# Sistema di Tipi Y di Small20

- **Sintassi Concreta**
  $\texttt{Type} ::= \texttt{Simple} \mid \texttt{void} \mid \texttt{Simple}\ [\texttt{Num}]$
  $\texttt{Simple} ::= \texttt{int} \mid \texttt{bool}$

- **Sintassi Astratta ed Espressioni di Tipo**
  $\texttt{Type} ::= [\texttt{int}] \mid [\texttt{bool}] \mid [\texttt{void}] \mid [\texttt{arr}]\ \texttt{Type Num} \mid [\texttt{mut}]\ \texttt{Type} \mid [\texttt{terr}]$
  $\mid [\texttt{abs}]\ \texttt{Type TypeSeq}$

- **Regole: Una Regola di** $\texttt{Y}_{\texttt{DCL}^{\texttt{Small20}}}$

$$\frac{\langle \texttt{e}, \texttt{Y}_\rho\rangle \rightarrow_{\texttt{Y}} (\texttt{t}', \texttt{Y}_\rho) \qquad \texttt{t}' \neq [\texttt{terr}] \qquad \texttt{t}' = \texttt{t}}{\langle [\texttt{const}]\ \texttt{t}\ \texttt{I}\ \texttt{e}, \texttt{Y}_\rho\rangle \rightarrow_{\texttt{Y}} (\texttt{Void}, [\texttt{I}/\texttt{t}]\circ \texttt{Y}_\rho)}$$

Sono inferenze di un **Sistema Deduttivo** che associa ad ogni struttura del programma, $\texttt{s}$, con tipi degli identificatori definiti da $\texttt{Y}_\rho$, una coppia $(\texttt{t}', \texttt{Y}'_\rho)$. Sopra, nella premessa della regola, nella formula:

$$\langle \texttt{e}, \texttt{Y}_\rho\rangle \rightarrow_{\texttt{Y}} (\texttt{t}, \texttt{Y}_\rho)$$

vediamo ciò. In particolare, il sistema ha dedotto che l'espressione $\texttt{e}$, dove i suoi identificatori liberi hanno tipi tutti legati in $\texttt{Y}_\rho$, abbia tipo $\texttt{t}$ (i.e. $\texttt{t}' = \texttt{t}$). Ovvero:

$$\texttt{Y}_\rho \vdash \texttt{e} : \texttt{t}.$$

Nella conclusione della regola sopra, la struttura di programma $\texttt{s}$ è:

$$[\texttt{const}]\ \texttt{t}\ \texttt{I}\ \texttt{e}.$$

Quando il sistema è in grado di inferire

$$\langle \texttt{s}, \texttt{Y}_\rho\rangle \rightarrow_{\texttt{Y}} (\texttt{Void}, [\texttt{I}/\texttt{t}]\circ \texttt{Y}_\rho)$$

allora il sistema deduce che la struttura $\texttt{s}$ è/introduce una corretta dichiarazione che estende in modo consistente $\texttt{Y}_\rho$ con il binding $[\texttt{I}/\texttt{t}]$. Ovvero:

$$\texttt{Y}_\rho, \texttt{I} : \texttt{t} \vdash \texttt{s} : \texttt{Void}.$$

# Sistema di Tipi: Regole, Strutture e Proprietà di Y

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho) \qquad \mathtt{t}' \neq [\mathtt{terr}] \qquad \mathtt{t}' = \mathtt{t}}{\langle [\mathtt{const}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{Void}, [\mathtt{I}/\mathtt{t}] \circ \mathtt{Y}_\rho)}$$

- **Regole.** Sono inferenze di un sistema deduttivo che associano ad ogni struttura del programma, $\mathtt{s}$, con tipi degli identificatori definiti da $\mathtt{Y}_\rho$, una coppia $(\mathtt{t}', \mathtt{Y}'_\rho)$.

