Exercise

Let G be the grammar whose productions are: S::= a S S | b.
Let L be the language:

$L=\{a^{n0} b^{m0} \dots a^{nk} b^{mk} \mid C1 \,\&\, C2 \,\&\, C3\} \cup \{b\}$

where: $C1 \equiv ni*mi>0 \,\&\, k \geq 0$

$C2 \equiv \Sigma_{0 \leq i \leq j<k}\, a^{ni} \geq \Sigma_{0 \leq i \leq j<k}\, b^{ni}$

$C3 \equiv \Sigma_{0 \leq i \leq k}\, a^{ni} = (\Sigma_{0 \leq i \leq k}\, b^{ni})-1$

Prove that L(G) = L. In particular, prove each of (1),(2),(3),(4) below:

(1) L(G) contains all the $\gamma \in L$;

(2) $\gamma \equiv a^{n0} b^{m0} \dots a^{nk} b^{mk} \in L(G)$ only if C1 holds

(3) $\gamma \equiv a^{n0} b^{m0} \dots a^{nk} b^{mk} \in L(G)$ only if C2 holds

(4) $\gamma \equiv a^{n0} b^{m0} \dots a^{nk} b^{mk} \in L(G)$ only if C3 holds

(1)Let Z = {a} x Z x Z + {b} be the equation defined by the grammar productions. We show that the following is an identity: L = {a} x L x L + {b}. To prove it: Let C≡C1 & C2 & C3

$\{a^{n0} b^{m0} \dots a^{nk} b^{mk} \mid C\} \cup \{b\} = \{a\}\ X\ \{a^{n0'} b^{m0'} \dots a^{nk'} b^{mk'} \mid C\}\ X\ \{a^{n0''} b^{m0''} \dots a^{nk''} b^{mk''} \mid C\} + \{b\}$

$= \{a^{n0'+1} b^{m0'} \dots a^{nk'} b^{mk'} a^{n0''} b^{m0''} \dots a^{nk''} b^{mk''} \mid C\} + \{b\}$

Putting k=k'+k" and n0 = n0'+1, m0=m0', …, nk=nk", mk=mk", makes conditions C true (as proved below*) and proof ends.

We show here, that conditions hold for the parameter assignment above. Condition C1: We have to prove only, (n0'+1)*m0'>0 (but it follows straightforwardly from n0'*m0'>0) and k'+k">0 (but it is a very trivial consequence of k',k">0). Condition C2: It trivially, follows from the fact that n0 = n0'+1 adds one "a" to the summation on the left hand side. Eventually, Condition3: It follows from the fact that $\Sigma_{0 \leq i \leq k'}\, a^{ni} = 1+ \Sigma_{0 \leq i \leq k'}\, a^{ni'}$.

(2)-(4) Complete the proofs for the necessary conditions.