \Delta$, where $\mathcal{R}$ is the set of all ranking problems, and $\Delta = \left\{ v_j : j \in S, v_j \geq 0, \sum_{j \in S} v_j = 1 \right\}$.

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They *derive* a ranking method by requiring a few simple properties:

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\[ \Phi(S, \Lambda J_J) = \Phi(S, J_J), \text{ for every non-negative diagonal matrix } \Lambda. \]
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Assiomatic Approach... continue

- Homogeneity for the two-journal problem: If two journals have the same number of cited references, the relative valuation of a journal should be proportional to the ratio of their mutual citations.
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Let \( R = \langle \{r, s\}, J_J \rangle \) be a two-journal problem such that

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Φ satisfies homogeneity for two-journal problem if there is $\alpha > 0$, such that for all such problems

$$\frac{\Phi_r(R)}{\Phi_s(R)} = \alpha \frac{J_J(s, r)}{J_J(r, s)}.$$
Consistency: If we know how to rank a small problem, we should be able to extend the ranking method to a big problem in a consistent way.
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- **Consistency**: If we know how to rank a small problem, we should be able to extend the ranking method to a big problem in a consistent way.

\[
\frac{\Phi_i(R)}{\Phi_j(R)} = \frac{\Phi_i(R \setminus \{k\})}{\Phi_j(R \setminus \{k\})} \quad \text{for all } i, j \in S \setminus \{k\}
\]
Invariant Method

Theorem:

There is a unique ranking function that satisfies the four properties described, and this is the so-called Invariant Method, i.e.

\[ \Phi(R) = v \in \Delta, \quad \text{where} \quad v^T D J J = v^T, \]

and \( D \) is a diagonal matrix so that \( J J \) becomes row-stochastic.
Some Comments:

- Very similar to PageRank, but without dumping factor
- Every journal cites at least another journal.
- We can apply the Invariant method for ranking papers and authors as well.
- It is static in the sense that the factor time is not present.
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They underline that
- The Impact Factor is a metric of popularity
- Weighted PageRank is a metric of prestige

By putting a threshold on the Weighted Page-Rank and on the ISI IF, one can identify Popular Journals versus Prestigious Journals.
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Ranking algorithms for scientific publications
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Prestigious Journals are journals that are not frequently cited, but their citations come from highly prestigious journals. They have very high Weighted Page-Rank and very low IF.

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Their proposal ...

\[ Y(j) = IF(j) \times PR_w(j). \]
The Eigenfactor Method uses the Page-Rank approach for ranking journals.

It considers a $d$-year cross-citation matrix for year $y$ as follows:

$$M(y, d) = \sum_{k=1}^{d} Z(y - k, y),$$

where

$Z_{kl}(y_i, y_j) = \text{Citations from journal } k \text{ in year } y_i \text{ to journal } l \text{ in year } y_j$

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and following Google’s PageRank approach,

$$P = \alpha N + (1 - \alpha)A,$$

where $A = ae^T$, and $a_j$ = article in journal $j$/(total articles).
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The leading eigenvector $\mathbf{f}$ of $P$ is the journal influence vector.

The eigenfactor $w_i$ of journal $i$ is the percentage of the total weighted citations that journal $i$ receives from the other journals.

$$w = \frac{100 \mathbf{Mf}}{\mathbf{e}^T \mathbf{Mf}}$$
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Desiderata

- A **paper** receives importance also from the **journal** in which it is published
- An important author gives importance to her co-authors

Mutual reinforcement between **papers**, **journals**, **authors**

Compare with the **Hubs and Authorities** approach
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The model

Many matrices play a role in our problem:

Let us rename the citation matrix as $P_P$

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A journal is important if:

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An author is important if:

- Has published important papers
- Has written on important journals
- Has written with important co-authors

The relative importance of authors, citations and papers for the attribution of a ranking score of an author depends on users selected weights.

The impact of co-authorship should be marginal respect to that of papers.
A paper is important if:

- Is cited by important papers
- Is published on important journals
- Is written by important authors

The relative importance of authors, citations and papers for the attribution of a ranking score of an author depends on users selected weights.

The impact of authors should be marginal respect to that of citations.
The model

Paper ranking

A paper is important if:

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- Is published on important journals
- Is written by important authors

The relative importance of authors, citations and papers for the attribution of a ranking score of an author depends on users selected weights.

