2) Productions as equations on languages $L(A_i)$

$\forall i \geq 0, \quad L(A_i) = L(e_i)$

- L(e) is an expression on $\mathbf{2}^{\Sigma^*}$ containing only:
  X (finite products)
  ∪ (possibly, denumerable unions)
- L(e) is continuos on $\mathbf{2}^{\Sigma^*}$

Whenever $L(A_i) = L(e_i)$ is recursive: $L(e_i) \equiv E(L(A_i))$

Recursive equations X=E(X), have to be solved in the variable $X \equiv L(A_i)$ on $\mathbf{2}^{\Sigma^*}$ using the **Tarski's (Fixpoint) Iteration** below:

$$X = \bigcup_{i \in \aleph} E(\bot)^i$$

$$E(\bot)^0 = E(X \leftarrow \bot)$$

$$E(\bot)^{k+1} = E(X \leftarrow E(\bot)^k)$$

What about a system of equations

A system of Recursive equations:

$$\{X_1=E_1(X_1,\ldots X_n),\ldots, X_n=E_n(X_1,\ldots X_n)\}$$

$X_i \equiv L(A_i)$ on $\mathbf{2}^{\Sigma^*}$ using the **Tarski's (Fixpoint) Iteration** below:

$$\mathbf{X_j = \cup_{i\in\aleph} E_j(\bot,\ldots,\bot)^i}$$

$$\mathbf{E_j(\bot,\ldots,\bot)^0 = E_j(X_1 \leftarrow \bot,\ldots,X_n \leftarrow \bot)}$$

$$\mathbf{E_j(\bot,\ldots,\bot)^{k+1} = E_j(X_1 \leftarrow E_1(\bot)^k,\ldots,X_n \leftarrow E_n(\bot)^k)}$$

Example1: Tarski's Fixpoint Iteration

S::=u S | ε → X= {u}×X∪{λ}

E(X)

$E(X \leftarrow \bot)^0 = \{u\} \times \bot \cup \{\lambda\} = \bot \cup \{\lambda\} = \{\lambda\}$

$E(X \leftarrow \bot)^1 = \{u\} \times \{\lambda\} \cup \{\lambda\} = \{u, \lambda\}$

$E(X \leftarrow \bot)^2 = \{u\} \times \{u, \lambda\} \cup \{\lambda\} = \{uu, u, \lambda\}$

$E(X \leftarrow \bot)^3 = \{u\} \times \{uu, u, \lambda\} \cup \{\lambda\} = \{u^3, u^2, u, \lambda\}$

↓

$E(X \leftarrow \bot)^n = \{u^n, u^{n-1}, \ldots, u, \lambda\}$

↓

$L(S) = \{u^n \mid n \in \aleph\}$ **= u***

Example2: Tarski's Fixpoint Iteration

$S ::= u\ S\ v \mid z$ → $X = \{u\} \times X \times \{v\} \cup \{z\}$

E(X)

$E(X \leftarrow \bot)^0 = \{u\} \times \bot \times \{v\} \cup \{z\} = \bot \cup \{z\} = \{z\}$

$E(X \leftarrow \bot)^1 = \{u\} \times \{z\} \times \{v\} \cup \{z\} = \{uzv, z\}$

$E(X \leftarrow \bot)^2 = \{u\} \times \{uzv, z\} \times \{v\} \cup \{z\} = \{u^2zv^2, uzv, z\}$

↓

$E(X \leftarrow \bot)^n = \{u^nzv^n, u^{n-1}zv^{n-1}, \ldots, z\}$

↓

$L(S) = \{u^nzv^n \mid n \geq 0\}$

Example3: Tarski's Fixpoint Iteration

A::= A+A
A::= A∗A
A::= id

→ X= $\{x+y \mid x,y \in X\} \cup \{x*y \mid x,y \in X\} \cup \{id\}$

E(X)

$E(X \leftarrow \bot)^0 = \{x+y \mid x,y \in \bot\} \cup \{x*y \mid x,y \in \bot\} \cup \{id\}$
$= \bot \cup \bot \cup \{id\} = \{id\}$

$E(X \leftarrow \bot)^1 = \{x+y \mid x,y \in \{id\}\} \cup \{x*y \mid x,y \in \{id\}\} \cup \{id\}$
$= \{id+id\} \cup \{id*id\} \cup \{id\} = \{id, id+id, id*id\}$

$E(X \leftarrow \bot)^2 = \{x+y \mid x,y \in E(X \leftarrow \bot)^1\} \cup \{x*y \mid x,y \in E(X \leftarrow \bot)^1\} \cup \{id\}$
$= \{id, id+id, id*id, id+id+id, id+id*id, id*id+id, id*id*id, \ldots, id*id*id*id\}$

↓

$E(X \leftarrow \bot)^n = \{id, id\ t^k \mid t \in \{+id, *id\}, k \in [1..2^n-1]\}$

↓

$L(A) = \{id\ t^n \mid t \in \{+id, *id\}, n \in \aleph\}$

How to do Syntactic Analysis

TOP-DOWN and BOTTOM-UP Parsers

Let $\mathbf{G=<V,\Sigma,s\in V,P>}$ be a (context free) grammar. Let **w** be a sequence of words in $\mathbf{\Sigma}$**.**

- Analysis has to answer to the following question:

  is **w $\in$L(G) or not** ?