  Sopra, in premessa $\langle \mathtt{e}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho)$ vediamo ciò. In particolare, il sistema ha dedotto che l'espressione $\mathtt{e}$, dove i suoi identificatori liberi siano tutti legati in $\mathtt{Y}_\rho$, abbia tipo $\mathtt{t}' = \mathtt{t}$. Ovvero: $\mathtt{Y}_\rho \vdash \mathtt{e} : \mathtt{t}$.
  Sopra, nella conclusione, la struttura di programma $\mathtt{s}$ è: $[\mathtt{const}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}$. Quando il sistema è in grado di inferire $\langle \mathtt{s}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{Void}, [\mathtt{I}/\mathtt{t}] \circ \mathtt{Y}_\rho)$ allora il sistema ha dedotto che la struttura $\mathtt{s}$ introduce una corretta dichiarazione che estende in modo consistente $\mathtt{Y}_\rho$ con il binding $[\mathtt{I}/\mathtt{t}]$. Ovvero: $\mathtt{Y}_\rho, \mathtt{I} : \mathtt{t} \vdash \mathtt{s} : \mathtt{Void}$.

- **Identificatori.** Tutti gli identificatori introdotti nel programma hanno un tipo $\mathtt{t}$ (definito da un'espressione di tipo del linguaggio). Questo tipo è associato all'identificatore nell'ambiente $\mathtt{Y}_\rho$ come il binding dell'identificatore in $\mathtt{Y}_\rho$.

- **Struttura $\mathtt{Y}_\rho$.** È una struttura di ambiente con le stesse operazioni viste per gli ambienti di denotazioni e con politica di inserimento ed accesso LIFO (quindi l'applicazione di Y in differenti parti di un programma può richiedere l'applicazione di regole che aggiungono a $\mathtt{Y}_\rho$ più volte uno stesso identificatore con binding diversi, ed operazioni, "$\circ$", per l'aggiunta di un binding, e applicazione, $\mathtt{Y}_\rho(\mathtt{I})$, per la selezione del tipo associato al più recente I introdotto).

- **Completezza** di Y richiede che il sistema sia dotato di un insieme di regole tali che: L'applicazione di Y ad ogni programma (sintatticamente corretto e legale) $\mathtt{P}$, nell'ambiente $\mathtt{Y}_{\mathtt{L}}$, contenente i binding per tutti e soli gli identificatori di primitive del Linguaggio, sia in grado di fornire una e una sola derivazione di tipi ovvero un albero dove ogni nodo è un la conclusione di una regola e i figli sono le premessa di tale regola sotto una stessa istanza di simboli della regola con strutture di P. L'albero così ottenuto ha:

  - radice: $\langle \mathtt{P}, \mathtt{Y}_{\mathtt{L}} \rangle \rightarrow_{\mathtt{Y}} (\mathtt{Void}, \mathtt{Y}_{\mathtt{L}})$. Ovvero: $\mathtt{Y}_{\mathtt{L}} \vdash \mathtt{P} : \mathtt{Void}$
  - discendenti: Per ogni termine $\mathtt{s}$ contenuto in $\mathtt{P}$, $\langle \mathtt{s}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}'_\rho)$. Ovvero: $\mathtt{Y}_{\mathtt{L}}, \mathtt{Y}_\rho \vdash \mathtt{s} : \mathtt{t}$

- **Ben tipato** in Y, è ogni termine (costrutto) $\mathtt{s}$ di ogni programma $\mathtt{P}$ (sintatticamente corretto e legale) che abbia una derivazione di tipi con radice come sopra e con discendenti come sopra e tali che $\mathtt{t} \neq [\mathtt{terr}]$.

# Sistema di Tipi: Regole, Strutture e Proprietà di Y - 2

$$\frac{\Gamma_1 \quad \ldots \quad \Gamma_n}{\langle \mathtt{s}, \mathtt{Y}_\rho \rangle \rightarrow_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}'_\rho)}$$

- **Stato di Stuck** in un Linguaggio con un sistema di tipi Y, è ogni stato inconsistente rispetto a Y. Definiamo inconsistente ogni stato che contiene:
  - **binding** tra un identificatore di tipo `t1` e un valore (denotabile o memorizzabile) di tipo `t2`, con $\mathtt{t1} \neq \mathtt{t2}$;
  - **valore atteso**, in qualche componente di un qualche AR (inclusi buffer di I/O), di tipo `t1` e un valore calcolato di tipo `t2`, con $\mathtt{t1} \neq \mathtt{t2}$

