301AA - Advanced Programming

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AP-2018-21: Constructor Classes and Monads in Haskell
Summary

- Type Constructor Classes
- **Functor** and **fmap**
- Towards monads: **Maybe** and partial functions
- Monads as contained and as computations
- Introducing side effects with the IO monad
- Control structures on monads
Type Constructor Classes

- **Type Classes** are predicates over *types*
- **[Type] Constructor Classes** are predicates over *type constructors*
- Allow to define overloaded functions common to several *type constructors*
- Example: *map* function useful on many Haskell types
  - Lists:

```
map:: (a -> b) -> [a] -> [b]
map f [] = []
map f (x:xs) = f x : map f xs
```

> map (\x->x+1) [1,2,4]
[2,3,5]
More examples of map function

```haskell
data Tree a = Leaf a | Node(Tree a, Tree a)
  deriving Show

mapTree :: (a -> b) -> Tree a -> Tree b
mapTree f (Leaf x) = Leaf (f x)
mapTree f (Node(l,r)) = Node (mapTree f l, mapTree f r)

> t1 = Node(Node(Leaf 3, Leaf 4), Leaf 5)
> mapTree (\x->x+1) t1
Node (Node (Leaf 4,Leaf 5),Leaf 6)
```

```haskell
data Maybe a = Nothing | Just a
  deriving Show

mapMaybe :: (a -> b) -> Maybe a -> Maybe b
mapMaybe f Nothing = Nothing
mapMaybe f (Just x) = Just (f x)

> o1 = Just 10
> mapMaybe (\x->x+1) o1
Just 11
```
Constructor Classes

• All map functions share the same structure

map :: (a -> b) -> [a] -> [b]
mapTree :: (a -> b) -> Tree a -> Tree b
mapMaybe :: (a -> b) -> Maybe a -> Maybe b

• They can all be written as:

fmap :: (a -> b) -> g a -> g b

  where g is:

  [-] for lists, Tree for trees, and Maybe for options

• Note that g is a function from types to types, i.e. a type constructor
Constructor Classes

• This pattern can be captured in a constructor class **Functor**:

  ```haskell
  class Functor g where
  fmap :: (a -> b) -> g a -> g b
  ```

• Simply a **type class** where the predicate is over a **type constructors** rather than on a **type**

• Compare with the definition of a standard type class:

  ```haskell
  class Eq a where
  (==) :: a -> a -> Bool
  ```
The **Functor** constructor class and some instances

```haskell
class Functor f where
    fmap :: (a -> b) -> f a -> f b

instance Functor [] where
    fmap f [] = []
    fmap f (x:xs) = f x : fmap f xs

instance Functor Tree where
    fmap f (Leaf x) = Leaf (f x)
    fmap f (Node(t1,t2)) = Node(fmap f t1, fmap f t2)

instance Functor Maybe where
    fmap f (Just s) = Just(f s)
    fmap f Nothing = Nothing
```
The **Functor** constructor class and some instances (2)

- Or by reusing the definitions `map`, `mapTree`, and `mapMaybe`:

```haskell
class Functor f where
  fmap :: (a -> b) -> f a -> f b

instance Functor [] where
  fmap = map

instance Functor Tree where
  fmap = mapTree

instance Functor Maybe where
  fmap = mapMaybe
```
Constructor Classes

- We can then use the **overloaded** symbol `fmap` to map over all three kinds of data structures:

```hs
*Main> fmap (\x->x+1) [1,2,3]
[2,3,4]
it :: [Integer]

*Main> fmap (\x->x+1) (Node(Leaf 1, Leaf 2))
Node (Leaf 2,Leaf 3)
it :: Tree Integer

*Main> fmap (\x->x+1) (Just 1)
Just 2
it :: Maybe Integer
```

- The **Functor** constructor class is part of the standard Prelude for Haskell
Towards Monads:
The **Maybe** type constructor
Towards **Monads**

- Often type constructors can be thought of as defining “boxes” for values
- **Functors** with **fmap** allow to apply functions inside “boxes”
- **Monads** are constructor classes introducing operations for
  - Putting a value into a “box” (**return**)
  - Compose functions that return “boxed” values (**bind**)
The **Maybe** type constructor