The impact of authors should be marginal respect to that of citations.
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Paper ranking

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- Is published on important journals
- Is **written** by important **authors**

The **relative** importance of authors, citations and papers for the attribution of a ranking score of an author depends on users selected **weights**.

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The model

The matrix

We come up with the following block-matrix

\[ M = \begin{bmatrix} J_J & J_A & J_P \\ A_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix} \]
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\[
M = \begin{bmatrix}
    J_J & J_A & J_P \\
    A_J & A_A & A_P \\
    P_J & P_A & P_P \\
\end{bmatrix}
\]
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  P_J & P_A & P_P 
\end{bmatrix} \]

The matrices in the first column contribute to the ranking of journals.
We come up with the following block-matrix

\[
M = \begin{bmatrix}
J_J & J_A & J_P \\
A_J & A_A & A_P \\
P_J & P_A & P_P
\end{bmatrix}
\]

The matrices in the second column contribute to the ranking of authors.
The model

The matrix

We come up with the following block-matrix

\[ M = \begin{bmatrix}
  J_J & J_A & J_P \\
  A_J & A_A & A_P \\
  P_J & P_A & P_P \\
\end{bmatrix} \]

The matrices in the third column contribute to the ranking of papers.
The model

The matrix

The matrices involved for the journals rank

\[
M = \begin{bmatrix}
  P^T_J & P_P & P_J & J_A & J_P \\
  P^T_A & P_J & A_A & A_P \\
  P_J & P_A & P_P
\end{bmatrix}
\]
The model

The matrix

The matrices involved for the journals rank

\[ M = \begin{bmatrix} P_J^T & P_P & P_J & J_A & J_P \\ P_A^T & P_J & A_A & A_P \\ P_J & P_A & P_P \end{bmatrix} \]
The matrix

The matrices involved for the authors rank

\[
M = \begin{bmatrix}
    P_J^T & P_P & P_J & P_J^T & P_A & J_P \\
    P_A^T & P_J & P_A & P_A^T & P_A & A_P \\
    P_J & P_A & P_P \\
\end{bmatrix}
\]
The model

The matrix

The matrices involved for the papers rank

\[
M = \begin{bmatrix}
P_J^T & P_P & P_J & P_J^T & P_A & P_J^T \\
P_A^T & P_J & P_A & P_J^T & P_A & P_A^T \\
P_J & P_A & P_P & P_P & P_P & P_P \\
\end{bmatrix}
\]
The model

Let \( \pi = [\pi_J, \pi_A, \pi_P] \) be the vector of the ranking scores of journals, authors and papers.

We can compute \( \pi \) with an iterative scheme using a matrix “derived” from \( M \).
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D.A. Bini, G. M. Del Corso, F. Romani

Ranking algorithms for scientific publications
The model

Let \( \pi = [\pi_J, \pi_A, \pi_P] \) be the vector of the ranking scores of journals, authors and papers.

We can compute \( \pi \) with an iterative scheme using a matrix “derived” from \( M \).

What does “derived” mean?
The model

The matrix $M$

We have to weight $M$ in order to emphasize the role of some matrices rather than others in the computation of the ranks.

$$M(w) = \begin{bmatrix}
w(1, 1) J_J & w(1, 2) J_A & w(1, 3) J_P \\
w(2, 1) A_J & w(2, 2) A_A & w(2, 3) A_P \\
w(3, 1) P_J & w(3, 2) P_A & w(3, 2) P_P
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 w(3, 1) P_J & w(3, 2) P_A & w(3, 2) P_P \end{bmatrix}$$

$$\pi_{J}^{(k+1)} = \pi_{J}^{(k)} w(1, 1) J_J + \pi_{A}^{(k)} w(2, 1) A_J + \pi_{P}^{(k)} w(3, 1) P_J$$

$$\pi_{A}^{(k+1)} = \pi_{J}^{(k)} w(1, 2) J_A + \pi_{A}^{(k)} w(2, 2) A_A + \pi_{P}^{(k)} w(3, 2) P_A$$

$$\pi_{P}^{(k+1)} = \pi_{J}^{(k)} w(1, 3) J_P + \pi_{A}^{(k)} w(2, 3) A_P + \pi_{P}^{(k)} w(3, 3) P_P$$
The model

The matrix \( M \)

We have to weight \( M \) in order to emphasize the role of some matrices rather than others in the computation of the ranks.

