Optimal nine node Diffusion for special torus graphs

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Abstract. The local Extrapolated Diffusion (EDF) method was studied in [5] for a torus network using communication among 4 adjacent neighbors. In the present paper we study the performance of the same method with the extension that we add four additional edges to the central node in position (i,j) of the torus network. We develop the EDF method for a 8 -regular torus network. The method uses two sets of parameters and for each node in order to increase its rate of convergence. The conventional way to analyze the convergence of the Diffusion method is to use matrix analysis. This approach depends heavily upon the property of the Laplacian matrix being circulant. However, the Laplacian matrix of our method does not have this property. To circumvent this problem we use Fourier analysis to determine optimum values for the set of parameters via a closed form formulae resulting in the maximization of its rate of convergence. It is shown that the optimum value of the convergence factor depends only upon the dimensions of the torus. Moreover, by keeping fixed the one dimension and increasing the other dimension of the torus the rate of convergence of the EDF method with 8 adjacent neighbors is also increased compared to the EDF method with 4 adjacent neighbors.

keywords Laplacian matrix, load balancing, regular graphs, iterative diffusion, Fourier analysis.

1 Introduction

The performance of a balancing algorithm can be measured in terms of number of iterations it requires to reach a balanced state and in terms of the amount of load moved over the edges of the underlying processor graph. In the Diffusion (DF) method [2], [3] a processor simultaneously sends workload to its neighbors with lighter workload and receives from its neighbors with heavier workload. More specifically DF is given by the following iterative scheme

$$u_i^{(n+1)} = u_i^{(n)} - \sum_{j \in N(i)} c_{ij}(u_i^{(n)} - u_j^{(n)}), \quad n = 0, 1, 2, \ldots, \tag{1}$$