- **Type constructor**: a generic type with one or more type variables
  
  ```haskell
data Maybe a = Nothing | Just a
```

- A value of type **Maybe a** is a possibly undefined value of type **a**

- A function  \( f :: a \rightarrow Maybe b \) is a partial function from **a** to **b**

  ```haskell
  max [] = Nothing
  max (x:xs) = Just
      (foldr (\y z \rightarrow if y > z then y else z) x xs)
  max :: Ord a => [a] \rightarrow Maybe a
  ```
father :: Person -> Maybe Person  -- partial function
mother :: Person -> Maybe Person  -- (lookup in a DB)

maternalGrandfather :: Person -> Maybe Person
maternalGrandfather p =
  case mother p of
    Nothing -> Nothing
    Just mom -> father mom  -- Nothing or a Person

bothGrandfathers :: Person -> Maybe (Person, Person)
bothGrandfathers p =
  case father p of
    Nothing -> Nothing
    Just dad ->
      case father dad of
        Nothing -> Nothing
        Just gf1 ->
          case mother p of
            Nothing -> Nothing
            Just mom ->
              case father mom of
                Nothing -> Nothing
                Just gf2 ->
                  Just (gf1, gf2)
                    -- found second grandfather

Composing partial functions

• We introduce a higher order operator to compose partial functions in order to “propagate” undefinedness automatically

\[ y >>= g = \text{case } y \text{ of } \]
\[ \quad \text{Nothing } \rightarrow \text{Nothing} \]
\[ \quad \text{Just } x \rightarrow g x \]

\((>>=) : \text{Maybe } a \rightarrow (a \rightarrow \text{Maybe } b) \rightarrow \text{Maybe } b\)

• The \textit{bind} operator will be part of the definition of a \textit{monad}. 
Use of **bind** of the **Maybe** monad to compose partial functions

```haskell
father :: Person -> Maybe Person  -- partial function
mother :: Person -> Maybe Person  -- (lookup in a DB)

maternalGrandfather :: Person -> Maybe Person
maternalGrandfather p =
  case mother p of
    Nothing -> Nothing
    Just mom -> father mom

(>>=)  :: Maybe a -> (a -> Maybe b) -> Maybe b
maternalGrandfather p = mother p >>= father

bothGrandfathers :: Person -> Maybe(Person, Person)
bothGrandfathers p =
  father p >>=
    (\dad -> father dad >>=
      (\gf1 -> mother p >>=
        (\mom -> father mom >>=
          (\gf2 -> return (gf1,gf2) ))))
```
The Monad type class and the Maybe monad

```
class Monad m where
  return :: a -> m a
  (>>=)  :: m a -> (a -> m b) -> m b  -- "bind"
  ... -- + something more
```

- \(m\) is a type constructor
- \(m a\) is the type of monadic values

```
instance Monad Maybe where
  return :: a -> Maybe a
  return x = Just x
  (>>=)  :: Maybe a -> (a -> Maybe b) -> Maybe b
  y >>= g  = case  y of
    Nothing -> Nothing
    Just x -> g x
```

- **bind (>>=)** shows how to “propagate” undefinedness
Alternative, imperative-style syntax: **do**

bothGrandfathers p =
father p >>=
  (\dad -> father dad >>=
       (\gf1 -> mother p >>=
           (\mom -> father mom >>=
               (\gf2 -> return (gf1,gf2) )))))

bothGrandfathers p = do {
  dad <- father p;
  gf1 <- father dad;
  mom <- mother p;
  gf2 <- father mom;
  return (gf1, gf2);
}

bothGrandfathers p = do {
  dad <- father p
  gf1 <- father dad
  mom <- mother p
  gf2 <- father mom
  return (gf1, gf2)
}

• **do** syntax is just syntactic sugar for **>>=**
### Some Haskell Monads

<table>
<thead>
<tr>
<th>Monad</th>
<th>Imperative semantics</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maybe</td>
<td>Exception (Anonymous)</td>
</tr>
<tr>
<td>Error</td>
<td>Exception (with error description)</td>
</tr>
<tr>
<td>State</td>
<td>Global state</td>
</tr>
<tr>
<td>IO</td>
<td>Input/output</td>
</tr>
<tr>
<td>[] (lists)</td>
<td>Non-determinism</td>
</tr>
<tr>
<td>Reader</td>
<td>Environment</td>
</tr>
<tr>
<td>Writer</td>
<td>Logger</td>
</tr>
</tbody>
</table>
Understanding Monads as containers

class Monad m where -- definition of Monad type class
    return :: a -> m a
    (>>=) :: m a -> (a -> m b) -> m b -- "bind"
    ...
    -- + something more + a few axioms

• The monadic constructor can be seen as a container: let’s see this for lists

• Getting bind from more basic operations

map :: (a -> b) -> [a] -> [b] -- seen. “fmap” for Functors

return :: a -> [a]  -- container with single element
return x = [x]

concat :: [[a]] -> [a] -- flattens two-level containers

Example: concat [[1,2],[],[4]] = [1,2,4]

(>>=) :: [a] -> (a -> [b]) -> [b]
xs >>= f = concat(map f xs)

Exercise: define map and concat using bind and return
Understanding Monads as computations

A value of type `m a` is a computation returning a value of type `a`.