\[
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    w(3, 1) P_J & w(3, 2) P_A & w(3, 2) P_P
\end{bmatrix}
\]

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\pi_J^{(k+1)} = \pi_J^{(k)} w(1, 1) J_J + \pi_A^{(k)} w(2, 1) A_J + \pi_P^{(k)} w(3, 1) P_J
\]

\[
\pi_A^{(k+1)} = \pi_J^{(k)} w(1, 2) J_A + \pi_A^{(k)} w(2, 2) A_A + \pi_P^{(k)} w(3, 2) P_A
\]

\[
\pi_P^{(k+1)} = \pi_J^{(k)} w(1, 3) J_P + \pi_A^{(k)} w(2, 3) A_P + \pi_P^{(k)} w(3, 3) P_P
\]
We have to weight $M$ in order to emphasize the role of some matrices rather than others in the computation of the ranks.

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$$\begin{align*}
\pi_J^{(k+1)} &= \pi_J^{(k)} w(1, 1) J_J + \pi_A^{(k)} w(2, 1) A_J + \pi_P^{(k)} w(3, 1) P_J \\
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\end{align*}$$
The model

The matrix $M$

$\pi$ is the eigenvector corresponding to the dominant eigenvalue of $M$.

$$\pi^T M = \lambda \pi^T$$

The matrix $M$ is non-negative, but not necessarily irreducible or primitive.

Perron-Frobenius guarantees only that there exist a $\lambda = \rho(M)$ and a corresponding non-negative eigenvector.
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The model

The matrix $M$

This model has two major problems

- We have to add *semantic* to the model

Some of the matrices should be normalized by row others by column

- We have to fix the math
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Adding Semantic

We introduce some normalization.

The normalization of some block or others depends on what one want to evaluate.
We introduce some normalization.

The normalization of some block or others depends on what one want to evaluate.
Some proposals:

- Subtract the diagonal to the diagonal blocks $J_j, A_A$ to avoid self-citations
- Invariance with respect to the number of authors

Blocks $A_J, A_P, P_J$, should be normalized by column.

- Invariance with respect to the length of the reference list
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Fixing the math: a first possibility

- Make every block row-stochastic, and choose the matrix of the weights $w$ also row-stochastic.
- The matrix $\tilde{M}(w)$ becomes then globally row-stochastic.
- We can compute $\pi$ as the PageRank vector of $\tilde{M}(w)$. 

D.A. Bini, G. M. Del Corso, F. Romani

Ranking algorithms for scientific publications
First model

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We do not force $M(w)$ to be stochastic, but we remain more “close” to the intuition behind the model.

Problems

- $M$ is not necessarily irreducible or primitive.
- In this case the power method will not converge, since there are $|\lambda_1| = |\lambda_2|$ but $\lambda_1 \neq \lambda_2$. 

Second model

Fixing the math: a second possibility

- We do not force $M(w)$ to be stochastic, but we remain more “close” to the intuition behind the model.

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Second model

A possible solution

To force $M$ to be irreducible and primitive consider the matrix

$$
\hat{M} = \begin{bmatrix}
M & \varepsilon \mathbf{e} \\
\varepsilon \mathbf{e}^T & \varepsilon
\end{bmatrix}
$$

$\hat{M}$ is irreducible and primitive

Under suitable hypothesis

$$
y(\varepsilon) = \hat{\pi} + \varepsilon \frac{\|\pi\|_1}{\lambda_1} \mathbf{e}_n + O(\varepsilon^2)
$$

The dominant eigenvector of $\hat{M}$ ranks well also the documents belonging to small components.
Second model

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D.A. Bini, G. M. Del Corso, F. Romani

Ranking algorithms for scientific publications
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These methods are not time aware!

- Newly published papers do not have yet received enough citations.
  Their rank is destined to be low.
- The same for junior researchers whose rank remains lower than that of senior researchers.
These methods are **not time aware!**

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Second model

A proposal for mostly recent cited papers

If we do not require the matrix to be row stochastic, we can scale the citation matrix $P_P$ with an exponential decay function.

The importance of a paper decays over the time.

A paper not cited recently looses its importance.

An old paper that is cited recently increases its importance.
If we do not require the matrix to be row stochastic, we can scale the citation matrix $P_p$ with an **exponential decay** function.

