UNIVERSITÀ DI PISA

A Critical Reassessment of the Saerens-Latinne-Decaestecker (SLD) Algorithm for Posterior Probability Adjustment

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The SLD Algorithm

- SLD is an instance of Expectation-Maximization, an algorithm for finding maximum likelihood estimates of parameters for models that depend on unobserved labels;

- We would like to leverage SLD in order to readjust our *a priori* and *a posteriori* probabilities (i.e. *priors* and *posteriors*) to new data;

- The algorithm key idea is to iteratively update the priors with the posteriors and vice versa.

A bit of Whys

- Why am I working on this daunting and, some could say, boring topic?
  Let’s say it is a consequence of my master thesis

- Why using the SLD algorithm?
  It looked like a good algorithm to use in my specific context, i.e.
  “transductive” dataset!

- Why did we decide to reassess an algorithm from the dinosaur era?
  Because it is one of the standard algorithms for probability
  readjustment!

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Before we start...
what is a *prior* probability?

- Let’s say you wake up with a really bad hangover: you don’t remember what day, month or year it is;
- Someone breaks into your room and asks you which do you think is the probability of rain today;
- You cannot base your prediction on anything else than your *a priori* knowledge, i.e. you know that in Pisa it rains 30% of the days over a year.

Before we start…
what is a *posterior* probability?

- Now you turn on your mobile phone and see it’s the first of january;
- You open the windows and notice it is very cloudy;
- Given these elements, you can now give a more accurate prediction of today probability of rain (eg. 70%).

Some details on the experiments setup

- Since my EAP presentation some things have changed: we dropped the 20NEWSGROUPS dataset and kept RCV1-V2 only;

- We tested SLD against posteriors coming from several machine learning classifiers: Support Vector Machines, Logistic Regression, Random Forests, Multinomial Naive Bayes;

- We evaluate the quality of the prior probabilities with the Normalized Absolute Error (NAE):

$$\mathrm{NAE}(p_U, \hat{p}_U) = \frac{\sum_{j=1}^{|\mathcal{Y}|} |\mathrm{Pr}(y_j) - \hat{\mathrm{Pr}}(y_j)|}{2(1 - \min_{y_j \in \mathcal{Y}} \mathrm{Pr}(y_j))}$$

Some details on the experiments setup

- We subsample and pre-process the RCV1-v2 dataset in order to allow for testing in different scenarios: in fact, we want to test the algorithm both in the binary and the single-label multi-class scenarios;

- We first extract from RCV1-v2 all and only the single-label documents, i.e. the documents which belong to one and only one class (that is, we keep more than 517k documents, and 37 out of 103 target labels);

- For the binary case, we run 500 experiments with all 37 classes, using one at a time as the positive label, and the remaining 36 as the negative examples.

Some details on the experiments setup

- For the single-label dataset, things get a little bit trickier: fix a *n* number of classes out of the 37 remaining;

- Generate two random arrays of probabilities of length *n*, where each element represent the probability of one of the *n* classes. One array is for the training set, the other for the test set;

- Draw a pre-fixed number of documents (1000 in our case) from the dataset, with the probabilities in the random array. We do this twice, once for the training set and a second time for the test set;

- We can now generate lots of subsamples (500 for each classifier in our case) from the original dataset!

So, how does SLD really work?

For every step *s*, we first update our posteriors as a ratio between priors at this step and the classifier priors, multiplied by the classifier posteriors

$$\hat{\Pr}^{(s)}(c_j|x) \leftarrow \frac{\frac{\hat{\Pr}^{(s)}(c_j)}{\hat{\Pr}_t(c_j)}\hat{\Pr}_t(y|x)}{\sum_{j\in C}\frac{\hat{\Pr}^{(s)}(c_j)}{\hat{\Pr}_t(c_j)}\hat{\Pr}_t(c_j|x)}$$

And then...we recompute the priors for next step as the average of the current step posteriors

$$\hat{\Pr}^{(s+1)}(c_j) \leftarrow \frac{1}{N}\sum_{x\in X}\hat{\Pr}^{(s)}(c|x)$$

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Or better, why should it work?

Probability distributions can often change between training and test sets!

Example

- Let's say that due to a huge catastrophe, it's now always winter and it truly rains a lot;
- But you still don't know and have your priors from before, 30% chance of rain. This does not represent the reality anymore!
- If you have posteriors based on observations during this never-ending winter, you could push your previous priors towards a more realistic value (eg. 70%).
- Theoretically, this could work in the other direction as well, for posteriors!

Or better, why should it work?