For any value, there is a computation which “does nothing” and produces that result. This is given by function `return`.

Given two computations `x` and `y`, one can form the computation `x >>= y` which intuitively “runs” `x`, throws away its result, then runs `y` returning its result.

Given computation `x`, we can use its result to decide what to do next. Given `f: a -> m b`, computation `x >>= f` runs `x`, then applies `f` to its result, and runs the resulting computation.

Note that we can define `then` using `bind`:

\[ x >> y = x >>= (\_ \to y) \]
Understanding Monads as computations (2)

class Monad m where -- definition of Monad type class
    return :: a -> m a
    (>>=) :: m a -> (a -> m b) -> m b -- "bind"
    (>>)  :: m a -> m b -> m b       -- "then"
    ...

- return, bind and then define basic ways to compose computations
- They are used in Haskell libraries to define more complex composition operators and control structures (sequence, for-each loops, ...)
- If a type constructor defining a library of computations is monadic, one gets automatically benefit of such libraries

Example: MAYBE
- \( f : a \rightarrow \text{Maybe } b \) is a partial function
- bind applies a partial function to a possibly undefined value, propagating undefinedness

Example: LISTS
- \( f : a \rightarrow [b] \) is a non-deterministic function
- bind applies a non-deterministic function to a list of values, collecting all possible results

Example: Parsing, handling errors, IO, backtracking....
Contaminating Haskell with side effects: Towards the IO monad
Pros of Functional Programming

• Functional programming is beautiful:
  – Concise and powerful abstractions
    • higher-order functions, algebraic data types, parametric polymorphism, principled overloading, ...
  – Close correspondence with mathematics
    • Semantics of a code function is the mathematical function
    • Equational reasoning: if \( x = y \), then \( f \times \) = \( f \times \)  
    • Independence of order-of-evaluation (Confluence, aka Church-Rosser)

The compiler can choose the best sequential or parallel evaluation order.
Problems...

• But to be *useful*, a language must be able to manage “impure features”:
  – Input/Output
  – Imperative update
  – Error recovery (eg, timeout, divide by zero, etc.)
  – Foreign-language interfaces
  – Concurrency control

The whole point of a running a program is to interact with the external environment and affect it
The Direct Approach

• Just add imperative constructs “the usual way”
  – I/O via “functions” with side effects:

    \[
    \text{putchar } 'x' + \text{putchar } 'y'
    \]

  – Imperative operations via assignable reference cells:

    \[
    z = \text{ref } 0; z := !z + 1;
    f(z);
    w = !z  \quad \text{(* What is the value of w? *)}
    \]

  – Error recovery via exceptions
  – Foreign language procedures mapped to “functions”
  – Concurrency via operating system threads

• Can work \textit{if language determines evaluation order}
  – Ocaml, Standard ML are good examples of this approach
But what if we are “lazy”?

In a lazy functional language, like Haskell, the order of evaluation is undefined.

• Example:
  - Output depends upon the evaluation order of (+).

• Example:
  - Output depends on how list is used
  - If only used in `length ls`, nothing will be printed because `length` does not evaluate elements of list
Fundamental question

• Is it possible to add imperative features without changing the meaning of pure Haskell expressions?

• Yes! Exploiting the concept of monad
  – The IO monad defines monadic values which are called actions, and prescribes how to compose them sequentially
Monadic Input and Output
The IO Monad

Problem

A functional program defines a pure function, with no side effects

The whole point of running a program is to have some side effect

The term “side effect” itself is misleading
Before Monads

• Streams
  – Program sends stream of requests to OS, receives stream of responses

• Continuations
  – User supplies continuations to I/O routines to specify how to process results

• Haskell 1.0 Report adopted Stream model
  – Stream and Continuation models were discovered to be inter-definable
Stream Model: Basic Idea

- Move “side effects” outside of functional program
- Haskell `main :: String -> String`

**Wrapper Program, written in some other language**

- standard input location (file or stdin)
- Haskell main program
- standard output location (file or stdin)

- But what if you need to read more than one file? Or delete files? Or communicate over a socket? ...
Stream Model

• Enrich argument and return type of `main` to include all input and output events.

```haskell
main :: [Response] -> [Request]
data Request = ReadFile Filename
  | WriteFile FileName String
  | ...
data Response = RequestFailed
  | ReadOK String
  | WriteOk
  | Success  | ...
```

• Wrapper program interprets requests and adds responses to input.