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Experimental results

- We apply our ideas to a set of more than 300,000 papers, 120,000 authors and 3500 math. journals.
- The references have been crawled from the AMS MathSciNet database, starting from the 2007 indexed papers.
- The database is incomplete. Only papers published after 1998 on specific journals have an expandable list of reference.
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The matrix

![Matrix Diagram]

$nz = 4640192$
## Top Journals

<table>
<thead>
<tr>
<th>Journal</th>
<th>num. cit</th>
<th>num. pap.</th>
<th>IF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trans. AMS</td>
<td>22796</td>
<td>5247</td>
<td>0.820</td>
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<tr>
<td>Inventiones Mathematicae</td>
<td>21181</td>
<td>2481</td>
<td>1.659</td>
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<tr>
<td>Annals of Mathematics</td>
<td>19365</td>
<td>2193</td>
<td>2.426</td>
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<tr>
<td>Proc. AMS</td>
<td>16722</td>
<td>6045</td>
<td>0.513</td>
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<tr>
<td>J. Algebra</td>
<td>15059</td>
<td>4457</td>
<td>0.568</td>
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<tr>
<td>Duke Math. J.</td>
<td>11939</td>
<td>2161</td>
<td>1.409</td>
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<tr>
<td>Mathematische Annalen</td>
<td>11248</td>
<td>2670</td>
<td>0.902</td>
</tr>
<tr>
<td>J. Functional Analysis</td>
<td>13778</td>
<td>2437</td>
<td>0.866</td>
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<tr>
<td>Comm. on Pure Appl. Math.</td>
<td>12111</td>
<td>1227</td>
<td>2.031</td>
</tr>
</tbody>
</table>

D.A. Bini, G. M. Del Corso, F. Romani

Ranking algorithms for scientific publications
These results are not in accordance with the IF

The rank is not the same of counting the citations received or the number of papers published
These results are not in accordance with the IF

The rank is not the same of counting the citations received or the number of papers published
### Top authors - time aware method

<table>
<thead>
<tr>
<th>Author</th>
<th>num. cit</th>
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</tr>
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<tbody>
<tr>
<td>Lions, Pierre-Louis (FM)</td>
<td>2641</td>
<td>199</td>
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<tr>
<td>Erdös, Paul</td>
<td>1358</td>
<td>377</td>
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<tr>
<td>Bourgain, Jean (FM)</td>
<td>1019</td>
<td>156</td>
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<tr>
<td>Simon, Barry</td>
<td>1502</td>
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<tr>
<td>Shelah, Saharonh</td>
<td>972</td>
<td>333</td>
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<td>Brezis, Haïm</td>
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<td>Lustzig, George</td>
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<tr>
<td>Caffarelli, Luis</td>
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<tr>
<td>Yau, Shing Tung (FM)</td>
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<tr>
<td>Connes, Alain (FM)</td>
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<tr>
<td>Arnold, Vladimir</td>
<td>551</td>
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</table>
# Top papers

<table>
<thead>
<tr>
<th>paper</th>
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<tr>
<td>Crandall, Ishii, Lions, P. L. <em>Bull. AMS</em> (1992)</td>
<td>2</td>
<td>221</td>
</tr>
<tr>
<td>Saad, Schultz, <em>SISC</em> (1986)</td>
<td>3</td>
<td>197</td>
</tr>
<tr>
<td>Ambrosetti, Rabinowitz, <em>J. Func. Anal.</em> (1973)</td>
<td>5</td>
<td>170</td>
</tr>
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<td>Hironaka, <em>Ann. of Math.</em> (1964)</td>
<td>19</td>
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<td>170</td>
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<tr>
<td>Aronszajn, <em>Trans. AMS</em> (1950)</td>
<td>82</td>
<td>89</td>
</tr>
<tr>
<td>Culler, Gordon, Luecke, Shalen <em>Ann. of Math.</em> (1987)</td>
<td>43</td>
<td>105</td>
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<tbody>
<tr>
<td>Kirkpatrick, Gelatt, Vecchi - Simulated Annealing</td>
<td>2</td>
<td>1337</td>
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<tr>
<td>R. Bryant - BDD</td>
<td>1</td>
<td>1636</td>
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<tr>
<td>Rivest, Shamir, Adleman - Public Key criptography</td>
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Conclusions

- We introduced a flexible model
- The nice matrix structure is a potential source of algorithmic and theoretical problems
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Thank you!