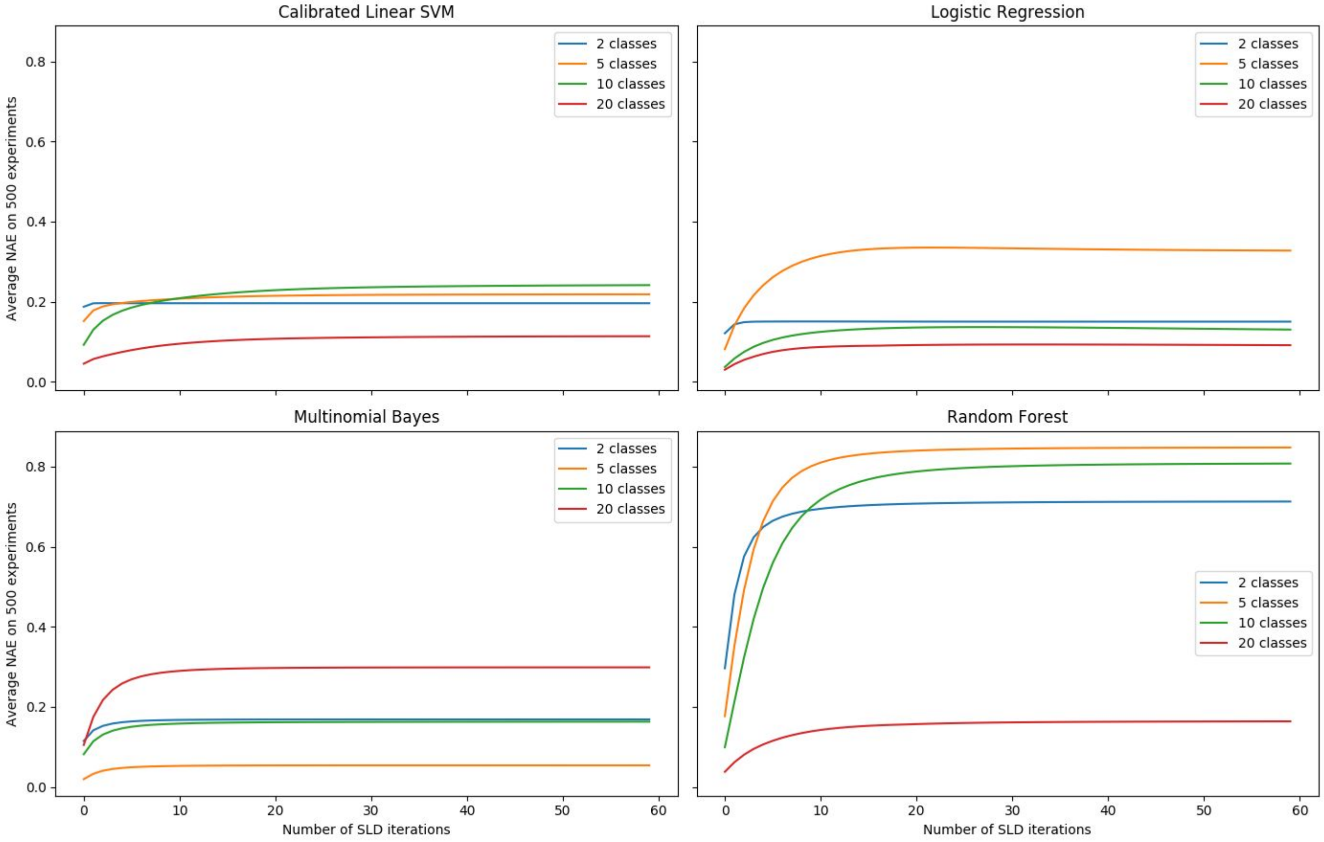
SLD could work really well on a “transductive” dataset, i.e.:

- With “transductive”, in this context, we mean that the dataset is fixed;
- That is, we do not expect to have new documents coming in the future. Yay, overfitting!
- Beware, we do not know the true labels of the set of documents we need to classify (of course), we just know that the dataset have and will keep having a fixed size

Great! But *does* it work?