• Move side effects outside of functional program
Stream Model: \texttt{main::[Response] \rightarrow [Request]}

- Problem: Laziness allows program to generate requests prior to processing any responses.
- Hard to extend
  - New I/O operations require adding new constructors to Request and Response types, modifying wrapper
- Does not associate Request with Response
  - easy to get “out-of-step,” which can lead to deadlock
- Not composable
  - no easy way to combine two “main” programs
- ... and other problems!!!
Monadic I/O: The Key Ideas

- **IO** is a type constructor, instance of **Monad**
- A value of type `(IO t)` is a computation or “action” that, when performed, may do some input/output before delivering a result of type `t`
- **return** returns the value without making I/O
- **then** `(>>)` [and also **bind** `(>>=)`] composes two actions sequentially into a larger action
- The only way to perform an action is to call it at some point, directly or indirectly, from **Main.main**
A value of type `(IO t)` is an "action." When performed, it may do some input/output before delivering a result of type `t`.

```
type IO t = World -> (t, World)
```

- An action is a first-class value
- **Evaluating** an action has no effect; **performing** the action has the effect
Implementation of the IO monad

- GHC uses “world-passing semantics” for the IO monad

\[
\text{type } \text{IO } t = \text{World} \rightarrow (t, \text{World})
\]

- It represents the “world” by an un-forgeable token of type \textbf{World}, and implements \textbf{bind} and \textbf{return} as:

\[
\begin{align*}
\text{return :: } & a \rightarrow \text{IO } a \\
\text{return } a &= \backslash w \rightarrow (a,w) \\
(\gg&=) :: \text{IO } a \rightarrow (a \rightarrow \text{IO } b) \rightarrow \text{IO } b \\
(\gg&=) m k &= \backslash w \rightarrow \text{case } m w \text{ of } (r,w') \rightarrow k r w'
\end{align*}
\]

- Using this form, the compiler can do its normal optimizations. The dependence on the world ensures the resulting code will still be single-threaded.

- The code generator then converts the code to modify the world “in-place.”
Simple I/O actions

getChar :: IO Char
putChar :: Char -> IO ()

main :: IO ()
main = putChar 'x'

Main program is an action of type IO ()
The Bind Combinator (\(\gg\gg=\))

\[(\gg\gg=) : : \text{IO } a \rightarrow (a \rightarrow \text{IO } b) \rightarrow \text{IO } b\]

- We have connected two actions to make a new, bigger action.

\[\text{getChar} :: \text{IO } \text{Char}\]
\[\text{putChar} :: \text{Char} \rightarrow \text{IO } ()\]
\[\text{echo} = \text{getChar} \gg\gg= \text{putChar}\]
The (>>=) Combinator

- Operator is called **bind** because it binds the result of the left-hand action in the action on the right.
- Performing compound action `a >>= \x->b`:
  - performs action `a`, to yield value `r`.
  - applies function `\x->b` to `r`.
  - performs the resulting action `b{x <- r}`.
  - returns the resulting value `v`.

```
(>>=) :: IO a -> (a -> IO b) -> IO b
```
The (>>) Combinator

- The “then” combinator (>>) does sequencing when there is no value to pass:

\[
(Seeder) :: \text{IO } a \to \text{IO } b \to \text{IO } b
\]

-- defined from bind

\[
(Sequencing) :: \text{IO } a \to (a \to \text{IO } b) \to \text{IO } b
\]

\[
m \gg n = m \gg (\_ \to n)
\]

\[
\begin{align*}
\text{echoDup} & :: \text{IO } () \\
\text{echoDup} & = \text{getChar} \gg (\_ \to \text{putChar } c) \gg \text{putChar } c
\end{align*}
\]

\[
\begin{align*}
\text{echoTwice} & :: \text{IO } () \\
\text{echoTwice} & = \text{echo} \gg \text{echo}
\end{align*}
\]
The return Combinator

• The action (return v) does no IO and immediately returns v:

\[ \text{return} :: a \rightarrow \text{IO} \ a \]

\[ \text{getTwoChars} :: \text{IO} \ (\text{Char},\text{Char}) \]
\[ \text{getTwoChars} = \text{getChar} \ >>> \ \backslash c1 \rightarrow \]
\[ \text{getChar} \ >>> \ \backslash c2 \rightarrow \]
\[ \text{return} \ (c1,c2) \]
The “do” Notation

• The “do” notation adds syntactic sugar to make monadic code easier to read.