  Ogni Linguaggio tipato ha una propria struttura di stato che include sempre l'ambiente dei bindings degli identificatori (introdotti nel programma), $\rho$, lo stack di AR e, secondo i casi, una (o più per heap, per variabili statiche) memoria $\mu$ per valori modificabili e ogni altra struttura (buffer di I/O, tabella delle classi, tabella dei moduli,..) che debba far parte dello stato. Il sistema di tipi provvede esplicitamente a definire la consistenza per la struttura di stato dello specifico Linguaggio tipato.
- **Stato NonStuck** in un Linguaggio con un sistema di tipi Y, è ogni stato che soddisfa la consistenza richiesta da Y.
- **Correttezza (Safety)** di Y richiede che esso sia definito in accordo alla semantica del Linguaggio garantendo le due proprietà sotto:
  - **Progress**: Ogni termine ben tipato `s` (avente $\mathtt{Y}_\rho \vdash \mathtt{s} : \mathtt{t}$) o è esso stesso un valore di tipo `t`[1] che occorre in uno stato NonStuck, oppure in accordo alla semantica la computazione contiene un successivo stato;
  - **Preservation**: Se `s` nel successivo stato è stato rimpiazzato con $\mathtt{s}'$ allora anche $\mathtt{s}'$è ben tipato ed anche per esso vale $\mathtt{Y}_\rho \vdash \mathtt{s}' : \mathtt{t}$
- **Theorem Completezza+Correttezza (locale)** Ogni Linguaggio tipato con sistema di tipi Completo e Safe è tale che tutti i suoi programmi ben tipati hanno computazioni prive di stati di Stuck.

[1] Ad ogni tipo è associato un insieme di valori non vuoto. Il tipo Void contiene il solo valore: `void`

# Sistema Y: Regole per DCL

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho) \qquad \mathtt{t} \in \mathtt{Simple} \qquad \mathtt{t}' = \mathtt{t}}{\langle [\mathtt{const}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{void}], [\mathtt{I}/\mathtt{t}] \circ \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho) \qquad \mathtt{t} \in \mathtt{Simple} \qquad \mathtt{t}' = \mathtt{t}}{\langle [\mathtt{var}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{void}], [\mathtt{I}/[\mathtt{mut}]\ \mathtt{t}] \circ \mathtt{Y}_\rho)} \qquad \frac{\mathtt{t} \in \mathtt{Simple}}{\langle [\mathtt{varN}]\ \mathtt{t}\ \mathtt{I}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{void}], [\mathtt{I}/[\mathtt{mut}]\ \mathtt{t}] \circ \mathtt{Y}_\rho)}$$

$$\frac{\mathtt{t} \in \mathtt{Simple}}{\langle [\mathtt{array}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{N}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{void}], [\mathtt{I}/[\mathtt{mut}]([\mathtt{arr}]\ \mathtt{t}\ \mathtt{N})] \circ \mathtt{Y}_\rho)}$$

Gestione Errori di Tipo:

$$\frac{\mathtt{t} \notin \mathtt{Simple}}{\langle [\mathtt{const}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho) \qquad \mathtt{t}' \neq \mathtt{t}}{\langle [\mathtt{const}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\mathtt{t} \notin \mathtt{Simple}}{\langle [\mathtt{var}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}', \mathtt{Y}_\rho) \qquad \mathtt{t}' \neq \mathtt{t}}{\langle [\mathtt{var}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\mathtt{t} \notin \mathtt{Simple}}{\langle [\mathtt{varN}]\ \mathtt{t}\ \mathtt{I}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\mathtt{t} \notin \mathtt{Simple}}{\langle [\mathtt{array}]\ \mathtt{t}\ \mathtt{I}\ \mathtt{N}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

**Notazione: Simboli e Strutture**

- N (anche con pedici, apici), Intero
- B (anche con pedici, apici), Booleans
- I (anche con pedici, apici), Identificatore
- t (anche con pedici, apici), (Espressione di) Tipo
- d (anche con pedici, apici), dichiarazione
- e (anche con pedici, apici), espressione
- c (anche con pedici, apici), comando
- m (anche con pedici, apici), statement
- $\mathtt{Simple} = \{[\mathtt{int}], [\mathtt{bool}]\}$