- or, equivalently:

  is **s** =>***w or not** ?

- Is this Decision Problem, decidable?

  -- Yes. It is decidable for all classes of **monotone grammars**.

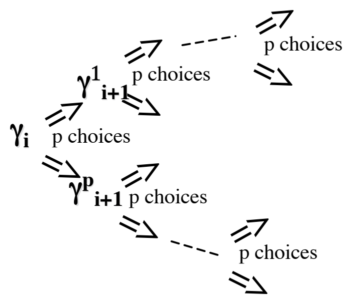
- The solution consists in defining a procedure (**The Parser Core**) able to construct a derivation $s \Rightarrow \gamma_1 \Rightarrow \ldots \Rightarrow \gamma_k \equiv w$, if one exists.

Construction of a Derivation

- The solution consists in defining a procedure (**The Parser Core**) able to construct a derivation $s \Rightarrow \gamma_1 \Rightarrow \ldots \Rightarrow \gamma_k \equiv w$, if one exists.

- The construction of a derivation could be done in a non-efficient way, and even worse, at a non-linear, up to exponential, complexity time (/space) cost.

Trying *p* optional productions at each γ_i leads to:

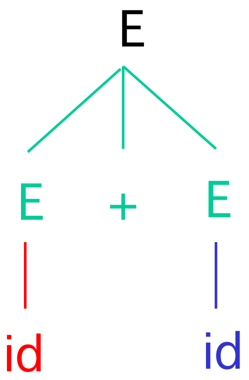


construction of (exponential) ($O(p^n)$) derivations to find the one right or to answer “no-accept”.

Top-Down

Simple for Handmade Constructions,
Few Grammars

E => E+E => id +E => id+id



Leftmost non-terminal of Left-Sentential-Form

First Applicable Production

Failure: Backward to the last alternative

Step 1
Step 2
Step 3

Top-Down = Leftmost

p1: E::= E+E
p2: E::= E*E
p3: E::= id

LSF forms a Complete Base for Context-Free Grammars

$G = \langle V, \Sigma, s \in V, P \rangle$

Left Sentential Form (of G): ***LSF_G***

$\alpha\beta\gamma \in LSF_G$ *iff* $s \;{}_{l}\!\Rightarrow^{+} \alpha\beta\gamma$

$\alpha A \beta \;{}_{l}\!\Rightarrow \alpha\gamma\beta$ *iff* $A ::= \gamma \in P \;\&\; \alpha \in \Sigma^*$

Only ***LSF_G***

$L(G) = \{ w \in \Sigma^* \mid s \Rightarrow^{+} w \}$

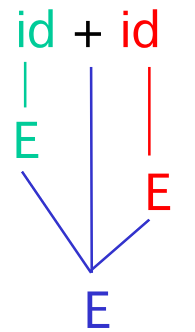
$= \{ w \in \Sigma^* \mid s \;{}_{l}\!\Rightarrow^{+} w \}$

Bottom-Up

More Complicated Techniques

Many More Grammars - Many More Languages

E =>r E+E =>r E+id =>r id+id



Looking for Handle

reduction

failure: backward for “true” Handle

Step 1
Step 2
Step 3

Bottom-Up = Rightmost Reversed

p1: E::= E+E
p2: E::= E*E
p3: E::= id

RSF forms a Complete Base for Context-Free Grammars

$G = \langle V, \Sigma, s \in V, P \rangle$

Right Sentential Form (of G): RSF_G

$\alpha\beta\gamma \in RSF_G$ *iff* $s \ {}_r\!\!=>^+ \alpha\beta\gamma$

$\boldsymbol{\alpha A \beta \ {}_r\!\!=> \alpha\gamma\beta}$ ***iff*** $A ::= \gamma \in P \ \& \ \beta \in \Sigma^*$

Only RSF

$L(G) = \{w \in \Sigma^* \mid s \ \mathbf{=>^+} \ w\}$

$= \{w \in \Sigma^* \mid s \ {}_r\!\!\mathbf{=>^+} \ w\}$

B::=β ∈P is **Handle** of **αβγ** $\in RSF_G$

if and only if $\alpha B \gamma \in RSF_G$

Top-down