-- Plain Syntax
getTwoChars :: IO (Char,Char)
getTwoChars = getChar >>= \c1 ->
              getChar >>= \c2 ->
              return (c1,c2)

-- Do Notation
getTwoCharsDo :: IO(Char,Char)
getTwoCharsDo = do { c1 <- getChar;
                    c2 <- getChar;
                    return (c1,c2) }

• do syntax designed to look imperative.
Desugaring “do” Notation

• The “do” notation only adds syntactic sugar:

\[
\begin{align*}
\text{do } \{ x \} &= x \\
\text{do } \{ x; \text{stmts} \} &= x >> \text{do } \{ \text{stmts} \} \\
\text{do } \{ v<-x; \text{stmts} \} &= x >>= \lambda v \to \text{do } \{ \text{stmts} \} \\
\text{do } \{\text{let } ds; \text{stmts} \} &= \text{let } ds \text{ in do } \{ \text{stmts} \}
\end{align*}
\]

The scope of variables bound in a generator is the rest of the “do” expression.

• The following are equivalent:

\[
\begin{align*}
\text{do } \{ x_1 <- p_1; \ldots; x_n <- p_n; \; q \} \\
\text{do } x_1 <- p_1; \ldots; x_n <- p_n; \; q
\end{align*}
\]
Bigger Example

• The `getline` function reads a line of input:

```haskell
generateLine :: IO [Char]
generateLine = do { c <- getChar;
  if c == '\n' then
    return []
  else
    do { cs <- generateLine;
         return (c:cs) }
```

Note the “regular” code mixed with the monadic operations
and the nested “do” expression.
Control Structures on Monads

• Exploiting the monadic combinators, we can define control structures that work for any monad

```haskell
repeatN 0 x = return ()
repeatN n x = x >> repeatN (n-1) x
repeatN :: (Num a, Monad m, Eq a) => a -> m a1 -> m ()
```

Main> repeatN 5 (putChar 'h')

```haskell
for []    fa = return ()
for (x:xs) fa = fa x >> for xs fa
for :: Monad m => [t] -> (t -> m a) -> m ()
```

Main> for [1..10] (\x -> putStrLn (show x))
Sequencing

A list of IO actions.

An IO action returning a list.

sequence :: [IO a] -> IO [a]
sequence [] = return []
sequence (a:as) = do { r <- a;
                      rs <- sequence as;
                      return (r:rs) }

sequence :: Monad m => [m a] -> m [a]

• Example use:

Main> sequence [getChar, getChar, getChar]
IO Provides Access to Files

• The IO Monad provides a large collection of operations for interacting with the “World.”
• For example, it provides a direct analogy to the Standard C library functions for files:

```haskell
openFile :: FilePath -> IOMode -> IO Handle
hPutStr :: Handle -> String -> IO ()
hGetLine :: Handle -> IO String
hClose :: Handle -> IO ()
```
References

• The IO operations let us write programs that do I/O in a strictly sequential, imperative fashion.
• Idea: We can leverage the sequential nature of the IO monad to do other imperative things

```
data IORef a   -- Abstract type
newIORef :: a -> IO (IORef a)
readIORef :: IORef a -> IO a
writeIORef :: IORef a -> a -> IO ()
```

• A value of type `IORef a` is a reference to a mutable cell holding a value of type `a`. 
import Data.IORef -- import reference functions

-- Compute the sum of the first n integers

count :: Int -> IO Int

count n = do
  { r <- newIORef 0;
    addToN r 1 }

where

  addToN :: IORef Int -> Int -> IO Int

  addToN r i | i > n = readIORef r
              | otherwise = do
    { v <- readIORef r
    ; writeIORef r (v + i)
    ; addToN r (i+1)}

This is terrible: Contrast with: sum [1..n].
The IO Monad as ADT

return :: a -> IO a
(>>=) :: IO a -> (a -> IO b) -> IO b

getchar :: IO Char
putChar :: Char -> IO ()
... more operations on characters ...
openFile :: [Char] -> IOMode -> IO Handle
... more operations on files ...
newIORef :: a -> IO (IORef a)
... more operations on references ...