# Sistema Y: Regole per EXP

$$\frac{}{\langle [\mathtt{num}]\,\mathtt{N}, \sigma\rangle \to_{\mathtt{Y}} ([\mathtt{int}], \mathtt{Y}_\rho)} \qquad \frac{}{\langle [\mathtt{truth}]\,\mathtt{B}, \sigma\rangle \to_{\mathtt{Y}} ([\mathtt{bool}], \mathtt{Y}_\rho)} \qquad \frac{}{\langle [\mathtt{emptyE}]\ , \sigma\rangle \to_{\mathtt{Y}} ([\mathtt{void}], \mathtt{Y}_\rho)}$$

$$\frac{\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]\,\mathtt{t} \qquad \mathtt{t} \in \mathtt{Simple}}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \to_{\mathtt{DY}} ([\mathtt{mut}]\,\mathtt{t}, \mathtt{Y}_\rho)} \qquad \frac{\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]\,\mathtt{t} \qquad \mathtt{t} \in \mathtt{Simple}}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)} \qquad \frac{\mathtt{Y}_\rho(\mathtt{I}) = \mathtt{t} \qquad \mathtt{t} \in \mathtt{Simple}}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c} \mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]([\mathtt{arr}]\,\mathtt{t}\,\mathtt{N}) \\ \langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{int}], \mathtt{Y}_\rho) \qquad \mathtt{t} \in \mathtt{Simple} \\ \textit{InBoundDinamycCheck} \end{array}}{\langle \mathtt{I}\,[\uparrow]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{DY}} ([\mathtt{mut}]\,\mathtt{t}, \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c} \mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]([\mathtt{arr}]\ \mathtt{t}\ \mathtt{N}) \\ \langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{int}], \mathtt{Y}_\rho) \qquad \mathtt{t} \in \mathtt{Simple} \\ \textit{InBoundDinamycCheck} \end{array}}{\langle \mathtt{I}\,[\uparrow]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c} \langle \mathtt{e}_1, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}_1, \mathtt{Y}_\rho) \qquad \langle \mathtt{e}_2, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}_2, \mathtt{Y}_\rho) \\ \mathtt{Y}_\rho(\mathtt{op}) = [\mathtt{abs}]\ \mathtt{t}\ \mathtt{t}_1'[\mathtt{x}]\mathtt{t}_2' \qquad \mathtt{op} \in \mathcal{O}_2 \\ \mathtt{t}_1' = \mathtt{t}_1 \qquad \mathtt{t}_2' = \mathtt{t}_2 \end{array}}{\langle \mathtt{e}_1\,[\mathtt{op}]\,\mathtt{e}_2, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c} \langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}_\mathtt{e}, \mathtt{Y}_\rho) \\ \mathtt{Y}_\rho(\mathtt{op}) = [\mathtt{abs}]\ \mathtt{t}\ \mathtt{t}' \qquad \mathtt{op} \in \mathcal{O}_1 \\ \mathtt{t}' = \mathtt{t}_\mathtt{e} \end{array}}{\langle [\mathtt{op}]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c} \langle \mathtt{e}_1, \mathtt{Y}_\rho\rangle \to_{\mathtt{DY}} (\mathtt{t}_1, \mathtt{Y}_\rho) \quad \langle \mathtt{e}_\mathtt{r}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}_\mathtt{r}, \mathtt{Y}_\rho) \\ \mathtt{t}_1 = [\mathtt{Mut}]\ \mathtt{t} \qquad \mathtt{t} = \mathtt{t}_\mathtt{r} \qquad \mathtt{t} \in \mathtt{Simple} \end{array}}{\langle \mathtt{e}_1\,[=]\,\mathtt{e}_\mathtt{r}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)}$$

**Notazione**

$\to_{\mathtt{DY}}$ è l'inferenza di Tipo del Valore Denotabile di espressioni con doppio significato.

$\mathcal{O}_2 = \{+, -, *, \mathtt{div}, ==, >, <, \mathtt{or}, \mathtt{and}\}$; $\quad \mathcal{O}_1 = \{-_1, \mathtt{not}\}$.