Not really

Results from 500 experiments on 20ng



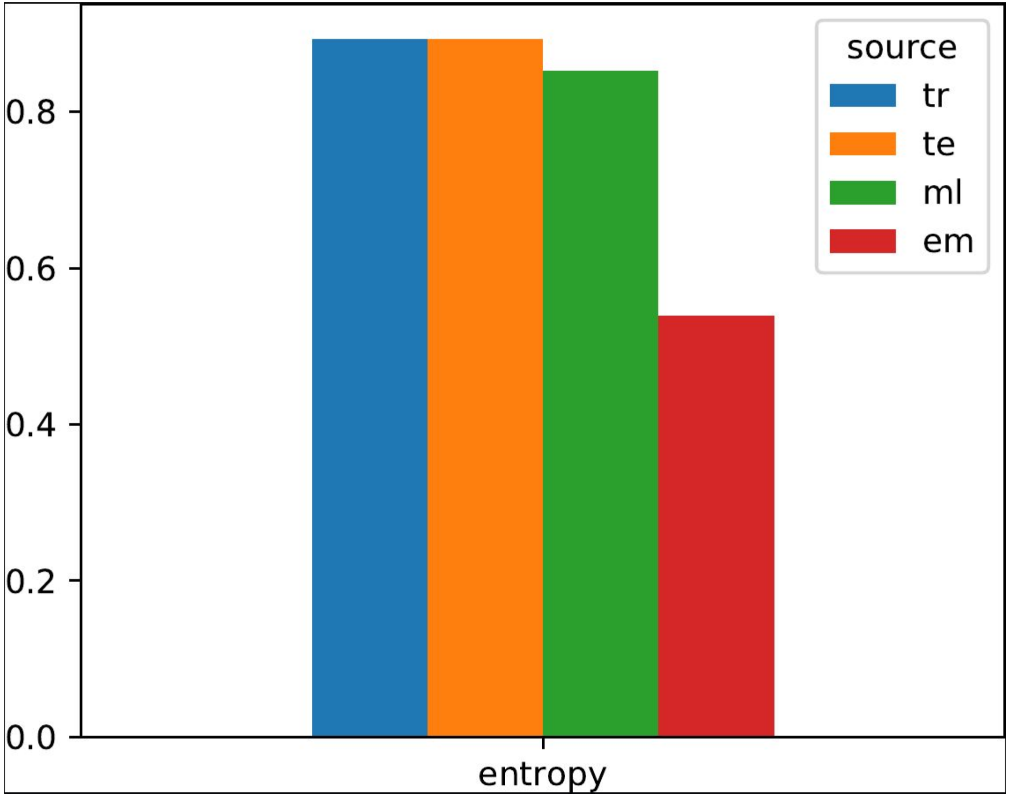
Entropy might be the answer

- In information theory, Entropy can be interpreted as the average level of “information”, “surprise” or “uncertainty” inherent in a random variable’s possible outcomes;

- In our case, this means that if our probabilities are always 1 (i.e. 100% chance of rain), then we are not really surprised to see the rain on any given day. The entropy would be 0;

- Conversely, if our probabilities are always 0.5, we never know what to expect and the entropy would be 1.

Entropy might be the answer



- Average entropy on 500 samples (4 classes);
- SLD (em) completely disrupts the entropy of the distribution;
- The algorithm is basically increasing the probability of one class, reducing probability of the others.

This is still a work in progress

- We are still studying the algorithm and carrying out many analysis. We hope to understand it better and give an answer as to why the algorithm gives such disappointing results;

- However, we are pretty confident SLD is not as good as it was supposed to be;

- It is still often used as one of the baselines in several tasks, eg. quantification.

References

- Saerens, M., Latinne, P., and Decaestecker, C. (2002). Adjusting the outputs of a classifier to new a priori probabilities: A simple procedure. *Neural Computation*, 14(1):21–41.

- Sebastiani, F. (2020). Evaluation measures for quantification: An axiomatic approach. *Information Retrieval Journal.* Forthcoming.

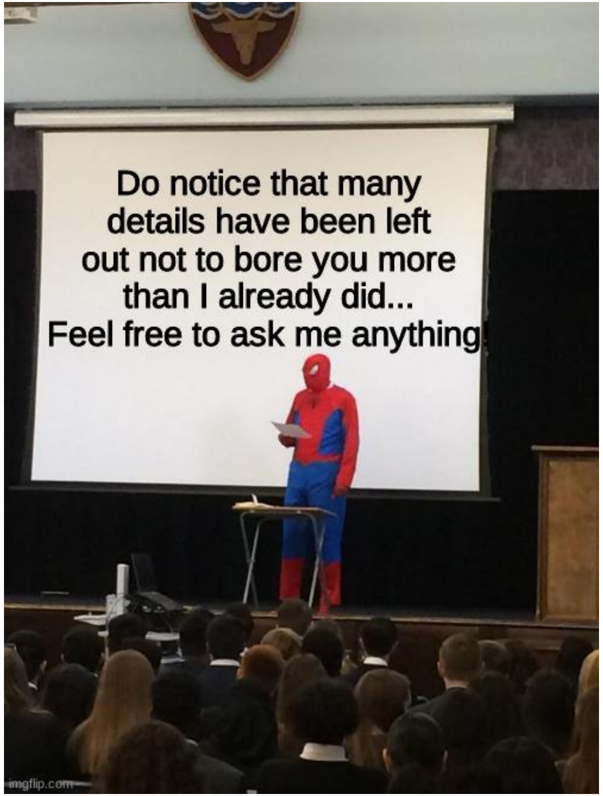
- Zadrozny, B. and Elkan, C. (2002). Transforming classifier scores into accurate multiclass probability estimates. In *Proceedings of the 8th ACM International Conference on Knowledge Discovery and Data Mining (KDD 2002)*, pages 694–699, Edmonton, CA.

- Dempster, A. P., Laird, N. M., and Rubin, D. B. (1977). Maximum likelihood from incomplete data via the EM algorithm. *Journal of the Royal Statistical Society,* B, 39(1):1–38.

- DeGroot, M. H. and Fienberg, S. E. (1983). The comparison and evaluation of forecasters. *The Statistician*, 32(1/2):12–22.

- Brier, G. W. (1950). Verification of forecasts expressed in terms of probability. *Monthly Weather Review*, 78(1):1–3.

Thank you!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