• All operations return an IO action, but only \textbf{bind} (>>=) takes one as an argument.

• \textbf{Bind} is the only operation that combines IO actions, which forces sequentiality.

• In \textbf{pure Haskell}, there is no way to transform a value of type \texttt{IO a} into a value of type \texttt{a}
Unreasonable Restriction?

• In **pure Haskell**, there is no way to transform a value of type \( \text{IO } a \) into a value of type \( a \)

• Suppose you wanted to read a **configuration file** at the beginning of your program:

```haskell
configFileContents :: [String]
configFileContents = lines (readFile "config") -- WRONG!
useOptimisation :: Bool
useOptimisation = "optimise" `elem` configFileContents
```

• The problem is that **readFile** returns an **IO String**, not a **String**.

• **Option 1**: Write entire program in **IO monad**. But then we lose the simplicity of pure code.

• **Option 2**: Escape from the **IO Monad** using a function from **IO String -> String**. But this is disallowed!
Type-Unsafe Haskell Programming

• Reading a file is an I/O action, so in general it matters when we read the file.

• But we know the configuration file will not change during the program, so it doesn’t matter when we read it.

• This situation arises sufficiently often that Haskell implementations offer one last unsafe I/O primitive: `unsafePerformIO`.

```
unsafePerformIO :: IO a -> a
configFileContents :: [String]
configFileContents = lines(unsafePerformIO(readFile "config"))
```
The operator has a deliberately long name to discourage its use.

Its use comes with a **proof obligation**: a promise to the compiler that the timing of this operation relative to all other operations doesn’t matter.
**unsafePerformIO**

- **Warning**: As its name suggests, *unsafePerformIO* breaks the soundness of the type system.

```haskell
r = unsafePerformIO (newIORef (error "urk"))
r :: IORef a  -- Type of the stored value is generic

cast x = unsafePerformIO (do {
  writeIORef r x;
  readIORef r
})
> :t (\x -> cast x)
(\x -> cast x) :: a1 -> a2
> cast 65:: Char
'A'
```

- So claims that Haskell is type safe only apply to programs that don’t use *unsafePerformIO*.
- Similar examples are what caused difficulties in integrating references with Hindley/Milner type inference in ML.
Summary on Mondas

• A complete Haskell program is a single IO action called `main`. Inside IO, code is **single-threaded**.
• Big IO actions are built by **gluing together** smaller ones with `bind (>>=)` and by converting **pure code** into actions with `return`.
• IO actions are first-class.
  – They can be passed to functions, returned from functions, and stored in data structures.
  – So it is easy to define new “glue” combinators.
• The IO Monad allows Haskell to be pure while efficiently supporting side effects.
• The type system separates the pure from the effectful code.
Comparison

• In languages like ML or Java, the fact that the language is in the IO monad is baked in to the language. There is no need to mark anything in the type system because it is everywhere.

• In Haskell, the programmer can choose when to live in the IO monad and when to live in the realm of pure functional programming.

• So it is not Haskell that lacks imperative features, but rather the other languages that lack the ability to have a statically distinguishable pure subset.
Appendix: Monad Laws

1) \( \text{return } x \triangleright\triangleright f = f x \)
2) \( m \triangleright\triangleright \text{return } = m \)
3) \( (x \triangleright\triangleright f) \triangleright\triangleright g = x \triangleright\triangleright (\lambda v \rightarrow f v \triangleright\triangleright g) \)

• In do-notation:

1) \[
\text{do } \{ w \leftarrow \text{return } v; f w \} = \text{do } \{ f v \}
\]
2) \[
\text{do } \{ v \leftarrow x; \text{return } v \} = \text{do } \{ x \}
\]
3) \[
\text{do } \{ x \leftarrow m1; y \leftarrow m2; m3 \} = \text{do } \{ y \leftarrow \text{do } \{ x \leftarrow m1; y \leftarrow m2; m3 \} \}
\]

\( x \) not in free vars of \( m3 \)
Derived Laws for (>>>) and done

\[ (\gg\gg) :: \text{IO } a \to \text{IO } b \to \text{IO } b \]

\[ m \gg n = m \gg\gg \ (\_ \to n) \]

done :: IO ()

done = return ()

done \gg m = m

m \gg done = m

m1 \gg (m2 \gg m3) = (m1 \gg m2) \gg m3