$\mathtt{Simple} = \{[\mathtt{int}], [\mathtt{bool}]\}$

# Sistema Y: Regole per EXP - Errori di tipo

$$\frac{\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]\,\mathtt{t} \qquad \mathtt{t} \notin \mathtt{Simple}}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\mathtt{Y}_\rho(\mathtt{I}) = \mathtt{t} \qquad \mathtt{t} \neq [\mathtt{mut}]\,\mathtt{t}'}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\mathtt{Y}_\rho(\mathtt{I}) \neq [\mathtt{mut}]\,\mathtt{t}}{\langle [\mathtt{val}]\,\mathtt{I}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c}\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]([\mathtt{arr}]\,\mathtt{t}\,\mathtt{N})\\ \langle \mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t_e}, \mathtt{Y}_\rho) \qquad \mathtt{t_e} \neq [\mathtt{int}]\end{array}}{\langle \mathtt{I}\,[\uparrow]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]([\mathtt{arr}]\,\mathtt{t}\,\mathtt{N})\\ \mathtt{t} \notin \mathtt{Simple}\end{array}}{\langle \mathtt{I}\,[\uparrow]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\mathtt{Y}_\rho(\mathtt{I}) = [\mathtt{mut}]([\mathtt{arr}]\,\mathtt{t}\,\mathtt{N})\\ \langle \mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{int}], \mathtt{Y}_\rho)\\ \mathit{OutOfBoundDinamycCheck}\end{array}}{\langle \mathtt{I}\,[\uparrow]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c}\langle \mathtt{e_1}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t_1}, \mathtt{Y}_\rho)\\ \mathtt{Y}_\rho(\mathtt{op}) = [\mathtt{abs}]\ \mathtt{t}\ \mathtt{t}_1'[\mathtt{x}]\mathtt{t}_2'\\ \mathtt{t}_1' \neq \mathtt{t_1}\end{array}}{\langle \mathtt{e_1}\,[\mathtt{op}]\,\mathtt{e_2}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\langle \mathtt{e_2}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t_2}, \mathtt{Y}_\rho)\\ \mathtt{Y}_\rho(\mathtt{op}) = [\mathtt{abs}]\ \mathtt{t}\ \mathtt{t}_1'[\mathtt{x}]\mathtt{t}_2'\\ \mathtt{t}_2' \neq \mathtt{t_2}\end{array}}{\langle \mathtt{e_1}\,[\mathtt{op}]\,\mathtt{e_2}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t_e}, \mathtt{Y}_\rho)\\ \mathtt{Y}_\rho(\mathtt{op}) = [\mathtt{abs}]\ \mathtt{t}\ \mathtt{t}'\\ \mathtt{t}' \neq \mathtt{t_e}\end{array}}{\langle [\mathtt{op}]\,\mathtt{e}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\begin{array}{c}\langle \mathtt{e_l}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} ([\mathtt{mut}]\,\mathtt{t}, \mathtt{Y}_\rho)\\ \mathtt{t} \notin \mathtt{Simple}\end{array}}{\langle \mathtt{e_l}\,[=]\,\mathtt{e_r}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\langle \mathtt{e_r}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} (\mathtt{t}, \mathtt{Y}_\rho)\\ \mathtt{t} \notin \mathtt{Simple}\end{array}}{\langle \mathtt{e_l}\,[=]\,\mathtt{e_r}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \qquad \frac{\begin{array}{c}\langle \mathtt{e_l}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{DY}} (\mathtt{t_l}, \mathtt{Y}_\rho)\\ \langle \mathtt{e_r}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t_r}, \mathtt{Y}_\rho)\\ \mathtt{t_l} = [\mathtt{Mut}]\ \mathtt{t} \qquad \mathtt{t} \neq \mathtt{t_r}\end{array}}{\langle \mathtt{e_l}\,[=]\,\mathtt{e_r}, \mathtt{Y}_\rho\rangle \rightarrow_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho)}$$

**Notazione**

$\rightarrow_{\mathtt{DY}}$ è l'inferenza di Tipo del Valore Denotabile di espressioni con doppio significato.
$\mathcal{O}_2 = \{+, -, *, \mathtt{div}, ==, >, <, \mathtt{or}, \mathtt{and}\}$; $\quad \mathcal{O}_1 = \{-_1, \mathtt{not}\}$.
$\mathtt{Simple} = \{[\mathtt{int}], [\mathtt{bool}]\}$

# Sistema Y: Regole per CMD, STM, PROG

$$\frac{\langle \texttt{e}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{bool}], \texttt{Y}_\rho) \quad \langle \texttt{c}_1, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho) \quad \langle \texttt{c}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}{\langle [\texttt{ifE}]\ \texttt{e}\ \texttt{c}_1\ \texttt{c}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{\langle \texttt{e}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{bool}], \texttt{Y}_\rho) \quad \langle \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}{\langle [\texttt{ifT}]\ \texttt{e}\ \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{\langle \texttt{c}_1, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho) \quad \langle \texttt{c}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}{\langle \texttt{c}_1\ [\texttt{seqC}]\ \texttt{c}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{\langle \texttt{e}_G, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{bool}], \texttt{Y}_\rho) \quad \langle \texttt{m}_S, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho) \quad \langle \texttt{m}_I, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho) \quad \langle \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}{\langle [\texttt{for}]\ \texttt{m}_S\ \texttt{e}_G\ \texttt{m}_I\ \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{\langle \texttt{e}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} (\texttt{t}, \texttt{Y}_\rho) \quad \texttt{t} \neq [\texttt{terr}]}{\langle [\texttt{cmd}]\ \texttt{e}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{}{\langle [\texttt{emptyC}], \texttt{Y}_\rho \rangle \to ([\texttt{void}], \texttt{Y}_\rho}$$

$$\frac{\langle \texttt{d}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)}{\langle [\texttt{stm1}]\ \texttt{d}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)} \qquad \frac{\langle \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)}{\langle [\texttt{stm2}]\ \texttt{c}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)}$$

$$\frac{\langle \texttt{m}_1, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho) \quad \langle \texttt{m}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}{\langle \texttt{m}_1\ [\texttt{seqM}]\ \texttt{m}_2, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)} \qquad \frac{}{\langle [\texttt{emptyM}], \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}_\rho)}$$

$$\frac{\langle \texttt{m}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)}{\langle [\texttt{prog}]\ \texttt{I}\ \texttt{m}, \texttt{Y}_\rho \rangle \to_{\texttt{Y}} ([\texttt{void}], \texttt{Y}'_\rho)}$$

**Notazione: Costruttori Aggiuntivi**
[cmd] : Iniettiva da Exp in Cmd; [stm1] : Iniettiva da Dcl in Stm; [stm2] : Iniettiva da Cmd in Stm.

# Sistema Y: CMD, STM, PROG - Errori di tipo

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{bool}]}{\langle [\mathtt{ifE}]\ \mathtt{e}\ \mathtt{c}\ \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle [\mathtt{ifE}]\ \mathtt{e}\ \mathtt{c}\ \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle [\mathtt{ifE}]\ \mathtt{e}\ \mathtt{c}\ \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{bool}]}{\langle [\mathtt{ifT}]\ \mathtt{e}\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle [\mathtt{ifT}]\ \mathtt{e}\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle \mathtt{c}\ [\mathtt{seqC}]\ \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle \mathtt{c}\ [\mathtt{seqC}]\ \mathtt{c}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{e}_{\mathtt{G}}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{bool}]}{\langle [\mathtt{for}]\ \mathtt{m}_{\mathtt{S}}\ \mathtt{e}_{\mathtt{G}}\ \mathtt{m}_{\mathtt{I}}\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{m}_{\mathtt{S}}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle [\mathtt{for}]\ \mathtt{m}_{\mathtt{S}}\ \mathtt{e}_{\mathtt{G}}\ \mathtt{m}_{\mathtt{I}}\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{m}_{\mathtt{I}}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} (\mathtt{t}, \mathtt{Y}_\rho) \quad \mathtt{t} \neq [\mathtt{void}]}{\langle [\mathtt{for}]\ \mathtt{m}_{\mathtt{S}}\ \mathtt{e}_{\mathtt{G}}\ \mathtt{m}_{\mathtt{I}}\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}{\langle [\mathtt{cmd}]\ \mathtt{e}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{d}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}'_\rho)}{\langle [\mathtt{stm1}]\ \mathtt{d}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}{\langle [\mathtt{stm2}]\ \mathtt{c}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{m}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}{\langle \mathtt{m}\ [\mathtt{seqM}]\ \mathtt{m}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)} \quad \frac{\langle \mathtt{m}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}{\langle \mathtt{m}\ [\mathtt{seqM}]\ \mathtt{m}', \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$

$$\frac{\langle \mathtt{m}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}{\langle [\mathtt{prog}]\ \mathtt{I}\ \mathtt{m}, \mathtt{Y}_\rho\rangle \to_{\mathtt{Y}} ([\mathtt{terr}], \mathtt{Y}_\rho)}$$